(76) $\int \frac{dx}{\sqrt{2ax - x^2}} = ...\quad (a > 0)$

(A) $\frac{1}{a} \sin^{-1}\left(\frac{x-a}{a}\right) + c$

(B) $\frac{1}{a} \log |(x - a) + \sqrt{2ax - x^2}| + c$

(C) $\sin^{-1}\left(\frac{x-a}{a}\right) + c$

(D) $\log |(x - a) + \sqrt{2ax - x^2}| + c$

ઉકેલ : $I = \int \frac{dx}{\sqrt{2ax - x^2}}$

$= \int \frac{dx}{\sqrt{a^2 - a^2 + 2ax - x^2}}$

$= \int \frac{dx}{\sqrt{(a)^2 - (x-a)^2}}$

$= \sin^{-1}\left(\frac{x-a}{a}\right) + c$ જવાબ : **(C)**

(77) $\int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right) dx = ...$

(A) $2(x + 1) \tan^{-1}\left\{\frac{2}{3}(x+1)\right\} + \frac{3}{4} \log \left|\sqrt{4 + 9(x+1)^2}\right| + c$

(B) $(x + 1) \tan^{-1}\left\{\frac{2}{3}(x+1)\right\} - \frac{3}{2} \log \left|\sqrt{9 + 4(x+1)^2}\right| + c$

(C) $(x + 1) \tan^{-1}\left\{\frac{3}{2(x+1)}\right\} + \frac{2}{3} \log \left|\sqrt{4 + 9(x+1)^2}\right| + c$

(D) $(x + 1) \tan^{-1}\left\{\frac{3}{2(x+1)}\right\} - \frac{4}{3} \log \left|\sqrt{9 + 4(x+1)^2}\right| + c$ (IIT : 2000)

ઉકેલ : $I = \int \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right) dx$

$= \int \sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+\frac{13}{4}}}\right) dx$

$= \int \sin^{-1}\left(\frac{x+1}{\sqrt{(x+1)^2+\left(\frac{3}{2}\right)^2}}\right) dx$

ધારો કે, $x + 1 = \frac{3}{2}\, tan\theta$. આથી, $dx = \frac{3}{2}\, sec^2\theta\ d\theta$

$$I = \int sin^{-1}\left(\frac{\frac{3}{2}\,tan\theta}{\frac{3}{2}\,sec\theta}\right) \frac{3}{2}\, sec^2\theta\ d\theta$$

$$= \int sin^{-1}(sin\theta) \cdot \frac{3}{2}\, sec^2\theta\ d\theta, \qquad \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$$

$$= \frac{3}{2} \int \theta \cdot sec^2\theta\ d\theta$$

$$= \frac{3}{2}\left[\theta\ tan\,\theta - \int 1 \cdot tan\theta\ d\theta\right] + c'$$

$$= \frac{3}{2}\left[\theta\ tan\,\theta - \log|sec\,\theta|\right] + c'$$

$$= \frac{3}{2}\left[tan^{-1}\left\{\frac{2}{3}(x+1)\right\} \cdot \frac{2}{3}(x+1) - \log\left|\sqrt{1+\left\{\frac{2}{3}(x+1)\right\}^2}\right|\right] + c'$$

$$= (x+1)\, tan^{-1}\left\{\frac{2}{3}(x+1)\right\} - \frac{3}{2}\log\left|\sqrt{9+4(x+1)^2}\right| + c \qquad \text{જ્યાં, } c = c' + \frac{3}{2}\log 3$$

જવાબ : (B)

(78) $\int x\, sin^{-1}\left(\sqrt{\frac{2a-x}{4a}}\right) dx = ...$ $\qquad (a > 0)$

(A) $\left(\frac{x^2-2a^2}{4}\right) cos^{-1}\left(\frac{x}{2a}\right) - \frac{x}{8}\sqrt{4a^2-x^2} + c$

(B) $\left(\frac{x^2-2a^2}{2}\right) cos^{-1}\left(\frac{x}{2a}\right) + \frac{x}{4}\sqrt{2a^2-x^2} + c$

(C) $\left(\frac{x^2-2a^2}{4}\right) sin^{-1}\left(\frac{x}{2a}\right) - \frac{x}{2}\sqrt{4a^2-x^2} + c$

(D) $\left(\frac{x^2-4a^2}{2}\right) sin^{-1}\left(\frac{x}{2a}\right) + \frac{x}{8}\sqrt{2a^2-x^2} + c$

ઉકેલ : $I = \int x\, sin^{-1}\left(\sqrt{\frac{2a-x}{4a}}\right) dx \qquad (a > 0)$

ધારો કે, $x = 2a\, cos\theta$. આથી, $dx = -2a\, sin\theta\ d\theta \qquad (0 < \theta < \pi)$

$$\sqrt{\frac{2a-x}{4a}} = \sqrt{\frac{2a-2a\,cos\theta}{4a}} = \sqrt{\frac{2\,sin^2\frac{\theta}{2}}{2}} = sin\frac{\theta}{2} \qquad \left(0 < \frac{\theta}{2} < \frac{\pi}{2}\right)$$

$$\therefore\quad I = \int 2a\, cos\theta \cdot sin^{-1}\left(sin\frac{\theta}{2}\right)(-2a\, sin\theta)\, d\theta$$

$$= -a^2 \int \theta \cdot sin\,2\theta\ d\theta$$

$$= -a^2\left[\theta\cdot\left(\frac{-cos2\theta}{2}\right) - \int 1\cdot\left(\frac{-cos2\theta}{2}\right)d\theta\right]$$

$$= \frac{a^2}{2}\left[\theta \cdot cos2\theta - \frac{sin2\theta}{2}\right] + c$$

$$= \frac{a^2}{2}\left[cos^{-1}\left(\frac{x}{2a}\right)\left(\frac{x^2-2a^2}{2a^2}\right) - \frac{x}{4a^2}\sqrt{4a^2 - x^2}\right] + c$$

$$\left(\begin{aligned} &\because\ cos\ 2\theta = cos^2\theta - sin^2\theta = \left(\frac{x}{2a}\right)^2 - \left(1 - \frac{x^2}{4a^2}\right) = \frac{x^2-2a^2}{2a^2} \\ &\text{અને } sin\ 2\theta = 2sin\ \theta \cdot cos\ \theta = 2\sqrt{1-\frac{x^2}{4a^2}} \cdot \frac{x}{2a} = \frac{x}{2a^2}\sqrt{4a^2 - x^2} \end{aligned}\right)$$

$$= \left(\frac{x^2-2a^2}{4}\right) cos^{-1}\left(\frac{x}{2a}\right) - \frac{x}{8}\sqrt{4a^2 - x^2} + c$$

જવાબ : **(A)**

(79) $\int cos\,2\theta \log\left|\frac{cos\theta + sin\theta}{cos\theta - sin\theta}\right| d\theta = ...$

(A) $\frac{cos2\theta}{2} \log\left|tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right| - \frac{1}{2}\log|sin\ 2\theta| + c$

(B) $\frac{cos2\theta}{2} \log\left|tan\left(\frac{\pi}{4} + \theta\right)\right| + \frac{1}{2}\log|sin\ 2\theta| + c$

(C) $\frac{sin2\theta}{2} \log\left|tan\left(\frac{\pi}{4} + \theta\right)\right| + \frac{1}{2}\log|cos\ 2\theta| + c$

(D) $\frac{sin2\theta}{2} \log\left|tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right| - \frac{1}{2}\log|cos\ 2\theta| + c$ (IIT : 1994)

ઉકેલ : $I = \int cos\,2\theta \log\left|\frac{cos\theta + sin\theta}{cos\theta - sin\theta}\right| d\theta$

$$= \int cos\,2\theta \cdot \log\left|\frac{1+tan\theta}{1-tan\theta}\right| d\theta$$

$$= \int cos\,2\theta \cdot \log\left|tan\left(\frac{\pi}{4} + \theta\right)\right| d\theta$$

$$= \log\left|tan\left(\frac{\pi}{4} + \theta\right)\right| \frac{sin2\theta}{2} - \int 2\ sec\ 2\theta \cdot \left(\frac{sin2\theta}{2}\right) d\theta$$

$$\left[\int sec\ \theta\ d\theta = \log\left|tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right| + c\right.$$

$$\int sec\ 2\theta\ d\theta = \frac{1}{2}\log\left|tan\left(\frac{\pi}{4} + \theta\right)\right| + c$$

$$\left.\therefore \frac{d}{d\theta}\left\{\log\left|tan\left(\frac{\pi}{4} + \theta\right)\right| + c\right\} = 2sec\ 2\theta\right]$$

$$\therefore I = \frac{sin2\theta}{2} \log\left|tan\left(\frac{\pi}{4} + \theta\right)\right| - \int tan\ 2\theta\ d\theta$$

$$= \frac{1}{2} sin\ 2\theta \log\left\{tan\left(\frac{\pi}{4} + \theta\right)\right\} + \frac{1}{2}\log|cos\ 2\theta| + c$$

જવાબ : **(C)**

(80) $\int e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right) dx = ...$

(A) $e^{\sin x}(x - \cos x) + c$ (B) $e^{\sin x}(2x + \cot x) + c$

(C) $e^{\sin x}(x - \sec x) + c$ (D) $e^{\sin x}(2x + \tan x) + c$

ઉકેલ : $I = \int e^{\sin x}\left(\frac{x\cos^3 x - \sin x}{\cos^2 x}\right) dx$

$= \int e^{\sin x}(x\cos x - \sec x\tan x)\,dx$

$= \int e^{\sin x}\,x\cos x\,dx - \int e^{\sin x}\sec x\tan x\,dx$

$I = I_1 - I_2$

હવે, $I_1 = \int x\cdot(e^{\sin x}\cos x)\,dx$

$= x\int e^{\sin x}\cos x\,dx - \int(1\cdot\int e^{\sin x}\cdot\cos dx)\,dx$

$= x\,e^{\sin x} - \int e^{\sin x}dx + c_1$ $(\int e^{\sin x}\cos x\,dx = e^{\sin x})$

અને $I_2 = \int e^{\sin x}(\sec x\tan x)\,dx$

$= e^{\sin x}\cdot\sec x - \int(e^{\sin x}\cos x\sec x)\,dx$

$I_2 = e^{\sin x}\cdot\sec x - \int e^{\sin x}dx + c_2$

$I = I_1 - I_2$

$= (x\cdot e^{\sin x} - \int e^{\sin x}dx + c_1 - e^{\sin x}\cdot\sec x + \int e^{\sin x}dx - c_2)$

$= xe^{\sin x} - e^{\sin x}\sec x + c$ $(c_1 - c_2 = c)$

$= e^{\sin x}(x - \sec x) + c$ **જવાબ : (C)**

(81) જો $I = \int \sin^{-1}x\,dx$ અને $J = \int \sin^{-1}\sqrt{1-x^2}\,dx$, તો ...... $(x > 0)$

(A) $J = \frac{\pi}{2}I$ (B) $I + J = \frac{\pi}{2}$ (C) $I + J = \frac{\pi}{2}x$ (D) $I - J = \frac{\pi}{2}x$

ઉકેલ : $J = \int \sin^{-1}\sqrt{1-x^2}\,dx = \int \cos^{-1}x\,dx$

$\therefore\ I + J = \int(\sin^{-1}x + \cos^{-1}x)\,dx = \frac{\pi}{2}\int dx = \frac{\pi}{2}x$ **જવાબ : (C)**

નોંધ : જો $x < 0$ તો $I - J = \frac{\pi}{2}x$. જો શરત ના આપી હોય, તો જવાબ (C), (D)

(82) $\int \frac{dt}{t^n(1+t^n)^{\frac{1}{n}}} = a\left(\frac{1}{t^n}+1\right)^b + c$, તો $a\cdot b = ...$

(A) 1 (B) −1 (C) $\frac{1}{n}$ (D) $-\frac{1}{n}$

(IIT : 2007)

ઉકેલ : $I = \int \frac{dt}{t^n(1+t^n)^{\frac{1}{n}}}$

$$= \int \frac{dt}{t^{n+1}\left(\frac{1}{t^n}+1\right)^{\frac{1}{n}}}$$

ધારો કે, $\frac{1}{t^n} + 1 = x$

$\therefore \quad -n \cdot t^{-n-1}\, dt = dx$

$-n\left(\frac{1}{t^{n+1}}\right) dt = dx$

$\frac{dt}{t^{n+1}} = \frac{-1}{n}\, dx$

$\therefore \quad I = \frac{-1}{n} \int x^{\frac{-1}{n}}\, dx$

$$= \frac{-1}{n} \frac{x^{-\frac{1}{n}+1}}{\frac{-1}{n}+1} + c$$

$$= \frac{1}{1-n}\, x^{1-\frac{1}{n}} + c$$

$$= \frac{1}{1-n}\left(\frac{1}{t^n}+1\right)^{\frac{n-1}{n}} + c$$

$\therefore \quad a = \frac{1}{1-n}, \quad b = \frac{n-1}{n}$

$\therefore \quad a \cdot b = \left(\frac{1}{1-n}\right)\left(\frac{n-1}{n}\right) = -\frac{1}{n}$

$\therefore \quad ab = -\frac{1}{n}$

જવાબ : **(D)**

(83) જો $\int \frac{(1+x^n)^{\frac{1}{n}}}{x^{n+2}}\, dx = a\left(1+\frac{1}{x^n}\right)^b + c$, તો $a + b =$ ...... (જ્યાં $n = 4$)

(A) $\frac{6}{5}$ (B) $\frac{16}{15}$ (C) $\frac{21}{20}$ (D) $\frac{11}{10}$

ઉકેલ : $I = \int \frac{(1+x^n)^{\frac{1}{n}}}{x^{n+2}}\, dx$

$$= \int \frac{\left(\frac{1}{x^n}+1\right)^{\frac{1}{n}} dx}{x^{n+1}}$$

ધારો કે, $1 + \frac{1}{x^n} = t$

$-n \cdot x^{-n-1}\, dx = dt$

$\frac{1}{x^{n+1}}\, dx = \frac{-1}{n}\, dt$

$I = \frac{-1}{n} \int t^{\frac{1}{n}}\, dt$

$= \frac{-1}{n} \frac{t^{\frac{1}{n}+1}}{\frac{1}{n}+1} + c$

$= \frac{-1}{n+1} (t)^{\frac{n+1}{n}} + c$

$= \frac{-1}{n+1} \left(1 + \frac{1}{x^n}\right)^{\frac{n+1}{n}} + c$

$\therefore\ n = 4,\ a = \frac{-1}{n+1} = \frac{-1}{5},\ b = \frac{n+1}{n} = \frac{5}{4}$

$\therefore\ a + b = \frac{21}{20}$ **જવાબ : (C)**

(84) જો $\int \left(x + \sqrt{x^2+1}\right)^{-5} dx = P\left(x + \sqrt{x^2+1}\right)^{-4} + Q\left(x + \sqrt{x^2+1}\right)^{-6} + c$, તો $P + Q = ...$

(A) $\frac{-5}{3}$ (B) $\frac{-5}{12}$ (C) $\frac{-5}{6}$ (D) $\frac{-5}{24}$

ઉકેલ : $I = \int \left(x + \sqrt{x^2+1}\right)^{-5} dx$

ધારો કે, $x + \sqrt{x^2+1} = t$

$\frac{1}{t} = \frac{1}{x + \sqrt{x^2+1}} \times \frac{x - \sqrt{x^2+1}}{x - \sqrt{x^2+1}}$

$= \frac{x - \sqrt{x^2+1}}{x^2 - x^2 - 1}$

$\frac{1}{t} = -x + \sqrt{x^2+1}$

$t + \frac{1}{t} = 2\sqrt{x^2+1}$. આથી, $\sqrt{x^2+1} = \frac{1}{2}\left(t + \frac{1}{t}\right)$

હવે, $x + \sqrt{x^2+1} = t$

$\therefore\ \left(1 + \frac{2x}{2\sqrt{x^2+1}}\right) dx = dt$

$$\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right) dx = dt$$

$$t dx = \sqrt{x^2+1}\ dt$$

$$dx = \frac{\sqrt{x^2+1}}{t}\ dt$$

$$\therefore \quad dx = \frac{1}{2}\left(t+\frac{1}{t}\right)\frac{dt}{t}$$

$$I = \frac{1}{2}\int\left(t+\frac{1}{t}\right) t^{-6}\ dt$$

$$= \frac{1}{2}\int (t^{-5} - t^{-7})\ dt$$

$$= \frac{1}{2}\left[\frac{t^{-4}}{-4}+\frac{t^{-6}}{-6}\right] + c$$

$$= \frac{-1}{8} t^{-4} - \frac{1}{12} t^{-6} + c$$

$$= \frac{-1}{8}(x+\sqrt{x^2+1})^{-4} - \frac{1}{12}(x+\sqrt{x^2+1})^{-6} + c$$

$$\therefore \quad P = \frac{-1}{8},\ Q = \frac{-1}{12}$$

$$\therefore \quad P + Q = \left(\frac{-1}{8}\right) + \left(\frac{-1}{12}\right) = \frac{-5}{24}$$

જવાબ : (D)

(85) $\int x^3\sqrt{\frac{1+x^2}{1-x^2}}\ dx = A\cos^{-1} x^2 + B\sqrt{1-x^4} + Cx^2\sqrt{1-x^4} + D$ તો $A + B + C =$ ......

(A) 0 (B) $\frac{-1}{2}$ (C) $\frac{1}{2}$ (D) -1

ઉકેલ : $I = \int x^3\sqrt{\frac{1+x^2}{1-x^2}}\ dx$

ધારો કે, $x^2 = \sin\theta$ $\quad (0 < \theta < \frac{\pi}{2})$

$2x\,dx = \cos\theta\ d\theta$. આથી, $x\,dx = \frac{1}{2}\cos\theta\ d\theta$

$$I = \int x^3\frac{(1+x^2)}{\sqrt{1-x^4}}\ dx$$

$$= \int \frac{x^2(1+x^2)}{\sqrt{1-x^4}}\ (x\,dx)$$

$$= \frac{1}{2}\int\frac{\sin\theta\,(1+\sin\theta)}{\cos\theta}\ \cos\theta\ d\theta$$

$$I = \frac{1}{2} \int (sin\theta + sin^2\theta)\, d\theta$$

$$= \frac{1}{2}\int\left(sin\theta + \frac{1 - cos2\theta}{2}\right)$$

$$= \frac{-1}{2}\, cos\theta + \frac{1}{4} \int (1 - cos2\theta)\, d\theta$$

$$= \frac{-1}{2}\, cos\theta + \frac{1}{4}\, \theta - \frac{sin2\theta}{4 \times 2} + c$$

$$= \frac{-1}{2}\, cos\theta + \frac{\theta}{4} - \frac{sin\theta\, cos\theta}{4} + c$$

$$= \frac{-1}{2}\sqrt{1 - x^4} + \frac{1}{4}\, sin^{-1}x^2 - \frac{1}{4}\, x^2\sqrt{1 - x^4} + c$$

$A = \frac{-1}{4}, B = \frac{-1}{2}, C = \frac{-1}{4}$ ($sin^{-1}x^2 = \frac{\pi}{2} - cos^{-1}x^2$ હોવાથી)

$A + B + C = \left(\frac{-1}{4}\right) + \left(\frac{-1}{2}\right) + \left(\frac{-1}{4}\right) = -1$ **જવાબ : (D)**

(86) $\int \frac{dx}{(4 + 3x^2)\sqrt{3 - 4x^2}} = ...$

(A) $\frac{1}{5}\, tan^{-1} \frac{2x}{\sqrt{3 - 4x^2}} + c$

(B) $\frac{1}{10}\, tan^{-1} \left(\frac{5x}{2\sqrt{3 - 4x^2}}\right) + c$

(C) $\frac{1}{5}\, tan^{-1} \frac{5x}{2\sqrt{3 - 4x^2}} + c$

(D) $\frac{1}{10}\, tan^{-1} \frac{5x}{\sqrt{3 - 4x^2}} + c$

ઉકેલ : $I = \int \frac{1}{(4 + 3x^2)\sqrt{3 - 4x^2}}\, dx$

ધારો કે, $x = \frac{\sqrt{3}}{2}\, sin\theta$ $\left(0 < \theta < \frac{\pi}{2}\right)$

$\therefore \quad dx = \frac{\sqrt{3}}{2}\, cos\theta\, d\theta$

$$\therefore \quad I = \int \frac{\frac{\sqrt{3}}{2} cos\theta\, d\theta}{\left(4 + \frac{9}{4} sin^2\theta\right)\sqrt{3 - 3sin^2\theta}}$$

$$I = 2\int \frac{d\theta}{9\, sin^2\theta + 16}$$

$$= 2\int \frac{sec^2\theta\, d\theta}{16 + 25\, tan^2\theta}$$

$= 2 \times \int \frac{dt}{(5t)^2 + (4)^2}$ (ધારો કે, $tan\theta = t$ અને $sec^2\theta\, d\theta = dt$)

$= 2 \times \frac{1}{5} \times \frac{1}{4} tan^{-1} \left(\frac{5t}{4}\right) + c$

$= \frac{1}{10} tan^{-1} \left(\frac{5}{4} tan\theta\right) + c$

$= \frac{1}{10} tan^{-1} \left(\frac{5x}{2\sqrt{3 - 4x^2}}\right) + c$ જવાબ : **(B)**

(87) $\int \frac{dx}{5+4cosx}$ = ...

(A) $\frac{1}{3} tan^{-1} \left(\frac{1}{3} tan\, x\right) + c$

(B) $\frac{1}{3} tan^{-1} \left(\frac{1}{3} tan \frac{x}{2}\right) + c$

(C) $\frac{2}{3} tan^{-1} \left(\frac{1}{3} tan\, x\right) + c$

(D) $\frac{2}{3} tan^{-1} \left(\frac{1}{3} tan \left(\frac{x}{2}\right)\right) + c$

ઉકેલ : $I = \int \frac{dx}{5+4cosx}$

ધારો કે, $tan \frac{x}{2} = t$

$\therefore \quad dx = \frac{2dt}{1+t^2}$, $cosx = \frac{1-t^2}{1+t^2}$

$I = \int \frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$

$= \int \frac{2dt}{5(1+t^2)+4(1-t^2)}$

$= 2\int \frac{dt}{(t^2)+(3)^2}$

$= \frac{2}{3} tan^{-1} \left(\frac{t}{3}\right) + c$

$= \frac{2}{3} tan^{-1} \left(\frac{1}{3} tan\frac{x}{2}\right) + c$ જવાબ : **(D)**

(88) જો $\int \frac{dx}{sin^4x + cos^4x} = \frac{1}{\sqrt{2}} tan^{-1} (f(x)) + c$ તો, ...

(A) $f(x) = tan\, x - cot\, x$

(B) $f\left(\frac{\pi}{4}\right) = 0$

(C) $f(x) = tan\, x + cot\, x$

(D) $f(x) = \frac{1}{2} (tan\, x - cot\, x)$

ઉકેલ : $I = \int \frac{dx}{sin^4x + cos^4x}$

$$I = \int \frac{sec^4 x\, dx}{tan^4 x + 1}$$ (અંશ અને છેદને $cos^4 x$ વડે ભાગતાં)

$$= \int \frac{(1 + tan^2 x)\, sec^2 x}{1 + tan^4 x}\, dx$$

$$= \int \frac{1 + t^2}{1 + t^4}\, dt$$ (ધારો કે, $tan\ x = t$, આથી, $sec^2 x\, dx = dt$)

$$= \int \frac{\left(1 + \frac{1}{t^2}\right)}{\left(t^2 + \frac{1}{t^2}\right)}\, dt$$

$$= \int \frac{du}{u^2 + (\sqrt{2})^2}$$ (ધારો કે, $t - \frac{1}{t} = u$, આથી, $\left(1 + \frac{1}{t^2}\right) dt = du$)

$$= \frac{1}{\sqrt{2}}\, tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c$$

$$= \frac{1}{\sqrt{2}}\, tan^{-1}\left(\frac{\left(t - \frac{1}{t}\right)}{\sqrt{2}}\right) + c$$

$$= \frac{1}{\sqrt{2}}\, tan^{-1}\left(\frac{(tan\, x - cot\ x)}{\sqrt{2}}\right) + c$$

$\therefore\ f(x) = \dfrac{tan\, x - cot\ x}{\sqrt{2}}$. તેથી $f\left(\frac{\pi}{4}\right) = 0$ **જવાબ : (B)**

(89) $\int \left[\log(\log x) + \dfrac{1}{(\log x)^2}\right] dx = x\, [f(x) - g(x)] + c$, તો ......

(A) $f(x) = \log(\log x),\ g(x) = \dfrac{1}{\log x}$

(B) $f(x) = \log x,\ g(x) = \dfrac{1}{\log x}$

(C) $f(x) = \dfrac{1}{\log x},\ g(x) = \log(\log x)$

(D) $f(x) = \dfrac{1}{x \log x},\ g(x) = \dfrac{1}{\log x}$

ઉકેલ : $\int \log(\log x)\, dx$

$$= \log(\log x) \int 1\, dx - \int \left(\frac{d}{dx} \log(\log x) \int 1\, dx\right) dx$$

$$= x \cdot \log(\log x) - \int \left(\frac{1}{\log x} \cdot \frac{1}{x} \cdot x\right) dx$$

$$= x \cdot \log(\log x) - \int \frac{1}{\log x}\, dx$$

$$= x \cdot \log(\log x) - x \cdot \frac{1}{\log x} + \int \left(\frac{(-1)}{(\log x)^2} \cdot \frac{1}{x} \cdot x \right) dx$$

$$\int \log(\log x)\, dx = x \cdot \log(\log x) - x \cdot \frac{1}{\log x} - \int \frac{1}{(\log x)^2}\, dx$$

$$\therefore \quad \int \log(\log x)\, dx + \int \frac{1}{(\log x)^2}\, dx = x \cdot \log(\log x) - x \cdot \frac{1}{\log x}$$

$$= x \left[\log(\log x) - \frac{1}{\log x} \right]$$

$\therefore \quad f(x) = \log(\log x),\ g(x) = \frac{1}{\log x}$ **જવાબ : (A)**

નોંધ : માંગેલ વિકલ્પો પ્રમાણે આ જવાબ છે. $f(x) = \log \log x + \sin x,\ g(x) = \frac{1}{\log x} + \sin x$ પણ લઈ શકાય. ખરેખર તો $f(x) = \log \log x + h(x),\ g(x) = \frac{1}{\log x} + h(x)$

(90) $\int \frac{x \tan^{-1}x}{\sqrt{1+x^2}}\, dx = \sqrt{1+x^2}\ f(x) + k \log(x + \sqrt{x^2+1}) + c$, તો ......

(A) $f(x) = \tan^{-1}x,\ k = -1$ (B) $f(x) = \tan^{-1}x,\ k = 1$

(C) $f(x) = 2\tan^{-1}x,\ k = -1$ (D) $f(x) = 2\tan^{-1}x,\ k = 1$

ઉકેલ : $\int \frac{f'(x)}{\sqrt{f(x)}}\, dx = 2\sqrt{f(x)} + c$

$$\therefore \quad \int \frac{x}{\sqrt{1+x^2}}\, dx = \frac{1}{2}\int \frac{2x}{\sqrt{1+x^2}}\, dx = \frac{1}{2}(2\sqrt{1+x^2}) = \sqrt{1+x^2}$$

હવે, $I = \int (\tan^{-1}x) \frac{x}{\sqrt{1+x^2}}\, dx$

$$= (\tan^{-1}x) \int \frac{x}{\sqrt{1+x^2}}\, dx - \int \left(\frac{d}{dx}(\tan^{-1}x) \int \frac{x}{\sqrt{1+x^2}}\, dx \right) dx$$

$$= \sqrt{1+x^2} \cdot \tan^{-1}x - \int \left(\frac{1}{1+x^2} \cdot \sqrt{1+x^2} \right) dx$$

$$= \sqrt{1+x^2} \cdot \tan^{-1}x - \int \frac{dx}{\sqrt{1+x^2}}$$

$$= \sqrt{1+x^2} \cdot \tan^{-1}x - \log|x + \sqrt{1+x^2}| + c$$

તેથી, $f(x) = \tan^{-1}x,\ k = -1$ **જવાબ : (A)**

(91) જો $\int \frac{dx}{\left(1+x^2\right)\sqrt{1-x^2}}$ = F(x) અને F(1) = 0, તો $x > 0$ માટે F(x) = ...

(A) $\frac{1}{\sqrt{2}}\ tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^2}}\right)+\frac{\pi}{2}$

(B) $\frac{1}{\sqrt{2}}\ tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)-\frac{\pi}{2\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}\ tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)+\frac{\pi}{2\sqrt{2}}$

(D) $\frac{1}{\sqrt{2}}\ tan^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2}x}\right)$

ઉકેલ : $I = \int \frac{dx}{\left(1+x^2\right)\sqrt{1-x^2}}$

$= \int \frac{-t\ dt}{(1+t^2)\sqrt{t^2-1}}$ (ધારો કે, $x = \frac{1}{t}$, $dx = \frac{-1}{t^2}\ dt$)

$= -\int \frac{u\ du}{(u^2+2)u}$ (ધારો કે, $t^2 - 1 = u^2$, આથી, $2t\ dt = 2u\ du$)

$= -\int \frac{du}{(u)^2+(\sqrt{2})^2}$

$= -\frac{1}{\sqrt{2}}\ tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c$

$= -\frac{1}{\sqrt{2}}\ tan^{-1}\left(\frac{\sqrt{t^2-1}}{\sqrt{2}}\right) + c$

$= -\frac{1}{\sqrt{2}}\ tan^{-1}\left(\frac{1}{\sqrt{2}}\frac{\sqrt{1-x^2}}{x}\right) + c$

હવે, F(1) = 0. આથી, $c = 0$

$\therefore$ $F(x) = \frac{-1}{\sqrt{2}}\left[tan^{-1}\frac{\sqrt{1-x^2}}{\sqrt{2}x}\right]$

$\therefore$ $F(x) = \frac{-1}{\sqrt{2}}\left[\frac{\pi}{2} - cot^{-1}\frac{\sqrt{1-x^2}}{\sqrt{2}x}\right]$

$= \frac{-\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}\ tan^{-1}\frac{\sqrt{2}x}{\sqrt{1-x^2}}$ ($tan^{-1}x + cot^{-1}x = \frac{\pi}{2}$, $cot^{-1}x = tan^{-1}\left(\frac{1}{x}\right)$, $x > 0$)

જવાબ : **(B)**

(92) $\int e^x \frac{2 - \sin 2x}{1 - \cos 2x}\, dx = ...$

(A) $-e^x \tan x + c$ (B) $-e^x \cot x + c$

(C) $e^x \tan x + c$ (D) $e^x \cot x + c$

ઉકેલ : $I = \int e^x \frac{2 - \sin 2x}{1 - \cos 2x}\, dx$

$= \int e^x \frac{2 - 2\sin x \cos x}{2\sin^2 x}\, dx$

$= \int e^x (\operatorname{cosec}^2 x - \cot x)\, dx$

$= -\int e^x (\cot x - \operatorname{cosec}^2 x)\, dx$

$= -e^x \cot x + c$ **જવાબ : (B)**

(93) જો $\int \frac{(2x^2 + 3)}{(x^2 - 1)(x^2 - 4)}\, dx = \log \left(\frac{x-2}{x+2}\right)^p \left(\frac{x+1}{x-1}\right)^q + c$, તો $p + q = ...$

(A) $\frac{7}{4}$ (B) $\frac{1}{12}$ (C) $\frac{-1}{12}$ (D) $\frac{-7}{4}$

ઉકેલ : $I = \int \frac{(2x^2 + 3)}{(x^2 - 1)(x^2 - 4)}\, dx$

ધારો કે, $\frac{2x^2 + 3}{(x^2 - 1)(x^2 - 4)} = \frac{A}{x^2 - 1} + \frac{B}{x^2 - 4}$

$\therefore\ 2x^2 + 3 = A(x^2 - 4) + B(x^2 - 1)$

સહગુણકો અને અચળ પદો સરખાવતાં,

$\therefore\ A + B = 2,\ -4A - B = 3$

ઉકેલતાં, $A = \frac{-5}{3},\ B = \frac{11}{3}$

$\therefore\ I = \int \frac{-\frac{5}{3}}{(x^2 - 1)}\, dx + \int \frac{\frac{11}{3}}{x^2 - 4}\, dx$

$= \frac{-5}{3} \times \frac{1}{2} \log \left|\frac{x-1}{x+1}\right| + \frac{11}{3} \times \frac{1}{2(2)} \log \left|\frac{x-2}{x+2}\right| + c$

$= \frac{-5}{6} \log \left|\frac{x-1}{x+1}\right| + \frac{11}{12} \log \left|\frac{x-2}{x+2}\right| + c$

$= \frac{5}{6} \log \left|\frac{x+1}{x-1}\right| + \frac{11}{12} \log \left|\frac{x-2}{x+2}\right| + c$

$= \log \left[\left(\frac{x+1}{x-1}\right)^{\frac{5}{6}} \cdot \left(\frac{x-2}{x+2}\right)^{\frac{11}{12}}\right] + c$

$\therefore\ p = \frac{11}{12}$ અને $q = \frac{5}{6}$. આથી, $p + q = \frac{11}{12} + \frac{10}{12} = \frac{21}{12} = \frac{7}{4}$ **જવાબ : (A)**

(94) $\int \frac{3x+2}{(x-2)^2(x-3)}\,dx = ...$

(A) $11\log\left|\frac{x+3}{x+2}\right| - \frac{8}{x-2} + c$ (B) $11\log\left|\frac{x-3}{x-2}\right| - \frac{8}{x-2} + c$

(C) $11\log\left|\frac{x-3}{x-2}\right| + \frac{8}{x-2} + c$ (D) $11\log\left|\frac{x+3}{x+2}\right| + \frac{8}{x-2} + c$

ઉકેલ : $I = \int \frac{3x+2}{(x-2)^2(x-3)}\,dx$

$\frac{3x+2}{(x-2)^2(x-3)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-3)}$ લો.

$\therefore\ (3x+2) = A(x-2)(x-3) + B(x-3) + C(x-2)^2$

જો $x = 2$, તો $8 = -B$ આથી, $B = -8$

જો $x = 3$, તો $11 = C$ આથી, $C = 11$

જો $x = 0$, તો $2 = 6A - 3B + 4C$

$2 = 6A - 3(-8) + 4(11)$

$\therefore\ A = -11,\ B = -8,\ C = 11$

$$\int \frac{3x+2}{(x-2)^2(x-3)}\,dx = \int \frac{-11}{x-2}\,dx + \int \frac{-8}{(x-2)^2}\,dx + \int \frac{11}{x-3}\,dx$$

$$= -11\log|x-2| + \frac{8}{x-2} + 11\log|x-3| + c$$

$$= 11\log\left|\frac{x-3}{x-2}\right| + \frac{8}{x-2} + c$$

જવાબ : (C)

(95) $\int \frac{dx}{x(x^5+1)} = ...$ $(x > 0)$

(A) $\frac{1}{5}\log x^4(x^5+1) + c$ (B) $\frac{1}{5}\log\left(\frac{1+x^5}{x^5}\right) + c$

(C) $\frac{1}{5}\log\left(\frac{x^4}{(x^5+2)}\right) + c$ (D) $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right) + c$

ઉકેલ : $I = \int \frac{dx}{x(x^5+1)} = \int \frac{dx}{x^6\left(1+\frac{1}{x^5}\right)}$

$\therefore\ I = \frac{-1}{5}\int \frac{1}{t}\,dt = \frac{-1}{5}\log|t| + c$ (ધારો કે, $1 + \frac{1}{x^5} = t$, આથી $\frac{-5}{x^6}\,dx = dt$)

$$= \frac{-1}{5}\log\left(1+\frac{1}{x^5}\right) + c = \frac{-1}{5}\log\left(\frac{x^5+1}{x^5}\right) + c = \frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right) + c$$

જવાબ : (D)

બીજી રીત : $\int \frac{dx}{x(x^5+1)}$

$$= \int \frac{x^4 dx}{x^5(x^5+1)}$$

$$= \frac{1}{5}\int \frac{dt}{t^2+t} \qquad (x^5 = t,\ 5x^4\ dx = dt)$$

$$= \frac{1}{5}\int \frac{dt}{\left(t+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{5}\ \frac{1}{2\left(\frac{1}{2}\right)} \log \left|\frac{t+\frac{1}{2}-\frac{1}{2}}{t+\frac{1}{2}+\frac{1}{2}}\right| + c$$

$$= \frac{1}{5} \log \left(\frac{x^5}{x^5+1}\right) \qquad (x > 0)$$

ત્રીજી રીત :

$$I = \frac{1}{5}\int \frac{dt}{t(t+1)}$$

$$= \frac{1}{5}\left[\int \frac{dt}{t} - \int \frac{dt}{t+1}\right]$$

$$= \frac{1}{5} \log \left|\frac{t}{t+1}\right|$$

$$= \frac{1}{5} \log \left|\frac{x^5}{x^5+1}\right|$$

(96) $\int \frac{3sin\ x + 2cos\ x}{3cos\ x + 2sin\ x}\ dx = ...$

(A) $\frac{13}{12} x + \frac{5}{13} \log |3cos\ x + 2sin\ x| + c$ (B) $\frac{12}{13} x + \frac{5}{13} \log |3cos\ x + 2sin\ x| + c$

(C) $\frac{12}{13} x - \frac{5}{13} \log |3cos\ x + 2sin\ x| + c$ (D) $\frac{13}{12} x - \frac{5}{13} \log |3cos\ x + 2sin\ x| + c$

ઉકેલ : $I = \int \frac{3sin\ x + 2cos\ x}{3cos\ x + 2sin\ x}\ dx$

ધારો કે $(3sin x + 2cos x) = l[(3cos x + 2sin x)] + m[\frac{d}{dx}(3cos x + 2sin x)]$

$\therefore$ $3sin\ x + 2cos\ x = l\ (3cos\ x + 2sin\ x) + m(-3sin\ x + 2cos\ x)$

$= (2l - 3m)\ sin\ x + (3l + 2m)\ cos\ x$

$sin\ x$ અને $cos\ x$ ના સહગુણકો સરખાવતાં,

$2l - 3m = 3,\ \ 3l + 2m = 2$

ઉકેલતાં, $l = \frac{12}{13},\ \ m = \frac{-5}{13}$

$$\therefore \quad I = \int \frac{\frac{12}{13}(3cos\ x + 2sin\ x) - \frac{5}{13}(-3sin\ x + 2cos\ x)}{(3cos\ x + 2sin\ x)}\ dx$$

$$= \frac{12}{13}\int dx - \frac{5}{13}\int \frac{-3sin\ x + 2cos\ x}{3cos\ x + 2sin\ x}\ dx$$

$$= \frac{12}{13} x - \frac{5}{13} \log |3cos\ x + 2sin\ x| + c$$

જવાબ : (C)

(97) $\int \frac{sin^8 x - cos^8 x}{1 - 2sin^2 x\ cos^2 x}\ dx = ...$

(A) $sin\ 2x + c$ (B) $-\frac{1}{2} sin\ 2x + c$ (C) $\frac{1}{2} sin\ 2x + c$ (D) $-sin\ 2x + c$

ઉકેલ : $I = \int \frac{sin^8x - cos^8x}{1-2sin^2x\, cos^2x}\, dx$

$= \int \frac{(sin^4x + cos^4x)(sin^4x - cos^4x)}{(sin^2x + cos^2x)^2 - 2sin^2x\, cos^2x}\, dx$

$= \int (sin^4x - cos^4x)\, dx$ $\quad (sin^4x + cos^4x = (sin^2x + cos^2x)^2 - 2sin^2xcos^2x)$

$= \int (sin^2x + cos^2x)(sin^2x - cos^2x)\, dx$

$= \int -cos\, 2x\, dx = -\frac{sin\, 2x}{2} + c$ જવાબ : **(B)**

(98) $\int \frac{x^2}{(a+bx)^2}\, dx = ...$

(A) $\frac{1}{b^2}\left[x + \frac{2a}{b}\log(a+bx) - \frac{a^2}{b}\cdot\frac{1}{a+bx}\right] + c$

(B) $\frac{1}{b^2}\left[x - \frac{2a}{b}\log(a+bx) + \frac{a^2}{b}\cdot\frac{1}{a+bx}\right] + c$

(C) $\frac{1}{b^2}\left[x + \frac{2a}{b}\log(a+bx) + \frac{a^2}{b}\cdot\frac{1}{a+bx}\right] + c$

(D) $\frac{1}{b^2}\left[x + \frac{a}{b} - \frac{2a}{b}\log(a+bx) - \frac{a^2}{b}\cdot\frac{1}{a+bx}\right] + c$ (IIT : 1979)

ઉકેલ : $I = \int \frac{x^2}{(a+bx)^2}\, dx$

ધારો કે, $a + bx = t$. આથી, $b\, dx = dt$ આથી, $dx = \frac{1}{b}\, dt$. હવે, $x = \frac{t-a}{b}$

$\therefore I = \int \left(\frac{t-a}{b}\right)^2 \cdot \frac{1}{t^2}\cdot\frac{1}{b}\, dt$

$= \frac{1}{b^3}\int\left(1 - \frac{2a}{t} + a^2t^{-2}\right) dt$

$= \frac{1}{b^3}\left[t - 2a\log|t| - \frac{a^2}{t}\right] + c$

$= \frac{1}{b^3}\left[(a+bx) - 2a\log|a+bx| - \frac{a^2}{a+bx}\right] + c$

$= \frac{1}{b^2}\left[x + \frac{a}{b} - \frac{2a}{b}\log(a+bx) - \frac{a^2}{b}\cdot\frac{1}{a+bx}\right] + c$ જવાબ : **(D)**

(99) $\int \frac{\sqrt{x^2 - a^2}}{x}\, dx = ...$

(A) $\sqrt{x^2 - a^2} - a\, tan^{-1}\left(\frac{\sqrt{x^2 - a^2}}{a}\right) + c$ (B) $\sqrt{x^2 - a^2} + a\, tan^{-1}\left(\frac{\sqrt{x^2 - a^2}}{a}\right) + c$

(C) $\sqrt{x^2 - a^2} + a\, tan^{-1}\left(\frac{\sqrt{x^2 - a^2}}{a}\right) + c$ (D) $cot^{-1}\left(\frac{x}{a}\right) + c$

ઉકેલ : $I = \int \frac{\sqrt{x^2 - a^2}}{x}\, dx$

ધારો કે, $x^2 - a^2 = t^2$. આથી, $x^2 = a^2 + t^2$ $(t > 0)$

$\therefore\ 2x\, dx = 2t\, dt$. આથી, $x\, dx = t\, dt$

$$I = \int \frac{\sqrt{x^2 - a^2}}{x}\, dx$$

$$= \int \frac{\sqrt{x^2 - a^2}}{x^2}\, x\, dx$$

$$= \int \frac{t}{a^2 + t^2}\, t\, dt$$

$$= \int \frac{t^2 + a^2 - a^2}{t^2 + a^2}\, dt$$

$$= \left(1 - \frac{a^2}{t^2 + a^2}\right) dt$$

$$= t - a^2 \cdot \frac{1}{a}\, tan^{-1}\left(\frac{t}{a}\right) + c$$

$$= \sqrt{x^2 - a^2} - a\, tan^{-1}\left(\frac{\sqrt{x^2 - a^2}}{a}\right) + c$$

બીજી રીત : $I = \int \frac{\sqrt{x^2 - a^2}}{x}\, dx$

ધારો કે $x = a\, sec\theta$. આથી, $dx = a\, sec\theta\, tan\theta\, d\theta$ $\left(0 < \theta < \frac{\pi}{2},\ a > 0\right)$

$$= \int \frac{\sqrt{a^2 sec^2\theta - a^2}}{a\, sec\theta}\, a\, sec\theta\, tan\theta\, d\theta$$

$$= a \int tan^2\theta\, d\theta$$

$$= a \int (sec^2\theta - 1)\, d\theta$$

$= a\ tan\theta - a\theta + c$

$= \sqrt{x^2 - a^2} - a\ sec^{-1}\frac{x}{a} + c$

$= \sqrt{x^2 - a^2} - a\ tan^{-1}\frac{\sqrt{x^2 - a^2}}{a} + c$ જવાબ : **(A)**

(100) $\int tan^3 2x \cdot sec\ 2x\ dx =$ ......

(A) $\frac{1}{6} sec^3 2x - \frac{1}{2} sec\ 2x + c$ (B) $\frac{1}{6} sec^3 2x + \frac{1}{2} sec\ 2x + c$

(C) $\frac{1}{9} sec^2 2x - \frac{1}{3} sec\ 2x + c$ (D) $\frac{1}{9} sec^2 2x + \frac{1}{3} sec\ 2x + c$

ઉકેલ : $I = \int tan^3 2x \cdot sec\ 2x\ dx$

ધારો કે, $sec\ 2x = t$, તો $sec\ 2x\ tan\ 2x\ dx = \frac{1}{2} dt$

$I = \frac{1}{2} \int (t^2 - 1)\ dt$

$= \frac{1}{2} \cdot \frac{t^3}{3} - \frac{1}{2} t + c$

$= \frac{1}{6} sec^3 2x - \frac{1}{2} sec\ 2x + c$ જવાબ : **(A)**

(101) $\int \sqrt{\frac{a-x}{x}}\ dx =$ ......

(A) $sin^{-1}\frac{x}{a} + \frac{x}{a}\sqrt{a^2 - x^2} + c$ (B) $a\left[sin^{-1}\sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a}}\sqrt{\frac{a-x}{a}}\right] + c$

(C) $sin^{-1}\frac{x}{a} - \frac{x}{a}\sqrt{a^2 - x^2} + c$ (D) $sin^{-1}\frac{x}{a} - \frac{x}{a}\sqrt{a^2 - x^2} + c$

ઉકેલ : $I = \int \sqrt{\frac{a-x}{x}}\ dx$

ધારો કે, $x = a\ sin^2\theta$ $(0 < \theta < \frac{\pi}{2})$

$\therefore$ $dx = 2a\ sin\theta\ cos\theta\ d\theta$

$\therefore$ $I = \int \sqrt{\frac{cos^2\theta}{sin^2\theta}}\ 2a\ sin\theta\ cos\theta\ d\theta$

$= a \int 2cos^2\theta\ d\theta$

$= a \int (1 + cos\ 2\theta)\ d\theta$

$= a\left[\theta + \frac{sin\ 2\theta}{2}\right] + c$

$= a\ [\theta + sin\theta\ cos\theta] + c$

$= a\left[sin^{-1}\sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a}}\sqrt{\frac{a-x}{a}}\right] + c$ જવાબ : **(B)**

(102) $\int \left(\frac{sin^{-1}\sqrt{x} - cos^{-1}\sqrt{x}}{sin^{-1}\sqrt{x} + cos^{-1}\sqrt{x}} \right) dx = ...$

(A) $\frac{2}{\pi} [\sqrt{x - x^2} - (1 - 2x)\ sin^{-1} \sqrt{x}\,] + x + c$

(B) $\frac{2}{\pi} [\sqrt{x - x^2} - (1 - 2x)\ sin^{-1} \sqrt{x}\,] - x + c$

(C) $\frac{2}{\pi} [\sqrt{x - x^2} + (1 - 2x)\ sin^{-1} \sqrt{x}\,] + x + c$

(D) $\frac{2}{\pi} [\sqrt{x - x^2} + (1 - 2x)\ sin^{-1} \sqrt{x}\,] - x + c$ (IIT : 1986)

ઉકેલ : $I = \int \left(\frac{sin^{-1}\sqrt{x} - cos^{-1}\sqrt{x}}{sin^{-1}\sqrt{x} + cos^{-1}\sqrt{x}} \right) dx$

$= \frac{2}{\pi} \int \left\{ sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - sin^{-1}\sqrt{x} \right) \right\} dx$ $\left(sin^{-1}\sqrt{x} + cos^{-1}\sqrt{x} = \frac{\pi}{2}\right)$

$= \frac{2}{\pi} \int \left(2\,sin^{-1}\sqrt{x} - \frac{\pi}{2} \right) dx$

$= \frac{4}{\pi} \int sin^{-1}\sqrt{x}\, dx - x$

$I = \frac{4}{\pi} I_1 - x,$ જ્યાં, $I_1 = \int sin^{-1}\sqrt{x}\ dx$

ધારો કે, $x = sin^2\theta$ $\left(0 < \theta < \frac{\pi}{2}\right)$

$\therefore$ $dx = 2sin\theta\ cos\theta\ d\theta$

$\therefore$ $I_1 = \int \theta \cdot sin2\theta\ d\theta$

$= \theta \int sin2\theta\ d\theta - \int \left(\frac{d}{d\theta}(\theta) \int sin\,2\theta\ d\theta \right) d\theta$

$= -\theta \cdot \frac{cos2\theta}{2} - \int 1 \cdot \left(\frac{-cos2\theta}{2} \right) d\theta$

$= -\frac{\theta}{2}\ cos2\theta + \frac{1}{4}\ sin\ 2\theta + c$

$= -\frac{1}{2}\ (1 - 2x)\ sin^{-1}\sqrt{x} + \frac{1}{2} \sqrt{x} \sqrt{1 - x} + c$

$(cos2\theta = 1 - 2sin^2\theta = 1 - 2x,\ sin2\theta = 2sin\theta\ \ cos\theta = 2\sqrt{x}\sqrt{1 - x})$

$= -\frac{1}{2}\ (1 - 2x)\ sin^{-1}\sqrt{x} + \frac{\sqrt{x - x^2}}{2} + c$

હવે, $I = \frac{4}{\pi} I_1 - x$

$$= \frac{4}{\pi}\left[-\frac{1}{2}(1-2x)\, sin^{-1}\sqrt{x} + \frac{\sqrt{x-x^2}}{2}\right] - x + c$$

$$= \frac{2}{\pi}\,[\sqrt{x-x^2} - (1-2x)\, sin^{-1}\sqrt{x}\,] - x + c$$ જવાબ : **(B)**

(103) $\int \frac{dx}{(2bx + x^2)^{\frac{3}{2}}} = ...$ $(b > 0)$

(A) $\frac{-1}{b^2}\frac{(x+2b)}{\sqrt{2bx+x^2}} + c$ (B) $\frac{-1}{b^2}\frac{(x-b)}{\sqrt{2bx+x^2}} + c$

(C) $\frac{-1}{b^2}\frac{2x}{\sqrt{2bx+x^2}} + c$ (D) $\frac{-1}{b^2}\frac{(x+b)}{\sqrt{2bx+x^2}} + c$

ઉકેલ : $I = \int \frac{1}{(2bx+x^2)^{\frac{3}{2}}} \cdot dx$

$$= \int \frac{1}{\left[(x^2+2bx+b^2)-(b)^2\right]^{\frac{3}{2}}}\, dx$$

$$= \int \frac{dx}{\left[(x+b)^2-(b)^2\right]^{\frac{3}{2}}}$$

ધારો કે, $x + b = b\, sec\theta$ $(0 < \theta < \frac{\pi}{2},\ b > 0)$

$\therefore\ dx = b\, sec\theta\ tan\theta\ d\theta$

$$I = \int \frac{b\, sec\theta\, tan\theta\, d\theta}{(b^2\, sec^2\theta - b^2)^{\frac{3}{2}}}$$

$$= \frac{1}{b^2}\int \frac{sec\theta\, tan\theta\, d\theta}{tan^3\theta}$$

$$= \frac{1}{b^2}\int \frac{cos\theta}{sin^2\theta}\, d\theta$$

$$= \frac{1}{b^2}\,\frac{(sin\theta)^{-1}}{-1} + c$$

$$= \frac{-1}{b^2 sin\theta} + c$$

$$= \frac{-1}{b^2}\,\frac{(x+b)}{\sqrt{x^2+2bx}} + c$$

$\left(sec\theta = \frac{x+b}{b} \Rightarrow cos\theta = \frac{b}{x+b}\right.$ આથી $\left. sin\theta = \sqrt{1-\frac{b^2}{(x+b)^2}} = \frac{\sqrt{x^2+2bx}}{x+b}\right)$ જવાબ : **(D)**

(104) $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}\, dx = Px + Q \log|9e^{2x} - 4| + c$, તો ......

(A) $P = \frac{-3}{2}, Q = \frac{29}{36}$ (B) $P = \frac{-3}{2}, Q = \frac{35}{36}$

(C) $P = \frac{-1}{2}, Q = \frac{17}{36}$ (D) $P = \frac{-1}{2}, Q = \frac{23}{36}$

(IIT : 1989)

ઉકેલ : $I = \int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}\, dx = \int \left(\frac{4e^{2x} + 6}{9e^{2x} - 4}\right) dx$

ધારો કે, $4e^{2x} + 6 = m(9e^{2x} - 4) + n\left\{\frac{d}{dx}(9e^{2x} - 4)\right\}$

$\therefore \quad 4e^{2x} + 6 = m(9e^{2x} - 4) + n\, 18e^{2x}$

$4e^{2x} + 6 = (9m + 18n)e^{2x} - 4m$

સહગુણકો સરખાવતાં $9m + 18n = 4$ અને $-4m = 6$. આથી, $m = \frac{-3}{2}, n = \frac{35}{36}$

$$\therefore \quad I = \int \frac{\frac{-3}{2}(9e^{2x} - 4) + \frac{35}{36}\left\{\frac{d}{dx}(9e^{2x} - 4)\right\}}{(9e^{2x} - 4)}\, dx$$

$$= \frac{-3}{2}\int dx + \frac{35}{36}\int \frac{\frac{d}{dx}(9e^{2x} - 4)}{(9e^{2x} - 4)}\, dx$$

$$= \frac{-3}{2}x + \frac{35}{36}\log(9e^{2x} - 4) + c$$

તેથી, $P = \frac{-3}{2}, Q = \frac{35}{36}$ **જવાબ : (B)**

બીજી રીત : $e^x = t$ લેતાં $e^x dx = dt,\ t dx = dt$

$$\therefore \quad I = \int \frac{4t^2 + 6}{9t^2 - 4}\left(\frac{dt}{t}\right)$$

$$= \int \frac{(4t^2 + 6)\, dt}{t(3t + 2)(3t - 2)}$$

$$= \int \frac{-\frac{3}{2}\, dt}{t} + \int \frac{\frac{35}{12}\, dt}{3t + 2} + \int \frac{\frac{35}{12}\, dt}{3t - 2}$$

$$= -\frac{3}{2}\log|t| + \frac{35}{36}\log|(3t + 2)(3t - 2)|$$

$$= -\frac{3}{2}x + \frac{35}{36}\log(9e^{2x} - 4)$$

$$\therefore P = \frac{-3}{2}, Q = \frac{35}{36}$$

(105) $\int \frac{2tan\,x + 3}{3tan\,x + 4}\,dx = ...$

(A) $\frac{24}{25}x + \frac{1}{25}\log|3sin\,x + 4cos\,x| + c$ (B) $\frac{18}{25}x + \frac{1}{25}\log|3sin\,x + 4cos\,x| + c$

(C) $\frac{14}{25}x + \frac{2}{25}\log|3sin\,x + 4cos\,x| + c$ (D) $\frac{4x}{25} + \frac{2}{25}\log|3sin\,x + 4cos\,x| + c$

ઉકેલ : $I = \int \frac{2tan\,x + 3}{3tan\,x + 4}\,dx$

$= \int \frac{2sin\,x + 3cos\,x}{3sin\,x + 4cos\,x}\,dx$

હવે, ધારો કે $2sin\,x + 3cos\,x = A(3sin\,x + 4cos\,x) + B\left[\frac{d}{dx}(3sin\,x + 4cos\,x)\right]$

$2sin\,x + 3cos\,x = (3A - 4B)\,sin\,x + (4A + 3B)\,cos\,x$

$\therefore$ $3A - 4B = 2,\ 4A + 3B = 3.$ આથી, $A = \frac{18}{25},\ B = \frac{1}{25}$

$\therefore$ $I = \frac{18}{25}\int dx + \frac{1}{25}\int \frac{\frac{d}{dx}(3sin\,x + 4cos\,x)}{3sin\,x + 4cos\,x}\,dx$

$= \frac{18}{25}x + \frac{1}{25}\log|3sin\,x + 4cos\,x| + c$ **જવાબ : (B)**

(106) $\int \sqrt{tan\,x}\,dx = A\,tan^{-1}\left(\frac{tanx - 1}{\sqrt{2tan\,x}}\right) + B\log\left|\frac{tanx - \sqrt{2tan\,x} + 1}{tanx + \sqrt{2tan\,x} + 1}\right| + c,$ તો ......

(A) $A = \frac{1}{\sqrt{2}},\ B = 2\sqrt{2}$ (B) $A = \frac{1}{\sqrt{2}},\ B = -\frac{1}{\sqrt{2}}$

(C) $A = \frac{1}{\sqrt{2}},\ B = \frac{1}{2\sqrt{2}}$ (D) $A = \frac{1}{\sqrt{2}},\ B = -\frac{1}{2\sqrt{2}}$

ઉકેલ : $I = \int \sqrt{tan\,x}\,dx$

ધારો કે, $tan\,x = t^2$

$\therefore$ $sec^2x\,dx = 2t\,dt$

$\therefore$ $dx = \frac{2t}{t^4 + 1}\,dt$

$\therefore$ $I = \int \frac{2t^2}{t^4 + 1}\,dt$

$= \int \frac{(t^2 + 1) + (t^2 - 1)}{(t^4 + 1)}\,dt$

$= \int \frac{t^2 + 1}{t^4 + 1}\,dt + \int \frac{t^2 - 1}{t^4 + 1}\,dt$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)}{t^2+\frac{1}{t^2}}\,dt + \int \frac{\left(1-\frac{1}{t^2}\right)}{t^2+\frac{1}{t^2}}\,dt$$

$$= \int \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}\,dt + \int \frac{\left(1-\frac{1}{t^2}\right)}{\left(t+\frac{1}{t}\right)^2-2}\,dt$$

ધારો કે, પ્રથમ સંકલ્ય માટે $t - \frac{1}{t} = u$ અને દ્વિતીય સંકલ્ય માટે $t + \frac{1}{t} = v$

$\therefore \quad \left(1+\frac{1}{t^2}\right) dt = du$ અને $\left(1-\frac{1}{t^2}\right) dt = dv$

$$\therefore \quad I = \int \frac{du}{u^2+2} + \int \frac{dv}{v^2-2}$$

$$= \frac{1}{\sqrt{2}}\, tan^{-1} \frac{u}{\sqrt{2}} + \frac{1}{2\sqrt{2}} \log \left|\frac{v-\sqrt{2}}{v+\sqrt{2}}\right| + c$$

$$= \frac{1}{\sqrt{2}}\, tan^{-1} \left(\frac{t^2-1}{\sqrt{2}t}\right) + \frac{1}{2\sqrt{2}} \log \left|\frac{t^2+1-\sqrt{2}t}{t^2+1+\sqrt{2}t}\right| + c$$

$$= \frac{1}{\sqrt{2}}\, tan^{-1} \left(\frac{tan\, x-1}{\sqrt{2 tan\, x}}\right) + \frac{1}{2\sqrt{2}} \log \left|\frac{tan\, x-\sqrt{2 tan\, x}+1}{tan\, x+\sqrt{2 tan\, x}+1}\right| + c$$

$\therefore \quad A = \frac{1}{\sqrt{2}},\ B = \frac{1}{2\sqrt{2}}$ **જવાબ : (C)**

(107) $\int \frac{(x^2-1)\ dx}{(x^2+1)\sqrt{x^4+1}}$ = ... $(x > 0)$

(A) $\frac{1}{\sqrt{2}}\, cos^{-1}\left(\frac{\sqrt{2}x}{x^2+1}\right) + c$

(B) $\frac{x}{\sqrt{2}}\, sin^{-1}\left(\frac{\sqrt{2}x}{x^2-1}\right) + c$

(C) $\frac{x}{\sqrt{2}}\, sin^{-1}\left(\frac{x-1}{x+1}\right) + c$

(D) $\frac{1}{\sqrt{2}}\, cos^{-1}\left\{\frac{1}{\sqrt{2}}\left(\frac{x+1}{x^2+1}\right)\right\} + c$

ઉકેલ : $I = \int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}}\,dx$

$$= \int \frac{(x^2-1)\,dx}{x(x^2+1)\sqrt{x^2+\frac{1}{x^2}}}$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}$$

$$= \int \frac{dt}{t\sqrt{t^2 - 2}}$$ $\left(\text{ધારો કે, } x + \frac{1}{x} = t \quad \text{આથી, } \left(1 - \frac{1}{x^2}\right) dx = dt\right)$

$$= \frac{1}{\sqrt{2}} sec^{-1} \left(\frac{t}{\sqrt{2}}\right) + c$$

$$= \frac{1}{\sqrt{2}} cos^{-1} \left(\frac{\sqrt{2}}{t}\right) + c$$

$$= \frac{1}{\sqrt{2}} cos^{-1} \left(\frac{\sqrt{2}x}{x^2 + 1}\right) + c$$

જવાબ : (A)

(108) $\int \frac{dx}{sin^6x + cos^6x} = tan^{-1} (f(x)) + c$, તો $f(x) =$ ......

(A) $tan\, x - 1$ (B) $tan^2x - 1$ (C) $tan\, x - cot\, x$ (D) $2tan^2x - 3cot^2x$

ઉકેલ : $I = \int \frac{dx}{sin^6x + cos^6x}$

$$= \int \frac{dx}{(sin^2x + cos^2x)\,(sin^4x + cos^4x - sin^2x\, cos^2x)}$$

$$= \int \frac{dx}{sin^4x + cos^4x - sin^2x\, cos^2x}$$

અંશ અને છેદને cos^4x વડે ભાગતાં,

$$= \int \frac{sec^4x\, dx}{tan^4x + 1 - tan^2x}$$

$$= \int \frac{(tan^2x + 1)\, sec^2x\, dx}{tan^4x + 1 - tan^2x}$$

$$= \int \frac{(t^2 + 1)\, dt}{t^4 + 1 - t^2}$$ (ધારો કે, $tan\, x = t$ આથી, $sec^2x\, dx = dt$)

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t^2 - 1 + \frac{1}{t^2}\right)}$$

$$= \int \frac{\left(1 + \frac{1}{t^2}\right) dt}{\left(t - \frac{1}{t}\right)^2 + 1}$$

$$= \int \frac{du}{u^2 + 1}$$ (ધારો કે, $t - \frac{1}{t} = u$ આથી, $\left(1 + \frac{1}{t^2}\right) dt = du$)

$= tan^{-1}(u) + c$

$= tan^{-1}\left(t - \frac{1}{t}\right) + c$

$= tan^{-1}(tan\ x - cot\ x) + c$

$\therefore\ f(x) = tan\ x - cot\ x$ જવાબ : **(C)**

(109) $\int e^{\frac{x}{2}} sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx = ...$

(A) $e^{\frac{x}{2}} sin\frac{x}{2} + c$ (B) $e^{\frac{x}{2}} cos\frac{x}{2} + c$ (C) $\sqrt{2}e^{\frac{x}{2}} cos\frac{x}{2} + c$ (D) $\sqrt{2}e^{\frac{x}{2}} sin\frac{x}{2} + c$

ઉકેલ : $I = \int e^{\frac{x}{2}} sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx$

$= 2sin\left(\frac{x}{2} + \frac{\pi}{4}\right) e^{\frac{x}{2}} - \int cos\left(\frac{x}{2} + \frac{\pi}{4}\right)\frac{1}{2} \cdot 2e^{\frac{x}{2}}\ dx$

$= 2sin\left(\frac{x}{2} + \frac{\pi}{4}\right) e^{\frac{x}{2}} - \int cos\left(\frac{x}{2} + \frac{\pi}{4}\right) e^{\frac{x}{2}}\ dx$

$I = 2 \cdot e^{\frac{x}{2}} sin\left(\frac{x}{2} + \frac{\pi}{4}\right) - \left[2e^{\frac{x}{2}} cos\left(\frac{x}{2} + \frac{\pi}{4}\right) + \int sin\left(\frac{x}{2} + \frac{\pi}{4}\right) e^{\frac{x}{2}}\ dx\right]$

$= 2e^{\frac{x}{2}}\left\{sin\left(\frac{x}{2} + \frac{\pi}{4}\right) - cos\left(\frac{x}{2} + \frac{\pi}{4}\right)\right\} - I$

$\therefore\ 2I = 2e^{\frac{x}{2}}\left\{sin\left(\frac{x}{2} + \frac{\pi}{4}\right) - cos\left(\frac{x}{2} + \frac{\pi}{4}\right)\right\} + c'$

$I = \sqrt{2}e^{\frac{x}{2}} sin\frac{x}{2} + c$ $\left(\frac{c'}{2} = c\right)$ જવાબ : **(D)**

(110) $\int \frac{x^2}{(x\,sin\ x + cos\ x)^2}\ dx = ...$

(A) $\frac{sin\ x + cos\ x}{x\,sin\ x + cos\ x}$ (B) $\frac{x\ sin\ x - cos\ x}{x\,sin\ x + cos\ x}$ (C) $\frac{sin\ x - x\ cos\ x}{x\,sin\ x + cos\ x}$ (D) $\frac{x\ sin\ x + cos\ x}{x\,sin\ x + cos\ x}$

ઉકેલ : $I = \int \frac{x^2}{(x\,sin\ x + cos\ x)^2}\ dx$

$= \int \frac{x\ cos\ x}{(x\,sin\ x + cos\ x)^2} \cdot \left(\frac{x}{cos\ x}\right) dx$

$= \frac{x}{cos\ x}\int \frac{x\ cos\ x}{(x\,sin\ x + cos\ x)^2}\ dx - \int\left(\frac{d}{dx}\left(\frac{x}{cos\ x}\right)\int \frac{x\ cos\ x}{(x\,sin\ x + cos\ x)^2}\,dx\right) dx$

$= \frac{x}{cos\ x} \cdot \frac{-1}{(x\,sin\ x + cos\ x)} + \int\left(\frac{cos\,x + x\,sin\,x}{cos^2 x} \times \frac{1}{(x\,sin\,x + cos\,x)}\right) dx$

$\left(\frac{d}{dx}(x\ sin\ x + cos\ x) = x\ cos\ x \text{ અને } \int \frac{1}{t^2}\ dt = \frac{-1}{t}\right)$

$$= \frac{-x}{(x \sin x + \cos x) \cos x} + \int \sec^2 x \, dx$$

$$= \frac{-x}{(x \sin x + \cos x) \cos x} + \frac{\sin x}{\cos x} + c$$

$$= \frac{-x + x \sin^2 x + \sin x \cos x}{(x \sin x + \cos x) \cos x} + c$$

$$= \frac{\sin x \cos x - x(1 - \sin^2 x)}{(x \sin x + \cos x) \cos x} + c$$

$$= \frac{\cos x (\sin x - x \cos x)}{(x \sin x + \cos x) \cos x} + c$$

$$= \frac{\sin x - x \cos x}{x \sin x + \cos x} + c$$

જવાબ : **(C)**

(111) $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} \, dx = ... + c$

(A) $\sin x - 6 \tan^{-1}(\sin x)$

(B) $\sin x - 2(\sin x)^{-1}$

(C) $\sin x - 2(\sin x)^{-1} - 6 \tan^{-1}(\sin x)$

(D) $\sin x - 2(\sin x)^{-1} - 5 \tan^{-1}(\sin x)$

(IIT : 1990)

ઉકેલ : $I = \int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} \, dx$

$$= \int \frac{(\cos^2 x + \cos^4 x) \cos x \, dx}{\sin^2 x + \sin^4 x}$$

$$= \int \frac{[(1 - \sin^2 x) + (1 - \sin^2 x)^2] \cos x \, dx}{\sin^2 x + \sin^4 x}$$

ધારો કે, $\sin x = t$. આથી, $\cos x \, dx = dt$

$$\therefore I = \int \frac{(1 - t^2) + (1 - t^2)^2}{t^2 + t^4} \, dt$$

$$= \int \frac{1 - t^2 + 1 - 2t^2 + t^4}{t^4 + t^2} \, dt$$

$$= \int \frac{(t^4 + t^2) - 4t^2 + 2}{(t^4 + t^2)} \, dt$$

$$= \int 1 \, dt - \int \frac{4t^2 - 2}{t^2(t^2 + 1)} \, dt$$

અહીંથી નીચે પ્રમાણે પણ આગળ વધી શકાય :

$$= t - \int\left(\frac{-2}{t^2} + \frac{6}{t^2+1}\right) dt \quad \text{(આંશિક અપૂર્ણાંક)}$$

$$= t - \frac{2}{t} - 6\, tan^{-1}\, t$$

$$= sin\, x - 2(sin\, x)^{-1} - 6\, tan^{-1}(sin\, x) + c$$

$$= \int 1\, dt - 4\int\frac{1}{t^2+1}\, dt + 2\int\frac{1}{t^2(t^2+1)}\, dt$$

$$= \int 1\, dt - 4\int\frac{1}{t^2+1^2}\, dt + 2\int\frac{(t^2+1)-(t^2)}{t^2(t^2+1)}\, dt$$

$$= \int 1\, dt - 4\int\frac{1}{t^2+1^2}\, dt + 2\int\frac{1}{t^2}\, dt - 2\int\frac{1}{t^2+1}\, dt$$

$$= t - 4\, tan^{-1}\, t + 2\,\frac{t^{-1}}{-1} - 2\, tan^{-1}\, t + c$$

$$= sin\, x - 2(sin\, x)^{-1} - 6\, tan^{-1}(sin\, x) + c$$

જવાબ : (C)

(112) $\int\frac{dx}{(x-a)\sqrt{(x-a)(x-b)}}\, dx = \text{......} + c.$

(A) $\frac{2}{a-b}\sqrt{\frac{x-a}{x-b}}$ (B) $\frac{-2}{a-b}\sqrt{\frac{x-b}{x-a}}$ (C) $\frac{1}{\sqrt{(x-a)(x-b)}}$ (D) $\frac{2}{a-b}\sqrt{\frac{x-b}{x-a}}$

(IIT : 1996)

ઉકેલ : $I = \int\frac{dx}{(x-a)\sqrt{(x-a)(x-b)}}\, dx$

$$= \int\frac{dx}{(x-a)^{\frac{3}{2}}(x-b)^{\frac{1}{2}}}$$

$$= \int\frac{dx}{(x-a)^2\sqrt{\frac{x-b}{x-a}}}$$

ધારો કે, $\sqrt{\frac{x-b}{x-a}} = t,$ આથી, $\frac{x-b}{x-a} = t^2$

$$\therefore \frac{(x-a)-(x-b)}{(x-a)^2}\, dx = 2t\, dt$$

$$\frac{b-a}{(x-a)^2}\, dx = 2t\, dt$$

$$\frac{1}{(x-a)^2}\, dx = \frac{2t}{b-a}\, dt$$

$$\therefore\ I = \frac{1}{b-a}\int \frac{2t}{t}\,dt$$

$$= \frac{2t}{b-a} + c$$

$$= \frac{-2}{a-b}\sqrt{\frac{x-b}{x-a}} + c$$

જવાબ : **(B)**

(113) $\int \frac{x+1}{x(1+xe^x)^2}\,dx = ... + c.$ (IIT : 1996)

(A) $\log\left|\frac{xe^x}{xe^x+1}\right|$

(B) $\log\left|\frac{xe^x}{xe^x+1}\right| - \frac{1}{xe^x+1}$

(C) $\log\left|\frac{xe^x}{xe^x+1}\right| + \frac{1}{xe^x+1}$

(D) $\frac{1}{1+xe^x} + \log|xe^x+1|$

ઉકેલ : $I = \int \frac{x+1}{x(1+xe^x)^2}\,dx$

$$= \int \frac{(x+1)\,e^x}{xe^x(1+xe^x)^2}\,dx$$

ધારો કે, $xe^x = t$ આથી, $(1+x)e^x\,dx = dt$

$$\therefore\ I = \int \frac{dt}{t(t+1)^2} = \int \frac{(t+1)-(t)}{t(t+1)^2}\,dt$$

$$= \int \frac{1}{t(t+1)}\,dt - \int \frac{1}{(t+1)^2}\,dt$$

$$= \int \frac{(t+1)-(t)}{t(t+1)}\,dt - \int (t+1)^{-2}\,dt$$

$$= \int \frac{1}{t}\,dt - \int \frac{1}{t+1}\,dt - \int (t+1)^{-2}\,dt$$

$$= \log|t| - \log|t+1| - \frac{(t+1)^{-1}}{-1} + c$$

$$= \log\left|\frac{t}{t+1}\right| + \frac{1}{t+1} + c$$

$$= \log\left|\frac{xe^x}{xe^x+1}\right| + \frac{1}{xe^x+1}$$

જવાબ : **(C)**