Quadrilateral

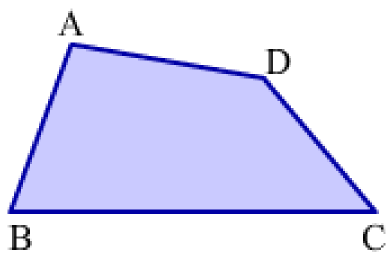
Angle Sum Property of Quadrilaterals

We have learnt about the angle sum property of triangles, which states that the sum of the measures of all three angles of a triangle is180°.Similarly, there is a property about the sum of all angles of a quadrilateral, which states that **the sum of the measures of the four angles of a quadrilateral is 360°.**

This property can be verified by two ways. They are as follows:

1. By measuring the angles of a quadrilateral

Let us consider a quadrilateral ABCD.



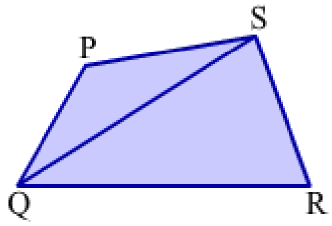
On measuring the angles of this quadrilateral, we found $m\angle A = 100°$, $m\angle B = 70°$, $m\angle C = 50°$and $m\angle D = 140°$.

$\therefore$Sum of four angles of quadrilateral ABCD = $100° + 70° + 50° + 140° = 360°$

Similarly, we can verify that the sum of the four angles of any quadrilateral is360°.

2. By dividing a quadrilateral into two triangles

Let us consider a quadrilateral PQRS with diagonal QS.



From the figure, it can be seen that the diagonal QS divides the quadrilateral PQRS into two triangles. We know that the sum of all three angles of a triangle is 180°.

So ,in ΔPQS, $m\angle P + m\angle PQS + m\angle PSQ = 180°$…(1)

Similarly, in ΔQRS, $m\angle SQR + m\angle R + m\angle QSR = 180°$…(2)

On adding (1) and (2):

$M\angle P + m\angle PQS + m\angle PSQ + m\angle SQR + m\angle R + m\angle QSR = 180° + 180°$

$\Rightarrow m\angle P + m\angle PQS + m\angle SQR + m\angle R + m\angle PSQ + m\angle QSR = 360°$

$\Rightarrow m\angle P + m\angle Q + m\angle R + m\angle S = 360°$

[$m\angle PQS + m\angle SQR = m\angle Q$ and $m\angle PSQ + m\angle QSR = m\angle S$]

This verifies that the sum of the four angles of a quadrilateral is 360°.

Now let us have a look at some examples based on this property.

Example1:

Two equal angles of a quadrilateral measure 75°each. What is the sum of the remaining two angles of the quadrilateral?

Solution:

We know that the sum of the measures of the four angles of a quadrilateral is 360°.

∴ Sum of measures of given two angles + Sum of measures of remaining two angles = 360°

⇒ 75°+ 75°+ Sum of measures of remaining two angles = 360°

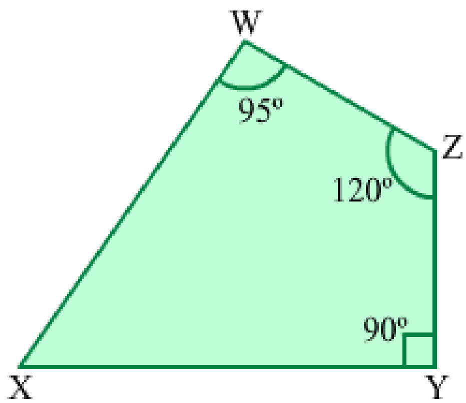
⇒ 150°+ Sum of measures of remaining two angles = 360°

⇒ Sum of measures of remaining two angles = 360°−150°

⇒ Sum of measures of remaining two angles = 210°

Example2:

In the given quadrilateral WXYZ, what is the measure of ∠X?



Solution:

In the given quadrilateral, $m\angle W = 95°$, $m\angle Y = 90°$ and $m\angle Z = 120°$.

Since the sum of the measures of the four angles of a quadrilateral is 360°,

$M\angle W + m\angle X + m\angle Y + m\angle Z = 360°$

$\Rightarrow 95° + m\angle X + 90° + 120° = 360°$

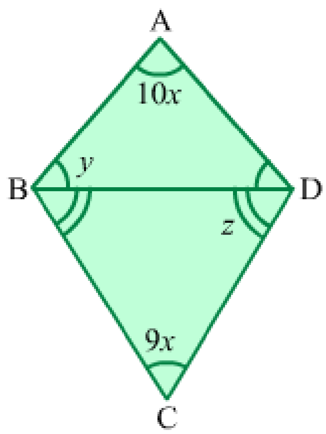
$\Rightarrow m\angle X + 305° = 360°$

$\Rightarrow m\angle X = 360° - 305°$

$\Rightarrow m\angle X = 55°$

Example3:

In the given figure, $\angle ABD = \angle ADB$ and $\angle CBD = \angle CDB$. If $y + z = 132.5°$, then find the measures of all angles of the quadrilateral ABCD.



Solution:

It is given that $\angle ABD = \angle ADB = y$, $\angle CBD = \angle CDB = z$ and $y + z = 132.5°$.

Since the sum of the measures of the four angles of a quadrilateral is360°,

$\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow \angle A + (\angle ABD + \angle DBC) + \angle C + (\angle CDB + \angle BDA) = 360°$

$\Rightarrow 10x + (y + z) + 9x + (z + y) = 360°$

$\Rightarrow 19x + 2(y + z) = = 360°$

$\Rightarrow 19x + 2 \times 132.5° = 360°$

$\Rightarrow 19x = 360° - 265°$

$\Rightarrow 19x = 95°$

$\Rightarrow x = 5°$

$\therefore \angle A = 10 \times 5° = 50°$ and $\angle C = 9 \times 5° = 45°$

Now, using angle sum property in ΔABD:

$\angle A + \angle ABD + \angle ADB = 180°$

$\Rightarrow 50° + y + y = 180°$

$\Rightarrow 2y = 180° - 50°$

$\Rightarrow 2y = 130°$

$\Rightarrow y = 65°$

Also, $y + z = 132.5°$

$\Rightarrow z = 132.5° - 65°$

$\Rightarrow z = 67.5°$

Now, $\angle B = \angle ABD + \angle DBC$

= $y + z$

= 132.5°

Similarly, ∠D = 132.5°

Thus, the measures of four angles of the quadrilateral ABCD are 50°,132.5°,45°and 132.5°.

Various Types of Quadrilaterals and Their Properties

Quadrilaterals are four-sided polygons. There are various types of quadrilaterals such as parallelograms, trapeziums, rectangles, and squares. Let us look at this video to learn what it is that makes these types of quadrilaterals different from one another.

Let us now look at following examples.

Example 1:

State whether the following statements are true or false.

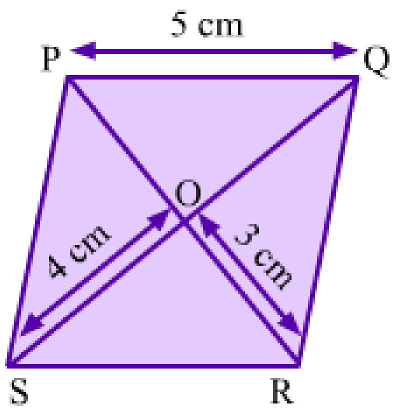
1. **Each angle of a parallelogram is a right angle.**
2. **All the sides of a rectangle are equal in length.**
3. **The opposite sides of a trapezium are parallel.**
4. **The diagonals of a rectangle are perpendicular to one another.**
5. **All the sides of a square are of equal lengths.**
6. **Opposite angles of a parallelogram are equal.**

Solution:

1. False
2. False
3. False
4. False
5. True
6. True

Example 2:

Observe the figure of given rhombus.



Find the following components of this rhombus.

(1) $m\angle SOR$

(2) $l(SQ)$

(3) $l(PO)$

(4) $l(QR)$, $l(RS)$ and $l(SP)$

Solution:

(1) Diagonals of a rhombus intersect each other at the angle of 90°.

$\therefore m\angle SOR = 90°$

(2) Diagonals of a rhombus bisect each other.

$\therefore l(SQ) = 2 \times l(SO) = (2 \times 4)$ cm = 8 cm

(3) Diagonals of a rhombus bisect each other.

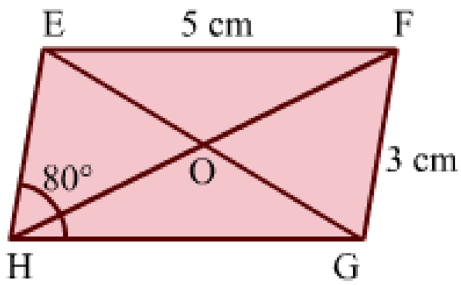
$\therefore l(PO) = l(RO) = 3$ cm

(4) All sides of a rhombus are of equal length.

$l(QR) = l(RS) = l(SP) = l(PQ) = 5$ cm

Example 3:

Observe the figure of given parallelogram.



Find the following components of this parallelogram.

(1) $m\angle$EFG

(2) l(GH)

(3) l(HE)

Solution:

(1) Opposite angles of a parallelogram are equal.

∴ $m\angle EFG = m\angle GHE = 80°$

(2) Opposite sides of a parallelogram are equal.

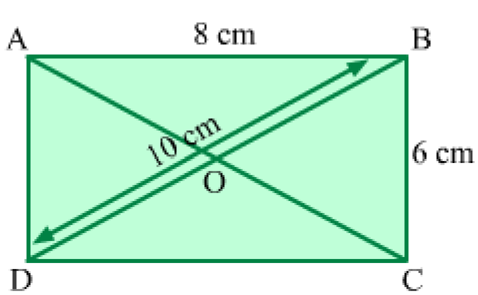
∴ $l(GH) = l(EF) = 5$ cm

(3) Opposite sides of a parallelogram are equal.

∴ $l(HE) = l(FG) = 3$ cm

Example 4:

Observe the figure of given rectangle.



Find the following components of this rectangle.

(1) *l*(AO)

(2) *m*∠ABC, *m*∠BCD, *m*∠CDA and *m*∠DAB

(3) *l*(CD)

(4) *l*(DA)

Solution:

(1) Diagonals of of rectangle are equal and bisect each other.

∴ *l*(AC) = *l*(BD) = 10 cm

∴ *l*(AO) = $\frac{1}{2}l(\mathrm{AC})$ = 5 cm

(2) All angles of a rectangle are of 90°.

∴ *m*∠ABC = *m*∠BCD = *m*∠CDA = *m*∠DAB = 90°

(3) Opposite sides of a rectangle are equal.

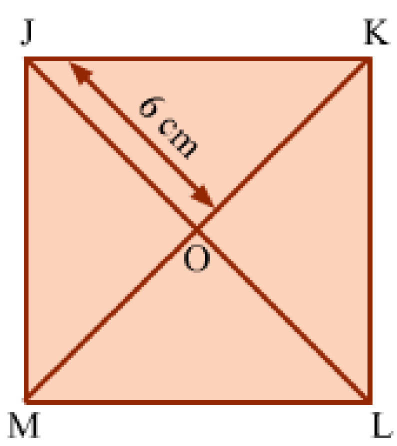
∴ *l*(CD) = *l*(AB) = 8 cm

(4) Opposite sides of a rectangle are equal.

∴ *l*(DA) = *l*(BC) = 6 cm

Example 5:

Observe the figure of given square.



Find the following components of this square.

(1) $m\angle KOL$

(2) $l(LO)$

(3) $l(MK)$

Solution:

(1) Diagonals of a square intersect each other at the angle of 90°.

$\therefore m\angle KOL = 90°$

(2) Diagonals of a square bisect each other.

$\therefore l(LO) = l(JO) = 6$ cm

(3) Diagonals of a square are equal.

$\therefore l(MK) = l(JL) = l(LO) + l(JO) = 6$ cm $+ 6$ cm $= 12$ cm