HOTS (Higher Order Thinking Skills)

Que 1. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

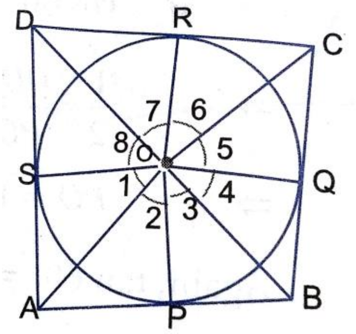

Fig. 8.51

Sol. Let a circle with centre O touches the sides AB, BC, CD and DA of a quadrilateral ABCD

at the points P, Q, R and S respectively. Then, we have to prove that

$\angle AOB + \angle COD = 180°$ and $\angle AOD + \angle BOC = 180°$

Now, Join OP, OQ, OR and OS.

Since the two tangents drawn from an external point to a circle subtend equal angles at

the centre.

$\therefore$ $\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6 \ and \ \angle 7 = \angle 8$ ….(i)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ ….(ii)

[sum of all the angles subtended at a point is 360 °]

$\Rightarrow$ $2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 360°$

$\Rightarrow$ $2(\angle 2 + \angle 3 + \angle 6 + \angle 7) = 180°$

$\angle AOB + \angle COD = 180°$

again $2(\angle 1 + \angle 8 + \angle 4 + \angle 5) = 360°$ [from (i) and (ii)]

$\therefore$ $(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$

$\Rightarrow$ $\angle AOD + \angle BOC = 180°$

Que 2. A triangle ABC [Fig.8.52] is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.

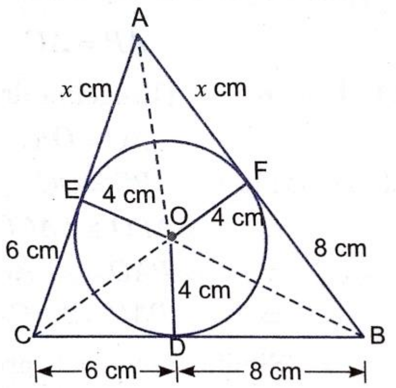

Fig. 8.52

Sol. Let ΔABC be drawn to circumscribe a circle with centre O and radius 4 cm and circle

touches the sides BC, CA and AB at D, E and F respectively.

We have given that CD = 6 cm and BD = 8 cm

∴ $BF = BD = 8\ cm$ and $CE = CD = 6\ cm$

{Length of two tangents drawn from an external point of circle are equal}

Now, let AF = AE = x cm

Then, AB = c = (x + 8) cm, BC = a = 14 cm, CA = b = (x + 6) cm

∴ 2s = (x + 8) + 14 + (x + 6)

⇒ 2s = 2x + 28 or s = x + 14

∴ s – a = (x + 14) – 14 = x

s – b = (x + 14) – (x + 6) = 8

s – c = (x + 14) – (x + 8) = 6

∴ $area\ of\ \Delta ABC = \sqrt{s(s-a)\ (s-b)\ (s-c)}$

$= \sqrt{(x+14)\ (x)\ (8)\ (6)} = \sqrt{48x(x+14)}$

Also, $area\ (\Delta ABC) = Area\ (\Delta OBC) + area(\Delta OCA) + area(\Delta OAB)$

$$= \frac{1}{2} \times BC \times OD + \frac{1}{2} \times CA \times OE + \frac{1}{2} \times AB \times OF$$

$$= \frac{1}{2} \times 14 \times 4 + \frac{1}{2} \times (x+6) \times 4 + \frac{1}{2} \times (x+8) \times 4$$

$$= 28 + 2x + 12 + 2x + 16 = 4x + 56$$

∴ $\sqrt{48x\ (x+14)} = 4x + 56 \quad \Rightarrow \quad \sqrt{48x\ (x+14)} = 4\ (x+14)$

Squaring both sides, we have

$48x\ (x+14) = 16\ (x+14)^2 \quad \Rightarrow \quad 48x\ (x+14) - 16\ (x+14)^2 = 0$

⇒ $16\ (x+14)\ [3x - (x+14)] = 0$

⇒ $16(x+14)\ (2x-14) = 0$

either 16 (x + 14) = 0 or 2x – 14 = 0

⇒ x = – 14 or 2x = 14

⇒ x = – 14 or x = 7

But x cannot be negative so x ≠ – 14

∴ x = 7 cm

Hence, the sides $AB = x + 8 = 7 + 8 = 15\ cm$

$AC = x + 6 = 7 + 6 = 13\ cm.$

Que 3. In Fig. 8.53, XY and X' Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X' Y' at B. Prove that $\angle AOB = 90°$

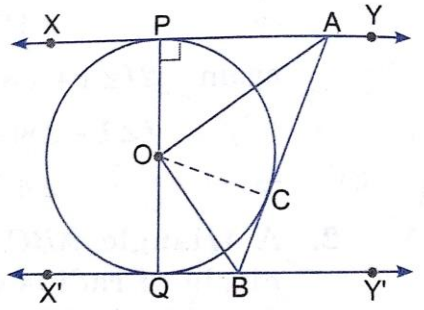

Fig. 8.53

Sol. Join OC. In $\Delta APO\ and\ \Delta ACO,$ we have

AP = AC

(Tangents drawn from external point A)

AO = OA (Common)

PO = OC (Radii of the same circle)

$\therefore$ $\Delta APO \cong \Delta ACO$ (By SSS criterion of congruence)

$\therefore$ $\angle PAO = \angle CAO$ (CPCT)

$\Rightarrow$ $\angle PAC = 2\,\angle CAO$

Similarly, we can prove that

$\Delta OQB \cong \Delta OCB$

$\therefore$ $\angle QBO = \angle CBO \quad \Rightarrow \quad \angle CBQ = 2\,\angle CBO$

Now, $\angle PAC + \angle CBQ = 180°$

[Sum of interior angle on the same side of transversal is 180°]

$\Rightarrow$ $2\,\angle CAO + 2\,\angle CBO = 180°$

$\Rightarrow$ $\angle CAO + \angle CBO = 90°$

$\Rightarrow$ $180° - \angle AOB = 90° \quad [\because\ \angle CAO + \angle CBO + \angle AOB = 180°]$

$\Rightarrow$ $180° - 90° = \angle AOB \quad \Rightarrow \quad \angle AOB = 90°$

Que 4. Let A be one point of intersection of two intersecting circles with centres O and Q. The tangents at A to the two circles meet the circles again at B and C respectively. Let the point P be located so that AOPQ is a parallelogram. Prove that P is the circumcentre of the triangle ABC.

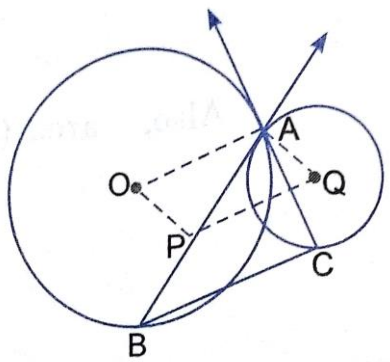

Fig. 8.54

Sol. In order to prove that P is the circumcentre of $\Delta ABC,$ it is sufficient to show that P is the point of intersection of perpendicular bisectors of the sides ofΔABC**,** i.e., OP and PQ are perpendicular bisectors of sides AB and AC respectively, Now, AC is tangent at A to the circle with centre at O and OA is its radius.

$\therefore$ OA ⊥ AC

⇒ PQ ⊥ AC $[\because\ OAQP\ is\ a\ parallel e\mathrm{log}ram\ \therefore\ OA||PQ]$

Also, Q is the centre of the circle

$\therefore$ QP bisects AC

[Perpendicular from the centre to the chord bisects the chord]

⇒ PQ is the perpendicular bisector of AC.

Similarly, BA is the tangent to the circle at A and AQ is its radius through A.

$\therefore$ BA ⊥ AQ

$\therefore$ BA ⊥ OP $\begin{bmatrix}\because\ AQPO\ is\ parall e\mathrm{log}ram \\ \therefore\ OP||AQ\end{bmatrix}$

Also, OP bisects AB $[\because\ O\ is\ the\ centre\ of\ the\ circle]$

⇒ OP is the perpendicular bisector of AB.

Thus, P is the point of intersection of perpendicular bisects PQ and PO of sides AC and AB respectively. Hence, P is the circumcentre of ΔABC.