Class-XII
Session - 2022-23
Subject - Mathematics (041)
Sample Question Paper - 30
With Solution

BLUE PRINT

Ch. No.	Chapter Name	Per Unit Marks	Section-A (1 Mark)		Section-B (2 Marks)	Section-C (3 Marks)	Section-D (5 Marks)	Section-E (4 Marks)	Total Marks
			MCQ	A/R	VSA	SA	LA	Case-Study	
1	Relations and Functions	8					Q.35		5
2	Inverse Trigonometry Functions			Q.20	Q.21				3
3	Matrices	10	Q.1,6						2
4	Determinants		Q.3,8,11				Q.32		8
5	Continuity and Differentiability	35	Q.2,14		Q.22				4
6	Applications of Derivatives				Q.23			Q.38,36	10
7	Integrals		Q.5,17			Q.26,27,28			11
8	Applications of Integrals						Q.34		5
9	Differential Equations		Q.9,16			Q.29			5
10	Vector Algebra	14	Q.7,13,18		Q.24,25				7
11	Three Dimensional Geometry		Q.15	Q.19			Q.33		7
12	Linear Programming	5	Q.10,12			Q.31			5
13	Probability	8	Q.4			Q.30		Q.37	8
	Total Marks (Total Questions)		18(18)	2(2)	10(5)	18(6)	20(4)	12(3)	80(38)

Note : The number given inside the bracket denotes question number, asked in the sample paper, while the number given outside the bracket are the number of questions from that particular chapter.

Time : 3 Hours **Max. Marks : 80**

General Instructions

1. *This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.*
2. *Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.*
3. *Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.*
4. *Section C has 6 Short Answer (SA)-type questions of 3 marks each.*
5. *Section D has 4 Long Answer (LA)-type questions of 5 marks each.*
6. *Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.*

SECTION-A (Multiple Choice Questions)

Each question carries 1 mark.

1. For a matrix A, $AI = A$ and $AA^T = I$ is true for

(a) If A is a square matrix. (b) If A is a non singular matrix.
(c) If A is symmetric matrix. (d) If A is any matrix.

2. The relationship between a and b, so that the function f defined by $f(x)=\begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$, is

(a) $a=b+\frac{2}{3}$ (b) $a-b=\frac{3}{2}$ (c) $a+b=\frac{2}{3}$ (d) $a+b=2$

3. The system $ax-3y+5z=4$, $x-ay+3z=2$, $9x-7y+8az=0$ has

(a) a unique solution for all a (b) no solution for all a
(c) unique solution if $4a^3-45a+58=0$ (d) no solution if $4a^3-45a+58=0$

4. Girl students constitute 10% of I year and 5% of II year at Roorkee University. During summer holidays 70% of the I year and 30% of II year students are given a project. The girls take turns on duty in canteen. The chance that I year girl student is on duty in a randomly selected day is

(a) $\frac{3}{17}$ (b) $\frac{14}{17}$ (c) $\frac{3}{10}$ (d) $\frac{7}{10}$

5. Value of $\int_0^{\pi} |\cos x|\, dx$ is

(a) 2 (b) −2 (c) 1 (d) None of these

6. The matrix X such that $X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$ is

(a) $\begin{bmatrix}1 & 2\\ 2 & 0\end{bmatrix}$ (b) $\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$ (c) $\begin{bmatrix}1 & 2\\ -2 & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 2\\ 0 & -2\end{bmatrix}$

7. If two vertices of a triangle are $i-j$ and $j+k$, then the third vertex can be

(a) $i+k$ (b) $i-2j-k$ and $-2i-j$ (c) $i-k$ (d) All the above

8. For $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, then $14\,\mathbf{A}^{-1}$ is given by :

(a) $14\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}$ (b) $\begin{bmatrix}4 & -2\\ 2 & 6\end{bmatrix}$ (c) $2\begin{bmatrix}2 & -1\\ 1 & -3\end{bmatrix}$ (d) $2\begin{bmatrix}-3 & -1\\ 1 & -2\end{bmatrix}$

9. The order of the differential equation satisfied by the curve $\sqrt{1+x}-a\sqrt{1+y}=1$, is

(a) 0 (b) 1 (c) 2 (d) 3

10. Corner points of feasible region of inequalities gives

(a) optional solution of L.P.P. (b) objective function
(c) constraints. (d) linear assumption

11. If c_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32}.c_{32}$

(a) 110 (b) 22 (c) – 110 (d) – 22

12. L.P.P. has constraints of

(a) one variables (b) two variables (c) one or two variables (d) two or more variables

13. If $|\vec{a}| = 5, |\vec{b}| = 4, |\vec{c}| = 3$, then the value of $|\overrightarrow{a}.\overrightarrow{b} + \overrightarrow{b}.\overrightarrow{c} + \overrightarrow{c}.\overrightarrow{a}|$, is equal to (given that $\vec{a} + \vec{b} + \vec{c} = 0$)

(a) 25 (b) 50 (c) –25 (d) –50

14. If $y = (\tan x)^{\sin x}$, then $\frac{dy}{dx}$ is equal to

(a) $\sec x + \cos x$ (b) $\sec x + \log \tan x$ (c) $(\tan x)^{\sin x}$ (d) None of these

15. If the angle θ between the lines $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and $\frac{x}{2} = \frac{y}{-1} = \frac{z}{\sqrt{\lambda}}$ is such that $\cos\theta = \frac{1}{3}$ then the value of λ is

(a) $\frac{5}{3}$ (b) $\frac{-3}{5}$ (c) $\frac{3}{4}$ (d) $\frac{-4}{3}$

16. For $y = \cos kx$ to be a solution of differential equation $\frac{d^2y}{dx^2} + 4y = 0$, the value of k is

(a) 2 (b) 4 (c) 6 (d) 8

17. $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx$ is equal to

(a) $10^x - x^{10} + C$ (b) $10^x + x^{10} + C$ (c) $(10^x - x^{10})^{-1} + C$ (d) $\log_e(\log^x + x^{10}) + C$

18. The two vectors $(x^2-1)\hat{i} + (x+2)\hat{j} + x^2\hat{k}$ and $2\hat{i} - x\hat{j} + 3\hat{k}$ are orthogonal

(a) for no real value of x (b) for $x = -1$ (c) for $x = \frac{1}{2}$ (d) for $x = -\frac{1}{2}$ and $x = 1$

(ASSERTION-REASON BASED QUESTIONS)

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) ***Both A and R are true and R is the correct explanation of A.***

(b) ***Both A and R are true but R is not the correct explanation of A.***

(c) ***A is true but R is false.***

(d) ***A is false but R is true.***

19. **Assertion:** Equation $\vec{r} = \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ represent a straight line.

Reason: Equation of the form

$$\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$$

represent a straight line passing through the point (α, β, γ) and having direction ratio proportional to l, m, n.

20. **Assertion:** The domain of the function $\sin^{-1}x.\cos^{-1}x.\tan^{-1}x$ is [–1, 1]

Reason: $\sin^{-1}x$, $\cos^{-1}x$ are defined for $|x| \le 1$ and $\tan^{-1}x$ is defined for all x.

SECTION-B

This section comprises of very short answer type-questions (VSA) of 2 marks each.

21. Find the value of $\cos\left[2\cos^{-1}x+\sin^{-1}x\right]$ at $x=\frac{1}{2}$.

OR

Find the value of $\cos\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)+\tan^{-1}\left(\sqrt{3}\right)\right]$.

22. If $y=\sqrt{\frac{(1+\cos 2\theta)}{(1-\cos 2\theta)}}$, then find $\frac{dy}{d\theta}$ at $\theta=\frac{3\pi}{4}$.

23. Find the intervals of functions $y=-5x^2+6x+7$ at which increasing and decreasing.

24. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}=0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a}=\pm 2(\vec{b}\times\vec{c})$.

OR

Let $\vec{a}$ and $\vec{b}$ be two given vectors such that $\left|\vec{a}\right|=2$, $\left|\vec{b}\right|=1$ and $\vec{a}.\vec{b}=1$. Find the angle between $\vec{a}$ and $\vec{b}$.

25. If $|\overline{a}|=3, |\overline{b}|=4$, then find the value of λ for which $\overline{a}+\lambda\overline{b}$ is perpendicular to $\overline{a}-\lambda\overline{b}$

SECTION-C

This section comprises of short answer type questions (SA) of 3 marks each.

26. Evaluate: $\int\left(\sqrt{\cot x}+\sqrt{\tan x}\right)dx$

OR

Evaluate: $\int\frac{1}{\cos^4 x+\sin^4 x}dx$

27. Evaluate : $\int\frac{x}{(x^2+1)(x-1)}dx$

28. Evaluate : $\int\frac{e^x(1+x)}{\cos^2\left(e^x x\right)}dx$

OR

Evaluate the value of integral, $\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$

29. $\frac{dy}{dx}+\frac{y}{x}=0$, where x denotes the percentage population living in a city & y denotes the area for living a healthy life of population. Find the particular solution when $x=100$, $y=1$.

30. Two balls are drawn at random from a bag containing 2 white, 3 red, 5 green, and 4 black balls one by one without replacement. Find the probability that both the balls are of different colours.

31. Solve the following linear programming problem graphically
Maximize $z=x+y$
Subjected to constraints

$\frac{x}{25}+\frac{y}{40}\le 1$

$2x+5y\le 100, x\ge 0, y,\ge 0$

OR

A dietician wishes to mix two types of foods in such a way that the vitamin contents of mixture contains atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C while food II contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C. It costs ₹ 5 per kg to purchase food I and ₹ 7 per kg to purchase food II. Find the minimum cost of such a mixture. Formulate above as LPP and solve graphically.

SECTION-D

This section comprises of long answer-type questions (LA) of 5 marks each.

32. Let the two matrices A and B be given by $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

Verify that AB = BA = 6I, where I is the unit matrix of order 3 and hence solve the system of equations. $x - y = 3$, $2x + 3y + 4z = 17$ and $y + 2z = 7$

OR

If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence, find A^{-1}.

33. Find the foot of the perpendicular drawn from the point whose position vector is $2\hat{i} - \hat{j} + 5\hat{k}$ to the line $\vec{r} = (11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + (-8 - 11\lambda)\hat{k}$. Also, find the length of the perpendicular.

34. Sketch the region lying in the first quadrant and bounded by $y = 9x^2$, $x = 0$, $y = 1$ and $y = 4$. Find the area of the region, using integration.

OR

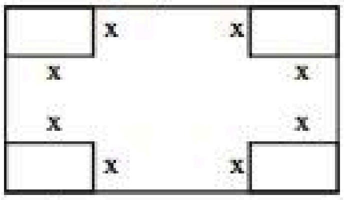
Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

35. Show that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b) : |a – b| is even}, is an equivalance relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of {2, 4}.

SECTION-E

This section comprises of 3 case study/passage - based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.

36. Case - Study 1: Read the following passage and answer the questions given below.
A student is given square card board of area 576 cm^2. He wishes to form a box without top by cutting a square from each corner and folding the flaps to form a box with maximum capacity and no wastage of the board.



If length of a side of the square cutout be x cm, then

(i) Find the length of a side of given square card board.

(ii) Find the value of x to maximize the volume.

(iii) Find the maximum capacity of box.

OR

Find the length of box

37. Case - Study 2: Read the following passage and answer the questions given below.
Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in the hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade. What is the probability that the student is a hostler? Grading system is better than numerical marking, explain.

(i) Find probability of all students reside in hostel.

(ii) Find probability of all students who reside in hostel attain A grade.

(iii) Find probability of all students who day scholars attain A grade.

OR

The probability of student is a hostler when selected student has an A grade.

38. **Case - Study 3:** Read the following passage and answer the questions given below.

A student of class 12 is given card board of area 27 square centimeters. He wishes to form a box with square base to have maximum capacity and no wastage of the board. If l be the length and h be the height of the square base of the box.

(i) Find the volume (V) in terms of length l.

(ii) Find the critical point of volume (V).

Solutions

SAMPLE PAPER-10

1. **(a)** It is obvious.

2. **(a)** Here, $f(x) = \begin{cases} ax+1, & \text{if } x \le 3 \\ bx+3, & \text{if } x > 3 \end{cases}$

$\text{LHL} = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (ax+1)$

Putting $x = 3 - h$ as $x \to 3^-, h \to 0$

$\therefore \lim_{h\to 0}[a(3-h)+1] = \lim_{h\to 0}(3a - ah + 1) = 3a + 1$

$\text{RHL} = \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (bx+3)$

Putting $x = 3 + h$ as $x \to 3^+, h \to 0$

$\therefore \lim_{h\to 0}[b(3+h)+3] = \lim_{h\to 0}(3b + bh + 3) = 3b + 3$

Also, $f(3) = 3a + 1$ $\quad [\because f(x) = ax + 1]$

Since, f(x) is continuous at x = 3.

$\therefore$ LHL = RHL = f(3)

$\Rightarrow 3a + 1 = 3b + 3 \Rightarrow 3a = 3b + 2 \Rightarrow a = b + \frac{2}{3}$

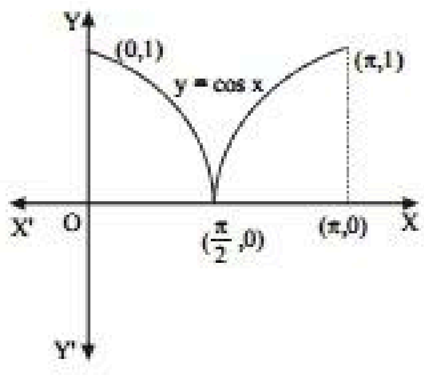
3. **(d)**

4. **(b)** The desired probability

$$= \frac{\frac{10}{100}\times\frac{70}{100}}{\frac{10}{100}\times\frac{70}{100}+\frac{5}{100}\times\frac{30}{100}} = \frac{14}{17}$$

5. **(a)** We have

$$|\cos x| = \begin{cases} \cos x & \text{when } 0 \le x \le \frac{\pi}{2} \\ -\cos x & \text{when } \frac{\pi}{2} \le x \le \pi \end{cases}$$



$$\therefore \int_0^{\pi} |\cos x|\,dx = \int_0^{\pi/2} |\cos x|\,dx + \int_{\pi/2}^{\pi} |\cos x|\,dx$$

$$= \int_0^{\pi/2} \cos x\,dx + \int_{\pi/2}^{\pi} (-\cos x)\,dx$$

$$= [\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi} = 1 + 1 = 2$$

6. **(b)** Here, $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Let $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$.

Therefore, we have

$\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

On equating the corresponding elements of the two matrices, we have

$a + 4c = -7, 2a + 5c = -8$ $3a + 6c = -9$

$b + 4d = 2, 2b + 5d = 4, 3b + 6d = 6$

Now, $a + 4c = -7 \Rightarrow a = -7 - 4c$

$2a + 5c = -8 \Rightarrow -14 - 8c + 5c = -8$

$\Rightarrow -3c = 6 \Rightarrow c = -2$

$\therefore a = -7 - 4(-2) = -7 + 8 = 1$

Now, $b + 4d = 2 \Rightarrow b = 2 - 4d$

$2b + 5d = 4$ and $4 - 8d + 5d = 4$

$\Rightarrow -3d = 0 \Rightarrow d = 0 \therefore b = 2 - 4(0) = 2$

Thus $a = 1, b = 2, c = -2, d = 0$

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

7. **(d)** Since, no vector given in options in collinear with the given vectors. Therefore all vectors can be third vertex of the triangle.

8. **(b)** $|A| = 7, adjA = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}, A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

$\therefore \quad 14A^{-1} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

9. **(b)** Differentiate the given equation has one arbitrary constant so, order is one.

10. **(a)**

11. **(a)** Let $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Here, $a_{32} = 5$ Then,

$c_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = (-1)^5(8 - 30) = -(-22) = 22$

$\therefore a_{32}.c_{32} = 5 \times 22 = 110$

12. **(d)**

13. **(a)** We have, $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow 25 + 16 + 9 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$

$\Rightarrow (\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -25 \; \therefore |\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}| = 25$

14. (d) We have, $y = (\tan x)^{\sin x}$
Taking logarithm on both sides
$\log y = \sin x \log (\tan x)$
Differentiating w.r.t. x

15. (a) Angle between lines is

$$\cos\theta = \frac{2-2+2\sqrt{\lambda}}{3\times\sqrt{5+\lambda}}$$

Where θ is angle between line and plane

$$\Rightarrow \cos\theta = \frac{2\sqrt{\lambda}}{3\sqrt{5+\lambda}} = \frac{1}{3} \Rightarrow \lambda = \frac{5}{3}.$$

16. (a) Given that $y = \cos kx$, therefore $\frac{dy}{dx} = -k \sin kx$ and $\frac{d^2y}{dx^2} = -k^2 \cos kx$ Putting this value of $\frac{d^2y}{dx^2}$ and $y = \cos kx$ in $\frac{d^2y}{dx^2} + 4y = 0$, we get

$-k^2 \cos kx + 4 \cos kx = 0$ or $k^2 = 4$ or $k = \pm 2$, or $k = 2$.

17. (d) Put $10^x + x^{10} = t$

$\therefore \quad \left(10^x \log_e 10 + 10x^9\right) dx = dt$

$\therefore \quad \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx = \int \frac{dt}{t}$

$= \log_e t + c = \log_e(10^x + x^{10}) + C$

18. (d) For orthogonality, the scalar product = 0

$\Rightarrow 2(x^2-1) + (-x)(x+2) + 3x^2 = 0$

$\Rightarrow 2(2x+1)(x-1) = 0 \Rightarrow x = -\frac{1}{2}, 1$

19. (a) Given equation in assertion represent a straight line and equation of the form

$\frac{x-\alpha}{\ell} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ represent a straight line passing through the point (α, β, γ) and having direction ratios proportional to ℓ, m, n. Thus, both statements are correct.

20. (a) **Assertion :** Domain of $\sin^{-1}x \to [-1, 1]$

$\cos^{-1}x \to [-1, 1]$

$\tan^{-1}x \to R$

Domain $= [-1, 1] \cap [-1, 1] \cap R = [-1, 1]$

21. $\cos\left[2\cos^{-1}\frac{1}{2} + \sin^{-1}\frac{1}{2}\right] = \cos\left[2\frac{\pi}{3} + \frac{\pi}{6}\right]$ [1 Mark]

$= \cos\left[\frac{5\pi}{6}\right] = \cos\left[\pi - \frac{\pi}{6}\right]$

$= -\cos\left[\frac{\pi}{6}\right] = -\frac{1}{2}$ [1 Mark]

OR

$\cos\left[\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sqrt{3}\right)\right]$

$= \cos\left(\frac{\pi}{6} + \frac{\pi}{3}\right) = \cos\frac{\pi}{2} = 0$ [2 Marks]

22. $y = \sqrt{\frac{2\cos^2\theta}{2\sin^2\theta}} = \sqrt{\cot^2\theta} = -\cot\theta$ [1 Mark]

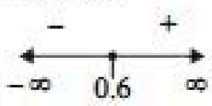
[$\because$ $\cot\theta$ is – ve in the neighbourhood of $\frac{3\pi}{4}$ i.e. in Q_2]

$\Rightarrow \left(\frac{dy}{d\theta}\right)_{\theta = 3\pi/4} (\text{cosec}^2\theta)_{3\pi/4} = 2.$ [1 Mark]

23. The equation of the given curve is
$y = -5x^2 + 6x + 7.$

$\Rightarrow \quad \frac{dy}{dx} = -10x + 6 = 0$ [1 Mark]

$\Rightarrow x = 0.6$



$\therefore$ Increasing at $[0.6, \infty)$

Decreasing at $(-\infty, 0.6]$ [1 Mark]

24. Given $|\vec{a}| = 1, |\vec{b}| = 1$ and $|\vec{c}| = 1$

$\vec{a}.\vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$ ...(i)

and $\vec{a}.\vec{c} = 0 \Rightarrow \vec{a} \perp \vec{c}$ ...(ii)

[½ Mark]

[From (i) and (ii)]

$\Rightarrow \quad \vec{a} \parallel \vec{b} \times \vec{c}$ [½ Mark]

$\Rightarrow \quad \vec{a} = \lambda\left(\vec{b} \times \vec{c}\right)$, λ is scalar ...(iii)

$\Rightarrow \quad |\vec{a}| = \lambda |\vec{b} \times \vec{c}|$

$\Rightarrow |\vec{a}| = \lambda |\vec{b}||\vec{c}| \sin\left(\pm\frac{\pi}{6}\right)$ [½ Mark]

$\Rightarrow \quad 1 = \pm\frac{\lambda}{2} \Rightarrow \lambda \pm 2$

Substituting in (iii), we get : $\vec{a} = \pm 2\left(\vec{b} \times \vec{c}\right)$ [½ Mark]

OR

Let θ be the angle between $\vec{a}$ and $\vec{b}$. Then,

$\vec{a} . \vec{b} = 1 \Rightarrow \left|\vec{a}\right|\left|\vec{b}\right| \cos\theta = 1$ [1 Mark]

$\Rightarrow \quad (2 \times 1)\cos\theta = 1$ $\left[\because \left|\vec{a}\right| = 2 \text{ and } \left|\vec{b}\right| = 1\right]$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$. [½ Mark]

Hence, the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$. [½ Mark]

25. If $\bar{a} + \lambda\bar{b}$ is perpendicular to $\bar{a} - \lambda\bar{b}$, then

$\Rightarrow (\bar{a} + \lambda\bar{b}).(\bar{a} - \lambda\bar{b}) = 0$ [1 Mark]

$\Rightarrow \bar{a}.\bar{a} - \lambda.\bar{a}.\bar{b} + \lambda.\bar{b}.\bar{a} - \lambda^2.\bar{b}.\bar{b} = 0$

$\Rightarrow \lambda = \frac{3}{4}$. [1 Mark]

26. $I = \int\left(\sqrt{\cot x} + \sqrt{\tan x}\right)dx$

$= \int\left[\sqrt{\tan x}(1 + \cot x)\right]dx$

Let $\tan x = t^2$

Differentiating both sides w.r.t x, we get

$\sec^2 x\ dx = 2tdt$

$\Rightarrow dx = \frac{2tdt}{1+t^4}$ [½ Mark]

$\therefore I = \int t\left(1+\frac{1}{t^2}\right) \times \frac{2t}{1+t^4}dt$ [½ Mark]

$= 2\int\frac{t^2+1}{t^4+1}dt = 2\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} = 2\int\frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+2}$ [½ Mark]

Let $t - \frac{1}{t} = y \Rightarrow \left(1+\frac{1}{t^2}\right)dt = dy$ [½ Mark]

$\therefore I = 2\int\frac{dy}{y^2+\left(\sqrt{2}\right)^2}$

$= 2\times\frac{1}{\sqrt{2}}\tan^{-1}\frac{y}{\sqrt{2}} + C = \sqrt{2}\tan^{-1}\frac{\left(t-\frac{1}{t}\right)}{\sqrt{2}} + C$ [½ Mark]

$= \sqrt{2}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right) + C = \sqrt{2}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + C$

[½ Mark]

OR

Let $I = \int\frac{1}{\cos^4 x + \sin^4 x}dx$

$= \int\frac{\sec^4 x.dx}{1+\tan^4 x}$

$= \int\frac{\sec^2 x \sec^2 x.dx}{1+\tan^4 x}$ [½ Mark]

Putting $\tan x = t \Rightarrow \sec^2 x\ dx = dt$

$= \int\frac{(1+\tan^2 x).dt}{1+t^4} = \int\frac{1+t^2}{1+t^4}dt$ [½ Mark]

$= \int\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt$

$= \int\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}+2-2}dt$ [½ Mark]

Putting $t - \frac{1}{t} = z \Rightarrow 1 + \frac{1}{t^2}dt = dz$ [½ Mark]

$= \int\frac{dz}{\left(t-\frac{1}{t}\right)^2+2} = \int\frac{dz}{z^2+2} = \int\frac{dz}{z^2+(\sqrt{2})^2}$

$= \frac{1}{2}\tan^{-1}\frac{z}{\sqrt{z}} + C$ [½ Mark]

$= \sqrt{2}\tan^{-1}\left(1+\frac{1}{t^2}\right)$

$= \sqrt{2}\tan^{-1}\left(\frac{1+t^2}{\sqrt{2}t^2}\right) + C$

$= \sqrt{2}\tan^{-1}\frac{(1+\tan^2 x)}{\sqrt{2}\tan^2 x} + C$ [½ Mark]

27. Let $\frac{x}{(x^2+1)(x-1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

$\Rightarrow x = A(x^2+1) + (Bx+C)(x-1)$ [1 Mark]

Let $x - 1 = 0 \Rightarrow x = 1 \Rightarrow A = \frac{1}{2}$

Now $x = 0 \Rightarrow C = \frac{1}{2}$

Comparing coefficient of x^2, we get $B = -\frac{1}{2}$ [1 Mark]

$\therefore \int\frac{x}{(x^2+1)(x-1)}dx$

$= \frac{1}{2}\int\frac{dx}{x-1} - \frac{1}{2}\int\frac{x}{x^2+1}dx + \frac{1}{2}\int\frac{dx}{x^2+1}$

$= \frac{1}{2}\log(x-1) - \frac{1}{4}\log(x^2+1) + \frac{1}{2}\tan^{-1}x + C.$

[1 Mark]

28. Let $xe^x = t \Rightarrow \left(xe^x + e^x\right) = \frac{dt}{dx}$ [1 Mark]

$\Rightarrow dx = \frac{dt}{e^x(x+1)}$

$\therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$

$= \int \frac{e^x(1+x)}{\cos^2 t} \times \frac{dt}{e^x(1+x)}$ [1 Mark]

$= \int \frac{1}{\cos^2 t}dt = \int \sec^2 t\,dt$

$= \tan t + C = \tan(xe^x) + C$ [1 Mark]

OR

$I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$

$\Rightarrow I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}dx \Rightarrow 2I = \int_3^6 dx = 3$ [2 Marks]

$\Rightarrow I = \frac{3}{2}$ [1 Mark]

29. Given $\frac{dy}{dx} + \frac{y}{x} = 0 \Rightarrow \frac{dy}{dx} + \left(\frac{1}{x}\right)y = 0$ ...(i)

Clearly, (i) is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$ where $P = \frac{1}{x}$ and $Q = 0$ [½ Mark]

Now, I.F. = $e^{\int Pdx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$ [½ Mark]

Multiply both sides of equation (i) by I.F. = x, we get :

$x\frac{dy}{dx} + y = 0$

Integrating both sides w.r.t. x, we get : yx = C ... (ii) [1 Mark]

When x = 100 and y = 1, then : 100 = C

Putting are value of C in equation (ii), we get : xy = 100 [1 Mark]

30. We have balls : 2W ; 3R ; 5G ; 4B

Total no of balls = 2 + 3 + 5 + 4 = 14

Two balls are to be drawn, one by one without replacement

	First ball	**Second ball**
Case I	White	not white
Case II	Red	not red
Case III	Green	not green
Case IV	Black	not black

$\therefore$ The reqd. prob. [1½ Marks]

$= \frac{2}{14}\times\frac{12}{13} + \frac{3}{14}\times\frac{11}{13} + \frac{5}{14}\times\frac{9}{13} + \frac{4}{14}\times\frac{10}{13}$

$= \frac{24+33+45+40}{102} = \frac{142}{182} = \frac{71}{91}$. [1½ Marks]

31. Now, consider the line $\frac{x}{25} + \frac{y}{40} = 1$

$\Rightarrow 8x + 5y = 200$ ...(i)

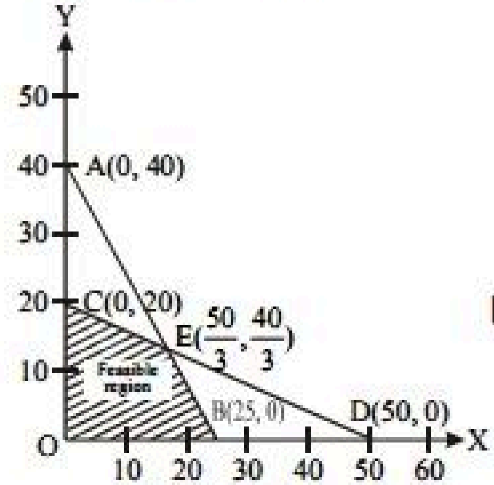
When x = 0, y = 40 and y = 0, x = 25

A (0, 40); B(25, 0)

Consider the line 2x + 5y = 100 ...(ii)

[½ Mark]

when x = 0, y = 20 and y = 0, x = 50



[1 Mark]

C(0, 20) ; D(50, 0) ; the lines (i) and (ii) intersect at $E\left(\frac{50}{3}, \frac{40}{3}\right)$.

Now, feasible region have corner points O(0, 0) B(25, 0), $E\left(\frac{50}{3}, \frac{40}{3}\right)$ and C (0, 20) [½ Mark]

Now, corner points of feasible region are examined for the maximum value of z.

At B(25, 0) ; z = 25 + 0 = 25

At $E\left(\frac{50}{3}, \frac{40}{3}\right)$; $z = \frac{50}{3} + \frac{40}{3} = 30$

AT C(0, 20) ; z = 0 + 20 = 20

Hence, z is maximum, when youngman travel $\frac{50}{3}$ km at a speed of 25 km/hour and $\frac{40}{3}$ km at a speed of 40 km/hour.

[1 Mark]

OR

The given data can be put in the tabular form as follows.

Food	Vitamin A	Vitamin C	Cost/Unit
I	2	1	₹ 5
II	1	2	₹ 7
Min requirement	8	10	

Suppose the diet contains x units of food I and y units of food II.

Then, the required LPP is

Minimize Z = 5x + 7y

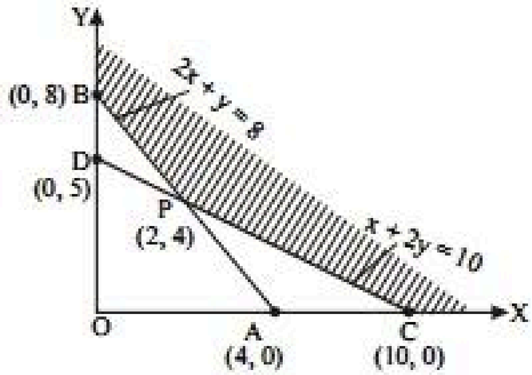
Subject to the constraints,

$2x + y \geq 8$

$x + 2y \geq 10$

$x \geq 0, y \geq 0$ [1 Mark]

Graph of above LPP is given as follows:



[1 Mark]

$\therefore$ We have, the table with corner point and value of Z.

Corner point	Value of objective function $Z = 5x + 7y$
C(10, 0)	5(10) + 7(0) = 50
P(2, 4)	5(2) + 7(4) = 10 + 28 = 38 (minimum)
B(0, 8)	5(0) + 7(8) = 0 + 56 = 56

[½ Mark]

Hence, the minimum cost is ₹ 38 when x = 2 and y = 4.

[½ Mark]

32. We have

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6I$$ [1 Mark]

Similarly, $BA = 6I$, Hence, $AB = 6I = BA$ [½ Mark]

As $AB = 6I$, $A^{-1}(AB) = 6A^{-1}I$. This gives

$$IB = 6A^{-1}, \text{ i.e., } A^{-1} = \frac{1}{6}B = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$ [1 Mark]

The given system of equations can be written as : $AX = C$, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \& C = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$ [1 Mark]

The solution of the given system AX = C is given by : X = $A^{-1}C$ [½ Mark]

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2, y = -1$ and $z = 4$. [1 Mark]

OR

$$\text{We have } A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$A^2 = AA = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$ [1½ Marks]

$$A^3 = A^2A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$ [1½ Marks]

Now, $A^3 - 6A^2 + 9A - 4I$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$+ 9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$ [1 Mark]

Hence, $A^3 - 6A^2 + 9A - 4I = 0$

$\Rightarrow$ $4I = A^3 - 6A^2 + 9A$

$$\Rightarrow A^{-1} = \frac{1}{4}A^2 - \frac{6}{4}A + \frac{9}{4}I$$ [½ Mark]

$$= \frac{1}{4}\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \frac{6}{4}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$+ \frac{9}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$ [½ Mark]

33. Let L be the foot of the perpendicular drawn from $P(2\hat{i} - \hat{j} + 5\hat{k})$ on the line.

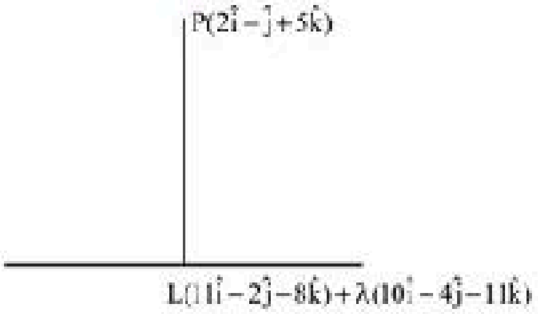
$\vec{r} = (11+10\lambda)\hat{i} + (-2-4\lambda)\hat{j} + (-8-11\lambda)\hat{k},$

$\Rightarrow \vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k}).$ [½ Mark]

Let the position vector of L be

$\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$

$= (11+10\lambda)\hat{i} + (-2-4\lambda)\hat{j} + (-8-11\lambda)\hat{k}.$ [½ Mark]



[½ Mark]

Then, $\overline{PL}$ = Position vector of L – Position vector of P

$\Rightarrow \overline{PL} = [(11+10\lambda)\hat{i} + (-2-4\lambda)\hat{j} + (-8-11\lambda)\hat{k}] - [2\hat{i} - \hat{j} + 5\hat{k}]$

$\Rightarrow \overline{PL} = (9+10\lambda)\hat{i} + (-1-4\lambda)\hat{j} + (-13-11\lambda)\hat{k}$ [1 Mark]

Since $\overline{PL}$ is perpendicular to the given line which is parallel to $\vec{b} = 10\hat{i} - 4\hat{j} - 11\hat{k}$

$\therefore \overline{PL} \perp \vec{b} \Rightarrow \overline{PL} \cdot \vec{b} = 0$ [½ Mark]

$\Rightarrow [(9+10\lambda)\hat{i} + (-1-4\lambda)\hat{j} + (-13-11\lambda)\hat{k}] \cdot (10\hat{i} - 4\hat{j} - 11\hat{k}) = 0$

$\Rightarrow 10(9+10\lambda) - 4(-1-4\lambda) - 11(-13-11\lambda) = 0$

$\Rightarrow 90 + 100\lambda + 4 + 16\lambda + 143 + 121\lambda = 0$

$\Rightarrow 237\lambda = -237 \Rightarrow \lambda = -1$ [1 Mark]

Putting the value of λ, we obtain the position vector of L as $\hat{i} + 2\hat{j} + 3\hat{k}$

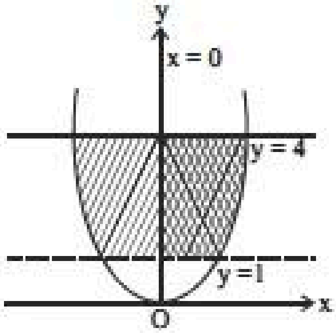
Now,

$\overline{PL} = (i + 2\hat{j} + 3\hat{k}) - (2\hat{i} - \hat{j} + 5\hat{k}) = -\hat{i} + 3\hat{j} - 2\hat{k}$

$\Rightarrow |\overline{PL}| = \sqrt{1+9+4} = \sqrt{14}.$ [1 Mark]

Hence, length of the perpendicular from P on the given line is 14 units.

34. The curve is $y = 9x^2$



[2 Marks]

$\Rightarrow \quad x^2 = \frac{1}{9}y. \Rightarrow x = \frac{1}{3}\sqrt{y}.$ [1 Mark]

The doubled shaded area

$= \int_1^4 x\,dy = \frac{1}{3}\int_1^4 \sqrt{y}\,dy$ [1 Mark]

$= \frac{1}{3} \times \frac{2}{3}\left[y^{3/2}\right]_1^4 = \frac{2}{9}\left[4^{3/2} - 1^{3/2}\right]$

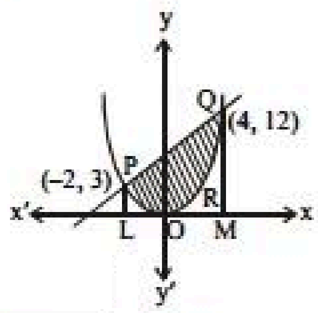
$= \frac{2}{9} \times 7 = \frac{14}{9}$ [1 Mark]

OR

The parabola and line are

$4y = 3x^2$ ...(i)

$2y = 3x + 12$ ...(ii)



[2 Marks]

Multiply in (ii) by 2 and subtracting from (i)

or $x^2 - 2x - 8 = 0$

or $(x-4)(x+2) = 0, x = 4, -2$

From (ii) y = 12, 3

The graph of parabola and lines are shown in the figure.

They intersect at P (–2, 3) and Q (4, 12) [1 Mark]

The area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$ = Area of the region PORQP

= Area of trapezium PLMQP – Area of the region LMQROP

$= \int_{-2}^{4} (y_1 - y_2)\,dx = \int_{-2}^{4}\left(\frac{3x+12}{2} - \frac{3x^2}{4}\right)dx$ [1 Mark]

[$\because y_1$ is for the line $y = \frac{3x+12}{2}$ and y_2 is for the parabola $y = \frac{3x^2}{4}$]

Area required $= \frac{1}{3}\int_{-2}^{4}(3x+12)\,dx - \frac{3}{4}\int_{-2}^{4} x^2\,dx$

$= \frac{1}{3}\left[\frac{3x^2}{2} + 12x\right]_{-2}^{4} - \frac{3}{4}\left[\frac{x^3}{3}\right]_{-2}^{4} = 27$ sq. units. [1 Mark]

35. A = {1, 2, 3, 4, 5} and R = {(a, b) : |a – b| is even}

R = {(1, 3), (1, 5), (3, 5), (2, 4)}

(a) (i) Let us take any element of a set A.

then |a – a| = 0 which is even.

⇒ R is reflexive. [1 Mark]

(ii) If |a – b| is even, then |b – a| is also even, where, R = {(a, b) : |a – b| is even} ⇒ R is symmetric. [1 Mark]

(iii) Further a – c = a – b + b – c

If |a – b| and |b – c| are even, then their sum

|a – b + b – c| is also even.

⇒ |a – c| is even, $\therefore$ R is transitive.

Hence R is an equivalence relation. [1 Mark]

(b) Elements of {1, 3, 5} are related to each other.

Since |1 – 3| = 2, |3 – 5| = 2, |1 – 5| = 4. All are even numbers.

= Elements of {1, 3, 5} are related to each other. Similarly elements of {2, 4} are related to each other. Since |2 – 4| = 2 an even number. No element of set {1, 3, 5} is related to any element of {2, 4}. [2 Marks]

36. (i) Area of square $= x^2$

$576 = x^2$

$x = \sqrt{576} = 24$ cm [1 Mark]

(ii) $l = 24 - 2x, b = 24 - 2x, \; h = x$

Volume, $V = (24 - 2x)^2 . x$

$$\frac{dV}{dx} = 2(24 - 2x)(-2)x + (24 - 2x)^2$$

$$= (24 - 2x)(24 - 6x) = 0$$

$$\therefore x = 4 \quad [\because x \neq 12]$$

$$\frac{d^2V}{dx^2} = -2(24 - 6x) + (-6)(24 - 2x)$$

$$\left(\frac{d^2V}{dx^2}\right)_{x=4} = 0 + (-6)(24 - 8) = -96 < 0$$

$\therefore$ Volume is maximum at x = 4 cm. [1 Mark]

(iii) Maximum capacity $= (24 - 8)^2 . 4$

$= 1024 \text{ cm}^3$ [2 Marks]

OR

Length of box = 24 – 2x = 24 – 8

= 16 cm [2 Marks]

37. (i) Probability of all students reside in hostel =

$\frac{60}{100} = \frac{3}{5}$. [1 Mark]

(ii) Probability of all students who reside in hostel

attain A grade $= \frac{30}{100} = \frac{3}{10}$ [1 Mark]

(iii) Probability of all student who day scholars

attain A grade $= \frac{20}{100} = \frac{1}{5}$. [2 Marks]

OR

$$P = \frac{\frac{3}{5} \times \frac{3}{10}}{\frac{3}{5} \times \frac{3}{10} + \frac{2}{5} \times \frac{1}{5}} = \frac{\frac{9}{50}}{\frac{9}{50} + \frac{4}{50}} = \frac{9}{13}.$$ [2 Marks]

38. (i) Surface area of the box

= Area of the given board

$2l^2 + 4lh = 27$ [1 Mark]

Volume of the box (V) $= l^2 h$

$\because 2l^2 + 4lh = 27$

$$\therefore h = \frac{27 - 2l^2}{4l}$$

$$\therefore V = l^2\left(\frac{27 - 2l^2}{4l}\right) = \frac{1}{4}(27l - 2l^3)$$ [1 Mark]

(ii) For critical point,

$$\because \frac{dV}{dt} = 0$$

$$\frac{1}{4}(27 - 6l^2) = 0 \Rightarrow l = \frac{3\sqrt{2}}{2}$$ [2 Marks]