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Oscillations

TOPIC 1 Displacement, Phase, Velocity and Acceleration in S.H.M.

1. The position co-ordinates of a particle moving in a 3-D coordinate system is given by **[9 Jan 2019, II]**

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is:

(a) $\sqrt{2}\,a\omega$ (b) $a\omega$ (c) $\sqrt{3}\,a\omega$ (d) $2a\omega$

2. Two simple harmonic motions, as shown, are at right angles. They are combined to form Lissajous figures.

$x(t) = A\sin(at + \delta)$

$y(t) = B\sin(bt)$

Identify the correct match below

[Online April 15, 2018]

(a) Parameters: $A = B$, $a = 2b$; $\delta = \frac{\pi}{2}$; Curve: Circle

(b) Parameters: $A = B$, $a = b$; $\delta = \frac{\pi}{2}$; Curve: Line

(c) Parameters: $A \neq B$, $a = b$; $\delta = \frac{\pi}{2}$; Curve: Ellipse

(d) Parameters: $A \neq B$, $a = b$; $\delta = 0$; Curve: Parabola

3. The ratio of maximum acceleration to maximum velocity in a simple harmonic motion is $10\ s^{-1}$. At, t = 0 the displacement is 5 m. What is the maximum acceleration ? The initial phase is $\frac{\pi}{4}$ **[Online April 8, 2017]**

(a) $500\ m/s^2$ (b) $500\sqrt{2}\ m/s^2$

(c) $750\ m/s^2$ (d) $750\sqrt{2}\ m/s^2$

4. A particle performs simple harmonic mition with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is : **[2016]**

(a) $A\sqrt{3}$ (b) $\frac{7A}{3}$ (c) $\frac{A}{3}\sqrt{41}$ (d) 3A

5. Two particles are performing simple harmonic motion in a straight line about the same equilibrium point. The amplitude and time period for both particles are same and equal to A and T, respectively. At time t = 0 one particle has displacement A while the other one has displacement $\frac{-A}{2}$ and they are moving towards each other. If they cross each other at time t, then t is:

[Online April 9, 2016]

(a) $\frac{5T}{6}$ (b) $\frac{T}{3}$ (c) $\frac{T}{4}$ (d) $\frac{T}{6}$

6. A simple harmonic oscillator of angular frequency 2 rad s^{-1} is acted upon by an external force $F = \sin t$ N. If the oscillator is at rest in its equilibrium position at $t = 0$, its position at later times is proportional to :

[Online April 10, 2015]

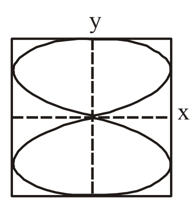
(a) $\sin t + \frac{1}{2}\cos 2t$ (b) $\cos t - \frac{1}{2}\sin 2t$

(c) $\sin t - \frac{1}{2}\sin 2t$ (d) $\sin t + \frac{1}{2}\sin 2t$

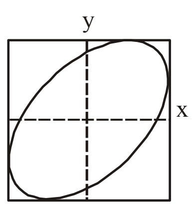
7. x and y displacements of a particle are given as $x(t) = a\sin\omega t$ and $y(t) = a\sin 2\omega t$. Its trajectory will look like :

[Online April 10, 2015]

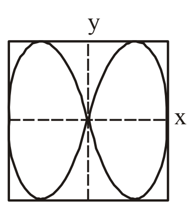
(a)



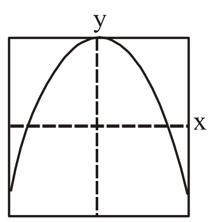
(b)



(c)



(d)



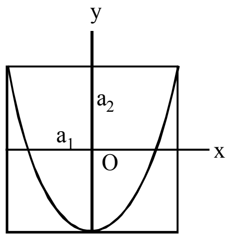
8. A body is in simple harmonic motion with time period half second (T = 0.5 s) and amplitude one cm (A = 1 cm). Find the average velocity in the interval in which it moves form equilibrium position to half of its amplitude. **[Online April 19, 2014]**

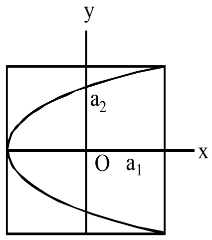
(a) 4 cm/s (b) 6 cm/s (c) 12 cm/s (d) 16 cm/s

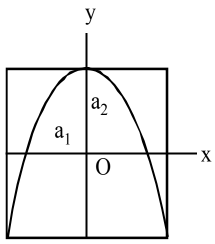
9. Which of the following expressions corresponds to simple harmonic motion along a straight line, where x is the displacement and a, b, c are positive constants? **[Online April 12, 2014]**

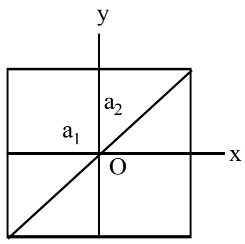
(a) $a + bx - cx^2$ (b) bx^2
(c) $a - bx + cx^2$ (d) $-bx$

10. A particle which is simultaneously subjected to two perpendicular simple harmonic motions represented by; $x = a_1 \cos \omega t$ and $y = a_2 \cos 2 \omega t$ traces a curve given by: **[Online April 9, 2014]**

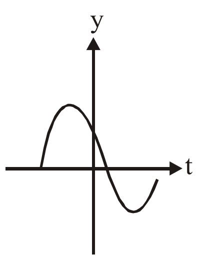
(a) 

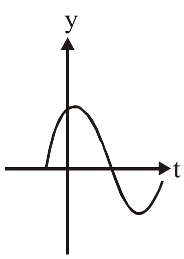
(b) 

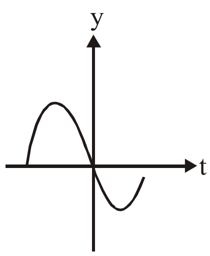
(c) 

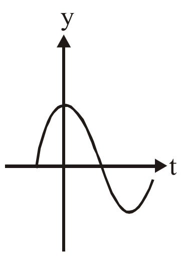
(d) 

11. The displacement $y(t) = A \sin (\omega t + \phi)$ of a pendulum for $\phi = \frac{2\pi}{3}$ is correctly represented by **[Online May 19, 2012]**

(a) 

(b) 

(c) 

(d) 

12. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0 (X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is: **[2011]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

13. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is: **[2011]**

(a) $\frac{M+m}{M}$ (b) $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$
(c) $\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$ (d) $\frac{M}{M+m}$

14. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then **[2007]**

(a) $A = x_0\omega^2, \ \delta = 3\pi/4$ (b) $A = x_0, \delta = -\pi/4$
(c) $A = x_0\omega^2, \ \delta = \pi/4$ (d) $A = x_0\omega^2, \ \delta = -\pi/4$

15. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time

(a) at the mean position of the platform **[2006]**
(b) for an amplitude of $\frac{g}{\omega^2}$
(c) for an amplitude of $\frac{g^2}{\omega^2}$
(d) at the highest position of the platform

16. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is **[2006]**

(a) 0.01 s (b) 10 s (c) 0.1 s (d) 100 s

17. The function $\sin^2(\omega t)$ represents **[2005]**

(a) a periodic, but not simple harmonic motion with a period $\frac{\pi}{\omega}$
(b) a periodic, but not simple harmonic motion with a period $\frac{2\pi}{\omega}$
(c) a simple harmonic motion with a period $\frac{\pi}{\omega}$
(d) a simple harmonic motion with a period $\frac{2\pi}{\omega}$

18. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is **[2005]**

(a) $\frac{\pi}{3}$ (b) $\frac{-\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $\frac{-\pi}{3}$

19. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillation, are equal, the ratio of amplitude of A and B is **[2003]**

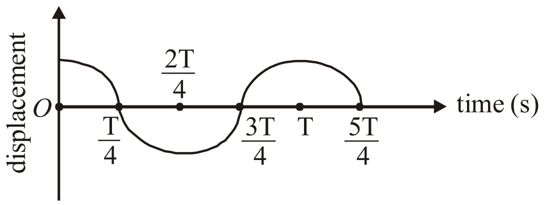
(a) $\sqrt{\frac{k_1}{k_2}}$ (b) $\frac{k_2}{k_1}$ (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_1}{k_2}$

20. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is **[2003]**

(a) -4 (b) 4 (c) $4\sqrt{2}$ (d) 8

TOPIC 2 Energy in Simple Harmonic Motion

21. The displacement time graph of a particle executing S.H.M. is given in figure : (sketch is schematic and not to scale)



Which of the following statements is/are true for this motion? **[Sep. 02, 2020 (II)]**

(1) The force is zero at $t = \frac{3T}{4}$

(2) The acceleration is maximum at $t = T$

(3) The speed is maximum at $t = \frac{T}{4}$

(4) The P.E. is equal to K.E. of the oscillation at $t = \frac{T}{2}$

(a) (1), (2) and (4) (b) (2), (3) and (4)
(c) (1), (2) and (3) (d) (1) and (4)

22. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A \sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210s will be : **[11 Jan 2019, I]**

(a) $\frac{1}{9}$ (b) 1 (c) 2 (d) $\frac{1}{3}$

23. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. **[11 Jan 2019, II]**

(a) $K_2 = 2K_1$ (b) $K_2 = \frac{K_1}{2}$

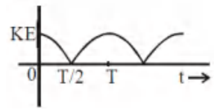
(c) $K_2 = \frac{K_1}{4}$ (d) $K_2 = K_1$

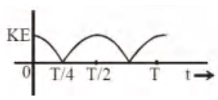
24. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about $x = 0$. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be: **[9 Jan 2019, II]**

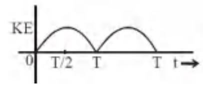
(a) $\frac{A}{2}$ (b) $\frac{A}{2\sqrt{2}}$

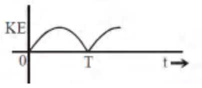
(c) $\frac{A}{\sqrt{2}}$ (d) A

25. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like: **[2017]**

(a) 

(b) 

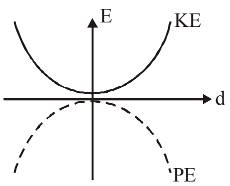
(c) 

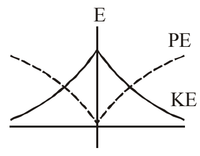
(d) 

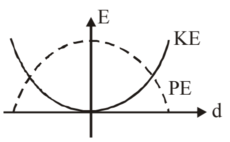
26. A block of mass 0.1 kg is connected to an elastic spring of spring constant 640 Nm^{-1} and oscillates in a medium of constant 10^{-2} kg s^{-1}. The system dissipates its energy gradually. The time taken for its mechanical energy of vibration to drop to half of its initial value, is closest to : **[Online April 9, 2017]**

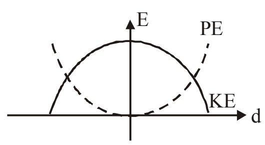
(a) 2 s (b) 3.5 s (c) 5 s (d) 7 s

27. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (*graphs are schematic and not drawn to scale*) **[2015]**

(a) 

(b) 

(c) 

(d) 

28. A pendulum with time period of 1s is losing energy. At certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant (in s^{-1}) is : **[Online April 11, 2015]**

(a) $\frac{1}{2}$ (b) $\frac{1}{30}\ln 3$ (c) 2 (d) $\frac{1}{15}\ln 3$

29. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

If two springs S_1 and S_2 of force constants k_1 and k_2 respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement 1 : If stretched by the same amount work done on S_1

Statement 2 : $k_1 < k_2$ **[2012]**

(a) Statement 1 is false, Statement 2 is true.
(b) Statement 1 is true, Statement 2 is false.
(c) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for Statement 1

30. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is **[2007]**

(a) $2\pi^2 ma^2\nu^2$ (b) $\pi^2 ma^2\nu^2$
(c) $\frac{1}{4}ma^2\nu^2$ (d) $4\pi^2 ma^2\nu^2$

31. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy? **[2006]**

(a) $\frac{1}{6}$s (b) $\frac{1}{4}$s (c) $\frac{1}{3}$s (d) $\frac{1}{12}$s

32. The total energy of a particle, executing simple harmonic motion is **[2004]**

(a) independent of x (b) $\propto x^2$
(c) $\propto x$ (d) $\propto x^{1/2}$

where x is the displacement from the mean position, hence total energy is independent of x.

33. A body executes simple harmonic motion. The potential energy (P.E), the kinetic energy (K.E) and total energy (T.E) are measured as a function of displacement x. Which of the following statements is true ? **[2003]**

(a) K.E. is maximum when $x = 0$
(b) T.E is zero when $x = 0$
(c) K.E is maximum when x is maximum
(d) P.E is maximum when $x = 0$

34. In a simple harmonic oscillator, at the mean position **[2002]**

(a) kinetic energy is minimum, potential energy is maximum
(b) both kinetic and potential energies are maximum
(c) kinetic energy is maximum, potential energy is minimum
(d) both kinetic and potential energies are minimum

TOPIC 3 Time Period, Frequency, Simple Pendulum and Spring Pendulum

35. An object of mass m is suspended at the end of a massless wire of length L and area of cross-section, A. Young modulus of the material of the wire is Y. If the mass is pulled down slightly its frequency of oscillation along the vertical direction is: **[Sep. 06, 2020 (I)]**

(a) $f = \frac{1}{2\pi}\sqrt{\frac{mL}{YA}}$ (b) $f = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$
(c) $f = \frac{1}{2\pi}\sqrt{\frac{mA}{YL}}$ (d) $f = \frac{1}{2\pi}\sqrt{\frac{YL}{mA}}$

36. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0 \sin^2\omega t$, where '$y$' is measured from the lower end of unstretched spring. Then ω is : **[Sep. 06, 2020 (II)]**

(a) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (b) $\sqrt{\frac{g}{y_0}}$
(c) $\sqrt{\frac{g}{2y_0}}$ (d) $\sqrt{\frac{2g}{y_0}}$

37. A block of mass m attached to a massless spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is : **[Sep. 03, 2020 (II)]**

(a) $\frac{1}{\sqrt{2}}$ (b) 1 (c) $\frac{1}{2}$ (d) $\sqrt{2}$

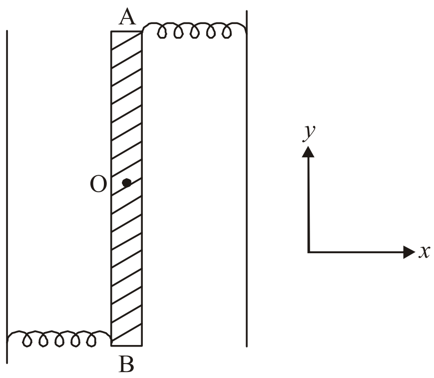
38. A person of mass M is, sitting on a swing of length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance l (l<<L), is close to : **[12 April 2019, II]**

(a) $mgl(1-\theta_0^2)$ (b) $mgl(1+\theta_0^2)$
(c) mgl (d) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$

39. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$ th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is : **[9 April 2019 I]**

(a) $2T\sqrt{\frac{1}{10}}$ (b) $2T\sqrt{\frac{1}{14}}$ (c) $4T\sqrt{\frac{1}{15}}$ (d) $4T\sqrt{\frac{1}{14}}$

40. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate frreely in horizontal plane. The other ends of two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is: **[12 Jan 2019, I]**



(a) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

41. A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. The M is given by : **[12 Jan 2019, I]**

(a) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (b) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

(c) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (d) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

42. A simple harmonic motion is represented by :

$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm

The amplitude and time period of the motion are :

[12 Jan 2019, II]

(a) 10 cm, $\frac{2}{3}$ s (b) 10 cm, $\frac{3}{2}$ s

(c) 5 cm, $\frac{3}{2}$ s (d) 5 cm, $\frac{2}{3}$ s

43. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by : **[11 Jan 2019, II]**

(a) 10^{-3} rad/s (b) 1 rad/s

(c) 10^{-1} rad/s (d) 10^{-5} rad/s

44. The mass and the diameter of a planet are three times the respective values for the Earth. The period of oscillation of a simple pendulum on the Earth is 2s. The period of oscillation of the same pendulum on the planet would be: **[11 Jan 2019, II]**

(a) $\frac{\sqrt{3}}{2}$s (b) $\frac{2}{\sqrt{3}}$s

(c) $\frac{3}{2}$s (d) $2\sqrt{3}$s

45. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is: **[10 Jan 2019, II]**

(a) $\frac{4\pi}{3}$ (b) $\frac{3}{8}\pi$

(c) $\frac{8\pi}{3}$ (d) $\frac{7}{3}\pi$

46. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω is close to: (density of water = 10^3 kg/m^3)

[10 Jan 2019, II]

(a) 3.75 rad s^{-1} (b) 1.25 rad s^{-1}

(c) 2.50 rad s^{-1} (d) 5.00 rad s^{-1}

47. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to : **[9 Jan 2019, II]**

(a) 0.77 (b) 0.57

(c) 0.37 (d) 0.17

48. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number $= 6.02\times10^{23}$ gm mole^{-1}) **[2018]**

(a) 6.4 N/m (b) 7.1 N/m (c) 2.2 N/m (d) 5.5 N/m

49. A particle executes simple harmonic motion and is located at x = a, b and c at times t_0, $2t_0$ and $3t_0$ respectively. The frequency of the oscillation is **[Online April 16, 2018]**

(a) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$ (b) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{3c}\right)$

(c) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$ (d) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

50. In an experiment to determine the period of a simple pendulum of length 1 m, it is attached to different spherical bobs of radii r_1 and r_2. The two spherical bobs have uniform mass distribution. If the relative difference in the periods, is found to be 5×10^{-4} s, the difference in radii, $|r_1 - r_2|$ is best given by: **[Online April 9, 2017]**

(a) 1 cm (b) 0.1 cm (c) 0.5 cm (d) 0.01 cm

51. A 1 kg block attached to a spring vibrates with a frequency of 1 Hz on a frictionless horizontal table. Two springs identical to the original spring are attached in parallel to an 8 kg block placed on the same table. So, the frequency of vibration of the 8 kg block is : **[Online April 8, 2017]**

(a) $\frac{1}{4}$Hz (b) $\frac{1}{2\sqrt{2}}$Hz (c) $\frac{1}{2}$Hz (d) 2 Hz

52. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20° C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively : **[2016]**

(a) 30°C; $\alpha = 1.85 \times 10^{-3}$/°C
(b) 55°C; $\alpha = 1.85 \times 10^{-2}$/°C
(c) 25°C; $\alpha = 1.85 \times 10^{-5}$/°C
(d) 60°C; $\alpha = 1.85 \times 10^{-4}$/°C

53. In an engine the piston undergoes vertical simple harmonic motion with amplitude 7 cm. A washer rests on top of the piston and moves with it. The motor speed is slowly increased. The frequency of the piston at which the washer no longer stays in contact with the piston, is close to : **[Online April 10, 2016]**

(a) 0.7 Hz (b) 1.9 Hz (c) 1.2 Hz (d) 0.1 Hz

54. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:

(g = *gravitational acceleration*) **[2015]**

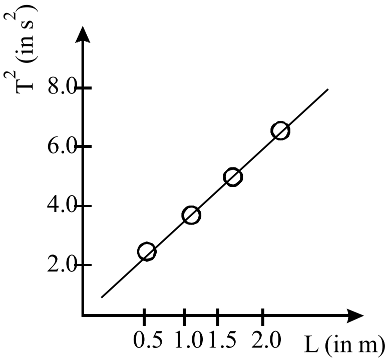
(a) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (b) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

(c) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{A}{Mg}$ (d) $\left[\left(\frac{T_M}{T}\right)^2-1\right]\frac{Mg}{A}$

55. A particle moves with simple harmonic motion in a straight line. In first τs, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then:

(a) amplitude of motion is 3a **[2014]**
(b) time period of oscillations is 8τ
(c) amplitude of motion is 4a
(d) time period of oscillations is 6τ

56. In an experiment for determining the gravitational acceleration g of a place with the help of a simple pendulum, the measured time period square is plotted against the string length of the pendulum in the figure. **[Online April 19, 2014]**



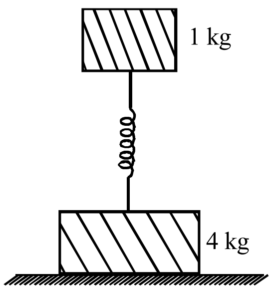
What is the value of g at the place?

(a) 9.81 m/s^2 (b) 9.87 m/s^2
(c) 9.91 m/s^2 (d) 10.0 m/s^2

57. The amplitude of a simple pendulum, oscillating in air with a small spherical bob, decreases from 10 cm to 8 cm in 40 seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3. The time in which amplitude of this pendulum will reduce from 10 cm to 5 cm in carbon dioxide will be close to ($\ln 5 = 1.601, \ln 2 = 0.693$). **[Online April 9, 2014]**

(a) 231 s (b) 208 s (c) 161 s (d) 142 s

58. Two bodies of masses 1 kg and 4 kg are connected to a vertical spring, as shown in the figure. The smaller mass executes simple harmonic motion of angular frequency 25 rad/s, and amplitude 1.6 cm while the bigger mass remains stationary on the ground. The maximum force exerted by the system on the floor is (take g = 10 ms^{-2}) **[Online April 9, 2014]**



(a) 20 N (b) 10 N (c) 60 N (d) 40 N

59. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **[2013]**

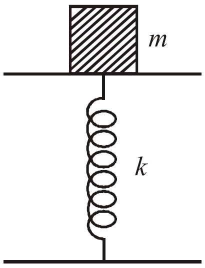
(a) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (b) $\frac{1}{2\pi}\frac{V_0 M P_0}{A^2\gamma}$

(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

60. A mass $m = 1.0$ kg is put on a flat pan attached to a vertical spring fixed on the ground. The mass of the spring and the pan is negligible. When pressed slightly and released, the mass executes simple harmonic motion. The spring constant is 500 N/m. What is the amplitude A of the motion, so that the mass m tends to get detached from the pan ?

(Take $g = 10$ m/s^2).

The spring is stiff enough so that it does not get distorted during the motion. **[Online April 22, 2013]**



(a) $A > 2.0$ cm (b) $A = 2.0$ cm
(c) $A < 2.0$ cm (d) $A = 1.5$ cm

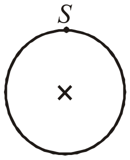
61. Two simple pendulums of length 1 m and 4 m respectively are both given small displacement in the same direction at the same instant. They will be again in phase after the shorter pendulum has completed number of oscillations equal to : **[Online April 9, 2013]**

(a) 2 (b) 7 (c) 5 (d) 3

62. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between $t = 0$s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum in second is (assuming damping is small) **[2012]**

(a) $\frac{0.693}{b}$ (b) b (c) $\frac{1}{b}$ (d) $\frac{2}{b}$

63. A ring is suspended from a point S on its rim as shown in the figure. When displaced from equilibrium, it oscillates with time period of 1 second. The radius of the ring is (take $g = \pi^2$) **[Online May 19, 2012]**



(a) 0.15 m (b) 1.5 m (c) 1.0 m (d) 0.5 m

64. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T' **[2011 RS]**

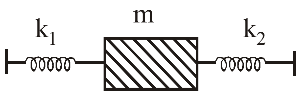
(a) $2\pi\sqrt{\frac{\ell d}{\rho g}}$ (b) $2\pi\sqrt{\frac{\ell \rho}{d g}}$

(c) $2\pi\sqrt{\frac{\ell d}{(\rho - d)g}}$ (d) $2\pi\sqrt{\frac{\ell \rho}{(\rho - d)g}}$

65. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time? **[2009]**

(a) aT/x (b) $aT + 2\pi v$
(c) aT/v (d) $a^2T^2 + 4\pi^2v^2$

66. Two springs, of force constants k_1 and k_2 are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes **[2007]**



(a) $2f$ (b) $f/2$ (c) $f/4$ (d) $4f$

67. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is **[2007]**

(a) 0.25 s (b) 0.5 s (c) 0.75 s (d) 0.125 s

68. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would **[2005]**

(a) first decrease and then increase to the original value
(b) first increase and then decrease to the original value
(c) increase towards a saturation value
(d) remain unchanged

69. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is **[2005]**

(a) $\frac{2\pi}{\sqrt{\alpha}}$ (b) $\frac{2\pi}{\alpha}$ (c) $2\pi\sqrt{\alpha}$ (d) $2\pi\alpha$

70. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3) \times 1000$ kg/m^3. Which relationship between t and t_0 is true? **[2004]**

(a) $t = 2t_0$ (b) $t = t_0/2$

(c) $t = t_0$ (d) $t = 4t_0$

71. A particle at the end of a spring executes S.H.M with a period t_1. while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T then **[2004]**

(a) $T^{-1} = t_1^{-1} + t_2^{-1}$ (b) $T^2 = t_1^2 + t_2^2$

(c) $T = t_1 + t_2$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

72. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $\frac{5T}{3}$. Then the ratio of $\frac{m}{M}$ is **[2003]**

(a) $\frac{3}{5}$ (b) $\frac{25}{9}$ (c) $\frac{16}{9}$ (d) $\frac{5}{3}$

73. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is **[2003]**

(a) 11% (b) 21% (c) 42% (d) 10%

74. If a spring has time period T, and is cut into n equal parts, then the time period of each part will be **[2002]**

(a) $T\sqrt{n}$ (b) $T/\sqrt{n}$ (c) nT (d) T

75. A child swinging on a swing in sitting position, stands up, then the time period if the swing will **[2002]**

(a) increase

(b) decrease

(c) remains same

(d) increases if the child is long and decreases if the child is short

TOPIC 4 Damped, Forced Oscillations and Resonance

76. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to : **[8 April 2019, II]**

(a) 50s (b) 100s (c) 20s (d) 10s

77. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}. \cos(10\pi t + \varphi)$. Here t is in seconds.

The time taken for its amplitude of vibration to drop to half of its initial value is close to : **[9 Jan 2019, II]**

(a) 4s (b) 7s (c) 13s (d) 27s

78. An oscillator of mass M is at rest in its equilibrium position in a potential $V = \frac{1}{2}k(x-X)^2$. A particle of mass m comes from right with speed u and collides completely inelastically with M and sticks to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is: (M = 10, m = 5, u = 1, k = 1). **[Online April 16, 2018]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{2}{3}$ (d) $\sqrt{\frac{3}{5}}$

79. The angular frequency of the damped oscillator is given by, $\omega = \sqrt{\left(\frac{k}{m} - \frac{r^2}{4m^2}\right)}$ where k is the spring constant, m is the mass of the oscillator and r is the damping constant. If the ratio $\frac{r^2}{mk}$ is 8%, the change in time period compared to the undamped oscillator is approximately as follows: **[Online April 11, 2014]**

(a) increases by 1% (b) increases by 8%

(c) decreases by 1% (d) decreases by 8%

80. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals **[2013]**

(a) 0.7 (b) 0.81 (c) 0.729 (d) 0.6

81. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. When the cylinder is given a downward push and released, it starts oscillating vertically with a small amplitude. The time period T of the oscillations of the cylinder will be : **[Online April 25, 2013]**

(a) Smaller than $2\pi\left[\frac{M}{(k + A\sigma g)}\right]^{1/2}$

(b) $2\pi\sqrt{\frac{M}{k}}$

(c) Larger than $2\pi\left[\frac{M}{(k + A\sigma g)}\right]^{1/2}$

(d) $2\pi\left[\frac{M}{(k + A\sigma g)}\right]^{1/2}$

82. Bob of a simple pendulum of length l is made of iron. The pendulum is oscillating over a horizontal coil carrying direct current. If the time period of the pendulum is T then : **[Online April 23, 2013]**

(a) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone.

(b) $T = 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone.

(c) $T > 2\pi\sqrt{\frac{l}{g}}$ and damping is smaller than in air alone.

(d) $T < 2\pi\sqrt{\frac{l}{g}}$ and damping is larger than in air alone.

83. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force while the energy is maximum for a frequency ω_2 of the force; then **[2004]**

(a) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large

(b) $\omega_1 > \omega_2$

(c) $\omega_1 = \omega_2$

(d) $\omega_1 < \omega_2$

84. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos\omega t(\omega \neq \omega_0)$ is applied to the oscillator. The displacement of the oscillator will be proportional to **[2004]**

(a) $\frac{1}{m(\omega_0^2 + \omega^2)}$

(b) $\frac{1}{m(\omega_0^2 - \omega^2)}$

(c) $\frac{m}{\omega_0^2 - \omega^2}$

(d) $\frac{m}{(\omega_0^2 + \omega^2)}$

Hints & Solutions

1. **(a)** Here, $v_x = -a\,\omega \sin \omega t$, $v_y = a\,\omega \cos \omega t$ and $v_z = a\omega$

$\Rightarrow v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$\Rightarrow v = \sqrt{(-a\omega \sin \omega t)^2 + (a\omega \cos \omega t)^2 + (a\omega)^2}$

$v = \sqrt{2}a\omega$

2. **(c)** From the two mutually perpendicular S.H.M.'s, the general equation of Lissajous figure,

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\delta = \sin^2\delta$$

$x = A \sin (at + \delta)$

$y = B \sin (bt + r)$

Clearly $A \neq B$ hence ellipse.

3. **(b)** Maximum velocity in SHM, $v_{max} = a\omega$
Maximum acceleration in SHM, $A_{max} = a\omega^2$
where a and ω are maximum amplitude and angular frequency.

Given that, $\frac{A_{max}}{v_{max}} = 10$

i.e., $\omega = 10\ s^{-1}$
Displacement is given by
$x = a \sin (\omega t + \pi/4)$
at $t = 0$, $x = 5$
$5 = a \sin \pi/4$

$5 = a \sin 45° \Rightarrow a = 5\sqrt{2}$

Maximum acceleration $A_{max} = a\omega^2 = 500\sqrt{2}\ m/s^2$

4. **(b)** We know that $V = \omega\sqrt{A^2 - x^2}$

Initially $V = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$

Finally $3V = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}$

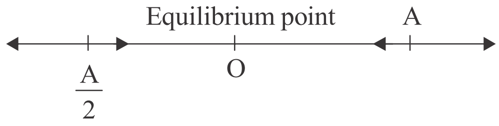
Where A'= final amplitude (Given at $x = \frac{2A}{3}$, velocity to trebled)

On dividing we get

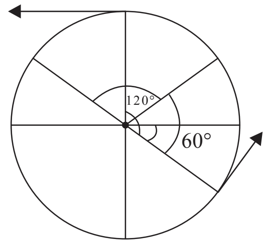
$$\frac{3}{1} = \frac{\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2}}{\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}}$$

$$9\left[A^2 - \frac{4A^2}{9}\right] = A'^2 - \frac{4A^2}{9} \quad \therefore\ A' = \frac{7A}{3}$$

5. **(d)**



(at time t = 0)



Angle covered to meet $\theta = 60° = \frac{\pi}{3}$ rad.
If they cross each other at time t then

$$t = \frac{\theta}{2\pi} = \frac{\pi}{3 \times 2\pi}T = \frac{T}{6}$$

6. **(c)** As we know,
$F = ma \Rightarrow a \propto F$
or, $a \propto \sin t$

$\Rightarrow \frac{dv}{dt} \propto \sin t$

$\Rightarrow \int_0^0 dV \propto \int_0^t \sin t\ dt$

$V \propto -\cos t + 1$

$\int_0^x dx = \int_0^t (-\cos t + 1)\ dt$

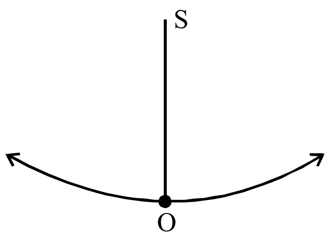
$x = \sin t - \frac{1}{2}\sin 2t$

7. **(c)** At $t = 0$, $x(t) = 0$; $y(t) = 0$
$x(t)$ is a sinusoidal function

At $t = \frac{\pi}{2\omega}$; $x(t) = a$ and $y(t) = 0$

Hence trajectory of particle will look like as (c).

8. **(c)** **Given:** Time period, T = 0.5 sec
Amplitude, A = 1 cm
Average velocity in the interval in which body moves from equilibrium to half of its amplitude, v = ?



Time taken to a displacement A/2 where A is the amplitude of oscillation from the mean position 'O' is $\frac{T}{12}$

Therefore, time, $t = \frac{0.5}{12}\text{sec}$

Displacement, $s = \frac{A}{2} = \frac{1}{2}\text{cm}$

$\therefore$ Average velocity, $v = \frac{\frac{A}{2}}{t} = \frac{\frac{1}{2}}{\frac{0.5}{12}} = 12\ \text{cm/s}$

9. **(d)** In linear S.H.M., the restoring force acting on particle should always be proportional to the displacement of the particle and directed towards the equilibrium position.

i.e., $F \propto x$

or $F = -bx$ where b is a positive constant.

10. **(b)** Two perpendicular S.H.Ms are

$x = a_1 \cos \omega t$ ....(1)

and $y = a_2\, 2 \cos \omega t$ ....(2)

From eqn (1)

$\frac{x}{a_1} = \cos wt$

and from eqn (2)

$\frac{y}{a_2} = 2\cos \omega t$

$\therefore \quad y = 2\frac{a_2}{a_1}x$

11. **(a)** Displacement $y(t) = A \sin (wt + \phi)$[Given]

For $\phi = \frac{2\pi}{3}$

at $t = 0$; $y = A \sin \phi = A \sin \frac{2\pi}{3}$

$= A \sin 120° = 0.87 A$ [$\because \sin 120° \simeq 0.866$]

Graph (a) depicts $y = 0.87A$ at $t = 0$

12. **(a)** Let, $x_1 = A \sin \omega t$ and $x_2 = A \sin (\omega t + \phi)$

$x_2 - x_1 = 2A \cos\left(\omega t + \frac{\phi}{2}\right)\sin\frac{\phi}{2}$

The above equation is SHM with amplitude $2A \sin \frac{\phi}{2}$

$\therefore\ 2A\sin\frac{\phi}{2} = A$

$\Rightarrow \sin\frac{\phi}{2} = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$

13. **(c)** At mean position, Fnet = 0

Therefore, by principal of conseruation of linear momentum.

$\therefore Mv_1 = (M+m)v_2$

M w, a, = $(M+m)$ w$_2$ a_2

$MA_1\sqrt{\frac{k}{M}} = (M+m)A_2\sqrt{\frac{k}{m+M}}$

$\therefore \left(V = A\sqrt{\frac{k}{M}}\right)$

$\Rightarrow A_1\sqrt{M} = A_2\sqrt{M+m}$

$\Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$

14. **(a)** Given,

Displacement, $x = x_0 \cos (\omega t - \pi/4)$

$\therefore$ Velocity, $v = \frac{dx}{dt} = -x_0\omega \sin\left(\omega t - \frac{\pi}{4}\right)$

Acceleration,

$a = \frac{dv}{dt} = -x_0\omega^2 \cos\left(\omega t - \frac{\pi}{4}\right)$

$= x_0\omega^2 \cos\left[\pi + \left(\omega t - \frac{\pi}{4}\right)\right]$

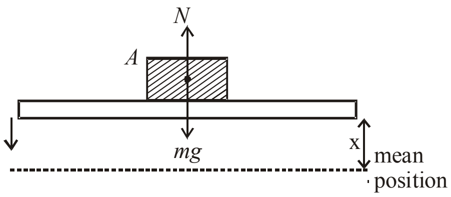
$= x_0\omega^2 \cos\left(\omega t + \frac{3\pi}{4}\right)$ ...(1)

Acceleration, $a = A \cos (\omega t + \delta)$ ...(2)

Comparing the two equations, we get

$A = x_0\omega^2$ and $\delta = \frac{3\pi}{4}$.

15. **(b)** For block A to move in SHM.



$mg - N = m\omega^2 x$

where x is the distance from mean position

For block to leave contact $N = 0$

$\Rightarrow mg = m\omega^2 x \Rightarrow x = \frac{g}{\omega^2}$

16. **(a)** Maximum velocity,

$v_{max} = a\omega$

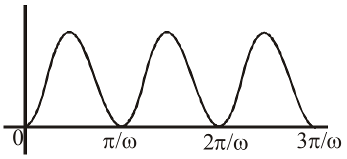
Here, a = amplitude of SHM

ω = angular velocity of SHM

$v_{max} = a \times \frac{2\pi}{T} \therefore \left(\because \omega = \frac{2\pi}{T}\right)$

$\Rightarrow T = \frac{2\pi a}{v_{max}} = \frac{2 \times 3.14 \times 7 \times 10^{-3}}{4.4} \approx 0.01\ s$

17. **(a)** Clearly $\sin 2\omega t$ is a periodic function with period $\frac{\pi}{\omega}$



For SHM $\frac{d^2y}{dt^2} \propto -y$

$y = \sin^2 \omega t = \frac{1-\cos 2\omega t}{2}$

$= \frac{1}{2} - \frac{1}{2}\cos 2\omega t$

$v = \frac{dy}{dt} = \frac{1}{2} \times 2\omega \sin 2\omega t = 2\omega \sin \omega t \cos \omega t$

$= \omega \sin 2\omega t$

Acceleration, a $= \frac{d^2y}{dt^2} = 2\omega^2 \cos 2\omega t$ which is not proportional to $-y$. Hence, it is not in SHM.

18. **(b)** Velocity of particle 1,

$v_1 = \frac{dy_1}{dt} = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right)$

Velocity of particle 2,

$v_2 = \frac{dy_2}{dt} = -0.1\pi \sin \pi t = 0.1\pi \cos\left(\pi t + \frac{\pi}{2}\right)$

$\therefore$ Phase difference of velocity of particle 1 with respect to the velocity of particle 2 is

$= \phi_1 - \phi_2 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi - 3\pi}{6} = -\frac{\pi}{6}$

19. **(c)** Maximum velocity during SHM, $V_{max} = A\omega$
But $k = m\omega^2$

$\therefore \omega = \sqrt{\frac{k}{m}}$

$\therefore$ Maximum velocity of the body in SHM

$= A\sqrt{\frac{k}{m}}$

As maximum velocities are equal

$\therefore A_1\sqrt{\frac{k_1}{m}} = A_2\sqrt{\frac{k_2}{m}}$

$\Rightarrow A_1\sqrt{k_1} = A_2\sqrt{k_2} \qquad \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$

20. **(c)** Displacement, $x = 4(\cos \pi t + \sin \pi t)$

$= \sqrt{2} \times 4\left(\frac{\sin \pi t}{\sqrt{2}} + \frac{\cos \pi t}{\sqrt{2}}\right)$

$= 4\sqrt{2}(\sin \pi t \cos 45° + \cos \pi t \sin 45°)$

$x = 4\sqrt{2}\sin(\pi t + 45°)$

On comparing it with standard equation $x = A\sin(\omega t + \phi)$ we get $A = 4\sqrt{2}$

21. **(c)** From graph equation of SHM

$X = A\cos \omega t$

(1) At $\frac{3T}{4}$ particle is at mean position.

$\therefore$ Acceleration = 0, Force = 0

(2) At T particle again at extreme position so acceleration is maximum.

(3) At $t = \frac{T}{4}$, particle is at mean position so velocity is maximum.

Acceleration = 0

(4) When KE = PE

$\Rightarrow \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$

Here, A = amplitude of SHM

x = displacement from mean position

$\Rightarrow A^2 = 2x^2 \Rightarrow x = \frac{+A}{\sqrt{2}}$

$\Rightarrow \frac{A}{\sqrt{2}} = A\cos \omega t \quad \Rightarrow t = \frac{T}{2}$

$\therefore x = -A$ which is not possible

$\therefore$ 1, 2 and 3 are correct.

22. **(d)** Kinetic energy, $k = \frac{1}{2}m\omega^2A^2\cos^2 \omega t$

Potential energy, $U = \frac{1}{2}m\omega^2A^2\sin^2 \omega t$

$\frac{k}{U} = \cot^2 \omega t = \cot^2 \frac{\pi}{90}(210) = \frac{1}{3}$

23. **(a)** $K = \frac{1}{2}m\omega^2x^2$

$\Rightarrow K_{max} = \frac{1}{2}m\omega^2A^2$

$A = L\theta$

$\omega = \sqrt{\frac{g}{L}}$

$\Rightarrow K = \frac{1}{2}m.\frac{g}{L}.L^2\theta^2$

$= \frac{1}{2}mgL\theta^2$

$\therefore \frac{K_1}{K_2} = \frac{L}{2L} = \frac{1}{2} \Rightarrow K_2 = 2K_1$

$\therefore \frac{K_1}{K_2} = \frac{L}{2L} = \frac{1}{2} \Rightarrow K_2 = 2K_1$

24. **(c)** Potential energy (U) = $\frac{1}{2}kx^2$

Kinetic energy (K) = $\frac{1}{2}kA^2 - \frac{1}{2}kx^2$

According to the question, U = k

$\therefore \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$\Rightarrow x^2 = A^2$ or, $x = \pm\frac{A}{\sqrt{2}}$

25. **(b)** For a particle executing SHM

At mean position; $t = 0, \omega t = 0, y = 0, V = V_{max} = a\omega$

$\therefore \quad K.E. = KE_{max} = \frac{1}{2}m\omega^2a^2$

At extreme position : $t = \frac{T}{4}, \omega t = \frac{\pi}{2}, y = A, V = V_{min} = 0$

$\therefore \quad K.E. = KE_{min} = 0$

Kinetic energy in SHM, $KE = \frac{1}{2}m\omega^2(a^2 - y^2)$

$= \frac{1}{2}m\omega^2a^2\cos^2\omega t$

Hence graph (b) correctly depicts kinetic energy time graph.

26. **(b)** Since system dissipates its energy gradually, and hence amplitude will also decreases with time according to

$a = a_0 e^{-bt/m}$ .... (i)

$\because$ Energy of vibration drop to half of its initial value (E_0), as $E \propto a^2 \Rightarrow a \propto \sqrt{E}$

$a = \frac{a_0}{\sqrt{2}} \Rightarrow \frac{bt}{m} = \frac{10^{-2}t}{0.1} = \frac{t}{10}$

From eq^n (i),

$\frac{a_0}{\sqrt{2}} = a_0e^{-t/10}$

$\frac{1}{\sqrt{2}} = e^{-t/10}$ or $\sqrt{2} = e^{\frac{t}{10}}$

$\ln\sqrt{2} = \frac{t}{10} \qquad \therefore \quad t = 3.5$ seconds

27. **(d)** K.E $= \frac{1}{2}k(A^2 - d^2)$

and P.E. $= \frac{1}{2}kd^2$

At mean position $d = 0$. At extreme positions $d = A$

28. **(d)** As we know, $E = E_0e^{-\frac{bt}{m}}$

$15 = 45e^{-\frac{b15}{m}}$

[As no. of oscillations = 15 so t = 15sec]

$\frac{1}{3} = e^{-\frac{b15}{m}}$

Taking log on both sides

$\frac{b}{m} = \frac{1}{15}\ell n\, 3$

29. **(b)** Work done, $w = \frac{1}{2}kx^2$

Work done by spring S_1, $w_1 = \frac{1}{2}k_1x^2$

Work done by spring S_2, $w_2 = \frac{1}{2}k_2x^2$

Since $w_1 > w_2$ Thus $(k_1 > k_2)$

30. **(b)** The kinetic energy of a particle executing S.H.M. at any instant t is given by

$K = \frac{1}{2}ma^2\omega^2\sin^2\omega t$

where, m = mass of particle
a = amplitude
ω = angular frequency
t = time

The average value of $\sin^2\omega t$ over a cycle is $\frac{1}{2}$.

$\therefore KE = \frac{1}{2}m\omega^2a^2\left(\frac{1}{2}\right) \quad \left(\because \langle\sin^2\theta\rangle = \frac{1}{2}\right)$

$= \frac{1}{4}m\omega^2a^2 = \frac{1}{4}ma^2(2\pi\nu)^2 \quad (\because \omega = 2\pi\nu)$

or, $\langle K\rangle = \pi^2ma^2\nu^2$

31. **(a)** K.E. of a body undergoing SHM is given by,

$K.E. = \frac{1}{2}ma^2\omega^2\cos^2\omega t$

Here, a = amplitude of SHM
ω = angular velocity of SHM

Total energy in S.H.M $= \frac{1}{2}ma^2\omega^2$

Given K.E. = 75% T.E.

$\frac{1}{2}ma^2\omega^2\cos^2\omega t = \frac{75}{100}\times\frac{1}{2}ma^2\omega^2$

$\Rightarrow 0.75 = \cos^2\omega t \Rightarrow \omega t = \frac{\pi}{6}$

$\Rightarrow t = \frac{\pi}{6\times\omega} \Rightarrow t = \frac{\pi\times 2}{6\times 2\pi} \Rightarrow t = \frac{1}{6}s$

32. **(a)** At any instant the total energy in SHM is

$\frac{1}{2}kA_0^2$ = constant,

where A_0 = amplitude
k = spring constant
hence total energy is independent of x.

33. **(a)** K.E. of simple harmonic motion

$= \frac{1}{2}m\omega^2(a^2 - x^2)$

34. **(c)** The kinetic energy (K. E.) of particle in SHM is given by,

$$K.E = \frac{1}{2}k(A^2 - x^2);$$

Potential energy of particle in SHM is $U = \frac{1}{2}kx^2$

Where A = amplitude and $k = m\omega^2$
x = displacement from the mean position
At the mean position $x = 0$

$$\therefore \text{ K.E.} = \frac{1}{2}kA^2 = \text{Maximum}$$

and U = 0

35. **(b)** An elastic wire can be treated as a spring and its spring constant.

$$k = \frac{YA}{L} \qquad \left[\because Y = \frac{F}{A} \Big/ \frac{\Delta l}{l_0}\right]$$

Frequency of oscillation,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$$

36. **(c)** $y = y_0 \sin^2 \omega t$

$$\Rightarrow y = \frac{y_0}{2}(1 - \cos 2\omega t) \qquad \left(\because \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}\right)$$

$$\Rightarrow y - \frac{y_0}{2} = \frac{-y_0}{2}\cos 2\omega t$$

$$\Rightarrow y = A\cos 2\omega t$$

$$\therefore \text{ Amplitude} = \frac{y_0}{2}$$

Angular velocity = 2ω

For equilibrium of mass, $\frac{ky_0}{2} = mg \Rightarrow \frac{k}{m} = \frac{2g}{y_0}$

Also, spring constant $k = m(2\omega)^2$

$$\Rightarrow 2\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{y_0}} \Rightarrow \omega = \frac{1}{2}\sqrt{\frac{2g}{y_0}} = \sqrt{\frac{g}{2y_0}}$$

37. **(a)** Potential energy of spring $= \frac{1}{2}kx^2$

Here, x = distance of block from mean position,
k = spring constant

At mean position, potential energy $= \frac{1}{2}kA^2$

At equilibrium position, half of the mass of block breaks off, so its potential energy becomes half.

$$\text{Remaining energy} = \frac{1}{2}\left(\frac{1}{2}kA^2\right) = \frac{1}{2}kA'^2$$

Here, A' = New distance of block from mean position

$$\Rightarrow A' = \frac{A}{\sqrt{2}}$$

38. **(b)**

39. **(c)** $T = 2\pi\sqrt{\frac{l}{g}}$

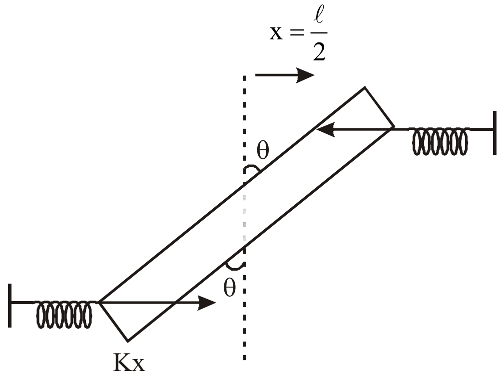
When immersed non viscous liquid

$$a_{mt} = \left(g - \frac{g}{16}\right) = \frac{15g}{16}$$

Now $T' = 2\pi\sqrt{\frac{l}{0_{net}}} = 2\pi\sqrt{\frac{l}{\sqrt{\frac{15g}{16}}}} = \frac{4}{\sqrt{15}}T$

40. **(c)** Net torque due to spring force:

$$\tau = -2Kx\frac{\ell}{2}\cos\theta$$

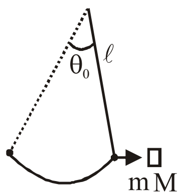


$$\Rightarrow \tau = \left(\frac{K\ell^2}{2}\right)\theta = -C\theta \qquad \left[\text{let } C = \frac{K\ell^2}{2}\right]$$

$\Rightarrow$ So, frequency of resulting oscillations

$$f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}} = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$$

41. **(b)**



Velocity before colision $\quad v = \sqrt{2g\ell(1 - \cos\theta_0)}$

Velocity after colision

$$v_1 = \sqrt{2g\ell(1 - \cos\theta_1)}$$

Using momentum conservation
$mv = MV_m - mV_1$

$$m\sqrt{2g\ell(1 - \cos\theta_0)} = MV_m - m\sqrt{2g\ell(1 - \cos\theta)}$$

$$\Rightarrow m\sqrt{2g\ell}\left\{\sqrt{1 - \cos\theta_0} + \sqrt{1 - \cos\theta_1}\right\} = MV_m$$

$$\text{and } e = 1 = \frac{V_m + \sqrt{2g\ell(1 - \cos\theta_1)}}{\sqrt{2g\ell(1 - \cos\theta_0)}}$$

$$\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1}\right)=V_m \quad ...(i)$$

$$m\sqrt{2g\ell}\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right)=MV_M \quad ...(ii)$$

Dividing (ii) by (i) we get

$$\frac{\left(\sqrt{1-\cos\theta_0}+\sqrt{1-\cos\theta_1}\right)}{\left(\sqrt{1-\cos\theta_0}-\sqrt{1-\cos\theta_1}\right)}=\frac{M}{m}$$

By componendo and dividendo rule

$$\frac{m-M}{m+M}=\frac{\sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0}}=\frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)}$$

$$\Rightarrow \frac{M}{m}=\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\Rightarrow M=m\frac{\theta_0-\theta_1}{\theta_0+\theta_1}$$

42. **(a)** Given : $y=5\left[\sin(3\pi t)+\sqrt{3}\cos(3\pi t)\right]$

$$\Rightarrow y=10\sin\left(3\pi t+\frac{\pi}{3}\right)$$

$\therefore$ Amplitude = 10 cm

Time period, $T=\frac{2\pi}{\omega}=\frac{2\pi}{3\pi}=\frac{2}{3}s$

43. **(a)** Angular frequency of pendulum $\omega=\sqrt{\frac{g}{\ell}}$

$\therefore$ relative change in angular frequency

$$\frac{\Delta\omega}{\omega}=\frac{1}{2}\frac{\Delta g}{g}$$ [as length remains constant]

$\Delta g=2A\omega_s^2$ [ω_s = angular frequency of support and, A = amplitude]

$$\frac{\Delta\omega}{\omega}=\frac{1}{2}\times\frac{2A\omega_s^2}{g}$$

$$\Delta\omega=\frac{1}{2}\times\frac{2\times 1^2\times 10^{-2}}{10}=10^{-3}\text{ rad/sec.}$$

44. **(d)** Acceleration due to gravity $g=\frac{GM}{R^2}$

$$\frac{g_p}{g_e}=\frac{M_p}{M_e}\left(\frac{R_e}{R_p}\right)^2=3\left(\frac{1}{3}\right)^2=\frac{1}{3}$$

Also $T\propto\frac{1}{\sqrt{g}}\Rightarrow\frac{T_p}{T_e}=\sqrt{\frac{g_e}{g_p}}=\sqrt{3}$

$$\Rightarrow T_p=2\sqrt{3}\text{ s}$$

45. **(c)** Velocity, $v=\omega\sqrt{A^2-x^2}$ ...(i)

acceleration, $a=-\omega^2x$ ...(ii)

and according to question,

$|v|=|a|$

$$\Rightarrow\omega\sqrt{A^2-x^2}=\omega^2x$$

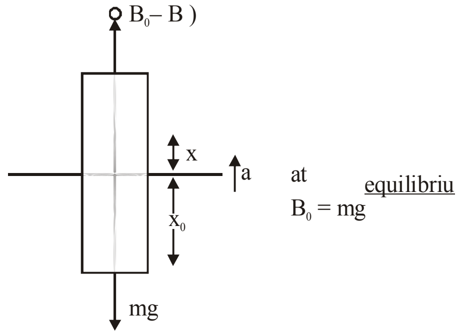
$$\Rightarrow A^2-x^2=\omega^2x^2$$

$$\Rightarrow 5^2-4^2=\omega^2\left(4^2\right)$$

$$3=\omega\times 4\Rightarrow\omega=\frac{3}{4}$$

$$\therefore T=2\pi/\omega=\frac{2\pi}{3/4}=\frac{8\pi}{3}$$

46. **(Bonus)**



Extra boyant force = ρAxg

$B_0+B=mg+ma$

$\therefore B=ma=\rho Axg=(\pi r^2\rho g)x$

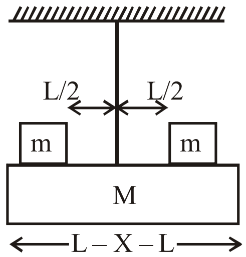
$$a=\frac{\left(\pi r^2\rho g\right)x}{m}$$

using, $a=\omega^2x$

$$\Rightarrow\omega=\sqrt{\frac{\pi r^2\rho g}{m}}$$

$W\simeq 7.95\text{ rads}^{-1}$

47. **(c)**



$$f_1=\frac{1}{2\pi}\sqrt{\frac{C}{1}} \quad ...(i)$$

$$=\frac{1}{2}\sqrt{\frac{3C}{ML^2}}$$

$$f_2=\frac{1}{2\pi}\sqrt{\frac{C}{L^2\left(\frac{M}{3}+\frac{M}{2}\right)}} \quad ...(ii)$$

As frequency reduces by 80%

$$\therefore f_2=0.8f_1\Rightarrow\frac{f_2}{f_1}=0.8 \quad ...(iii)$$

Solving equations (i), (ii) & (iii)

$$\text{Ratio }\frac{m}{M}=0.37$$

As frequency reduces by 80%

$\therefore\ f_2 = 0.8\ f_1 \Rightarrow \frac{f_2}{f_1} = 0.8$ ...(iii)

Solving equations (i), (ii) & (iii)

Ratio $\frac{m}{M} = 0.37$

48. (b) As we know, frequency in SHM

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 10^{12}$$

where m = mass of one atom

Mass of one atom of silver, $= \frac{108}{(6.02\times10^{23})}\times10^{-3}$ kg

$$\frac{1}{2\pi}\sqrt{\frac{k}{108\times10^{-3}}\times6.02\times10^{23}} = 10^{12}$$

Solving we get, spring constant,

K = 7.1N/m

49. (d) Using y = A sin ωt

$a = A\sin\omega t_0$

$b = A\sin 2\omega t_0$

$c = A\sin 3\omega t_0$

$a + c = A[\sin\omega t_0 + A\sin 3\omega t_0] = 2A\sin 2\omega t_0\cos\omega t_0$

$$\frac{a+c}{b} = 2\cos\omega t_0$$

$$\Rightarrow \omega = \frac{1}{t_0}\cos^{-1}\left(\frac{a+c}{2b}\right) \Rightarrow f = \frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$$

50. (b) As we know, Time-period of simple pendulum, $T \propto \sqrt{l}$

differentiating both side, $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$

$\because$ change in length $\Delta l = r_1 - r_2$

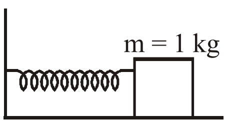
$$5\times10^{-4} = \frac{1}{2}\frac{r_1 - r_2}{1}$$

$r_1 - r_2 = 10\times10^{-4}$

10^{-3} m = 10^{-1} cm = 0.1 cm

51. (c) Frequency of spring (f) $= \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1$ Hz

$$\Rightarrow 4\pi^2 = \frac{k}{m}$$

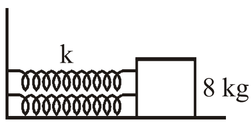


If block of mass m = 1 kg is attached then,

$k = 4\pi^2$

Now, identical springs are attached in parallel with mass m = 8 kg. Hence,

$k_{eq} = 2k$



$$F = \frac{1}{2\pi}\sqrt{\frac{k\times2}{g}} = \frac{1}{2}\text{Hz}$$

52. (c) Time lost/gained per day $= \frac{1}{2}\propto\Delta\theta\times86400$ second

$$12 = \frac{1}{2}\alpha(40-\theta)\times86400 \quad \text{.... (i)}$$

$$4 = \frac{1}{2}\alpha(\theta-20)\times86400 \quad \text{....(ii)}$$

On dividing we get, $3 = \frac{40-\theta}{\theta-20}$

$3\theta - 60 = 40 - \theta$

$4\theta = 100 \Rightarrow \theta = 25°C$

53. (b) Washer contact with piston $\Rightarrow$ N = 0

Given Amplitude A = 7 cm = 0.07 m.

$a_{max} = g = \omega^2 A$

The frequency of piston

$$f = \frac{\omega}{2\pi} = \sqrt{\frac{g}{A}}\frac{1}{2\pi} = \sqrt{\frac{1000}{7}}\frac{1}{2\pi} = 1.9\text{ Hz.}$$

54. (c) As we know, time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

When additional mass M is added then

$$T_M = 2\pi\sqrt{\frac{\ell+\Delta\ell}{g}}$$

$$\frac{T_M}{T} = \sqrt{\frac{\ell+\Delta\ell}{\ell}}$$

$$\Rightarrow\left(\frac{T_M}{T}\right)^2 = \frac{\ell+\Delta\ell}{\ell}$$

$$\text{or, }\left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY} \qquad \left[\because \Delta\ell = \frac{Mg\ell}{AY}\right]$$

$$\therefore \frac{1}{Y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

55. (d) In simple harmonic motion, starting from rest,

At $t = 0$, $x = A$

$x = A\cos\omega t$ ..... (i)

When $t = \tau$, $x = A - a$

When $t = 2\tau$, $x = A - 3a$

From equation (i)

$A - a = A\cos\omega\tau$ ......(ii)

$A - 3a = A\cos 2\omega\tau$ ....(iii)

As $\cos 2\omega\tau = 2\cos^2\omega\tau - 1$ ...(iv)

From equation (ii), (iii) and (iv)

$$\frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\Rightarrow \frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$\Rightarrow \quad A^2 - 3aA = A^2 + 2a^2 - 4Aa$

$\Rightarrow \quad 2a^2 = aA$

$\Rightarrow \quad A = 2a$

$\Rightarrow \quad \frac{a}{A} = \frac{1}{2}$

Now, $A - a = A\cos\omega\tau$

$\Rightarrow \quad \cos\omega\tau = \frac{A-a}{A}$

$\Rightarrow \quad \cos\omega\tau = \frac{1}{2}$ or $\frac{2\pi}{T}\tau = \frac{\pi}{3}$

$\Rightarrow \quad T = 6\tau$

56. (b) From graph it is clear that when $L = 1m$, $T^2 = 4s^2$

As we know,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\Rightarrow \quad g = \frac{4\pi^2 L}{T^2}$$

$$= 4\times\left(\frac{22}{7}\right)^2 \times \frac{1}{4} = \left(\frac{22}{7}\right)^2$$

$$\therefore \quad g = \frac{484}{49} = 9.87 m/s^2$$

57. (d) As we know,

$x = x_0 e^{-bt/2m}$

From question,

$$8 = 10e^{-\frac{40b}{2m}} \quad ....(i)$$

Similarly, $5 = 10e^{-\frac{bt}{2m}}$ ....(ii)

Solving eqns (i) and (ii) we get

$t \cong 142$ s

58. (c) Mass of bigger body M = 4 kg

Mass of smaller body m = 1 kg

Smaller mass (m = 1 kg) executes S.H.M of angular frequency $\omega = 25$ rad/s

Amplitude $x = 1.6$ cm $= 1.6 \times 10^{-2}$

As we know,

$$T = 2\pi\sqrt{\frac{m}{K}}$$

or, $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$

or, $\frac{1}{25} = \sqrt{\frac{1}{K}}$ $[\because m = 1kg;\ \omega = 25\ rad/s]$

or, $K = 625\ Nm^{-1}$.

The maximum force exerted by the system on the floor

$= Mg + Kx + mg$

$= 4 \times 10 + 625 \times 1.6 \times 10^{-2} + 1 \times 10$

$= 40 + 10 + 10$

$= 60$ N

59. (c) $\frac{Mg}{A} = P_0$

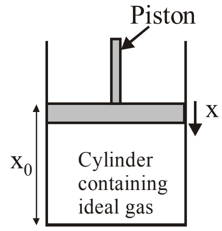
$P_0 V_0^\gamma = PV^\gamma$

$Mg = P_0 A$ ...(1)

Let piston is displaced by distance x

$$P_0 A x_0^\gamma = PA(x_0 - x)^\gamma$$

$$P = \frac{P_0 x_0^\gamma}{(x_0 - x)^\gamma}$$



$$Mg - \left(\frac{P_0 x_0^\gamma}{(x_0 - x)^\gamma}\right)A = F_{restoring}$$

$$P_0 A\left(1 - \frac{x_0^\gamma}{(x_0 - x)^\gamma}\right) = F_{restoring} \quad [x_0 - x \approx x_0]$$

$$F = -\frac{\gamma P_0 A x}{x_0}$$

$\therefore$ Frequency with which piston executes SHM.

$$f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A}{x_0 M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0 A^2}{M V_0}}$$

60. (c) As $F = -kx$

61. (a) Let T_1 and T_2 be the time period of the two pendulums

$$T_1 = 2\pi\sqrt{\frac{1}{g}} \text{ and } T_2 = 2\pi\sqrt{\frac{4}{g}}$$

As $\ell_1 < \ell_2$ therefore $T_1 < T_2$

Let at t = 0 they start swinging together. Since their time periods are different, the swinging will not be in unison always. Only when number of completed oscillations differ by an integer, the two pendulums will again begin to swing together

Let longer length pendulum complete n oscillation and shorter length pendulum complete (n + 1) oscillation. For unison swinging

$$(n+1)T_1 = nT_2$$

$$(n+1)\times 2\pi\sqrt{\frac{l}{g}} = (n)\times 2\pi\sqrt{\frac{4}{g}}$$

$\Rightarrow n = 1$

$\therefore n + 1 = 1 + 1 = 2$

62. **(d)** The equation of motion for the pendulum, for damped harmonic motion

$F = -kx - bv$

$\Rightarrow ma + kx + bv = 0$

$$\Rightarrow \quad m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{k}{m}x + \frac{b}{m}\frac{dx}{dt} = 0$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0 \quad \ldots (1)$$

Let $x = e^{\lambda t}$ is the solution of the equation (1)

$$\frac{dx}{dt} = \lambda e^{\lambda t} \Rightarrow \frac{d^2x}{dt^2} = \lambda^2 e^{\lambda t}$$

Substituting in the equation (1)

$$\lambda^2 e^{\lambda t} + \frac{b}{m}\lambda e^{\lambda t} + \frac{k}{m}e^{\lambda t} = 0$$

$$\lambda^2 + \frac{b}{m}\lambda + \frac{k}{m} = 0$$

$$\lambda = \frac{-\frac{b}{m} \pm \sqrt{\frac{b^2}{m^2} - 4\frac{k}{m}}}{2} = \frac{-b \pm \sqrt{b^2 - 4km}}{2m}$$

Solving the equation (1) for x, we have

$$x = e^{\frac{-b}{2m}t}$$

$$\omega = \sqrt{\omega_0{}^2 - \lambda^2} \text{ where } \omega_0 = \frac{k}{m},\ \lambda = \frac{+b}{2}$$

The average life $= \frac{1}{\lambda} = \frac{2}{b}$

63. **(a)**

64. **(a)** Let the cube be at a depth x from the equilibrium position.

Force acting on the cube = up thrust on the portion of length x.

$F = -\rho\ell^2 xg$ [$\therefore$ mass density X volume] ....(i)

Clearly $F \propto -x$, Hence it is a SHM.

Equation of SHM is $F = -kx$ ....(ii)

Comparing equation (i) and (ii) we have

$k = \rho\ell^2 g$

Now, Time period, T = $2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow T = 2\pi\sqrt{\frac{\ell^3 d}{\rho\ell^2 g}}$$

$$= 2\pi\sqrt{\frac{\ell d}{\rho g}}$$

Comparing the above equation with $a = -\omega^2 x$, we get

$$\therefore \quad \omega = \sqrt{\frac{\rho g}{d\ell}} \Rightarrow T = 2\pi\sqrt{\frac{\ell d}{\rho g}}$$

65. **(a)** For an SHM, the acceleration

$a = -\omega^2 x$ where ω^2 is a constant.

$$a = \frac{-4\pi^2 x}{T^2} \Rightarrow \frac{aT}{x} = \frac{-4\pi^2}{T}$$

The time period T is also constant. Therefore, $\frac{aT}{x}$ is a constant.

66. **(a)** The two springs are in parallel.

$\therefore$ Effective spring constant,

$k = k_1 + k_2$

Initial frequency of oscillation is given by

$$v = \frac{1}{2p}\sqrt{\frac{k_1 + k_2}{m}} \quad ....(i)$$

When both k_1 and k_2 are made four times their original values, the new frequency is given by

$$v' = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 4k_2}{m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{4(k_1 + 4k_2)}{m}} = 2\left(\frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}\right) = 2v$$

67. **(b)** Here, Displacement, $x = 2 \times 10^{-2} \cos \pi t$

Velocity is given by

$$v = \frac{dx}{dt} = 2 \times 10^{-2}\,\pi \sin \pi t$$

For the first time, the when velocity becomes maximum, $\sin \pi t = 1$

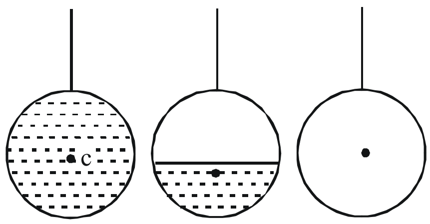
$$\Rightarrow \sin \pi t = \sin\frac{\pi}{2}$$

$$\Rightarrow \pi t = \frac{\pi}{2} \text{ or, } t = \frac{1}{2} = 0.5 \text{ sec.}$$

68. **(b)** When plugged hole near the bottom of the oscillating bob gets suddenly unplugged, centre of mass of combination of liquid and hollow portion (at position ℓ), first goes down (to $\ell + \Delta\ell$) and when total water is drained out, centre of mass regain its original position (to ℓ),

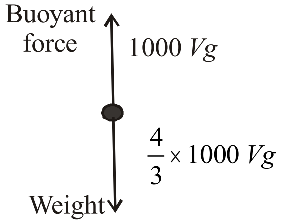
Time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\therefore$ 'T' first increases and then decreases to original value.



70. **(a)** Time period, $t = 2\pi\sqrt{\frac{\ell}{g_{\text{eff}}}}$;

In air, $t_0 = 2\pi\sqrt{\frac{\ell}{g}}$



Net force $= \left(\frac{4}{3} - 1\right) \times 1000\,Vg = \frac{1000}{3} Vg$

$g_{\text{eff}} = \frac{1000\,Vg}{3 \times \frac{4}{3} \times 1000 V} = \frac{g}{4}$

$\therefore t = 2\pi\sqrt{\frac{\ell}{g/4}} = 2 \times 2\pi\frac{\ell}{g}$

$t = 2t_0$

71. **(b)** Time period for first spring, $t_1 = 2\pi\sqrt{\frac{m}{k_1}}$,

Time period for second spring, $t_2 = 2\pi\sqrt{\frac{m}{k_2}}$

Force constant of the series combination $k_{\text{eff}} = \frac{k_1 k_2}{k_l + k_2}$

$\therefore$ Time period of oscillation for series combination

$T = 2\pi\sqrt{\frac{m(k_l + k_2)}{k_1 k_2}}$

$\therefore T = 2\pi\sqrt{\frac{m}{k_2} + \frac{m}{k_1}} = 2\pi\sqrt{\frac{t_2^2}{(2\pi)^2} + \frac{t_1^2}{(2\pi)^2}}$

$\Rightarrow T^2 = t_1^2 + t_2^2$

where *x* is the displacement from the mean position

72. **(c)** With mass M, the time period of the spring.

$T = 2\pi\sqrt{\frac{M}{k}}$

With mass $M + m$, the time period becomes,

$T' = 2\pi\sqrt{\frac{M+m}{k}} = \frac{5T}{3}$

$\therefore\ 2\pi\sqrt{\frac{M+m}{k}} = \frac{5}{3} \times 2\pi\sqrt{\frac{M}{k}}$

$\Rightarrow \quad M + m = \frac{25}{9} \times M$

$\Rightarrow 1 + \frac{m}{M} = \frac{25}{9}$

$\Rightarrow \frac{m}{M} = \frac{25}{9} - 1 = \frac{16}{9}$

73. **(d)** Time period, $T = 2\pi\sqrt{\frac{\ell}{g}}$

New length, $\ell' = \ell + 21\%$ of ℓ

$\ell' = \ell + 0.21\,\ell$

$\Rightarrow \quad \ell' = 1.21\,\ell$

$T' = 2\pi\sqrt{\frac{1.21\ell}{g}}$

% increase in length $= \frac{T' - T}{T} \times 100$

$= \frac{\sqrt{1.21\ell} - \sqrt{\ell}}{\sqrt{\ell}} \times 100 = \left(\sqrt{1.21} - \sqrt{1}\right) \times 100$

$= (1.1 - 1) \times 100 = 10\%$

74. **(b)** Let *k* be the spring constant of the original spring.

Time period $T = 2\pi\sqrt{\frac{m}{k}}$ where m is the mas s of oscillating body.

When the spring is cut into *n* equal parts, the spring constant of one part becomes *nk*. Therefore the new time period,

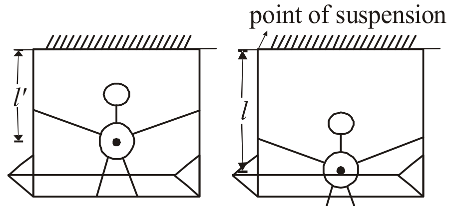
$T' = 2\pi\sqrt{\frac{m}{nk}} = \frac{T}{\sqrt{n}}$

75. **(b)** The time period $T = 2\pi\sqrt{\frac{\ell}{g}}$ where ℓ = distance between the point of suspension and the centre of mass of the child.

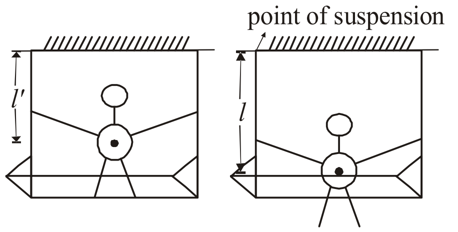
As the child stands up, her centre of mass is raised. The distance between point of suspension and centre of mass decreases ie length ℓ decreases.

$\therefore\ \ell' < \ell$

$\therefore\ T' < T$ *i.e.*, the period decreases.



Case (ii) child standing Case (i) child sitting



76. **(c)** Time of half the amplitude is = 2*s*

Using, $A = A_0 e^{-kt}$

$$\frac{A_0}{2} = A_e\, e^{-k\times 2} \quad ...(i)$$

and $$\frac{A_0}{1000} = A_e\, e^{-kt} \quad ...(ii)$$

Dividing (i) by (ii) and solving, we get

$t = 20$ s

77. **(b)** Amplitude of vibration at time t = 0 is given by

$$A = A_0 e^{-0.1\times 0} = 1 \times A_0 = A_0$$

also at t = t, if $A = \frac{A_0}{2}$

$$\Rightarrow \frac{1}{2} = e^{-0.1t}$$

$t = 10 \ln 2 \simeq 7s$

78. **(b)** In first collision mu momentum will be imparted to system, in second collision when momentum of (M + m) is in opposite direction mu momentum of particle will make its momentum zero.

On 13[th] collision, $\boxed{m} \rightarrow \boxed{M+12}$; $\boxed{M+13m} \rightarrow V$

$$mu = (M + 13m)v \Rightarrow v = \frac{mu}{M+13m} = \frac{u}{15}$$

$$v = \omega A \Rightarrow \frac{u}{15} = \sqrt{\frac{K}{M-13m}} \times A$$

Putting value of M, m, u and K we get amplitude

$$A = \frac{1}{15}\sqrt{\frac{75}{1}} = \frac{1}{\sqrt{3}}$$

79. **(b)** The change in time period compared to the undamped oscillator increases by 8%.

80. **(c)** $\because A = A_0 e^{-\frac{bt}{2m}}$

(where, A_0 = maximum amplitude)

According to the questions, after 5 second,

$$0.9A_0 = A_0 e^{-\frac{b(5)}{2m}} \quad \dots (i)$$

After 10 more second,

$$A = A_0 e^{-\frac{b(15)}{2m}} \quad \dots (ii)$$

From eq[n]s (i) and (ii)

$A = 0.729\, A_0$

$\therefore\ \alpha = 0.729$

81. **(a)**

82. **(d)** When the pendulum is oscillating over a current carrying coil, and when the direction of oscillating pendulum bob is opposite to the direction of current. Its instantaneous acceleration increases.

Hence time period $T < 2\pi\sqrt{\frac{\ell}{g}}$

and damping is larger than in air alone due energy dissipation.

83. **(c)** As energy $\propto$ (Amplitude)^2, the maximum for both of them occurs at the same frequency and this is only possible in case of resonance.

In resonance state $\omega_1 = \omega_2$

84. **(b)** Equation of displacement in forced oscillation is given by

$$y = \frac{F_0}{m\,(\omega_0^2 - \omega^2)^2}$$

$$= \frac{F_0}{m(\omega_0^2 - \omega^2)}$$

Here damping effect is considered to be zero

$$\therefore x \propto \frac{1}{m(\omega_0^2 - \omega^2)}$$