GATE 2023
Civil Engineering
Exam Held on : 12-2-2023
Forenoon Session

SECTION - A GENERAL APTITUDE

Q.1 - Q.5 carry ONE mark Each

Q.1 "I have not yet decided what I will do this evening; I ______ visit a friend."

(a) mite (b) would

(c) might (d) didn't

(MCQ)

Ans. (c)

End of Solution

Q.2 Eject : Insert : : Advance : _______

(By word meaning)

(a) Advent (b) Progress

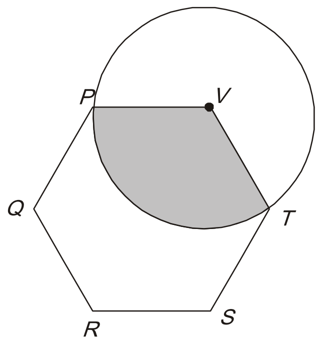
(c) Retreat (d) Loan

(MCQ)

Ans. (c)

End of Solution

Q.3 In the given figure, PQRSTV is a regular hexagon with each side of length 5 cm. A circle is drawn with its centre at V such that it passes through P. What is the area (in cm^2) of the shaded region? (The diagram is representative)



(a) $\frac{25\pi}{3}$ (b) $\frac{20\pi}{3}$

(c) 6π (d) 7π

(MCQ)

Ans. (a)

Interior angle sum = $(n-2) \times 180°$

Each angle of regular Hexagon.

$$\frac{(n-2)\times 180°}{n} = 120°$$

$$\text{Required area} = \frac{\theta}{360°}\times\pi\times 25 = \frac{25\pi}{3}$$

End of Solution

Q.4 A duck named Donald Duck says "All ducks always lie."

Based only on the information above, which one of the following statements can be logically inferred with certainty?

(a) Donald Duck always lies. (b) Donald Duck always tells the truth.

(c) Donald Duck's statement is true. (d) Donald Duck's statement is false.

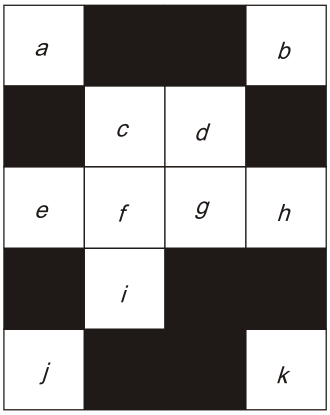
(MCQ)

Ans. **(d)**

End of Solution

Q.5 A line of symmetry is defined as a line that divides a figure into two parts in a way such that each part is a mirror image of the other part about that line.

The figure below consists of 20 unit squares arranged as shown. In addition to the given black squares, upto 5 more may be coloured black. Which one among the following options depicts the minimum number of boxes that must be coloured black to achieve two lines of symmetry? (The figure is representative)



(a) *d* (b) *c, d, i*

(c) *c, i* (d) *c, d, i, f, g*

(MCQ)

Ans. **(b)**

If box *d* or *i* is coloured block we get one line of symmetry, i.e., horizontal or vertical. But for two line of symmetry, we need to atleast colour 3 box, i.e., *c*, *d* and *i*.

End of Solution

Q.6 - Q.10 carry TWO mark Each

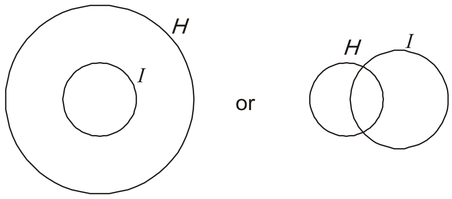
Q.6 Based only on the truth of the statement 'Some humans are intelligent', which one of the following options can be logically inferred with certainty?

(a) No human is intelligent. (b) All humans are intelligent.

(c) Some non-humans are intelligent. (d) Some intelligent beings are humans.

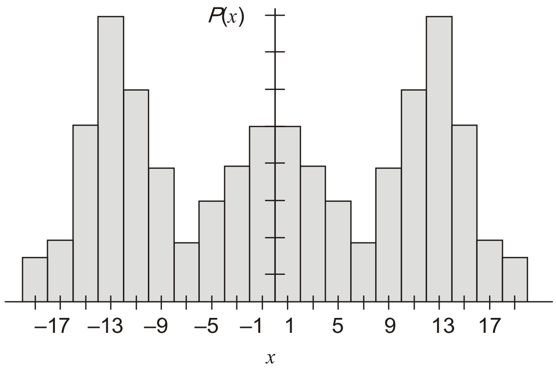
(MCQ)

Ans. (d)



End of Solution

Q.7 Which one of the options can be inferred about the mean, median, and mode for the given probability distribution (i.e. probability mass function), $P(x)$, of a variable x?



(a) mean = median ≠ mode
(b) mean = median = mode
(c) mean ≠ median = mode
(d) mean ≠ mode = median

(MCQ)

Ans. (a)

End of Solution

Q.8 The James Webb telescope, recently launched in space, is giving humankind unprecedented access to the depths of time by imaging very old stars formed almost 13 billion years ago. Astrophysicists and cosmologists believe that this odyssey in space may even shed light on the existence of dark matter. Dark matter is supposed to interact only via the gravitational interaction and not through the electromagnetic-, the weak- or the strong-interaction. This may justify the epithet "dark" in dark matter.

Based on the above paragraph, which one of the following statements is FALSE?

(a) No other telescope has captured images of stars older than those captured by the James Webb telescope.
(b) People other than astrophysicists and cosmologists may also believe in the existence of dark matter.
(c) The James Webb telescope could be of use in the research on dark matter.
(d) If dark matter was known to interact via the strong-interaction, then the epithet "dark" would be justified.

(MCQ)

Ans. (d)

End of Solution

Q.9 Let a = 30! b = 50! , and c =100! . Consider the following numbers:

$\log_a c, \log_c a, \log_b a, \log_a b$

Which one of the following inequalities is CORRECT?

(a) $\log_c a < \log_b a < \log_a c < \log_a c$ (b) $\log_c a < \log_a b < \log_b a < \log_b c$

(c) $\log_c a < \log_b a < \log_a c < \log_a b$ (d) $\log_b a < \log_c a < \log_a b < \log_a c$

(MCQ)

Ans. **(a)**

Given, $a = 30!,\ b = 50!$ and $c = 100!$

We know that $\log_n m = \dfrac{\log m}{\log n}$

$\log_a c, \log_c a, \log_a b, \log_b a$

$$\frac{\log 100!}{\log 30!}, \frac{\log 30!}{\log 100!}, \frac{\log 50!}{\log 30!}, \frac{\log 30!}{\log 50!}$$

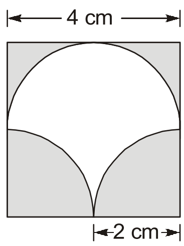
$$\frac{\log 30!}{\log 100!} < \frac{\log 30!}{\log 50!} < \frac{\log 50!}{\log 30!} < \frac{\log 100!}{\log 30!}$$

$$\log_c a < \log_b a < \log_a b < \log_a c$$

End of Solution

Q.10 A square of side length 4 cm is given. The boundary of the shaded region is defined by one semi-circle on the top and two circular arcs at the bottom, each of radius 2 cm, as shown.

The area of the shaded region is _______ cm^2.

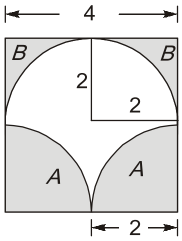


(a) 8 (b) 4

(c) 12 (d) 10

(MCQ)

Ans. (a)



$$\text{Area of quadrant, } A = \frac{1}{4}\pi(2)^2 = \pi$$

$$\begin{aligned}\text{Area of sector, } B &= \text{Area of square} - \text{Area of quadrant}\\ &= 4 - \pi\end{aligned}$$

So,
$$\begin{aligned}\text{total shaded area} &= 2A + 2B = 2 \times \pi + 2 \times (4 - \pi)\\ &= 2\pi + 8 - 2\pi = 8 \text{ unit}^2\end{aligned}$$

End of Solution

■■■■

SECTION - B TECHNICAL

Q.11 - Q.35 carry ONE mark Each

Q.11 For the integral

$$I = \int_{-1}^{1} \frac{1}{x^2} dx$$

which of the following statements is TRUE?

(a) $I = 0$

(b) $I = 2$

(c) $I = -2$

(d) The integral does not converge

(MCQ)

Ans. (d)

$$f(x) = \frac{1}{x^2} \text{ is not defined at } x = 0$$

So,

$$I = \int_{-1}^{0} \frac{1}{x^2} dx + \int_{0}^{1} \frac{1}{x^2} dx$$

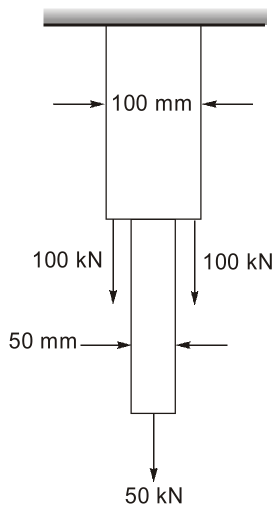
$$= \left(-\frac{1}{x}\right)_{-1}^{0} + \left(-\frac{1}{x}\right)_{0}^{1} = \infty$$

$\frac{1}{x}$ is not defined at $x = 0$

Therefore, the integral does not converge.

End of Solution

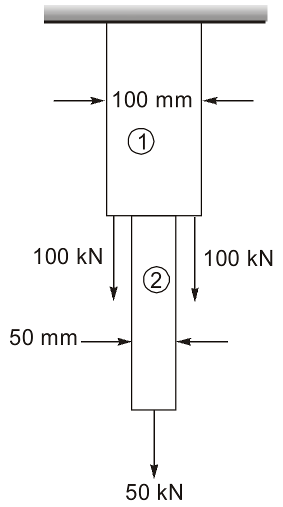
Q.12 A hanger is made of two bars of different sizes. Each bar has a square cross-section. The hanger is loaded by three-point loads in the mid vertical plane as shown in the figure. Ignore the self-weight of the hanger. What is the maximum tensile stress in N/mm^2 anywhere in the hanger without considering stress concentration effects?



(a) 15.0
(b) 25.0
(c) 35.0
(d) 45.0

(MCQ)

Ans. (b)



Stress in member (1),

$$(\sigma_1) = \frac{P_1}{A_1} = \frac{250 \times 10^3}{100 \times 100} = 25 \text{ N/mm}^2$$

Stress in member (2),

$$(\sigma_2) = \frac{P_2}{A_2} = \frac{50 \times 10^3}{50 \times 50} = 20 \text{ N/mm}^2$$

$$\therefore \quad \sigma_{max} = 25 \text{ N/mm}^2$$

End of Solution

Q.13 Creep of concrete under compression is defined as the ____.

(a) increase in the magnitude of strain under constant stress
(b) increase in the magnitude of stress under constant strain
(c) decrease in the magnitude of strain under constant stress
(d) decrease in the magnitude of stress under constant strain

(MCQ)

Ans. (a)

increase in the magnitude of strain under constant stress

End of Solution

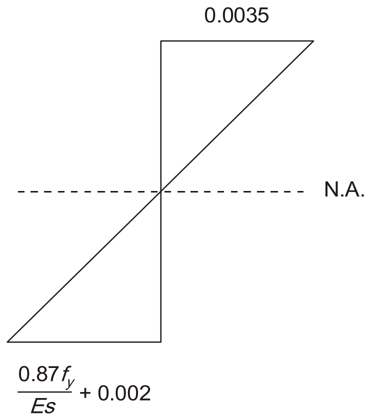
Q.14 A singly reinforced concrete beam of balanced section is made of M20 grade concrete and Fe415 grade steel bars. The magnitudes of the maximum compressive strain in concrete and the tensile strain in the bars at ultimate state under flexure, as per IS 456: 2000 are ________, respectively. (round off to four decimal places)

(a) 0.0035 and 0.0038
(b) 0.0020 and 0.0018
(c) 0.0035 and 0.0041
(d) 0.0020 and 0.0031

(MCQ)

Ans. (a)

In a balanced section, strain diagram under flexure is



So, strain in concrete = 0.0035

$$\text{Strain in steel} = \frac{0.87 \times 415}{2 \times 10^5} + 0.002 = 0.0038$$

End of Solution

Q.15 In cement concrete mix design, with the increase in water-cement ratio, which one of the following statements is TRUE?

(a) Compressive strength decreases but workability increases
(b) Compressive strength increases but workability decreases
(c) Both compressive strength and workability decrease
(d) Both compressive strength and workability increase

(MCQ)

Ans. (a)

End of Solution

Q.16 The specific gravity of a soil is 2.60. The soil is at 50% degree of saturation with a water content of 15%. The void ratio of the soil is _____.

(a) 0.35
(b) 0.78
(c) 0.87
(d) 1.28

(MCQ)

Ans. (b)

Given:Specific gravity, $G = 2.6$

Saturation, $S = 0.5$

Water content, $w = 0.15$

As we know

$$Se = Gw$$

$$\Rightarrow \quad 0.5 \times e = 2.6 \times 0.15$$

$$\Rightarrow \quad e = \frac{2.6 \times 0.15}{0.5} = 0.78$$

End of Solution

Q.17 A group of 9 friction piles are arranged in a square grid maintaining equal spacing in all directions. Each pile is of diameter 300 mm and length 7 m. Assume that the soil is cohesionless with effective friction angle $\phi' = 32°$. What is the center-to-center spacing of the piles (in m) for the pile group efficiency of 60%?

(a) 0.582
(b) 0.486
(c) 0.391
(d) 0.677

(MCQ)

Ans. **(b)**

By Converse – Labarre formula

$$\eta_g = \left[1 - \frac{\theta}{90}\left(\frac{m(n-1)+n(m-1)}{mn}\right)\right] \times 100$$

$$\Rightarrow \quad 0.6 = 1 - \frac{\theta}{90}\left(\frac{3\times2+3\times2}{3\times3}\right)$$

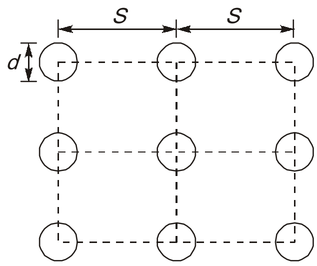
$$\Rightarrow \quad \frac{\theta}{90}\left(\frac{6+6}{9}\right) = 0.4$$

$$\Rightarrow \quad \theta = 27°$$

Now, $\tan\theta = \dfrac{d}{S}$ where d is diameter and S is spacing.

So, Spacing, $S = \dfrac{d}{\tan\theta} = \dfrac{0.3}{\tan 27°} = 0.587$

Alternatively,



$$\text{Group efficiency} = \frac{Q_{ug}}{nQ_{us}}$$

$$\Rightarrow \qquad 0.6 = \frac{\frac{1}{2}(\gamma l)\tan\delta(2S+d)\times l\times 4}{9\times\frac{1}{2}(\gamma l)\tan\delta(pdl)}$$

$$\Rightarrow \qquad 0.6 = \frac{(2S+d)\times 4}{9\times\pi d}$$

$$\Rightarrow \qquad S = \frac{1}{2}\left[\frac{0.6\times 9\times\pi\times 0.3}{4} - 0.3\right] = 0.486$$

NOTE: As the exact answer of spacing is not matching with the given options by Converse-Labarre formula, therefore we will use alternative method to find spacing.

End of Solution

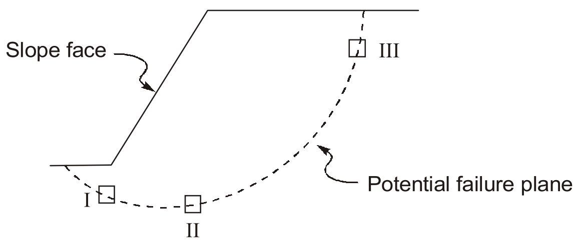
Q.18 A possible slope failure is shown in the figure. Three soil samples are taken from different locations (I, II and III) of the potential failure plane. Which is the most appropriate shear strength test for each of the sample to identify the failure mechanism? Identify the correct combination from the following options:

P: Triaxial compression test

Q: Triaxial extension test

R: Direct shear or shear box test

S: Vane shear test



(a) I-Q, II-R, III-P
(b) I-R, II-P, III-Q
(c) I-S, II-Q, III-R
(d) I-P, II-R, III-Q

(MCQ)

Ans. (a)

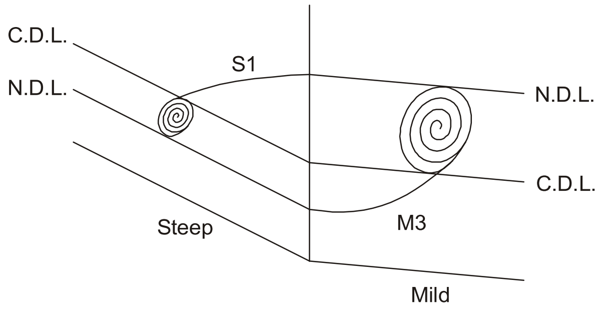
End of Solution

Q.19 When a supercritical stream enters a mild-sloped (M) channel section, the type of flow profile would become _______.

(a) M_1
(b) M_2
(c) M_3
(d) M_1 and M_2

(MCQ)

Ans. (c)



End of Solution

Q.20 Which one of the following statements is TRUE for Greenhouse Gas (GHG) in the atmosphere?

(a) GHG absorbs the incoming short wavelength solar radiation to the earth surface, and allows the long wavelength radiation coming from the earth surface to pass through

(b) GHG allows the incoming long wavelength solar radiation to pass through to the earth surface, and absorbs the short wavelength radiation coming from the earth surface

(c) GHG allows the incoming long wavelength solar radiation to pass through to the earth surface, and allows the short wavelength radiation coming from the earth surface to pass through

(d) GHG allows the incoming short wavelength solar radiation to pass through to the earth surface, and absorbs the long wavelength radiation coming from the earth surface

(MCQ)

Ans. (d)

End of Solution

Q.21 G_1 and G_1 are the slopes of the approach and departure grades of a vertical curve, respectively.

Given $|G_1| < |G_2|$ and $|G_2| \neq |G_2| \neq 0$

Statement 1: $+G_1$ followed by $+G_2$ results in a sag vertical curve.

Statement 2: $-G_1$ followed by $-G_2$ results in a sag vertical curve.

Statement 3: $+G_1$ followed by $-G_2$ results in a crest vertical curve.

Which option amongst the following is true?

(a) Statement 1 and Statement 3 are correct; Statement 2 is wrong

(b) Statement 1 and Statement 2 are correct; Statement 3 is wrong

(c) Statement 1 is correct; Statement 2 and Statement 3 are wrong

(d) Statement 2 is correct; Statement 1 and Statement 3 are wrong

(MCQ)

Ans. (a)

Statement (1) and (3) are correct.

Condition for summit curve (crest vertical curve),

+ve to +ve; +ve to flat

($G_1 > G_2$)

+ve to –ve; –ve to –ve

($G_1 < G_2$)

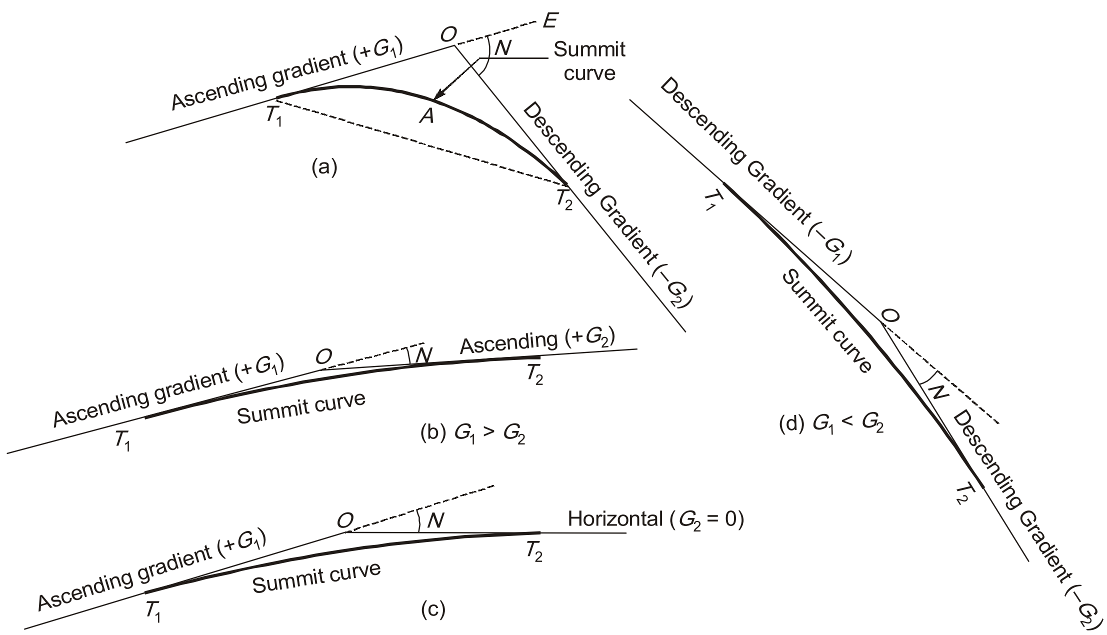


Fig. Summit curve, (showing deflection angle)

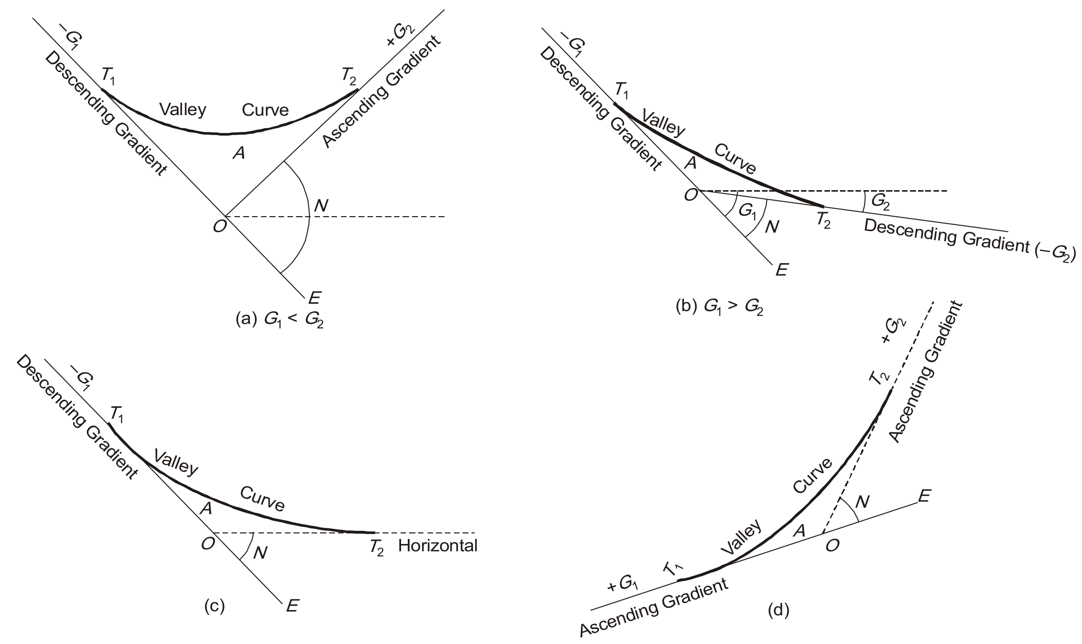
Condition for valley curve (sag vertical curve),

–ve to –ve; –ve to flat

($G_1 > G_2$)

–ve to +ve; +ve to +ve

($G_1 < G_2$)



End of Solution

Q.22 The direct and reversed zenith angles observed by a theodolite are 56° 00′ 00″ and 303° 00′ 00″, respectively. What is the vertical collimation correction?

(a) +1° 00′ 00″ (b) –1° 00′ 00″
(c) –0° 30′ 00″ (d) +0° 30′ 00″

(MCQ)

Ans. (d)

$$\phi_1 = \text{Observed value of direct zenith angle}$$
$$= 56° \; 00' \; 00''$$
$$\phi_2 = \text{Observed value of reversed zenith angle}$$
$$= 303° \; 00' \; 00''$$

$$\text{Error} = \left[\frac{(\phi_1 + \phi_2) - 360°}{2}\right]$$

$$= \frac{359°00'00'' - 360°00'00''}{2}$$

$$= -0° \; 30' \; 00''$$

So, $$\text{Correction} = +0° \; 30' \; 00''$$

End of Solution

Q.23 A student is scanning his 10 inch × 10 inch certificate at 600 dots per inch (dpi) to convert it to raster. What is the percentage reduction in number of pixels if the same certificate is scanned at 300 dpi?

(a) 62 (b) 88
(c) 75 (d) 50

(MCQ)

Ans. **(c)**

Number of dots as per 600 dpi = $(10 \times 600)^2$ = 36000000

Number of dots as per 300 dpi = $(10 \times 300)^2$ = 9000000

Percentage reduction in number of pixels

$$= \frac{36000000 - 9000000}{36000000} \times 100 = \frac{27}{36} \times 100 = 75\%$$

End of Solution

Q.24 If M is an arbitrary real $n \times n$ matrix, then which of the following matrices will have non-negative eigenvalues?

(a) M^2

(b) MM^T

(c) M^TM

(d) $(M^T)^2$

(MSQ)

Ans. **(b, c)**

Let us take $M = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ then E. values are $\lambda = \pm i$

So, Eigen values of M^2 are i^2 and $(-i)^2 = -1, -1$

Eigen values of $(M^T)^2$ are same i.e. $-1, -1$

Now, MM^T and M^TM are symmetric matrix, as well as the semi definite matrix and it is the property of +ve semi definite matrix that it's eigen values are non –ve.

Hence correct answer (b) and (c).

End of Solution

Q.25 The following function is defined over the interval $[-L, L]$:

$$F(x) = px^4 + qx^5$$

If it is expressed as a Fourier series,

$$F(x) = a_0 + \sum_{n=1}^{\infty} \left\{ a_n \sin\left(\frac{\pi x}{L}\right) + b_n \cos\left(\frac{\pi x}{L}\right) \right\}$$

which options amongst the following are true?

(a) a_n, $n = 1, 2, \ldots, \infty$ depend on p

(b) a_n, $n = 1, 2, \ldots, \infty$ depend on q

(c) b_n, $n = 1, 2, \ldots, \infty$ depend on p

(d) b_n, $n = 1, 2, \ldots, \infty$ depend on q

(MSQ)

Ans. (b, c)

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{(n\pi x)}{l}dx = \frac{1}{l}\int_{-l}^{l}\left(px^4 + qx^5\right)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{l}\int_{-l}^{l} px^4 \cos\frac{(n\pi x)}{l}dx + 0 \qquad (\because 2^{nd} \text{ integral is an odd functions})$$

Thus, b_n depend on p

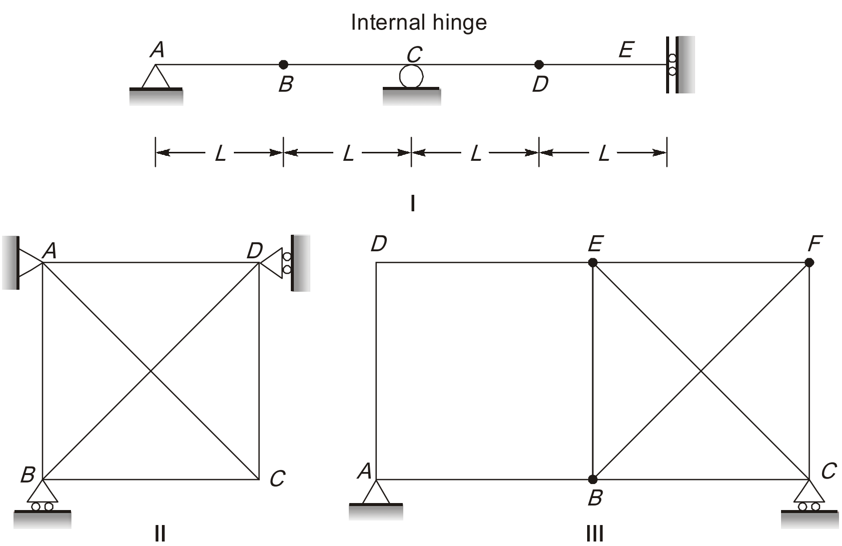
Similarly, $a_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{(n\pi x)}{l}dx = \frac{1}{l}\int_{-l}^{l}\left(px^4 + qx^5\right)\sin\left(\frac{n\pi x}{l}\right)dx$

$$= 0 + \frac{1}{l}\int_{-l}^{l} qx^5 \sin\frac{(n\pi x)}{l}dx \qquad (\because 1^{st} \text{ integral is an odd functions})$$

Thus, a_n depend on q.

End of Solution

Q.26 Consider the following three structures:



Structure I: Beam with hinge support at A, roller at C, guided roller at E, and internal hinges at B and D

Structure II: Pin-jointed truss, with hinge support at A, and rollers at B and D

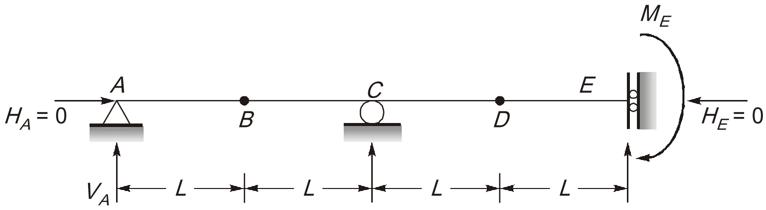
Structure III: Pin-jointed truss, with hinge support at A and roller at C

Which of the following statements is/are TRUE?

(a) Structure I is unstable
(b) Structure II is unstable
(c) Structure III is unstable
(d) All three structures are stable

(MSQ)

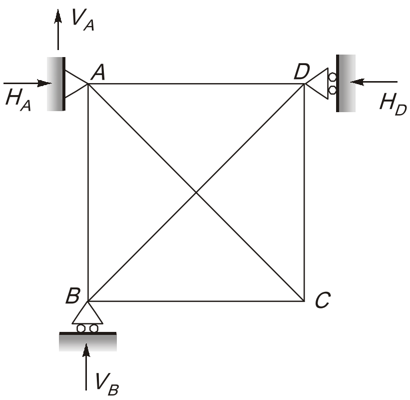
Ans. (a, b, c)



$r = 3\ (V_A,\ V_C,\ M_E)$

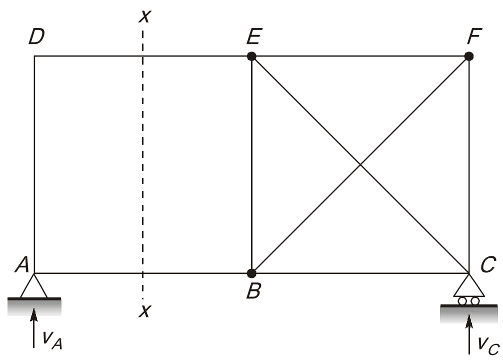
$s = 4\ (\Sigma F_y = 0,\ \Sigma M = 0,\ BM_B = 0,\ BM_D = 0)$

$D_s = r - s = 3 - 4 = -1$ $\Rightarrow$ negative implies unstable



Rigid body rotation is possible since all the four reactions are concurrent at *A*.

Unstable,



Shear force at x-x = Panel shear force = V_A

There is no member to resist panel shear force.

So unstable.

End of Solution

Q.27 Identify the waterborne diseases caused by viral pathogens:

(a) Acute anterior poliomyelitis
(b) Cholera
(c) Infectious hepatitis
(d) Typhoid fever

(MSQ)

Ans. (a, c)

End of Solution

Q.28 Which of the following statements is/are TRUE for the Refuse-Derived Fuel (RDF) in the context of Municipal Solid Waste (MSW) management?

(a) Higher Heating Value (HHV) of the unprocessed MSW is higher than the HHV of RDF processed from the same MSW
(b) RDF can be made in the powdered form
(c) Inorganic fraction of MSW is mostly converted to RDF
(d) RDF cannot be used in conjunction with oil

(MSQ)

Ans. (b)

End of Solution

Q.29 The probabilities of occurrences of two independent events A and B are 0.5 and 0.8, respectively. What is the probability of occurrence of at least A or B (rounded off to one decimal place)? ____________.

(NAT)

Ans. (0.9)(0.9 – 0.9)

Given, $P(A) = 0.5$ and $P(B) = 0.8$

and A and B are independents.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A)P(B)$$

$$= 0.5 + 0.8 - 0.5 \times 0.8 = 0.9$$

End of Solution

Q.30 In the differential equation $\frac{dy}{dx} + \alpha xy = 0$, α is a positive constant. If $y = 1.0$ at $x = 0.0$, and $y = 0.8$ at $x = 1.0$, the value of α is ______________(rounded off to three decimal places).

(NAT)

Ans. **(0.446)(0.445 – 0.447)**

Given, $\frac{dy}{dx} + \alpha xy = 0$

Using variable separable method.

$$\frac{dy}{dx} = -dxy$$

$$\int \frac{dy}{y} = -\alpha \int x dx + \ln C$$

$$\ln y = -\alpha \frac{x^2}{2} + \ln C$$

$$\ln\left(\frac{y}{c}\right) = -\frac{\alpha}{2} x^2$$

$$y = Ce^{\frac{-\alpha}{2}x^2}$$

By putting $y(0) = 1$

we get $C = 1$

By putting $y(1) = 0.8$

$$e^{\frac{-\alpha}{2}} = 0.8$$

$$\ln e^{-\frac{\alpha}{2}} = \ln 0.8$$

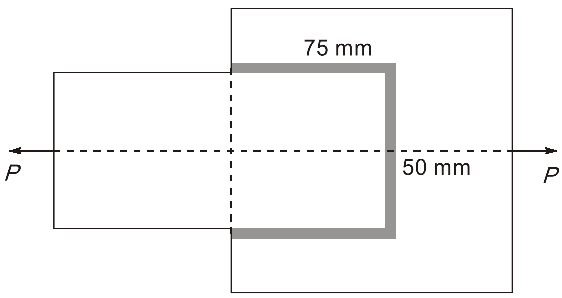
$$-\frac{\alpha}{2} = \ln(0.8)$$

$$\alpha = -2 \ln 0.8 = 0.446$$

End of Solution

Q.31 Consider the fillet-welded lap joint shown in the figure (not to scale). The length of the weld shown is the effective length. The welded surfaces meet at right angle. The weld size is 8 mm, and the permissible stress in the weld is 120 MPa. What is the safe load P (in kN, rounded off to one decimal place) that can be transmitted by this welded joint?

____



(NAT)

Ans. **(134.4) (134 – 136)**

Given: $S = 8$ mm, $\tau_{vw} = 120$ MPa, $L_w = 2 \times 75 + 50 = 200$ mm

The safe load is calculated as per WSM

$$\therefore \quad \rho_{safe} = \tau_{vw} \times A_w$$

$$= \tau_{vw} \times L_w \times (0.7 \times s) \qquad [\because t_t = 0.7s]$$

$$= 120 \times [2 \times 75 + 50] \times 0.7 \times 8 \times 10^{-3} \text{ kN} = 134.4 \text{ kN}$$

End of Solution

Q.32 A drained direct shear test was carried out on a sandy soil. Under a normal stress of 50 kPa, the test specimen failed at a shear stress of 35 kPa. The angle of internal friction of the sample is ________degree (round off to the nearest integer).

(NAT)

Ans. **(35)(35 – 35)**

Given, Normal stress, $\sigma_n = 50$ kPa

Shear stress, $\tau = 35$ kPa

$$\tau = c + \sigma_n \tan\phi$$

$$\Rightarrow \quad 35 = 50 \tan\phi \qquad (\because c = 0, \text{ sandy soil})$$

$$\Rightarrow \quad \tan\phi = 0.7$$

$$\Rightarrow \quad \phi = 35°$$

End of Solution

Q.33 A canal supplies water to an area growing wheat over 100 hectares. The duration between the first and last watering is 120 days, and the total depth of water required by the crop is 35 cm. The most intense watering is required over a period of 30 days and requires a total depth of water equal to 12 cm. Assuming precipitation to be negligible and neglecting all losses, the minimum discharge (in m^3/s, rounded off to three decimal places) in the canal to satisfy the crop requirement is ______________.

(NAT)

Ans. **(0.046)(0.045 – 0.047)**

Given: Area : 100 ha

Total depth of water, $\Delta_2 = 35$ cm

Total time period, $B_2 = 120$ days

Kor depth, $\Delta_1 = 12$ cm

Kor period, $B_1 = 30$ days

(i) $$\text{Kor duty, } \Delta_1 = \frac{8.64 \times B_1}{\Delta_1} = \frac{8.64 \times 30}{0.12} = 2160 \text{ ha/m}^3\text{/sec}$$

$$\text{Corresponding discharge} = \frac{\text{Area}}{\text{Duty}} = \frac{100}{2160} = 0.046 \text{m}^3/\text{sec}$$

(ii) $$\text{Overall duty, } \Delta_2 = \frac{8.64 \times B_2}{\Delta_2} = \frac{8.64 \times 120}{0.35} = 2962.28 \text{ ha/m}^3\text{/sec}$$

Corresponding discharge $= \frac{\text{Area}}{\text{Duty}} = \frac{100}{2962.28} = 0.034\ m^3/sec$

Minimum discharge required will be maximum of the above two.

$\therefore \qquad Q_{req.} = 0.046\ m^3/sec$

End of Solution

Q.34 The ordinates of a one-hour unit hydrograph for a catchment are given below:

t(hour)	0	1	2	3	4	5	6	7
$Q(m^3/s)$	0	9	21	18	12	5	2	0

Using the principle of superposition, a D-hour unit hydrograph for the catchment was derived from this one-hour unit hydrograph. The ordinates of the D-hour unit hydrograph were obtained as 3 m^3/s at t = 1 hour and 10 m^3/s at t = 2 hour. The value of D (in integer) is ___________.

(NAT)

Ans. (3)(3 –3)

Time	1hr-UH	1hr UH lagged by 1 hr	1hr-UH lagged by 1 more hr	3 hr DRH	Ordinate of 3hr UH $= \frac{\text{Ordinate of 3hr DRH}}{3\text{cm}}$
(1)	(2)	(3)	(4)	(5)	(6)
0	0	–	–	0	0
1	9	0	–	9	3
2	21	9	0	30	10
3	18	21	9	48	16
4	12	18	21	51	17
5	5	12	18	35	11.67
6	2	5	12	19	6.33
7	0	2	5	7	2.33
		0	2	2	0.67
			0	0	0

- Firstly lag the UH by 1 hr. By lagging 1 hr. We obtained a 2 hr UH of ordinate 4.5 m^3/sec. at 1 hr and 15 m^3/sec at 2 hr.
- So, further lag the UH by 1 more hr we obtained a 3 hr, UH of ordinates 3 m^3/sec at 1 hr and 10 m^3/sec at 2 hr. Therefore D = 3 hr.

End of Solution

Q.35 For a horizontal curve, the radius of a circular curve is obtained as 300 m with the design speed as 15 m/s. If the allowable jerk is 0.75 m/s^3, what is the minimum length (in m, in integer) of the transition curve?

(NAT)

Ans. **(15)(15 – 15)**

Given: $V = 15$ m/s

$R = 300$ m

Jerk, $C = 0.75$ m^3/s

So, height of transition curve,

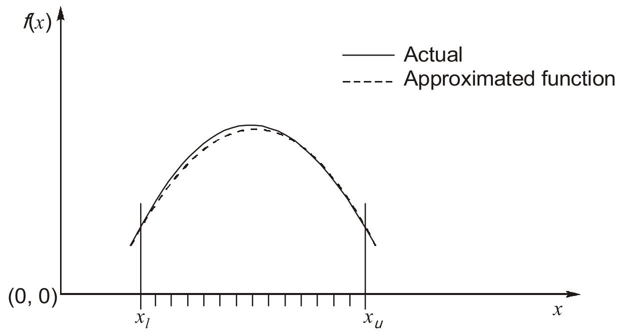
$$L_t = \frac{V^3}{RC} = \frac{15^3}{300 \times 0.75} = 15 \text{ m}$$

Note: We should also find out the length by formula given by IRC but terrain is not mentioned.

End of Solution

Q.36 - Q.65 carry TWO mark Each

Q.36 A function $f(x)$, that is smooth and convex-shaped between interval (x_l, x_u) is shown in the figure. This function is observed at odd number of regularly spaced points. If the area under the function is computed numerically, then .



(a) the numerical value of the area obtained using the trapezoidal rule will be less than the actual

(b) the numerical value of the area obtained using the trapezoidal rule will be more than the actual

(c) the numerical value of the area obtained using the trapezoidal rule will be exactly equal to the actual

(d) with the given details, the numerical value of area cannot be obtained using trapezoidal rule

(MCQ)

Ans. **(a)**

Using the standard property of trapezoidal rule for converse shaped parabola, the numerical value of the area obtained using the trapezoidal rule will be less than the actual.

End of Solution

Q.37 Consider a doubly reinforced RCC beam with the option of using either Fe250 plain bars or Fe500 deformed bars in the compression zone. The modulus of elasticity of steel is 2×10^5 N/mm^2. As per IS 456 : 2000, in which type(s) of the bars, the stress in the compression steel (f_{sc}) can reach the design strength ($0.87f_y$) at the limit state of collapse?

(a) Fe250 plain bars only

(b) Fe500 deformed bars only

(c) Both Fe250 plain bars and Fe500 deformed bars

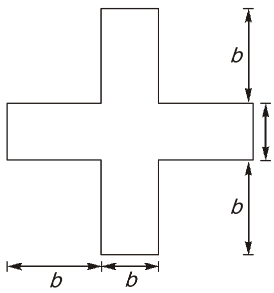
(d) Neither Fe250 plain bars nor Fe500 deformed bars

(MCQ)

Ans. **(a)**

End of Solution

Q.38 Consider the horizontal axis passing through the centroid of the steel beam cross-section shown in the figure. What is the shape factor (rounded off to one decimal place) for the cross-section?



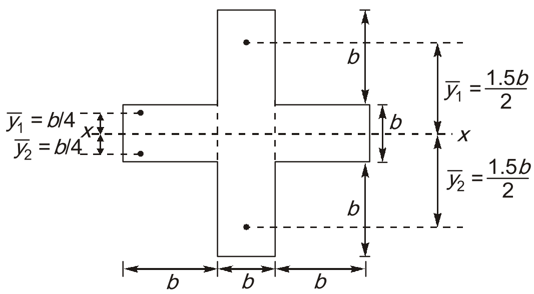
(a) 1.5 (b) 1.7

(c) 1.3 (d) 2.0

(MCQ)

Ans. **(b)**

As we know,



$$Z_p = \frac{A}{2}[\bar{y}_1 + \bar{y}_2]$$

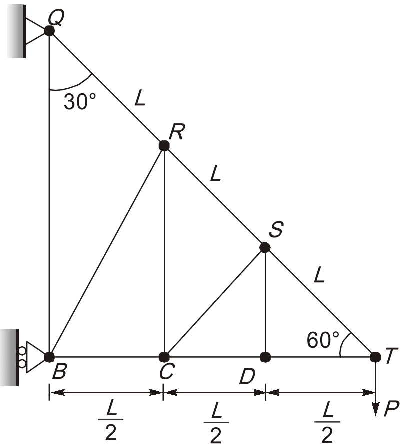
$$= (b\times1.5b)\left[\frac{1.5b}{2}+\frac{1.5b}{2}\right]+\left[2b\times\frac{b}{2}\right]\times\left[\frac{b}{4}+\frac{b}{4}\right] = 2.75b^3$$

$$Z = \frac{I_{xx}}{y} = \frac{\left[\frac{b\times(3b)^3}{12} + \frac{2b\times b^3}{12}\right]}{1.5b} = \frac{29}{18}b^3$$

Shape factor, $\quad \text{S.F.} = \frac{Z_p}{Z} = \frac{2.75b^3}{\frac{29}{18}b^3} = 1.7$

End of Solution

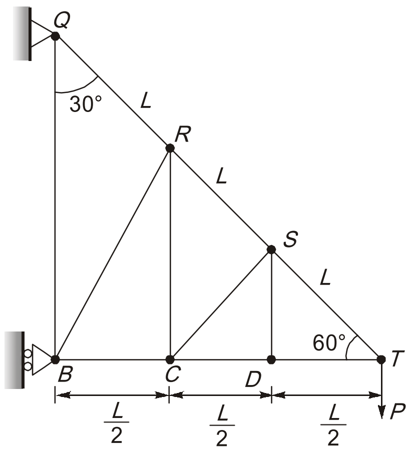
Q.39 Consider the pin-jointed truss shown in the figure (not to scale). All members have the same axial rigidity, AE. Members QR, RS, and ST have the same length L. Angles QBT, RCT, SDT are all 90°. Angles BQT, CRT, DST are all 30°. The joint T carries a vertical load P. The vertical deflection of joint T is $k\frac{PL}{AE}$. What is the value of k?



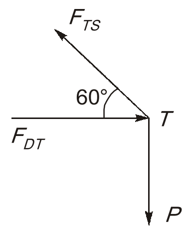
(a) 1.5 (b) 4.5
(c) 3.0 (d) 9.0

(MCQ)

Ans. (b)



At joint T,



$$\Sigma F_y = 0$$

$$\Rightarrow \qquad F_{TS}\sin 60^\circ = P$$

$$F_{TS} = \frac{2P}{\sqrt{3}}(T)$$

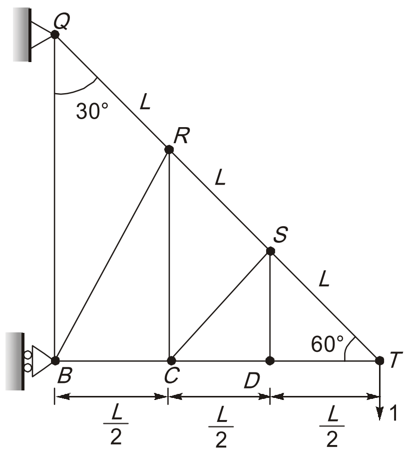
$$\Sigma F_x = 0$$

$$F_{TS}\cos 60^\circ = F_{DT}$$

$$F_{DT} = \frac{2P}{\sqrt{3}}\left(\frac{1}{2}\right) = \frac{P}{\sqrt{3}}(C)$$

$$F_{TS} = F_{SR} = F_{RQ} = \frac{2P}{\sqrt{3}}(T)$$

$$F_{DT} = F_{CD} = F_{BL} = \frac{P}{\sqrt{3}}(C)$$



Similarly,

$$k_{ST} = k_{SR} = k_{RQ} = \frac{2}{\sqrt{3}}(T)$$

$$k_{DT} = k_{CD} = k_{BC} = \frac{1}{\sqrt{3}}(T)$$

$$\therefore \qquad \delta_{VT} = \Sigma\frac{PkL}{AE} = \frac{\left(\frac{2P}{\sqrt{3}}\right)\left(\frac{2}{\sqrt{3}}\right)L\times 3}{AE} + \frac{\left(\frac{P}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\frac{L}{2}\times 3}{AE}$$

$$\delta_{VT} = \frac{4PL}{AE} + \frac{PL}{2AE} = \frac{4.5PL}{AE}$$

$$\Rightarrow \qquad k = 4.5$$

End of Solution

Q.40 With reference to the compaction test conducted on soils, which of the following is INCORRECT?

(a) Peak point of the compaction curve gives the maximum dry unit weight and optimum moisture content

(b) With increase in the compaction effort, the maximum dry unit weight increases

(c) With increase in the compaction effort, the optimum moisture content decreases

(d) Compaction curve crosses the zero-air-voids curve

(MCQ)

Ans. (d)

Zero air void line do not cross compaction curve.

End of Solution

Q.41 Consider that a force P is acting on the surface of a half-space (Boussinesq's problem). The expression for the vertical stress (σ_z) at any point (r, z), within the half-space is given as,

$$\sigma_z = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{\frac{5}{2}}}$$

where, r is the radial distance, and z is the depth with downward direction taken as positive. At any given r, there is a variation of σ_z along z, and at a specific z, the value of σ_z will be maximum. What is the locus of the maximum σ_z?

(a) $z^2 = \frac{3}{2}r^2$

(b) $z^3 = \frac{3}{2}r^2$

(c) $z^5 = \frac{5}{2}r^2$

(d) $z^3 = \frac{5}{2}r^2$

(MCQ)

Ans. (a)

At given r

For maximum σ_z

$$\frac{d\sigma_z}{dz} = 0$$

$$\sigma_z = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

$$\Rightarrow \quad \frac{d\sigma_z}{dz} = \frac{3P}{2\pi}\left[\frac{(r^2+z^2)^{5/2}\,3z^2 - z^3\cdot\frac{5}{2}(r^2+z^2)^{3/2}\cdot 2z}{(r^2+z^2)^5}\right] = 0$$

$$\Rightarrow \left(r^2+z^2\right)^{3/2} z^2\left[3\left(r^2+z^2\right)-5z^2\right] = 0$$

$$\Rightarrow \quad 3r^2 + 3z^2 - 5z^2 = 0$$

$$\Rightarrow \quad 3r^2 = 2z^2$$

$$\Rightarrow \quad z^2 = \frac{3r^2}{2}$$

End of Solution

Q.42 A square footing of size 2.5 m × 2.5 m is placed 1.0 m below the ground surface on a cohesionless homogeneous soil stratum. Considering that the groundwater table is located at the base of the footing, the unit weights of soil above and below the groundwater table are 18 kN/m^3 and 20 kN/m^3, respectively, and the bearing capacity factor N_q is 58, the net ultimate bearing capacity of the soil is estimated as 1706 kPa (unit weight of water = 10 kN/m^3).

Earlier, a plate load test was carried out with a circular plate of 30 cm diameter in the same foundation pit during a dry season, when the water table was located beyond the plate influence zone. Using Terzaghi's bearing capacity formulation, what is the ultimate bearing capacity (in kPa) of the plate?

(a) 110.16 (b) 61.20

(c) 204.00 (d) 163.20

(MCQ)

Ans. (a)

$$q_{nu} = q_u - \bar{\sigma} = q_u - \gamma D_f$$

$$1706 = \left\{1.3CN_C + \gamma D_f N_q + 0.4B\gamma N_\gamma\right\} - \gamma D_f$$

$$1706 = 18 \times 1 \times 58 + 0.4 \times 2.5 \times 10\ N_\gamma - 18\gamma_f$$

$$N_\gamma = 68$$

For Plate: [Surcharge at the plate level is zero]

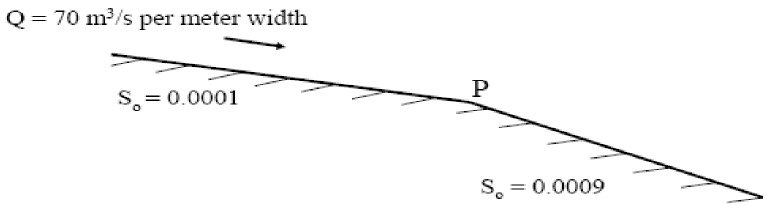
$$q_u = 1.3CN_c + \gamma D_f N_q + 0.3B\gamma N_\gamma$$

$$q_u = 0 + 0.3 \times 0.3 \times 18 \times 68$$

$$q_u = 110.16$$

End of Solution

Q.43 A very wide rectangular channel carries a discharge (Q) of 70 m^3/s per meter width. Its bed slope changes from 0.0001 to 0.0009 at a point P, as shown in the figure (not to scale). The Manning's roughness coefficient of the channel is 0.01. What water surface profile(s) exist(s) near the point P?



(a) M_2 and S_2 (b) M_2 only

(c) S_2 only (d) S_2 and hydraulic jump

(MCQ)

Ans. (a)

For a wide Rectangular channel

$$R = \frac{A}{P} = \frac{By}{B+2y} \approx \frac{By}{b} = y$$

Dischare per unit width, q is given by,

$$\Rightarrow \qquad q = \frac{1}{n} y\, y^{2/3} S^{1/2}$$

$$\Rightarrow \qquad q = \frac{1}{n} y^{5/3} S^{1/2}$$

Calculation of S_c,

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$\Rightarrow \qquad q = \frac{1}{n} y_c^{5/3} S_c^{1/2}$$

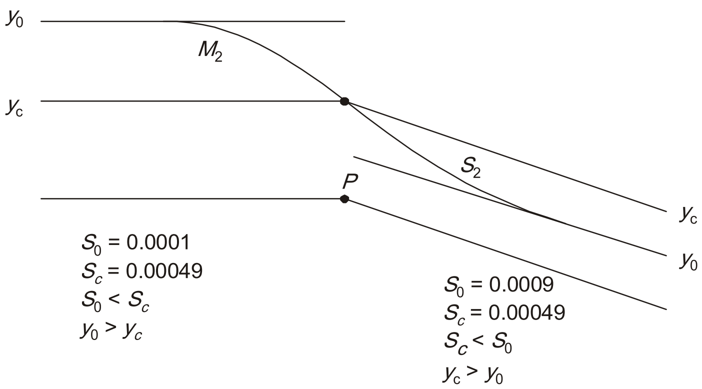
$$\Rightarrow \qquad q = \frac{1}{n}\left(\frac{q^2}{g}\right)^{5/3} S_c^{1/2}$$

$$\Rightarrow \qquad q = \frac{1}{n}\frac{q^{10/9}}{g^{5/3}} S_s^{1/2}$$

$$\Rightarrow \qquad S_c^{1/2} = \frac{nqg^{5/9}}{q^{10/9}}$$

$$\Rightarrow \qquad S_c = \left(\frac{ng^{5/9}}{q^{1/9}}\right)^2 = \left(\frac{0.01 \times 9.8^{5/9}}{70^{1/9}}\right)^2$$

$$= 0.00049 \quad (> 0.0001 \text{ and } < 0.0009)$$



End of Solution

Q.44 A jet of water having a velocity of 20 m/s strikes a series of plates fixed radially on a wheel revolving in the same direction as the jet at 15 m/s. What is the percentage efficiency of the plates? (round off to one decimal place)

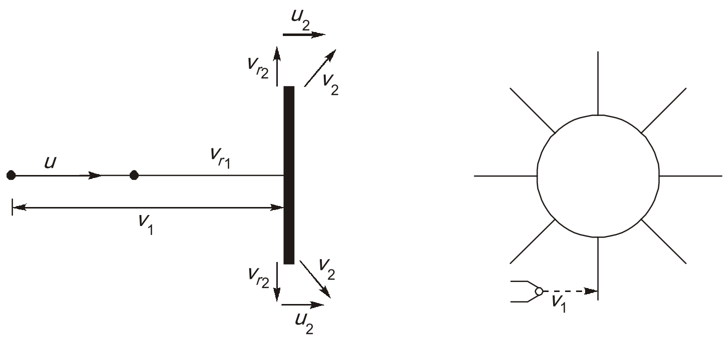
(a) 37.5
(b) 66.7
(c) 50.0
(d) 88.9

(MCQ)

Ans. **(a)**

Given:

$$V_1 = 20 \text{ m/s}$$
$$u = 15 \text{ m/s} \qquad [u_1 = u_2 = u]$$



$$\text{Efficiency, } \eta = \frac{\text{Work done per second}}{\text{Imput water power}}$$

$$= \frac{F_x \cdot u}{\frac{1}{2}\dot{m}V_1^2} = \frac{\dot{m}[V_1 - u] \times u}{\frac{1}{2}\dot{m}V_1^2}$$

$$= \frac{[20 - 15] \times 15}{\frac{1}{2} \times (20)^2} = 0.375 \simeq 37.5\%$$

End of Solution

Q.45 In the following table, identify the correct set of associations between the entries in Column-1 and Column-2.

Column-1	Column-2
P: Reverse Osmosis	I: Ponding
Q: Trickling Filter	II: Freundlich Isotherm
R: Coagulation	III: Concentration Polarization
S: Adsorption	IV: Charge Neutralization

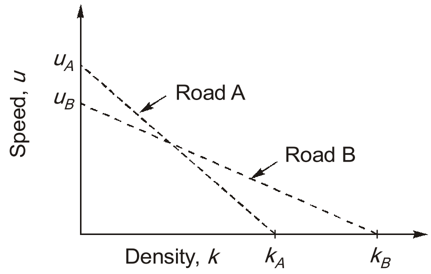
(a) P-II, Q-I, S-III
(b) Q-III, R-II, S-IV
(c) P-IV, R-I, S-II
(d) P-III, Q-I, R-IV

(MCQ)

Ans. **(d)**

End of Solution

Q.46 A plot of speed-density relationship (linear) of two roads (Road A and Road B) is shown in the figure.



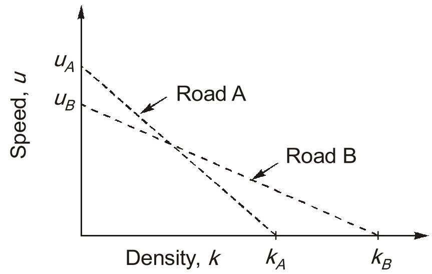
If the capacity of Road A is C_A and the capacity of Road B is C_B, what is $\frac{C_A}{C_B}$?

(a) $\frac{k_A}{k_B}$ (b) $\frac{u_A}{u_B}$

(c) $\frac{k_A u_A}{k_B u_B}$ (d) $\frac{k_A u_B}{k_B u_A}$

(MCQ)

Ans. **(c)**



Now, capacity of road *A*,

$$q_{\max\ A} = \frac{1}{4} k_A u_A$$

Similarly, capacity of road *B*,

$$q_{\max\ B} = \frac{1}{4} k_B u_B$$

Now,
$$\frac{C_A}{C_B} = \frac{\frac{1}{4} \times k_A \times u_A}{\frac{1}{4} \times k_B \times u_B} = \frac{k_A u_A}{k_B u_B}$$

End of Solution

Q.47 For the matrix

$$[A] = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Which of the following statements is/are TRUE?

(a) The eigenvalues of $[A]^T$ are same as the eigenvalues of $[A]$

(b) The eigenvalues of $[A]^{-1}$ are the reciprocals of the eigenvalues of $[A]$

(c) The eigenvectors of $[A]^T$ are same as the eigenvectors of $[A]$

(d) The eigenvectors of $[A]^{-1}$ are same as the eigenvectors of $[A]$

(MSQ)

Ans. **(a, b, d)**

Using standard properties of eigen values eigen vectors,

- Eigen values of A^T and A are same.
- Eigen values of A^{-1} is reciprocal of Eigen value of A.
- Eigen vectors of A^{-1} are same as the eigenvectors of A.

Row space and column space for any matrix are not same. Thus option (c) is false.

End of Solution

Q.48 For the function $f(x)=.e^x \left|\sin x\right|_{\text{¸}}\, x \in \mathbb{R}$ which of the following statements is/are TRUE?

(a) The function is continuous at all x

(b) The function is differentiable at all x

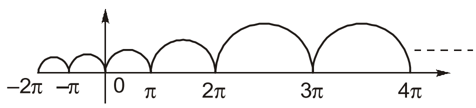
(c) The function is periodic

(d) The function is bounded

(MSQ)

Ans. **(a)**

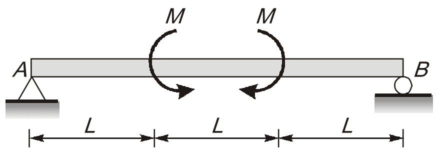
Given: $f(x) = e^x \left|\sin x\right|$



By observing graph we can easily say given function is neither periodic, nor bounded and nor differentiable, but is continuous at all x.

End of Solution

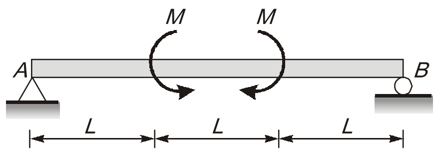
Q.49 Consider the beam shown in the figure (not to scale), on a hinge support at end A and a roller support at end B. The beam has a constant flexural rigidity, and is subjected to the external moments of magnitude M at one-third spans, as shown in the figure. Which of the following statements is/are TRUE?



(a) Support reactions are zero
(b) Shear force is zero everywhere
(c) Bending moment is zero everywhere
(d) Deflection is zero everywhere

(MSQ)

Ans. (a, b)



$$\Sigma F_y = 0$$

$\therefore$ $$V_A + V_B = 0 \quad \text{...(i)}$$

$$\Sigma M_A = 0$$

$\Rightarrow$ $$V_B \times L - M + M = 0$$

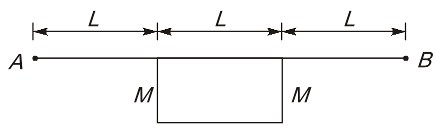
$\therefore$ $$V_B = 0$$

from equation (i), we get

$$V_A = 0$$

$\therefore$ shear force is zero everywhere in the beams, and reactions are also zero.

Bending moment diagram is shown below



So, bending moment is not zero everywhere. Also deflection is not zero everywhere.

End of Solution

Q.50 Which of the following statements is/are TRUE in relation to the Maximum Mixing Depth (or Height) 'D_{max}' in the atmosphere?

(a) D_{max} is always equal to the height of the layer of unstable air

(b) Ventilation coefficient depends on D_{max}

(c) A smaller Dmax will have a smaller air pollution potential if other meteorological conditions remain same

(d) Vertical dispersion of pollutants occurs up to D_{max}

(MSQ)

Ans. (b, d)

Maximum Mixing Depth (MMD)

- The dispersion of pollutants in the lower atmosphere is greatly aided by the convective and turbulent mixing that takes place.
- The vertical extent to which this mixing takes place depends on the environmental lapse rate which varies diurnally, from season to season and is also affected by topographical features.
- The depth of the convective mixing layer in which vertical movement of pollutants is possible, is called the maximum mixing depth (MMD).
- The ventilation coefficient is the product of mixing depth and the average wind speed.

End of Solution

Q.51 Which of the following options match the test reporting conventions with the given material tests in the table?

Test reporting convention	Material test
(P) Reported as ratio	(I) Solubility of bitumen
(Q) Reported as percentage	(II) Softening point of bitumen
(R) Reported in temperature	(III) Los Angeles abrasion test
(S) Reported in length	(IV) Flash point of bitumen
	(V) Ductility of bitumen
	(VI) Specific gravity of bitumen
	(VII) Thin film oven test

(a) (P) - (VI); (Q) - (I); (R) - (II); (S) - (VII)

(b) (P) - (VI); (Q) - (III); (R) - (IV); (S) - (V)

(c) (P) - (VI); (Q) - (I); (R) - (II); (S) - (V)

(d) (P) - (VI); (Q) - (III); (R) - (IV); (S) - (VII)

(MSQ)

Ans. (b, c)

Specific gravity of bitumen is the ratio of mass of given volume of substance to the mass of equal volume of water, the temperature of both being specified.

(P) - (VI); (Q) - (I, III, VII); (R) - (II, IV); (S) - (V)

End of Solution

Q.52 The differential equation,

$$\frac{du}{dt} + 2tu^2 = 1,$$

is solved by employing a backward difference scheme within the finite difference framework. The value of u at the $(n-1)^{th}$ time-step, for some n, is 1.75. The corresponding time (t) is 3.14 s. Each time step is 0.01 s long. Then, the value of $(u_n - u_{n-1})$ is ________. (round off to three decimal places).

(NAT)

Ans. (–0.151) (–0.152 to –0.149)

Given differential equation

$$\frac{du}{dt} + 2tu^2 = 1$$

$$\frac{du}{dt} = f(t,\ u) = 1 - 2tu^2$$

By backward Euler Iterative scheme,

$$u_n = u_{n-1} + hf(t_n,\ u_n)$$

$$u_n = u_{n-1} + h[1 - 2t_n\ u_n^2] \qquad \text{...(i)}$$

Given, $h = 0.01,\ u_{n-1} = 1.75,\ t_{n-1} = 3.14$

So, $t_n = t_{n-1} + h = 3.15$

Putting these values in (i)

$$u_n = 1.75 + 0.01[1 - 2 \times 3.15 \times u_n^2]$$

$$u_n = 1.75 + 0.01 - 0.628u_n^2$$

or $0.628u_n^2 + u_n - 1.75 = 0$

$62.8u_n^2 + 100u_n + 175 = 0$

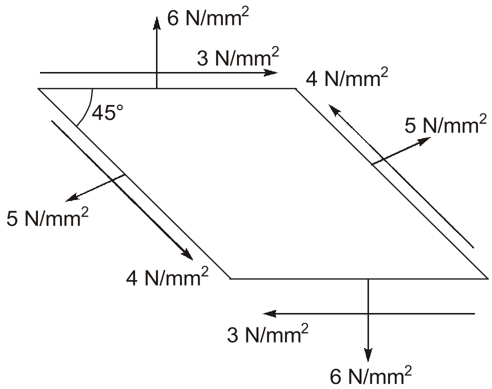
$\Rightarrow$ $u_n = 1.599\ –17.472$

So, for $u_n = 1.599$ and $u_{n-1} = 1.75$

We have, $u_n - u_{n-1} = 1.599 - 1.75 = –0.151$

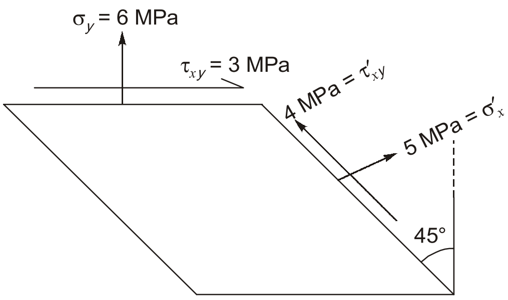
End of Solution

Q.53 The infinitesimal element shown in the figure (not to scale) represents the state of stress at a point in a body. What is the magnitude of the maximum principal stress (in N/mm^2, in integer) at the point?



(NAT)

Ans. (7)(7 – 7)



Now, $\sigma'_x = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + \tau_{xy}\sin 2\theta$ ...(i)

here $\theta = 45°$

$\sigma'_x = 5$ MPa

$\sigma_y = 6$ MPa

$\tau_{xy} = 3$ MPa

from eq. (i), we get, $\sigma_x = -2$ MPa

Now, $$\sigma_{1/2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau xy^2}$$

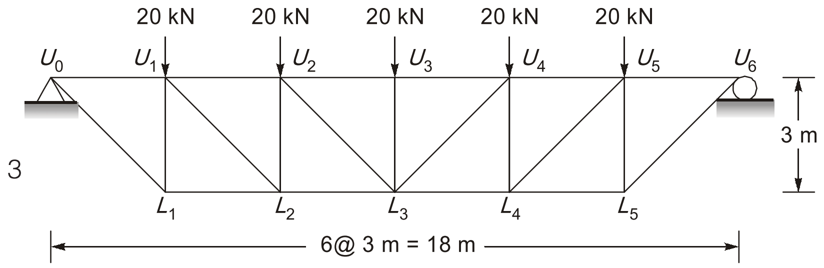
$$= \frac{-2+6}{2} \pm \frac{1}{2}\sqrt{(6+2)^2 + 4 \times 3^2}$$

$\sigma_1 = 7$ MPa

$\sigma_2 = -3$ MPa

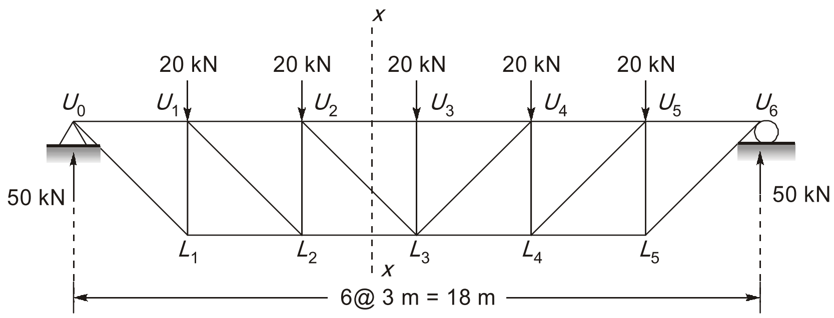
End of Solution

Q.54 An idealised bridge truss is shown in the figure. The force in Member U_2L_3 is ________kN (round off to one decimal place).



(NAT)

Ans. (14.1) (13.5 – 14.5)

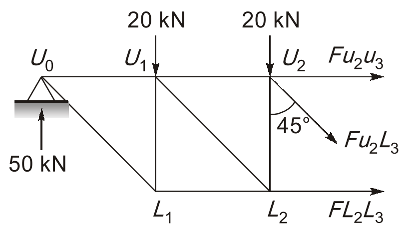


Reactions at supports:

Because of symmetry vertical reaction at supports

$$= \frac{100}{2} = 50 \text{ kN}$$

Applying method of sections and taking the section as shown in the figure above,



$$\Sigma F_y = 0$$

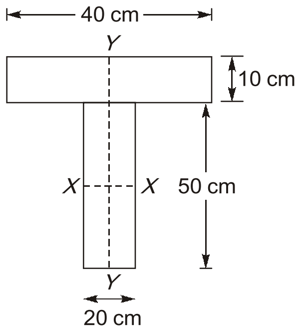
$\Rightarrow Fu_2L_3\cos 45° + 20 + 20 - 50 = 0$

$$F_{u2}L_3\cos 45° = 10$$

$$F_{u2}L_3 = 10\sqrt{2} \text{ kN} = 14.12 \text{ kN} \quad \text{(Tension)}$$

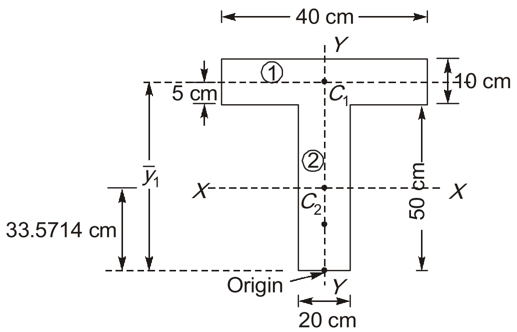
End of Solution

Q.55 The cross-section of a girder is shown in the figure (not to scale). The section is symmetric about a vertical axis (Y-Y). The moment of inertia of the section about the horizontal axis (X-X) passing through the centroid is ____________ cm^4. (round off to nearest integer).



(NAT)

Ans. (468810) (464000 – 472000)



$$A_1 = 40 \times 10 = 400 \text{ cm}^2$$

$$A_2 = 20 \times 50 = 1000 \text{ cm}^2$$

$$\bar{y}_1 = 50 + 5 = 55 \text{ cm}$$

$$\bar{y}_2 = \frac{50}{2} = 25 \text{ cm}$$

Centroid of composite shape, from bottom fibre,

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2}$$

$$\bar{y} = \frac{(400 \times 55) + (1000 \times 25)}{400 + 1000}$$

$$\bar{y} = 33.5714 \text{ cm}$$

$$I_{xx} = \left[\frac{40 \times 10^3}{12} + 400(\bar{y}_1 - \bar{y})^2\right] + \left[\frac{20 \times 50^3}{12} + 1000(\bar{y} - \bar{y}_2)^2\right]$$

$$= \left[\frac{40\times10^3}{12} + 400(55-33.5714)^2\right] + \left[\frac{20\times50^3}{12} + 1000(33.5714-25)^2\right]$$

$$= 187007.2925 + 281802.2313$$

$$I_{xx} = 468809.5238 \text{ cm}^4 \simeq 468810 \text{ cm}^4$$

End of Solution

Q.56 A soil having the average properties, bulk unit weight = 19 kN/m^3; angle of internal friction = 25° and cohesion = 15 kPa, is being formed on a rock slope existing at an inclination of 35° with the horizontal. The critical height (in m) of the soil formation up to which it would be stable without any failure is ________ (round off to one decimal place). [Assume the soil is being formed parallel to the rock bedding plane and there is no ground water effect.]

(NAT)

Ans. **(5.0) (4.8 – 5.2)**

For critical height of slope

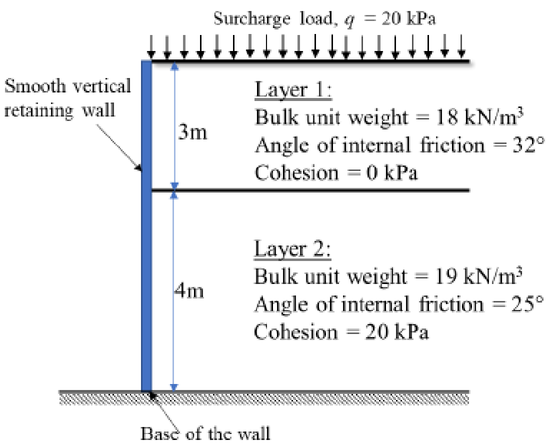
$$C + \gamma H_C \cos^2\beta \tan\phi = \gamma H_C \sin\beta \cos\beta$$

So, $15 + 19 \times H_C \times \cos^2 35° \tan 25° = 19 \times H_C \times \sin 35° \cos 35°$

$\Rightarrow \quad H_C = 5.03$ m

End of Solution

Q.57 A smooth vertical retaining wall supporting layered soils is shown in figure. According to Rankine's earth pressure theory, the lateral active earth pressure acting at the base of the wall is ________kPa (round off to one decimal place).



(NAT)

Ans. **(35.4) (34.0 – 37.0)**

Active earth pressure at base,

$$\therefore \qquad \sigma = k_a(\gamma z + q) - 2C\sqrt{ka}$$

where, $$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 25°}{1+\sin 25°} = 0.406$$

$$q = 20 + 18 \times 3 = 74 \text{ kN/m}^2$$

$$\gamma z = 19 \times 4 = 76 \text{ kN/m}^2$$

$$C = 20 \text{ kN/m}^2 \qquad [\because \text{Given}]$$

So, $$\sigma = 0.406 \times (76 + 74) - 2 \times 20 \times \sqrt{0.406}$$

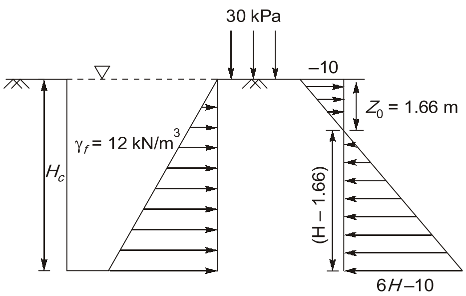
$$= 35.413 \text{ kN/m}^2 \simeq 35.4 \text{ kN/m}^2$$

End of Solution

Q.58 A vertical trench is excavated in a clayey soil deposit having a surcharge load of 30 kPa. A fluid of unit weight 12 kN/m^3 is poured in the trench to prevent collapse as the excavation proceeds. Assume that the fluid is not seeping through the soil deposit. If the undrained cohesion of the clay deposit is 20 kPa and saturated unit weight is 18 kN/m^3, what is the maximum depth of unsupported excavation (in m, rounded off to two decimal places)?

(NAT)

Ans. **(3.33) (3.30 – 3.35)**



Now, active pressure at depth z,

$$\sigma_z = k_a\gamma z - 2C\sqrt{k_a} - 12z + kq$$

At $z = 0$

$[\because k = 1 \text{ for } \phi = 0°,\ q = 30 \text{ kN/m}^2;\ C = 20 \text{ kN/m}^2]$

$$\sigma_z = -2 \times C + q = 2 \times 20 + 30$$

$$= -40 + 30$$

$$= -10 \text{ kN/m}^2$$

At z_0, $\qquad \sigma_z = 0$

$$18z_0 - 40 - 12z_0 + 30 = 0$$

$$\Rightarrow \qquad 6z_0 - 10 = 0$$

$$\Rightarrow \qquad z_0 = 1.66 \text{ m}$$

At depth H, $$\sigma_H = k_a\gamma H - 2C\sqrt{k_a} - 12H + k_q$$

So, $\sigma_H = 18H - 40 - 12H + 30$

$= 6H - 10$

For unsupported depth of excavation, total active thrust must be zero.

So, $$\frac{1}{2}\times 10\times 1.66 = \frac{1}{2}\times(H-1.66)\times(6H-10)$$

$$\Rightarrow \quad H = \frac{10}{3} = 3.33\text{ m}$$

Alternatively:

$$\sigma_v = q + \gamma z$$
$$= 30 + 18z$$

$$p_a = k_a\sigma_v - 2c\sqrt{k_a}$$

$$p_a = 30 + 18z - 2 \times 20 = 18z - 10$$

For critical depth, $p_a = 0$

$$18z_c - 10 = 0$$

$$z_c = \frac{10}{18}\text{m} = 0.556\text{ m}$$

For the maximum depth of unsupported excavation, the active earth pressure force at that depth should be zero.

$$\therefore \quad P_a = \frac{1}{2}(18H_c - 10)(H_c - 0.56) - \frac{1}{2}10\times 0.56 - \frac{1}{2}\gamma_f H_c^2$$

$$0 = (9H_c - 5)(H_c - 0.56) - 5 \times 0.56 - 6H_c^2$$

$$0 = 3H_c^2 - 10.04H_c + 5 \times 0.56 - 5 \times 0.56$$

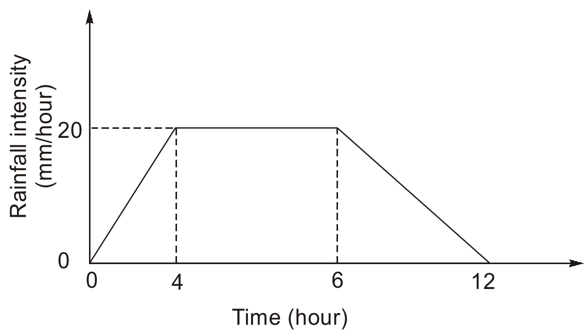
$$3H_c^2 - 10.04H_c = 0$$

$$H_c = 0, 3.33\text{ m}$$

Therefore, the maximum depth of unsupported excavation is 3.33 m.

End of Solution

Q.59 A 12-hour storm occurs over a catchment and results in a direct runoff depth of 100 mm. The time-distribution of the rainfall intensity is shown in the figure (not to scale). The ϕ-index of the storm is (in mm, rounded off to two decimal places) ___________.



(NAT)

Ans. **(3.60) (3.59 – 3.61) or (39.9 - 40.1)**

Given: Time = 12 hr

Runoff, R = 100 mm

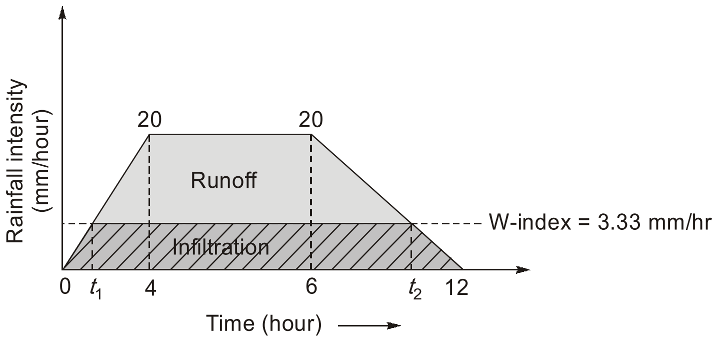
Total depth of precipitation,

$$P = \frac{1}{2}[20\times 4]+2\times 20+\frac{1}{2}\times 20\times 6 = 140 \text{ mm}$$

$\therefore$ Total infiltration, $I = P - R$

$= 140 - 100 = 40$ mm

No, $W\text{-Index} = \frac{I}{t} = \frac{140-100}{12} = 3.33$ mm/hr



Now, $\frac{20}{4} = \frac{3.33}{t_1}$

$\Rightarrow$ $t_1 = 0.67$ hr

Similarly, $\frac{20}{6} = \frac{3.33}{t_2}$

$\Rightarrow$ $t_2 = 0.99$ hr

Now, $\phi\text{-Index} = \frac{\text{Total infiltration in which rainfall excess occur}}{\text{Time period in rainfall which excess occur}}$

$$= \frac{I - \text{infiltration in which no runoff}}{\text{Time excess}}$$

$$\phi\text{-Index} = \frac{140-\left[\frac{1}{2}\times 3.33\times 0.67+\frac{1}{2}\times 3.33\times 0.99\right]}{12-0.67-0.99}$$

$= 3.6$ mm/hr

Check: Runoff = Area of hatched portion

$$= \frac{1}{2}\times(20-3.6)\times(4-0.67)+(20-3.6)\times 2+\frac{1}{2}\times(20-3.6)\times(6-0.99)$$

$= 100$ mm (Hence, OK)

$\therefore$ ϕ-Index = 3.60 mm/hr

NOTE: This question is challengeable since ϕ-index unit is mm/hr not mm.

End of Solution

Q.60 A hydraulic jump occurs in a 1.0 m wide horizontal, frictionless, rectangular channel, with a pre-jump depth of 0.2 m and a post-jump depth of 1.0 m. The value of g may be taken as 10 m/s^2. The values of the specific force at the pre-jump and post-jump sections are same and are equal to (in m^3, rounded off to two decimal places) ________.

(NAT)

Ans. **(0.62) (0.60 – 0.64)**

Given, $y_1 = 0.2$ m, $y_2 = 1$ m

As we know

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$\Rightarrow \quad \frac{2q^2}{10} = 0.2 \times 1(0.2+1)$$

$$\Rightarrow \quad q^2 = \frac{10}{2} \times 0.2 \times 1.2 = 1.2$$

$$q = 1.095 \text{ m}^3/\text{s/m}$$

Now, specific force $= \dfrac{Q^2}{Ag} + A\bar{y}$

$$= \frac{q^2 B^2}{B y_1 g} + \frac{B y_1 y_1}{2}$$

$$= \left(\frac{1.095^2}{0.2 \ \times 102} + \frac{0.2^2}{2}\right) \times 1 = 0.6195 \text{ m}^3 \simeq 0.62 \text{ m}^3$$

So,Specific force per unit width = 0.62 m^3/m

End of Solution

Q.61 In Horton's equation fitted to the infiltration data for a soil, the initial infiltration capacity is 10 mm/h; final infiltration capacity is 5 mm/h; and the exponential decay constant is 0.5 /h. Assuming that the infiltration takes place at capacity rates, the total infiltration depth (in mm) from a uniform storm of duration 12 h is ______________. (round off to one decimal place)

(NAT)

Ans. **(70.0) (69.7 – 70.1)**

Initial infiltration capacity, f_0 = 10 mm/h

Final infiltration capacity, f_c = 5 mm/h

Horton's Decay constant, k_h = 0.5/h

Now, $$F_P = \int_0^T \left[f_c + [f_0 - f_c] e^{-k_h t}\right] dt$$

$$= \int_0^{12} \left(5 + (10-5) e^{-0.5t}\right) dt$$

$$= [5t]_0^{12} + \left[\frac{5e^{-0.5t}}{(-0.5)}\right]_0^{12}$$

$$= [5\times12] + \left[-10e^{-0.5\times12} - \left[-10e^{-0.5\times0}\right]\right]$$

$$= 60 + [9.975]$$

$$= 69.97 \text{ mm} \simeq 70 \text{ mm}$$

Alternatively:

For large value of 't' cumulative infiltration capacity is given by,

$$F_P = f_c t + \frac{(f_0 - f_c)}{k_h}$$

$$= (5\times12) + \frac{(10-5)}{0.5} = 70 \text{ mm}$$

End of Solution

Q.62 The composition and energy content of a representative solid waste sample are given in the table. If the moisture content of the waste is 26%, the energy content of the solid waste on dry-weight basis is MJ/kg (round off to one decimal place).

Components	% by Mass	Energy content as - discarded basis [MJ/kg]
Food waste	20	4.5
Paper	45	16
Card board	5	14
Plastic	10	32
Other	20	8.0

(NAT)

Ans. **(18.4) (18 – 19)**

Energy content (as discarded basis):

[0.2 × 4.5 + 0.45 × 16 + 0.05 × 14 + 0.1 × 32 + 0.2 × 8] = 13.6 MJ/kg

Energy content (on dry basis) for moisture content of 26%:

$$= \frac{13.6 \text{ MJ/kg} \times 100\%}{(100-26)\%} = 18.38 \text{ MJ/kg} \simeq 18.4 \text{ MJ/kg}$$

End of Solution

Q.63 A flocculator tank has a volume of 2800 m^3. The temperature of water in the tank is 15°C, and the average velocity gradient maintained in the tank is 100/s. The temperature of water is reduced to 5°C, but all other operating conditions including the power input are maintained as the same. The decrease in the average velocity gradient (in %) due to the reduction in water temperature is (round off to nearest integer).

[Consider dynamic viscosity of water at 15°C and 5°C as 1.139×10^{-3} N-s/m^2 and 1.518×10^{-3} N-s/m^2, respectively]

(NAT)

Ans. **(13) (12 – 15)**

$$G_1 = \sqrt{\frac{P}{\mu_1 \times V}} \quad \text{...(i)}$$

$$G_2 = \sqrt{\frac{P}{\mu_2 \times V}} \quad \text{...(ii)}$$

Divide (i) by (ii), we get

$$\frac{G_1^2}{G_2^2} = \frac{\mu_2}{\mu_1}$$

$$G_2{}^2 = \left[\frac{(100s^{-1})^2 \times 1.139 \times 10^{-3}\,\text{N-s/m}^2}{1.518 \times 10^{-3}\ \text{N-s/m}^2}\right]^{1/2} = 86.62\,\text{S}^{-1}$$

$$\%\text{ change in } G = \frac{G_1 - G_2}{G_1} \times 100 = \frac{100 - 86.62}{100} \times 100 = 13.38\%$$

End of Solution

Q.64 The wastewater inflow to an activated sludge plant is 0.5 m^3/s, and the plant is to be operated with a food to microorganism ratio of 0.2 mg/mg-d. The concentration of influent biodegradable organic matter of the wastewater to the plant (after primary settling) is 150 mg/L, and the mixed liquor volatile suspended solids concentration to be maintained in the plant is 2000 mg/L. Assuming that complete removal of biodegradable organic matter in the tank, the volume of aeration tank (in m^3, in integer) required for the plant is ________.

(NAT)

Ans. **(16200) (16200 – 16200)**

Given:

$$Q = 0.5\ \text{m}^3/\text{sec}$$

$$\frac{F}{M} = 0.2\ \text{mg/mg–d}$$

$$S_0 = 150\ \text{mg/L}$$

$$X = 2000\ \text{mg/L}$$

Now,

$$\frac{F}{M} = \frac{Q_0 S_0}{V \cdot X}$$

$$0.2\,d^{-1} = \frac{0.5\dfrac{\text{m}^3}{\text{sec.}} \times 86400\dfrac{\text{sec}}{d} \times 150\dfrac{\text{mg}}{L}}{V \times 2000\,\dfrac{\text{mg}}{L}}$$

$$V = 16200\ \text{m}^3$$

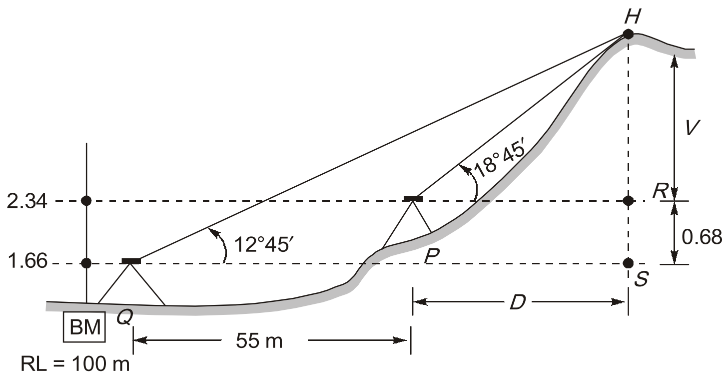
End of Solution

Q.65 Trigonometric levelling was carried out from two stations P and Q to find the reduced level (R. L.) of the top of hillock, as shown in the table. The distance between Stations P and Q is 55 m. Assume Stations P and Q, and the hillock are in the same vertical plane. The R. L. of the top of the hillock (in m) is ________ (round off to three decimal places).

Station	Vertical angle of top of hilllock	Staff reading on B.M	R.L of BM
P	18° 45′	2.340 m	100.000 m
Q	12° 45′	1.660 m	

(NAT)

Ans. **(137.627) (137.500 – 137.700)**



Now, $\tan 12°45' = \dfrac{V+0.68}{D+55}$ ...(i)

Also, $\tan 18°45' = \dfrac{V}{D}$ ...(ii)

On solving (i) and (ii), we get

$$V = 35.287 \text{ m}$$

$$D = 103.954 \text{ m}$$

Now, R.L. of hill top = R.L. of BM + 2.34 + V

= 100 + 2.34 + 35.287 = 137.627 m

End of Solution

■■■■