

13. HEAT & THERMODYNAMICS

Total translational K.E. of gas = $\frac{1}{2} M \langle V^2 \rangle = \frac{3}{2} PV = \frac{3}{2} nRT$

$$\langle V^2 \rangle = \frac{3P}{\rho} \quad V_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_{mol}}} = \sqrt{\frac{3KT}{m}}$$

Important Points :

$$V_{rms} \propto \sqrt{T} \quad \bar{V} = \sqrt{\frac{8KT}{\pi m}} = 1.59 \sqrt{\frac{KT}{m}} \quad V_{ms} = 1.73 \sqrt{\frac{KT}{m}}$$

Most probable speed $V_g = \sqrt{\frac{2KT}{m}} = 1.41 \sqrt{\frac{KT}{m}}$

$$\therefore V_{rms} > \bar{V} > V_g$$

Degree of freedom :

Mono atomic $f = 3$

Diatomic $f = 5$

polyatomic $f = 6$

Maxwell's law of equipartition of energy :

Total K.E. of the molecule = $\frac{1}{2} f KT$

For an ideal gas :

Internal energy $U = \frac{f}{2} nRT$

Workdone in isothermal process : $W = [2.303 nRT \log_{10} \frac{V_f}{V_i}]$

Internal energy in isothermal process : $\Delta U = 0$

Work done in isochoric process : $dW = 0$

Change in int. energy in isochoric process :

$$\Delta U = n \frac{f}{2} R \Delta T = \text{heat given}$$

Isobaric process :

Work done $\Delta W = nR(T_f - T_i)$

change in int. energy $\Delta U = nC_V \Delta T$

heat given $\Delta Q = \Delta U + \Delta W$

Specific heat : $C_V = \frac{f}{2} R \quad C_p = \frac{f}{2} R + R$

Molar heat capacity of ideal gas in terms of R :

(i) for monoatomic gas : $\frac{C_p}{C_V} = 1.67$

(ii) for diatomic gas : $\frac{C_p}{C_V} = 1.4$

(iii) for triatomic gas : $\frac{C_p}{C_v} = 1.33$

In general : $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$

Mayer's eq. $C_p - C_v = R$ for ideal gas only

Adiabatic process :

Work done $\Delta W = \frac{nR(T_i - T_f)}{\gamma - 1}$

In cyclic process :

$\Delta Q = \Delta W$

In a mixture of non-reacting gases :

Mol. wt. = $\frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$

$$C_v = \frac{n_1 C_{v_i} + n_2 C_{v_{ii}}}{n_1 + n_2}$$

$$\gamma = \frac{C_{p(mix)}}{C_{v(mix)}} = \frac{n_1 C_{p_i} + n_2 C_{p_{ii}} + \dots}{n_1 C_{v_i} + n_2 C_{v_{ii}} + \dots}$$

Calorimetry and thermal expansion

Types of thermometers :

(a) Liquid Thermometer : $T = \frac{\ell - \ell_0}{\ell_{100} - \ell_0} \times 100$

(b) Gas Thermometer :

Constant volume : $T = \frac{P - P_0}{P_{100} - P_0} \times 100$; $P = P_{atm} + \rho g h$

Constant Pressure : $T = \frac{V}{V - V_0} \times T_0$

(c) Electrical Resistance Thermometer :

$$T = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

Thermal Expansion :

(a) Linear :

$$\alpha = \frac{\Delta L}{L_0 \Delta T} \quad \text{or} \quad L = L_0 (1 + \alpha \Delta T)$$

(b) Area/superficial :

$$\beta = \frac{\Delta A}{A_0 \Delta T} \quad \text{or} \quad A = A_0 (1 + \beta \Delta T)$$

(c) volume/ cubical :

$$r = \frac{\Delta V}{V_0 \Delta T} \quad \text{or} \quad V = V_0 (1 + \gamma \Delta T)$$

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

Thermal stress of a material :

$$\frac{F}{A} = Y \frac{\Delta \ell}{\ell}$$

Energy stored per unit volume :

$$E = \frac{1}{2} K (\Delta L)^2 \quad \text{or} \quad E = \frac{1}{2} \frac{AY}{L} (\Delta L)^2$$

Variation of time period of pendulum clocks :

$$\Delta T = \frac{1}{2} \Delta \theta T$$

$T < T_0$ - clock-fast : time-gain

$T > T_0$ - clock slow : time-loss

CALORIMETRY :

$$\text{Specific heat } S = \frac{Q}{m \cdot \Delta T}$$

$$\text{Molar specific heat } C = \frac{\Delta Q}{n \cdot \Delta T}$$

$$\text{Water equivalent} = m_w S_w$$

HEAT TRANSFER

$$\text{Thermal Conduction :} \quad \frac{dQ}{dt} = KA \frac{dT}{dx}$$

$$\text{Thermal Resistance :} \quad R = \frac{\ell}{KA}$$

Series and parallel combination of rod :

$$(i) \text{ Series :} \quad \frac{\ell_{eq}}{K_{eq}} = \frac{\ell_1}{K_1} + \frac{\ell_2}{K_2} + \dots \quad (\text{when } A_1 = A_2 = A_3 = \dots)$$

$$(ii) \text{ Parallel :} \quad K_{eq} A_{eq} = K_1 A_1 + K_2 A_2 + \dots \quad (\text{when } \ell_1 = \ell_2 = \ell_3 = \dots)$$

for absorption, reflection and transmission

$$r + t + a = 1$$

Emissive power : $E = \frac{\Delta U}{\Delta A \Delta t}$

Spectral emissive power : $E_{\lambda} = \frac{dE}{d\lambda}$

Emissivity : $e = \frac{\text{E of a body at T temp.}}{\text{E of a black body at T temp.}}$

Kirchoff's law : $\frac{E(\text{body})}{a(\text{body})} = E(\text{black body})$

Wein's Displacement law : $\lambda_m \cdot T = b.$
 $b = 0.282 \text{ cm-k}$

Stefan Boltzmann law :

$$u = \sigma T^4 \quad s = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$$

$$\Delta u = u - u_0 = e \sigma A (T^4 - T_0^4)$$

Newton's law of cooling : $\frac{d\theta}{dt} = k (\theta - \theta_0) ; \theta = \theta_0 + (\theta - \theta_0) e^{-kt}$