Chapter 6 Polynomials and Polynomial Functions

Ex 6.1

Answer 1e.

We know that nth root of a is written as $\sqrt[n]{a}$, where a is the radicand and n is the index.

In the given expression, a is 10,000 and n is 4.

Thus, in the expression $\sqrt[4]{10,000}$, the number 4 is called the <u>index</u>.

Answer 1gp.

We know that the nth root of a can be written as $\sqrt[n]{a}$, where a is the radicand and n is the index.

We have n as 4 and a as 625.
The fourth root of 625 can be written as $\sqrt[4]{625}$.

If $a > 0$, and n is an even integer then there will be two real nth roots, which are $\pm\sqrt[n]{a}$, or $\pm a^{1/n}$.
Since $625 > 0$ and 4 is an even number, 625 has two real square roots $\pm\sqrt[4]{625}$, or $\pm 625^{1/4}$.

The number 625 can be written as either 5^4 or $(-5)^4$.
$\pm\sqrt[4]{625} = \pm 5$, or $\pm 625^{1/4} = \pm 5$

Thus. the two real roots of 625 are 5 and -5.

Answer 2e.

We know that for an integer $n > 0$, if $b^n = a$, then b is an n^{th} root of a.
It is written as $\sqrt[n]{a}$ where n is the index of the radical.
The number of real fourth root of a is $\sqrt[4]{a}$.

The sign of a determines the number of real fourth root of a is $\sqrt[4]{a}$
The number of real fifth root of a is $\sqrt[5]{a}$
The sign of a determine the number of real fifth root of a is $\boxed{\sqrt[5]{a}}$

Answer 2gp.

Let us consider that $n = 6$ and $a = 64$

We know that an n^{th} root of a is written as $\sqrt[n]{a}$

The sixth root of 64 is written as $\sqrt[6]{64}$

Because $n = 6$ is even and $64 > 0$, 64 has two real square roots.

Since $2^6 = 64$ and $(-2)^6 = 64$, so $\sqrt[6]{64} = \pm 2$

$\boxed{\therefore \sqrt[6]{64} = \pm 2}$

Answer 3e.

We know that the radical notation of the expression $a^{1/n}$ is equal to $\sqrt[n]{a}$, where a is the radicand and n is the index.

From the given expression, we get a as 2 and n as 3.

$2^{1/3} = \sqrt[3]{2}$

The answer matches with **choice C**.

Answer 3gp.

We know that the nth root of a can be written as $\sqrt[n]{a}$, where a is the radicand and n is the index.

We have n as 3 and a as -64.

The cube root of -64 can be written as $\sqrt[3]{-64}$.

If $a < 0$, and n is an odd integer then there will be one real nth root, which is $\sqrt[n]{a}$, or $a^{1/n}$.

Since $-64 < 0$ and 3 is a odd number, -64 has one real cube root $\sqrt[3]{-64}$, or $-64^{1/3}$.

The number -64 can be written as $(-4)^3$.

$\sqrt[3]{-64} = -4$, or $-64^{1/3} = -4$

Thus, the real root of -64 is -4.

Answer 4e.

Let us consider the rational exponent $2^{3/2}$

$2^{\frac{3}{2}} = 2^{\frac{1}{2}3}$ $\left[\text{Use the rule } a^{\frac{m}{n}} = a^{\frac{1}{n}\cdot m}\right]$

$= \left[2^{\frac{1}{2}}\right]^3$

$= \left(\sqrt{2}\right)^3$ $\left[\text{Use the rule } a^{m\cdot n} = \left(a^m\right)^n\right]$

$\therefore 2^{3/2} = \left(\sqrt{2}\right)^3$

Hence the given expression in rational exponent notation is $2^{3/2}$ with the equivalent expression in radical notation is $\left(\sqrt{2}\right)^3$

The Answer is $\boxed{A}$

Answer 4gp.

Let us consider that $n = 5$ and $a = 243$

We know that n$^{\text{th}}$ root of a is written as $\sqrt[n]{a}$

The fifth root of 243 is written as $\sqrt[5]{243}$

Because $n = 5$ is odd and $a = 243 > 0$, 243 has one real cube roots.

Since $3^5 = 243$, so $\sqrt[5]{243} = 3$

$\boxed{\therefore \sqrt[5]{243} = 3}$

Answer 5e.

If $a^{1/n}$ is the nth root of a and m is a positive integer, then $a^{m/n}$ can be written as $\left(\sqrt[n]{a}\right)^m$.

On comparing the given expression with $a^{m/n}$, we get a as 2, n as 3, and m as 2.
Write the expression in radical notation.

$\left(2^{1/3}\right)^2 = \left(\sqrt[3]{2}\right)^2$

The answer matches with **choice** D.

Answer 5gp.

If $a^{1/n}$ is the nth root of a and m is a positive integer, then $a^{m/n}$ can be written as $\left(a^{1/n}\right)^m$ or $\left(\sqrt[n]{a}\right)^m$.

From the given expression, we get a as 4, n as 2, and m as 5.

The rational exponent form of the given expression is $\left(4^{1/2}\right)^5$, and the radical form of the expression is $\left(\sqrt[2]{4}\right)^5$, or $\left(\sqrt{4}\right)^5$.

We have $4^{1/2} = \sqrt{4} = 2$.

$$\begin{array}{l|l} \left(4^{1/2}\right)^5 = 2^5 & \left(\sqrt{4}\right)^5 = 2^5 \\ \qquad\quad = 32 & \qquad\quad = 32 \end{array}$$

Thus, the expression evaluates to 32.

Answer 6e.

Let us consider the rational exponent $2^{3/2}$

$$\begin{aligned} 2^{1/2} &= 2^{\frac{1}{2}\cdot 1} && \left[\because\ a^{m/n} = a^{\frac{1}{n}\cdot m}\right] \\ &= \left[2^{1/2}\right]^1 && \left[\because\ a^{m\cdot n} = \left(a^m\right)^n\right] \\ &= \left(\sqrt{2}\right)^1 && \left[\because a^{1/n} = \sqrt[n]{a} \text{ for any integer } n > 1\right] \\ &= \sqrt{2} \end{aligned}$$

$\therefore 2^{1/2} = \sqrt{2}$

Hence the given expression in rational exponent notation is $2^{1/2}$ with the equivalent expression in radical notation is $\sqrt{2}$

The Answer is $\boxed{B}$

Answer 6gp.

Let us consider the expression $9^{-\frac{1}{2}}$

$$9^{-\frac{1}{2}} = \frac{1}{9^{\frac{1}{2}}} \qquad \left[\because a^{-n} = \frac{1}{a^n}\right]$$

$$\Rightarrow \quad 9^{\frac{-1}{2}} = \frac{1}{\sqrt{9}} \qquad \left[\because a^{1/n} = \sqrt[n]{a}\right]$$

$$\Rightarrow \quad 9^{\frac{-1}{2}} = \frac{1}{\sqrt{3\cdot 3}} \qquad \left[\begin{array}{l}\text{Write the radicand } 9 \text{ as} \\ \text{a prime factor } 3\cdot 3\end{array}\right]$$

$$\Rightarrow \quad 9^{\frac{-1}{2}} = \frac{1}{\sqrt{3^2}} \qquad \left[\because a^m \cdot a^n = a^{m+n}\right]$$

$$\Rightarrow \quad 9^{\frac{-1}{2}} = \pm\frac{1}{3} \qquad \left[\because \sqrt[n]{a^n} = \pm a \text{ if } n \text{ is even number}\right]$$

$$\therefore \boxed{9^{\frac{-1}{2}} = \pm\frac{1}{3}}$$

Answer 7e.

We know that the rational exponent notation of the radical expression $\sqrt[n]{a}$ is equal to $a^{1/n}$, where a is the radicand and n is the index.

From the given expression, we get a as 12, and n as 3.
Write the expression in rational exponent notation.
$\sqrt[3]{12} = 12^{1/3}$

Thus, the rational exponent notation of the given expression is $12^{1/3}$.

Answer 7gp.

If $a^{1/n}$ is the nth root of a and m is a positive integer, then $a^{m/n}$ can be written as $\left(a^{1/n}\right)^m$ or $\left(\sqrt[n]{a}\right)^m$.

From the given expression, we get a as 81, n as 4, and m as 3.
The rational exponent form of the given expression is $\left(81^{1/4}\right)^3$, and the radical form of the expression is $\left(\sqrt[4]{81}\right)^3$.

We have $81^{1/4} = \sqrt[4]{81} = 3$.

$\left(81^{1/4}\right)^3 = 3^3$ | $\left(\sqrt[4]{81}\right)^3 = 3^3$

$= 27$ | $= 27$

Thus, the expression evaluates to 27.

Answer 8e.

Let us consider the rational exponent $\sqrt[5]{8}$

We know that $\sqrt[n]{a} = a^{1/n}$ for any integer $n > 1$

$\therefore \quad \sqrt[5]{8} = (8)^{1/5}$

Hence the expression in radical notation $\sqrt[5]{8}$ with the equivalent expression in rational exponent notation is $\boxed{(8)^{1/5}}$

Answer 8gp.

Let us consider an expression $1^{7/8}$

$$1^{7/8} = 1^{\frac{1}{8}\cdot 7} \qquad \left[\text{Use the rule } a^{\frac{m}{n}} = a^{\frac{1}{n}\cdot m}\right]$$

$$\Rightarrow \quad 1^{7/8} = \left(1^{\frac{1}{8}}\right)^7 \qquad \left[\text{Use the rule } a^{m\cdot n} = \left(a^m\right)^n\right]$$

$$\Rightarrow \quad 1^{7/8} = \left(\sqrt[8]{1}\right)^7 \qquad \left[\text{Use the rule } a^{1/n} = \sqrt[n]{a}\right]$$

$$\Rightarrow \quad 1^{7/8} = (\pm 1)^7 \qquad \left[\text{Use the rule } \sqrt[n]{a^n} = \pm a \text{ if } n \text{ is even number}\right]$$

$$\Rightarrow \quad 1^{7/8} = \pm 1$$

$$\therefore \boxed{1^{7/8} = \pm 1}$$

Answer 9e.

The rational exponent notation of $\left(\sqrt[n]{a}\right)^m$ is $\left(a^{1/n}\right)^m$, where a is the radicand, n is the index, and m is a positive integer.

From the given expression, we get a as 10, n as 3, and m as 7. We can rewrite the given expression as $\left(10^{1/3}\right)^7$.

Apply the power of a power property of exponents.
$\left(10^{1/3}\right)^7 = 10^{7/3}$

Thus, the rational exponent notation of the given expression is $10^{7/3}$.

Answer 9gp.

First, enter the number 4 on the calculator. Then press the key $\boxed{\wedge}$.

4 $\boxed{\wedge}$

Next, press the key $\boxed{(}$, enter the number 2, press the key $\boxed{\div}$, enter 5, and then press key $\boxed{)}$.

4 $\boxed{\wedge}$ $\boxed{(}$ 2 $\boxed{\div}$ 5 $\boxed{)}$

Finally, press the $\boxed{\text{ENTER}}$ key to view the result.

4 $\boxed{\wedge}$ $\boxed{(}$ 2 $\boxed{\div}$ 5 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display will be 1.741101127.

Round the answer to two decimal places.
$1.741101127 \approx 1.74$

Thus, the given expression evaluates to about 1.74.

Answer 10e.

Let us consider the rational exponent $\left[\sqrt[8]{15}\right]^3$

$$\left[\sqrt[8]{15}\right]^3 = \left[15^{1/8}\right]^3 \qquad \left[\text{Use the rule } \left(\sqrt[n]{a}\right)^m = \left(a^{\frac{1}{n}}\right)^m\right]$$

$$= 15^{\frac{1}{8}\cdot 3} \qquad \left[\text{Use the rule } \left(a^m\right)^n = a^{m\cdot n}\right]$$

$$= 15^{3/8} \qquad [\text{Multiply}]$$

$$\therefore \left[\sqrt[8]{15}\right]^3 = 15^{3/8}$$

Hence the expression in rational notation $\left(\sqrt[8]{15}\right)^3$ with the equivalent expression in rational exponent is $\boxed{15^{3/8}}$

Answer 10gp.

Let us consider an expression $64^{-2/3}$

Approximate roots with a calculator

Expression	Key strokes	Display
$64^{-2/3}$	64 $\boxed{\wedge}$ $\boxed{(}$ $-$2 $\boxed{\div}$ 3 $\boxed{)}$ $\boxed{\text{ENTER}}$	0.0625

$\therefore \boxed{64^{-2/3} = 0.0625}$

Answer 11e.

We know that the radical notation of the expression $a^{1/n}$ is $\sqrt[n]{a}$, where a is the radicand and n is the index.

From the given expression, we get a as 5 and n as 4.

$5^{1/4} = \sqrt[4]{5}$

Thus, the radical notation of the given expression is $\sqrt[4]{5}$.

Answer 11gp.

We know that the exponent notation of the radical expression $\left(\sqrt[n]{a}\right)^m$ is $a^{m/n}$, where a is the radicand, m is a positive integer, and n is the index.

From the given expression, we get a as 16, m as 5, and n as 4.

$\left(\sqrt[4]{16}\right)^5 = 16^{5/4}$

First, enter the number 16 on the calculator. Then press the key $\boxed{\wedge}$.

16 $\boxed{\wedge}$

Next, press the key $\boxed{(}$, enter the number 5, press the key $\boxed{\div}$, enter 4, and then press key $\boxed{)}$.

16 $\boxed{\wedge}$ $\boxed{(}$ 5 $\boxed{\div}$ 4 $\boxed{)}$

Finally, press the $\boxed{\text{ENTER}}$ key to view the result.

16 $\boxed{\wedge}$ $\boxed{(}$ 5 $\boxed{\div}$ 4 $\boxed{)}$ $\boxed{\text{ENTER}}$

Thus, the expression evaluates to 32.

Answer 12e.

Let us consider the rational exponent $7^{1/3}$

We know that $a^{1/n} = \sqrt[n]{a}$ for any integer $n > 1$

$\therefore \quad 7^{\frac{1}{3}} = \sqrt[3]{7}$

Hence the expression in rational exponent notation with the equivalent expression in radical notation is $\boxed{\sqrt[3]{7}}$

Answer 12gp.

Let us consider an expression $\left(\sqrt[3]{-3}\right)^2$

$\left[\sqrt[3]{-30}\right]^2 = (-30)^{2/3} \qquad \left[\because \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}\right]$

Approximate roots with a calculator

Expression	Key strokes	Display
$(-30)^{2/3}$	-30 $\boxed{\wedge}$ $\boxed{(}$ 2 $\boxed{\div}$ 3 $\boxed{)}$ $\boxed{\text{ENTER}}$	9.655

$\therefore \boxed{(-30)^{2/3} = 9.655}$

Answer 13e.

If $a^{1/n}$ is the nth root of a and m is a positive integer, then the radical notation $a^{m/n}$ is equivalent to $\left(\sqrt[n]{a}\right)^m$.

From the given expression, we get a as 14, n as 3, and m as 2.

$14^{2/5} = \left(\sqrt[5]{14}\right)^2$

Thus, the radical notation of the given expression is $\left(\sqrt[5]{14}\right)^2$.

Answer 13gp.

Take the cube root of each side.

$\sqrt[3]{x^3} = \sqrt[3]{64}$

$x = 4$

Therefore, the solution is 4.

Answer 14e.

Let us consider the rational exponent $21^{9/4}$

$21^{9/4} = 21^{\frac{1}{4}\cdot 9}$ $\left[\text{Use the rule } a^{m/n} = a^{\frac{1}{n}\cdot m}\right]$

$= \left(21^{\frac{1}{4}}\right)^9$ $\left[\text{Use the rule } a^{m\cdot n} = \left(a^m\right)^n\right]$

$= \left[\sqrt[4]{21}\right]^9$ $\left[\text{Use the rule } \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m\right]$

$\therefore \quad 21^{9/4} = \left(\sqrt[4]{21}\right)^9$

Hence the expression $\boxed{\left(\sqrt[4]{21}\right)^9}$

Answer 14gp.

Let us consider an expression $\frac{1}{2}x^5 = 512$

$\therefore \quad \frac{1}{2}x^5 = 512$ [Write the original equation]

$\Rightarrow \quad x^5 = 512 \times 2$ [Multiply "2" to each side]

$\Rightarrow \quad x^5 = 1024$

$\Rightarrow \quad x = \sqrt[5]{1024}$ [Take fifth root to each side]

$\Rightarrow \quad x = \sqrt[5]{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}$ [Write the radicand 1024 as a product of a factor $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$]

$\Rightarrow \quad x = \sqrt[5]{4^5}$ $\left[\text{Use the rule } a^m \cdot a^n = a^{m+n}\right]$

$\Rightarrow \quad x = 4$ $\left[\text{Use the rule } \sqrt[n]{a^n} = a \text{ if } n \text{ is odd}\right]$

$\boxed{x = 4}$ is the solution of the equation $\frac{1}{2}x^5 = 512$

Answer 15e.

We know that the nth root of a can be written as $\sqrt[n]{a}$, where a is the radicand and n is the index.

We have n as 2 and a as 64.

The square root of 64 can be written as $\sqrt[2]{64}$ or $\sqrt{64}$.

If $a > 0$, and n is an even integer then there will be two real nth roots, which are $\pm\sqrt[n]{a}$, or $\pm a^{1/n}$.

Since $64 > 0$ and 2 is an even number, 64 has two real square roots $\pm\sqrt{64}$, or $\pm 64^{1/2}$.

The number 64 can be written as either 8^2 or $(-8)^2$.
$\pm\sqrt{64} = \pm 8$, or $\pm 64^{1/2} = \pm 8$

Thus, the two real roots of 64 are 8 and -8.

Answer 15gp.

Divide each side by 3.

$$\frac{3x^2}{3} = \frac{108}{3}$$
$$x^2 = 36$$

Take the square root of each side.

$$\sqrt{x^2} = \sqrt{36}$$
$$x = \pm 6$$

Therefore, the solution is ± 6.

Answer 16e.

Let use consider that $n = 3$ and $a = -27$

We know that n^{th} root of a is written as $\sqrt[n]{a}$.

The cube root of "-27" is written as $\sqrt[3]{-27}$.

Since $n = 3$ is odd, $a = -27 < 0, -27$ has one real cube root and $(-3)^3 = -27$

$\therefore \quad \boxed{\sqrt[3]{-27} = -3}$

Answer 16gp.

Let us consider the expression $\frac{1}{4}x^3 = 2$

$\therefore$	$\frac{1}{4}x^3 = 2$	[Write the original equation]
$\Rightarrow$	$x^3 = 4 \times 2$	[Multiply "4" to each side]
$\Rightarrow$	$x^3 = 8$	
$\Rightarrow$	$x = \sqrt[3]{8}$	[Take cube root to each side]
$\Rightarrow$	$x = \sqrt[3]{2 \cdot 2 \cdot 2}$	[Write the radicand "8" as a product of a prime factor]
$\Rightarrow$	$x = \sqrt[3]{2^3}$	[Use the rule $a^m \cdot a^n = a^{m+n}$]
$\Rightarrow$	$x = 2$	[Use the rule $\sqrt[n]{a^n} = a$ if n is odd]

$\boxed{x = 2}$ is the solution of the equation $\frac{1}{4}x^3 = 2$

Answer 17e.

We know that the nth root of a can be written as $\sqrt[n]{a}$, where a is the radicand and n is the index.

If a is 0 and n any nonzero real number, then the nth root of a is 0 itself.
Thus, the fourth root of 0 will be 0 itself.

Answer 17gp.

Take the cube root of each side of the equation.

$$\sqrt[3]{(x-2)^3} = \sqrt[3]{-14}$$
$$x - 2 = \sqrt[3]{-14}$$

Add 2 to each side and evaluate x.

$$x - 2 + 2 = \sqrt[3]{-14} + 2$$
$$x \approx -2.41 + 2$$
$$\approx -0.41$$

The solution for the given equation is approximately -0.41.

Answer 18e.

Let use consider that $n=3$ and $a=343$

The n^{th} root of a is written as $\sqrt[n]{a}$.

The cube root of 343 is written as $\sqrt[3]{343}$.

Since $n=3$ is odd and $343=7^3$ so it has one real cube root 7.

$\therefore \quad \boxed{\sqrt[3]{343}=7}$

Answer 18gp.

Let us consider the expression $(x+5)^4=16$

$\therefore$	$(x+5)^4=16$		[Write the original equation]
$\Rightarrow$	$x+5=\sqrt[4]{16}$		[Take fourth root to each side]
$\Rightarrow$	$x+5=\sqrt[4]{2\cdot 2\cdot 2\cdot 2}$		[Write the radicand 16 as a product of a prime factor]
$\Rightarrow$	$x+5=\sqrt[4]{2^4}$		[Use the rule $a^m\cdot a^n=a^{m+n}$]
$\Rightarrow$	$x+5=\pm 2$		[Use the rule $\sqrt[n]{a^n}=\pm a$ if n is even]
$\Rightarrow$	$x+5=+2$	or	$x+5=-2$
$\Rightarrow$	$x=+2-5$	or	$x=-2-5$
$\Rightarrow$	$x=-3$	or	$x=-7$
$\therefore$	$\boxed{x=-3,-7}$		

$x=-3,-7$ are the solution of the given equation $(x+5)^4=16$

Answer 19e.

We know that the nth root of a can be written as $\sqrt[n]{a}$, where a is the radicand and n is the index.

We have n as 4 and a as -16.

The fourth root of -16 can be written as $\sqrt[4]{-16}$.

If $a<0$ and n is an even integer, then there will be no real nth roots.

Since $-16<0$ and 4 is an even number. -16 has no real roots.

Answer 19gp.

The weight w of a coral cod is given by the formula, $w = 0.0167l^3$, where l is the length of the coral cod.

(a) Substitute 275 for w in the formula.

$275 = 0.0167l^3$ or $0.0167l^3 = 275$

Divide each side by 0.0167, and simplify.

$$\frac{0.0167l^3}{0.0167} = \frac{275}{0.0167}$$

$$l^3 \approx 16{,}467.06$$

Take the cube root of each side.

$$\sqrt[3]{l^3} \approx \sqrt[3]{16{,}467.06}$$

Use a calculator to find the cube root to find l.

$$l \approx 25.44$$

The length of the coral cod will be approximately 25.44 centimeters for the given weight.

(b) In order to find the length of the coral cod, substitute 340 for w in the formula.

$340 = 0.0167l^3$ or $0.0167l^3 = 340$

Divide each side by 0.0167, and simplify.

$$\frac{0.0167l^3}{0.0167} = \frac{340}{0.0167}$$

$$l^3 \approx 20359.28$$

Take the cube root of each side.

$$\sqrt[3]{l^3} \approx \sqrt[3]{20359.28}$$

$$l \approx 27.30$$

Use a calculator to find the cube root to find l.

$$l \approx 27.30$$

The length of the coral cod will be approximately 27.30 centimeters for the given weight.

(c) In order to find the length of the coral cod, substitute 450 for w in the formula.

$450 = 0.0167l^3$ or $0.0167l^3 = 450$

Divide each side by 0.0167, and simplify.

$$\frac{0.0167l^3}{0.0167} = \frac{450}{0.0167}$$

$$l^3 \approx 26946.11$$

Take the cube root of each side and find l using a calculator.

$$\sqrt[3]{l^3} \approx \sqrt[3]{26946.11}$$

$$l \approx 29.98$$

The length of the coral cod will be approximately 29.98 centimeters for the given weight.

Answer 20e.

Let use consider the $n=5$ and $a=-32$

The n^{th} root of a is written as $\sqrt[n]{a}$.

$\therefore$ The fifth root of -32 is written as $\sqrt[5]{-32}$.

Since $n=5$ is odd and $-32<0$ and $(-2)^5=-32$, so it has one real fifth root

$\therefore$ $\boxed{\sqrt[5]{-32}=-2}$

Answer 21e.

We know that the exponent notation of the radical expression $\sqrt[n]{a}$ is equal to $a^{1/n}$, where a is the radicand and n is the index.

In the given expression a is 64 and n is 6. Write the expression in rational exponent notation.

$$\sqrt[6]{64} = 64^{1/6}$$

Write 64 as 2^6 and apply the power of a power property of exponents.

$$64^{1/6} = \left(2^6\right)^{1/6}$$

$$= 2$$

Thus, the given expression evaluates to 2.

Answer 22e.

Let us consider the rational exponent $8^{1/3}$

$8^{1/3} = \sqrt[3]{8}$ $\quad \left[\begin{array}{l}\text{Use the rule } a^{1/n} = \sqrt[n]{a} \text{ for} \\ \text{any integer } n \text{ greater than}\end{array}\right]$

$8^{1/3} = \sqrt[3]{2 \cdot 2 \cdot 2}$ $\quad \left[\begin{array}{l}\text{Write the radicand 8} \\ \text{as a prime product of prime} \\ \text{factor}\end{array}\right]$

$8^{1/3} = \sqrt[3]{2^3}$ $\quad \left[\text{Use the rule } a^m \cdot a^n = a^{m+n}\right]$

$8^{1/3} = 2$ $\quad \left[\because \sqrt[n]{a^n} = a \text{ if } n \text{ is odd integer}\right]$

$\therefore \quad \boxed{8^{1/3} = 2}$

Answer 23e.

If $a^{1/n}$ is the nth root of a and m is a positive integer, then $a^{m/n}$ can be written as $\left(a^{1/n}\right)^m$, or $\left(\sqrt[n]{a}\right)^m$.

From the given expression, we get a as 16, m as 3, and n as 2.

The rational exponent form of the given expression is $\left(16^{1/2}\right)^3$, and the radical form of the expression is $\left(\sqrt[2]{16}\right)^3$, or $\left(\sqrt{16}\right)^3$.

We have $16^{1/2} = \sqrt{16} = 4$.

$\left(16^{1/2}\right)^3 = 4^3$	$\left(\sqrt{16}\right)^3 = 4^3$
$= 64$	$= 64$

Thus, the expression evaluates to 64.

Answer 24e.

Let use consider the rational exponent $\sqrt[3]{-125}$

$\sqrt[3]{-125} = \sqrt[3]{-5 \cdot -5 \cdot -5}$ $\quad \left[\begin{array}{l}\text{Write the radicand } -125 \\ \text{as a prime product of prime} \\ \text{factor}\end{array}\right]$

$\sqrt[3]{-125} = \sqrt[3]{(-5)^3}$ $\quad \left[\text{Use the rule } a^m \cdot a^n = a^{m+n}\right]$

$\sqrt[3]{-125} = -5$ $\quad \left[\begin{array}{l}\text{Use the rule} \\ \sqrt[n]{a^n} = a \text{ if } n \text{ is odd integer}\end{array}\right]$

$\therefore \boxed{\sqrt[3]{-125} = -5}$

Answer 25e.

If $a^{1/n}$ is the nth root of a and m is a positive integer, then $a^{m/n}$ can be written as $\left(a^{1/n}\right)^m$ or $\left(\sqrt[n]{a}\right)^m$.

From the given expression, we get a as 27, m as 2 and n as 3.

The rational exponent form of the given expression is $\left(27^{1/3}\right)^2$, and the radical form of the expression is $\left(\sqrt[3]{27}\right)^2$.

We have $27^{1/3} = \sqrt[3]{27} = 3$.

$$\left(27^{1/3}\right)^2 = 3^2 = 9 \qquad \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$$

Thus, the expression evaluates to 9.

Answer 26e.

Let use consider the rational exponent $(-243)^{1/5}$

$$
\begin{aligned}
(-243)^{1/5} &= \sqrt[5]{-243} && \left[\begin{array}{l}\text{Use the rule } a^{1/n} = \sqrt[n]{a} \\ \text{for any integer } n \text{ greater than } 1\end{array}\right] \\
&= \sqrt[5]{-3\cdot -3\cdot -3\cdot -3\cdot -3} && \left[\begin{array}{l}\text{Write the radicand } -243 \text{ as a} \\ \text{product of prime factors}\end{array}\right] \\
&= \sqrt[5]{(-3)^5} && \left[\text{Use the rule } a^m \cdot a^n = a^{m+n}\right] \\
&= -3 && \left[\begin{array}{l}\text{Use the rule} \\ \sqrt[n]{a^n} = a \text{ if } n \text{ is odd integer}\end{array}\right]
\end{aligned}
$$

$\therefore \boxed{(-243)^{1/5} = -3}$

Answer 27e.

We know that the exponent notation of the radical expression $\left(\sqrt[n]{a}\right)^{-m}$ is equivalent to $\frac{1}{a^{m/n}}$ or $\frac{1}{\left(a^{1/n}\right)^m}$, where a is the radicand, m is a positive integer, and n is the index.

From the given expression, we get a as 8, m as 3, and n as 2. Write the expression in rational exponent notation.

$$
\begin{aligned}
\left(\sqrt[3]{8}\right)^{-2} &= \frac{1}{8^{2/3}} \\
&= \frac{1}{\left(8^{1/3}\right)^2}
\end{aligned}
$$

We have $8^{1/3} = 2$.

$$\frac{1}{\left(8^{1/3}\right)^2} = \frac{1}{2^2}$$

$$= \frac{1}{4}$$

Thus, the given expression evaluates to $\frac{1}{4}$.

Answer 28e.

Let us consider the rational exponent $\left(\sqrt[3]{-64}\right)^4$

$$\left(\sqrt[3]{-64}\right)^4 = \left(\sqrt[3]{-4\cdot -4\cdot -4}\right)^4 \quad \left[\begin{array}{l}\text{Write the radicand } -64 \text{ as a}\\ \text{product of prime factors}\end{array}\right]$$

$$= \left(\sqrt[3]{(-4)^3}\right)^4 \quad \left[\text{Use the rule } a^m \cdot a^n = a^{m+n}\right]$$

$$= (-4)^4 \quad \left[\begin{array}{l}\text{Use the rule}\\ \sqrt[n]{a^n} = a \text{ if } n \text{ is odd integer}\end{array}\right]$$

$$= -4\cdot -4\cdot -4\cdot -4$$
$$= 256$$

$$\therefore \boxed{\left(\sqrt[3]{-64}\right)^4 = 256}$$

Answer 29e.

We know that the exponent notation of the radical expression $\left(\sqrt[n]{a}\right)^{-m}$ is equivalent to $\frac{1}{a^{m/n}}$ or $\frac{1}{\left(a^{1/n}\right)^m}$, where a is the radicand, m is a positive integer, and n is the index.

From the given expression, we get a as 16, m as 7, and n as 4.

$$\left(\sqrt[4]{16}\right)^{-7} = \frac{1}{16^{7/4}}$$

$$= \frac{1}{\left(16^{1/4}\right)^7}$$

We have $16^{1/4} = 2$.

$$\frac{1}{\left(16^{1/4}\right)^7} = \frac{1}{2^7}$$

$$= \frac{1}{128}$$

Thus, the given expression evaluates to $\frac{1}{128}$.

Answer 30e.

Let us consider the rational exponent $25^{3/2}$

$$25^{3/2} = 25^{\frac{1}{2}\cdot 3} \qquad \left[\text{Use the rule } a^{m/n} = a^{m\cdot\frac{1}{n}}\right]$$

$$= \left[25^{1/2}\right]^3 \qquad \left[\text{Use the rule } a^{m/n} = a^{\frac{1}{n}\cdot m}\right]$$

$$= \left[\sqrt{25}\right]^3 \qquad \left[\begin{array}{l}\text{Use the rule} \\ \left(a^{1/n}\right)^m = \left(\sqrt[n]{a}\right)^m \text{ for any integer greater than 1}\end{array}\right]$$

$$= \left(\sqrt{5\cdot 5}\right)^3 \qquad \left[\begin{array}{l}\text{Write the radicand 25 as a} \\ \text{product of prime factors}\end{array}\right]$$

$$= \left(\sqrt{5^2}\right)^3 \qquad \left[\text{Use the rule } a^m \cdot a^n = a^{m+n}\right]$$

$$= (\pm 5)^3 \qquad \left[\begin{array}{l}\text{Use the rule } \sqrt[n]{a^n} = \pm a \text{ if } a \text{ is} \\ \text{an even integer}\end{array}\right]$$

$$= \pm 125$$

$$\therefore \boxed{25^{3/2} = \pm 125}$$

Answer 31e.

If $a^{1/n}$ is the nth root of a and m is a positive integer, then $a^{m/n}$ can be written as $\dfrac{1}{\left(a^{1/n}\right)^m}$ or $\dfrac{1}{\left(\sqrt[n]{a}\right)^m}$.

From the given expression, we get a as 64, m as -2, and n as 3.

The rational exponent form of the given expression is $\dfrac{1}{\left(64^{1/3}\right)^2}$, and the radical form of the expression is $\dfrac{1}{\left(\sqrt[3]{64}\right)^2}$.

We have $64^{1/3} = \sqrt[3]{64} = 4$.

$$\frac{1}{\left(64^{1/3}\right)^2} = \frac{1}{4^2} = \frac{1}{16} \qquad \Bigg| \qquad \frac{1}{\left(\sqrt[3]{64}\right)^2} = \frac{1}{4^2} = \frac{1}{16}$$

Thus, the expression evaluates to $\dfrac{1}{16}$.

Answer 32e.

Let use consider the rational exponent $\frac{1}{81^{-3/4}}$

$$\frac{1}{81^{-3/4}} = 81^{3/4} \quad \left[\text{Use the rule } \frac{1}{a^{-m/n}} = a^{m/n}\right]$$

$$= 81^{\frac{1}{4}\cdot 3} \quad \left[\text{Use the rule } a^{m/n} = a^{\frac{1}{n}\cdot m}\right]$$

$$= \left(81^{1/4}\right)^3 \quad \left[\text{Use the rule } \left(a^{m/n}\right) = \left(a^{1/n}\right)^m\right]$$

$$= \left(4\sqrt{81}\right)^3 \quad \left[\text{Use the rule } \left(a^{1/n}\right)^m = \sqrt[n]{a}\right]$$

$$= \left(\sqrt[4]{3\cdot 3\cdot 3\cdot 3}\right)^3 \quad \left[\begin{array}{l}\text{Write the radicand 81 as a}\\ \text{product of prime factors}\end{array}\right]$$

$$= \left(\sqrt[4]{3^4}\right)^3 \quad \left[\text{Use the rule } a^m \cdot a^n = a^{m+n}\right]$$

$$= (\pm 3)^3 \quad \left[\begin{array}{l}\text{Use the rule } \sqrt[n]{a^n} = \pm a \text{ for}\\ n \text{ is an even integers}\end{array}\right]$$

$$= \pm 27$$

$$\therefore \boxed{\frac{1}{81^{-3/4}} = \pm 27}$$

Answer 33e.

If $a^{1/n}$ is the nth root of a and m is a positive integer, then $a^{m/n}$ can be written as or $\left(a^{1/n}\right)^m$ or $\left(\sqrt[n]{a}\right)^m$.

From the given expression, we get a as 128, m as 5, and n as 7.

The rational exponent form of the given expression is $\left(128^{1/7}\right)^5$, and the radical form of the expression is $\left(\sqrt[7]{128}\right)^5$.

We have $128^{1/7} = \sqrt[7]{128} = 2$.

$$\left(128^{1/7}\right)^5 = 2^5 \qquad \left(\sqrt[7]{128}\right)^5 = 2^5$$
$$= 32 \qquad\qquad = 32$$

Thus, the expression evaluates to 32, which matches with choice **C**.

Answer 34e.

Let use consider the rational exponent $\sqrt[5]{32,768}$

$\sqrt[5]{32,768} = (32,768)^{1/5}$ [Use the rule $\sqrt[n]{a} = a^{1/n}$, for any integer $n > 1$]

Approximate roots with a calculator expression

Expression	Keystrokes	Display
$(32,768)^{1/5}$	32, 768 ^ (1 ÷ 5) ENTER	8

$\boxed{\sqrt[5]{32,768} = 8}$

Answer 35e.

We know that the exponent notation of the radical expression $\sqrt[n]{a}$ is equal to $a^{1/n}$, where a is the radicand and n is the index.

From the given expression, we get a as 1695, and n as 7.

$\sqrt[7]{1695} = 1695^{1/7}$

First, enter the number 1695 on the calculator. Then press the key ^.

1695 ^

Next, press the key (, enter the number 1, press the key ÷, enter 7, and then press key).

1695 ^ (1 ÷ 7)

Finally, press the ENTER key to view the answer.

1695 ^ (1 ÷ 7) ENTER

The display will be 2.892743707.

Round your answer to two decimal places.

$2.892743707 \approx 2.89$

Thus, the expression evaluates to about 2.89.

Answer 36e.

Let use consider the rational exponent $\sqrt[9]{-230}$

$\sqrt[9]{-230} = (-230)^{1/9}$ $\left[\text{Use the rule } \sqrt[n]{a} = a^{1/n}, \text{ for any integer } n > 1\right]$

Expression	Keystrokes	Display
$(-230)^{1/9}$	-230 $\boxed{\wedge}$ $\boxed{(}$ 1 $\boxed{\div}$ 9 $\boxed{)}$ $\boxed{\text{ENTER}}$	-1.83

$\boxed{\sqrt[9]{-230} = -1.83}$

Answer 37e.

First, enter the number 85 on the calculator. Then press the key $\boxed{\wedge}$.

85 $\boxed{\wedge}$

Next, press the key $\boxed{(}$, enter the number 1, press the key $\boxed{\div}$, enter 6, and then press key $\boxed{)}$.

85 $\boxed{\wedge}$ $\boxed{(}$ 1 $\boxed{\div}$ 6 $\boxed{)}$

Finally, press the $\boxed{\text{ENTER}}$ key to view the answer.

85 $\boxed{\wedge}$ $\boxed{(}$ 1 $\boxed{\div}$ 6 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display will be 2.096861863.

Finally, press the $\boxed{\text{ENTER}}$ key to view the answer.

85 $\boxed{\wedge}$ $\boxed{(}$ 1 $\boxed{\div}$ 6 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display will be 2.096861863.

Answer 38e.

Let use consider the rational exponent $25^{1/23}$

Approximate roots with a calculator

Expression	Keystrokes	Display
$25^{-1/3}$	25 $\boxed{\wedge}$ $\boxed{(}$ -1 $\boxed{\div}$ 3 $\boxed{)}$ $\boxed{\text{ENTER}}$	0.342

$\boxed{25^{-1/3} = 0.342}$

Answer 39e.

First, enter the number 20736 on the calculator. Then press the key $\boxed{\wedge}$.

20736 $\boxed{\wedge}$

Next, press the key $\boxed{(}$, enter the number 1, press the key $\boxed{\div}$, enter 4, and then press key $\boxed{)}$.

20736 $\boxed{\wedge}$ $\boxed{(}$ 1 $\boxed{\div}$ 4 $\boxed{)}$

Finally, press the $\boxed{\text{ENTER}}$ key to view the result.

20736 $\boxed{\wedge}$ $\boxed{(}$ 1 $\boxed{\div}$ 4 $\boxed{)}$ $\boxed{\text{ENTER}}$

The result will be 12.

Answer 40e.

Let us consider the radical expression $\left(\sqrt[4]{187}\right)^3$

$$\left(\sqrt[4]{187}\right)^3 = \left(187^{1/4}\right)^3 \qquad \left[\text{Use the rule } \left(\sqrt[n]{a}\right)^m = \left(a^{1/n}\right)^m\right]$$

$$= (187)^{\frac{1}{4}\cdot 3} \qquad \left[\text{Use the rule } \left(a^m\right)^n = a^{m\cdot n}\right]$$

$$= 187^{3/4}$$

$$\left(\sqrt[4]{187}\right)^3 = 187^{3/4}$$

Approximate roots with a calculator

Expression	Keystrokes	Display
$187^{3/4}$	187 $\boxed{\wedge}$ $\boxed{(}$ 3 $\boxed{\div}$ 4 $\boxed{)}$ $\boxed{\text{ENTER}}$	50.57

$$\therefore \boxed{\left(\sqrt[4]{187}\right)^3 = 50.57}$$

Answer 41e.

We can rewrite the given expression as $\left(\sqrt[2]{6}\right)^{-5}$.

We know that the exponent notation of the radical expression $\left(\sqrt[n]{a}\right)^{-m}$ is equivalent to $\frac{1}{a^{m/n}}$, where a is the radicand, m is a positive integer, and n is the index.

From the given expression, we get a as 6, m is 5, and n as 2.

$$\left(\sqrt[2]{6}\right)^{-5} = \frac{1}{6^{5/2}}$$

First, enter the number 1 on the calculator. Then, press the key $\boxed{\div}$, enter the key $\boxed{(}$.

1 $\boxed{\div}$ $\boxed{(}$

Next, enter the number 6, press the key $\boxed{\wedge}$, enter the key $\boxed{(}$.

1 $\boxed{\div}$ $\boxed{(}$ 6 $\boxed{\wedge}$ $\boxed{(}$

Enter the number 5, press the key $\boxed{\div}$, enter the key $\boxed{)}$, and then again press the key $\boxed{)}$

1 $\boxed{\div}$ $\boxed{(}$ 6 $\boxed{\wedge}$ $\boxed{(}$ 5 $\boxed{\div}$ 2 $\boxed{)}$ $\boxed{)}$

Finally, press the $\boxed{\text{ENTER}}$ key to view the answer.

1 $\boxed{\div}$ $\boxed{(}$ 6 $\boxed{\wedge}$ $\boxed{(}$ 5 $\boxed{\div}$ 2 $\boxed{)}$ $\boxed{)}$ $\boxed{\text{ENTER}}$

The display will be 0.01134023.

Round your answer to two decimal places.

$0.01134023 \approx 0.01$

Thus, the expression evaluates to about 0.01.

Answer 42e.

Let us consider the radical expression $\left(\sqrt[5]{-8}\right)^8$

$$\left(\sqrt[5]{-8}\right)^8 = \left(\left(-8\right)^{1/5}\right)^8 \qquad \left[\text{Use the rule } \left(\sqrt[n]{a}\right)^m = \left(a^{1/n}\right)^m\right]$$

$$= \left(-8\right)^{\frac{1}{5}\cdot 8} \qquad \left[\text{Use the rule } \left(a^m\right)^n = a^{m\cdot n}\right]$$

$$= \left(-8\right)^{8/5}$$

$$\left(\sqrt[5]{-8}\right)^8 = \left(-8\right)^{8/5}$$

Approximate roots with a calculator

Expression	Keystrokes	Display
$(-8)^{8/5}$	-8 $\boxed{\wedge}$ $\boxed{(}$ 8 $\boxed{\div}$ 5 $\boxed{)}$ $\boxed{\text{ENTER}}$	-27.86

$\therefore \boxed{\left(\sqrt[5]{-8}\right)^8 = -27.86}$

Answer 43e.

First, enter the number 86 on the calculator. Then press the key $\boxed{\wedge}$.

86 $\boxed{\wedge}$

Next, press the key $\boxed{(}$, enter the number -5, press the key $\boxed{\div}$, enter 6, and then press key $\boxed{)}$.

86 $\boxed{\wedge}$ $\boxed{(}$ -5 $\boxed{\div}$ 6 $\boxed{)}$

Finally, press the $\boxed{\text{ENTER}}$ key to view the answer.

86 $\boxed{\wedge}$ $\boxed{(}$ -5 $\boxed{\div}$ 6 $\boxed{)}$ $\boxed{\text{ENTER}}$

The display will be 0.02442969.

Round your answer to two decimal places.
$0.02442969 \approx 0.02$

Thus, the expression evaluates to about 0.02.

Answer 44e.

Let us consider the radical expression $1974^{2/7}$
Approximation roots with a calculator

Expression	Keystrokes	Display
$1974^{2/7}$	1974 $\boxed{\wedge}$ $\boxed{(}$ 2 $\boxed{\div}$ 7 $\boxed{)}$ $\boxed{\text{ENTER}}$	8.74

$\therefore \boxed{(1974)^{2/7} = 8.74}$

Answer 45e.

We can rewrite the expression as $(-17)^{-3/5}$.

First, enter the number -17 on the calculator. Then press the key $\boxed{\wedge}$.

-17 $\boxed{\wedge}$

Next, press the key $\boxed{(}$, enter the number -3, press the key $\boxed{\div}$, enter 5, and then press key $\boxed{)}$.

-17 $\boxed{\wedge}$ $\boxed{(}$ -3 $\boxed{\div}$ 5 $\boxed{)}$

Finally, press the $\boxed{\text{ENTER}}$ key to view the answer.

$-17\ \boxed{\wedge}\ \boxed{(}\ -3\ \boxed{\div}\ 5\ \boxed{)}\ \boxed{\text{ENTER}}$

The display will be –0.182696676.

Round your answer to two decimal places.
$-0.182696676 \approx -0.18$

Thus, the expression evaluates to about –0.18.

Answer 46e.

Approximation roots with a calculator

	Expression	Keystrokes	Display
A.	$27^{3/5}$	$27\ \boxed{\wedge}\ \boxed{(}\ 3\ \boxed{\div}\ 5\ \boxed{)}\ \boxed{\text{ENTER}}$	7.23
B.	$5^{3/2}$	$5\ \boxed{\wedge}\ \boxed{(}\ 3\ \boxed{\div}\ 2\ \boxed{)}\ \boxed{\text{ENTER}}$	11.18
C.	$\sqrt[3]{81} = (81)^{1/3}$	$81\ \boxed{\wedge}\ \boxed{(}\ 1\ \boxed{\div}\ 3\ \boxed{)}\ \boxed{\text{ENTER}}$	4.33
D.	$\left(\sqrt[3]{2}\right)^8 = 2^{8/3}$	$2\ \boxed{\wedge}\ \boxed{(}\ 8\ \boxed{\div}\ 3\ \boxed{)}\ \boxed{\text{ENTER}}$	6.35

$\boxed{5^{3/2} \text{ is the greatest value}}$

Answer 47e.

We have $a^{1/n} = 3$.
Take the nth power of both side of the equation.

$$\left(a^{1/n}\right)^n = 3^n$$
$$a = 3^n$$

Substitute any value greater than 1 for n in the equation and find the corresponding value of a.
Let the value of n be 3.

$$a = 3^3$$
$$= 27$$

We know that $27^{1/3} = 3$.

Thus, one of the expression is $27^{1/3}$.

Similarly, substitute 4 for n and find a.

$$a = 3^4$$
$$= 81$$

The two expressions are $27^{1/3}$ and $81^{1/4}$.
Other answers are also possible.

Answer 48e.

Let use consider the equation $x^3 = 27$

$x = (27)^{1/3}$ [Use the rule if $a^n = b$ then $a = b^{1/n}$]

$\Rightarrow \quad x = \sqrt[3]{27}$ [Use the rule $a^{1/n} = \sqrt[n]{a}$ for any integer $n > 1$]

$\Rightarrow \quad x = \sqrt[3]{3 \cdot 3 \cdot 3}$ [Write the radicand as a product of a prime factor]

$\Rightarrow \quad x = \sqrt[3]{3^3}$ [Use the rule $a^m \cdot a^n = a^{m+n}$]

$\Rightarrow \quad x = 3$ [Use the rule $\sqrt[n]{a^n} = a$ if n is an odd integer]

$\therefore \quad \boxed{x = 3}$

Hence $\boxed{x = 3}$ is a solution of the given equation $x^3 = 27$

Answer 49e.

Take the fourth root of both sides $x^4 = 81$.

$$\sqrt[4]{x^4} = \pm\sqrt[4]{81}$$

Since 81 is equal to 3^4 and $(-3)^4$, x will be equal to ± 3.

Thus, for the given equation there are two real solutions.

Answer 50e.

Let us consider the equation $x^3 = 125$

$x^3 = (125)^{1/3}$ [Use the rule if $a^n = b$ then $a = b^{1/n}$]

$\Rightarrow \quad x = \sqrt[3]{125}$ [Use the rule $a^{1/n} = \sqrt[n]{a}$ for any integer $n > 1$]

$\Rightarrow \quad x = \sqrt[3]{5 \cdot 5 \cdot 5}$ [Write the radicand as a product of a prime factor]

$\Rightarrow \quad x = \sqrt[3]{5^3}$ [Use the rule $a^m \cdot a^n = a^{m+n}$]

$\Rightarrow \quad x = 5$ [Use the rule $\sqrt[n]{a^n} = a$ if n is an odd integer]

$\therefore \quad \boxed{x = 5}$

Hence $x = 5$ is a solution of the given equation $x = 5$

Answer 51e.

Divide each side by 5.

$$\frac{5x^3}{5} = \frac{1080}{5}$$
$$x^3 = 216$$

Take the cube root of each side and simplify.

$$\sqrt[3]{x^3} = \sqrt[3]{216}$$
$$x = 6$$

Therefore. the solution is 6.

Answer 52e.

Let use consider the equation $x^6 + 36 = 100$

$$x^6 = 100 - 36 \qquad [\text{Substrat 36 from both sides}]$$
$$\Rightarrow \quad x^6 = 64$$
$$\Rightarrow \quad x = (64)^{1/6} \qquad [\text{Use the rule if } a^n = b \text{ then } a = b^{1/n}]$$
$$\Rightarrow \quad x = \sqrt[6]{64} \qquad [\text{Use the rule } a^{1/n} = \sqrt[n]{a} \text{ for any integer } n > 1]$$
$$\Rightarrow \quad x = \sqrt[6]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} \qquad [\text{Write the radicand as a product of a prime factor}]$$
$$\Rightarrow \quad x = \sqrt[6]{2^6} \qquad [\text{Use the rule } a^m \cdot a^n = a^{m+n}]$$
$$\Rightarrow \quad x = \pm 2 \qquad [\text{Use the rule if } \sqrt[n]{a^n} = \pm a \text{ for } n \text{ is any even integer}]$$
$$\therefore \quad \boxed{x = \pm 2}$$

Hence $x = \pm 2$ is a solution of the given equation $x^6 + 36 = 100$

Answer 53e.

Take the fourth root of each side of the equation.

$$\sqrt[4]{(x-5)^4} = \sqrt[4]{256}$$
$$x - 5 = \pm 4$$

Add 5 to each side.

$$5 + x - 5 = 5 \pm 4$$
$$x = 5 \pm 4$$

Find x.

$$x = 5 - 4 \quad \text{or} \quad x = 5 + 4$$
$$x = 1 \quad \text{or} \quad x = 9$$

Therefore, the solutions for the given equation are 1 and 9.

Answer 54e.

Let use consider the equation $x^5 = -48$

$\Rightarrow \quad x = (-48)^{1/5}$ [Use the rule if $a^n = b$ then $a = b^{1/n}$]

Approximate roots with a calculator

Expression	Keystroke	Display
$(-48)^{1/5}$	-48 $\boxed{\wedge}$ $\boxed{(}$ 1 $\boxed{\div}$ 5 $\boxed{)}$ $\boxed{\text{ENTER}}$	-2.168

$\therefore \quad \boxed{x = -2.16}$

Hence $x = -2.16$ is a solution of the given equation $x^5 = -48$

Answer 55e.

Divid each side of the equation by 7.

$$\frac{7x^4}{7} = \frac{56}{7}$$

$$x^4 = 8$$

Take the fourth root of each side.

$$\sqrt[4]{x^4} = \pm\sqrt[4]{8}$$

$$x = \pm\sqrt[4]{8}$$

Use a calculator to find the fourth root.

$$x \approx \pm 1.68$$

Therefore, the solutions are approximately -1.68 and 1.68.

Answer 56e.

Let use consider the equation $x^3 + 40 = 25$

$\Rightarrow \quad x^3 = 25 - 40$ Substract 40 from both sides

$\Rightarrow \quad x^3 = -15$

$\Rightarrow \quad x = (-15)^{1/3}$ Use the rule if $a^n = b$ then $a = b^{1/n}$

$\therefore \quad x = (-15)^{1/3}$

Approximate roots with a calculator

Expression	Keystroke	Display
$(-15)^{1/3}$	-15 $\boxed{\wedge}$ $\boxed{(}$ 1 $\boxed{\div}$ 3 $\boxed{)}$ $\boxed{\text{ENTER}}$	-2.47

$\therefore \quad \boxed{x = -2.47}$

Hence $x = -2.47$ is a solution of the given equation $x^3 + 40 = 25$

Answer 57e.

Take the fifth root of each side of the equation.

$$\sqrt[5]{(x+10)^5} = \sqrt[5]{70}$$

$$x+10 \approx \sqrt[5]{70}$$

Subtract 10 from each side.

$$x+10-10 \approx \sqrt[5]{70}-10$$

$$x \approx \sqrt[5]{70}-10$$

Use a calculator to find x.

$x \approx -7.66$

The solution for the given equation is approximately −7.66.

Answer 58e.

Let use consider the equation $x^6 - 34 = 181$

$\Rightarrow \quad x^6 = 181 + 34 \quad$ Add 34 from both sides

$\Rightarrow \quad x^6 = 215 \quad$ add

$\Rightarrow \quad x = (215)^{1/6} \quad$ Use the rule if $a^n = b$ then $a = b^{1/n}$

$\therefore \quad x = (215)^6$

Approximate roots with a calculator

Expression	Keystroke	Display
$(215)^{1/6}$	215 $\boxed{\wedge}$ $\boxed{(}$ $\boxed{1}$ $\boxed{\div}$ $\boxed{6}$ $\boxed{)}$ $\boxed{\text{ENTER}}$	2.44

$\therefore \quad \boxed{x = 2.44}$

Hence $x = 2.44$ is a solution of the given equation.

Answer 59e.

(a) We know that if y is x^n, then x is $\sqrt[n]{y}$, or $y^{1/n}$.

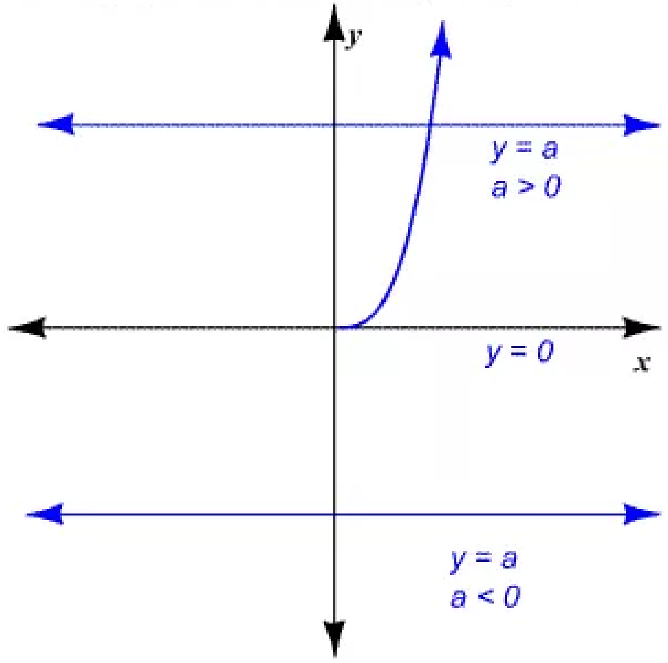
From the graph, we can say that for the same value of y, there are be two values of x. In other words, for $y = a$ and $a > 0$, we have $x = -1$ and $x = 1$.

In the Key Concept box, it is said that that if n is an even integer and $a > 0$, there exists two real nth roots, which is $\pm\sqrt[n]{a}$, or $\pm a^{1/n}$.

In the graph, when the y-value is 0, x-value is also 0.
The nth root of 0, $\sqrt[n]{0}$, is 0 itself.

Also, there is no graph below the x-axis. This means you will not find any $y = a$ such that $a > 0$ for a particular x-value.
This justifies the concept of no real nth roots in the case $a < 0$.

(b) When n is odd and $a > 0$, there will be one real nth root: $\sqrt[n]{a} = a^{1/n}$.
In the graph, there will be only one y-value for an x-value.



Answer 61e.

The surface area of a sphere is given by the formula, $S = 4\pi r^2$.

Substitute 232 for S in the formula.
$232 = 4\pi r^2$ or $4\pi r^2 = 232$

Divide each side by 4π, and simplify.

$$\frac{4\pi r^2}{4\pi} = \frac{232}{4\pi}$$
$$r^2 \approx 18.46$$

Take the square root of each side.

$$\sqrt{r^2} \approx \sqrt{18.46}$$
$$r \approx 4.30$$

Thus, the radius of the bowling ball is approximately 4.3 inches for the given surface area.

Answer 62e.

Let us consider the annual rate of inflation is $r=\left(\frac{P_2}{P_1}\right)^{1/n}-1$

To find the rate of inflation for the Butter

$$\therefore r=\left(\frac{P_2}{P_1}\right)^{1/n}-1$$

Here $P_1=\$0.7420$

$P_2=\$2.195$

$n=4$

$$r=\left(\frac{2.195}{0.7420}\right)^{1/4}-1 \qquad \begin{bmatrix}\text{Substitute } 0.7420 \text{ for } P_1,\ 2.195 \text{ for} \\ P_2 \text{ and } 4 \text{ for } n\end{bmatrix}$$

$$=(2.9583)^{1/4}-1$$

$$=1.311468-1$$

$$=0.311468$$

$r=0.311468$

The rate of inflation for the butter is 31.14%

To find rate of inflation for chicken

$$r=\left[\frac{P_2}{P_1}\right]^{1/n}-1$$

Here $P_1=\$0.4430$

$P_2=\$1.087$

$n=4$

$$r=\left(\frac{1.087}{0.4430}\right)^{1/4}-1 \qquad \begin{bmatrix}\text{Substitute } 0.4430 \text{ for } P_1,\ 1.087 \text{ for} \\ P_2 \text{ and } 4 \text{ for } n\end{bmatrix}$$

$$=(2.4538)^{1/4}-1$$

$$=(1.2516)-1$$

$$=0.2516$$

$r=0.2516$

The rate of inflation for chicken is 25.15%.

To find the rate of inflation for Eggs:

$$\therefore \quad r=\left[\frac{P_2}{P_1}\right]^{1/n}-1$$

Here $P_1=\$0.6710$

$P_2=\$1.356$

$n=4$

$$r=\left(\frac{1.356}{0.6710}\right)^{1/4}-1 \qquad \left[\begin{array}{l}\text{Substitute } 0.6710 \text{ for } P_1,\ 1.356 \text{ for} \\ P_2 \text{ and } 4 \text{ for } n\end{array}\right]$$

$$=(2.0209)^{1/4}-1 \qquad \left[\text{Simplify } \frac{1.356}{0.6710}=1.1923\right]$$

$$=(1.1923)-1$$

$$r=0.1923$$

The rate of inflation for eggs is 19.23%

To find the rate of inflation for sugar

$$\therefore \quad r=\left[\frac{P_2}{P_1}\right]^{1/n}-1$$

Here $P_1=\$0.936$

$P_2=\$0.4560$

$n=4$

$$r=\left(\frac{0.4560}{0.936}\right)^{1/4}-1 \qquad \left[\begin{array}{l}\text{Substitute } 0.936 \text{ for } P_1,\ 0.4560 \text{ for} \\ P_2 \text{ and } 4 \text{ for } n\end{array}\right]$$

$$=(0.48718)^{1/4}-1$$

$$=0.8354-1$$

$$r=-0.1646$$

The rate of inflation for sugar is $\boxed{-16.46\%}$

Answer 63e.

The formula for the power used by a fan is given as $p=ks^3$.

Substitute 1.2 for p, 1700 for s in the formula.

$1.2=k(1700)^3$

Divide both the sides by 1700^3.

$$\frac{1.2}{1700^3}=k$$

The value of k is $\frac{1.2}{1700^3}$, or approximately 2.44×10^{-10}.

Now, we have to find the speed of the fan with 1.5 horsepower.

Substitute 1.5 for p, $\frac{1.2}{1700^3}$ for k in the formula.

$$1.5 = \frac{1.2}{1700^3} \cdot s^3 \quad \text{or} \quad \frac{1.2}{1700^3} \cdot s^3 = 1.5$$

Multiply each side by $\frac{1700^3}{1.2}$.

$$s^3 = \frac{1700^3}{1.2}(1.5)$$

Take the cube root of each side.

$$\sqrt[3]{s^3} \approx \sqrt[3]{\frac{1700^3}{1.2}(1.5)}$$

$$s \approx 1831$$

The speed of the fan is approximately 1831 revolutions per minute if it uses 1.5 horsepower.

Answer 64e.

Let us consider the flow rate of a weir can be calculating using the formula is $Q = 3.367 \cdot l \cdot h^{3/2}$

The length of the bottom of the spill way $l = 20$ feet.

The depth of the bottom of the spill way $h = 5$ feet

To find the flow rate of a weir with a spillway

$$Q = 3.367(20)(5)^{3/2} \quad [\text{substitute 20 for } h \text{ and 5 for } h]$$

$$= 3.367 \cdot (20)(11.1804)$$

$$\boxed{Q = 752.89}$$

The flow rate of a weir with spill way is 752.89 cubic feet per second.

Answer 65e.

(a) The volume v of a cube is given by the formula $V = x^3$, where x is the length of each side.

Substitute 16 for a in the formula, and simplify.

$$V = 16^3$$

$$= 4096$$

The volume of the cube is 4096 mm^3 for the given value of edge length.

(b) Since you are designing a dice having the same volume as that of cube, the volume of each polyhedron will be 4096 mm^3.

Find the edge length of tetrahedron.
Substitute 4096 for v in the formula $V = 0.118x^3$.
$4096 = 0.118x^3$ or $0.118x^3 = 4096$

Divide each side by 0.118 and simplify.

$$\frac{0.118x^3}{0.118} = \frac{4096}{0.118}$$
$$x^3 \approx 34{,}711.86$$

Take the cube roots of each side, and simplify.

$$\sqrt[3]{x^3} \approx \sqrt[3]{34{,}711.86}$$
$$x \approx 32.62$$

The edge length of tetrahedron will be approximately 32.62 mm.

Find the edge length of octahedron.

Substitute 4096 for v in the formula $V = 0.471x^3$.
$4096 = 0.471x^3$ or $0.471x^3 = 4096$

Divide each side by 0.471 and simplify.

$$\frac{0.471x^3}{0.471} = \frac{4096}{0.471}$$
$$x^3 \approx 8{,}696.39$$

Take the cube roots of each side and simplify.

$$\sqrt[3]{x^3} \approx \sqrt[3]{8{,}696.39}$$
$$x \approx 20.56$$

The edge length of octahedron will be approximately 20.56 millimeters.

Similarly, we will get the edge length of the dodecahedron as 8.12 mm and the icosahedrons as 12.3 mm.

(c) We know that icosahedrons have the greatest number of faces, which is 20. Its edge length is 12.3 mm.

The dodecahedron which has 12 faces has the edge length 8.12 mm which is lesser than the icosahedron's edge length.

Thus, we can conclude that the polyhedron with the greatest number of faces will not have the smallest edge length.

Answer 66e.

Let us consider the mass of the particles that a river can transport is m.
Assume speed of the river is s.

Given that the mass of the particles that a river can transform is proportional to the sixth power of the speed of the river.

i.e. $m \propto s^6$

$\Rightarrow \quad m = cs^6 \quad \ldots\ldots (1)$ Where "c" is any arbitrary constant

A certain river normally flows at a speed of 1 meter per second.

$\therefore \quad m = c(1)^6$ Substitute s for 1

$m = c$

To find that must its speed be in order to transport particles that are twice as mass

$$2m = ms^6 \quad \left[\begin{array}{l}\text{Substitute } 2m \text{ for } m \text{ and m} \\ \text{for } c \text{ in equatin (1)}\end{array}\right]$$

$$\Rightarrow \quad \frac{2m}{m} = s^6 \quad [\text{Divide } m]$$

$$\Rightarrow \quad s^6 = 2$$

$$\Rightarrow \quad s = (2)^{1/6} \quad \left[\text{ If } a^n = b \text{ then } a = b^{1/n}\right]$$

$$\Rightarrow \quad s = 1.122462048$$

1.22462048 speed in order to transport particle that are twice as mass.

To find what must its speed be in order to transport particle. That is 10 times massive.

$$10m = ms^6 \quad [\text{Substitute } 10m \text{ for } m \text{ and m for } c \text{ in equation (1)}]$$

$$\Rightarrow \quad \frac{10m}{m} = s^6 \quad [\text{Divide } m]$$

$$\Rightarrow \quad 10 = s^6$$

$$\Rightarrow \quad s^6 = 10$$

$$\Rightarrow \quad s = (10)^{1/6} \quad \left[\text{Use the rule if } a^n = b \text{ then } a = b^{1/n}\right]$$

$$s = 1.467799268$$

1.467799268 speed in order to transport particle that are 10 times as massiv.

To find what must its speed be in order to transport particle that are 100 times as massive.

$$100m = ms^6 \quad \begin{bmatrix}\text{Substitute } 100m \text{ for } m \text{ and } M \text{ for } c \\ \text{in equation (1)}\end{bmatrix}$$

$$\Rightarrow \quad \frac{100m}{m} = s^6 \quad [\text{Divide } m]$$

$$\Rightarrow \quad \frac{100m}{m} = s^6$$

$$\Rightarrow \quad s^6 = 100$$

$$\Rightarrow \quad s = (100)^{1/6} \quad \left[\text{use the rule if } a^n = b \text{ then } a = b^{1/n}\right]$$

$$\Rightarrow \quad \boxed{s = 2.15443469}$$

2.15443469 speed in order to transport particle that are 100 times as massive.

Answer 67e.

Substitute 3 for x, and 5 for y.

$$\frac{3+3(5)}{3-5} = \frac{3+15}{3-5}$$

The expression can be simplified using the order of operations. The fraction bar in the expression acts as a grouping symbol. The numerator and the denominator are to be evaluated separately.

Take the expression on the numerator first.
Since multiplications have more priority than additions and subtractions, perform the multiplication first.
$3 + 3(5) = 3 + 15$

Add.
$3 + 15 = 18$

The numerator simplifies to 18.

Now, perform subtraction in the expression in the denominator.
$3 - 5 = -2$

Replace the numerator and the denominator with the simplified values.

$$\frac{3+15}{3-5} = \frac{18}{-2}$$
$$= -9$$

Thus, the expression evaluates to -9 when $x = 3$ and $y = 5$.

Answer 68e.

Let us consider the given equation $\frac{4x-4}{x-24}$

$\frac{4x-4}{x-24}=\frac{4(6)-(-2)}{6-2(-2)}$ [Substitute 6 for "x" and -2 for y]

$\frac{4x-4}{x-24}=\frac{24+2}{6+4}$ [Do the multiplication]

$\frac{4x-4}{x-24}=\frac{26}{10}$ [Do the addition]

$\frac{4x-4}{x-24}=\frac{13}{5}$ $\left[\text{Simplify } \frac{26}{10}=\frac{13}{5}\right]$

$\frac{4x-4}{x-24}=2.6$ $\left[\text{Simplify } \frac{13}{5}=2.6\right]$

$\therefore \boxed{\frac{4x-4}{x-24}=2.6}$ when $x=6$ and $y=-2$

Answer 69e.

Factor the right side of the given function. For this, you have to find two numbers with product -35 and sum -2. Two such numbers are 5 and -7.

Rewrite the given function.
$f(x)=(x+5)(x-7)$

We know that the x-intercepts of the graph of $y=a(x-p)(x-q)$ are the zeros of the function. This means that the function's value is zero when $x=p$ and $x=q$. Thus, p and q are zeros of the function.

The value of the function will be zero when x takes the value -5 and 7.

Therefore, the zeros of the given function are -5 and 7.

Answer 70e.

Let us consider the given equation $x^2 - 8x + 25 = 0$

We know that, $ax^2 + bc + c = 0$ roots are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\therefore x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 25}}{2 \cdot 1} \qquad \left[\begin{array}{l}\text{Substitute 1 for } a, \ -8 \text{ for } b \text{ and} \\ 25 \text{ for c in the formula}\end{array}\right]$$

$$x = \frac{8 \pm \sqrt{64 - 100}}{2}$$

$$x = \frac{8 \pm \sqrt{-36}}{2} \qquad \left[\sqrt{-36} = 6i\right]$$

$$x = \frac{8 \pm 6i}{2}$$

$$x = \frac{2(4 \pm 3i)}{2}$$

$$\boxed{x = 4 \pm 3i}$$

$4 + 3i$ or $4 - 3i$ are zeros of the function $f(x) = x^2 - 8x + 25$

Answer 71e.

According to rational zero theorem, if $f(x) = a_n x^n + \ldots\ldots + a_1 x + a_0$ the every rational zero of f will be of the form $\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leadingcoeffecient } a_n}$.

STEP1 **List** the possible rational zeros. We know that the leading coefficient is 1, and the constant term is -32.

The factors of 32 are $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$ and ± 32.

The possible rational zeros are

$$\frac{\pm 1}{1}, \frac{\pm 2}{1}, \frac{\pm 4}{1}, \frac{\pm 8}{1}, \pm\frac{16}{1}, \frac{\pm 32}{1} \quad \text{or} \quad \pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32.$$

STEP1 **Test** the zeros using synthetic division.

First, test the zero 1.

Write the terms in the dividend without the variables.

Bring down the first coefficient, 1. Multiply 1 by 1 to get 1 and then place this number under the next coefficient -8. Add -8 and 1.

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 4 & -32 \\ & & 1 & & \\ \hline & 1 & -7 & & \end{array}$$

Similarly, multiply and add till we get the remainder.

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 4 & -32 \\ & & 1 & -7 & -3 \\ \hline & 1 & -7 & -3 & -35 \end{array}$$

Since the remainder is not 0, 1 is not a zero of the given polynomial.

Test the zero -2 using synthetic division.

$$\begin{array}{r|rrrr} -2 & 1 & -8 & 4 & -32 \\ & & -2 & 20 & -48 \\ \hline & 1 & -10 & 24 & -80 \end{array}$$

The remainder is not 0. Thus, -2 is not a zero of the given polynomial.

Test the zero 4 using synthetic division.

$$\begin{array}{r|rrrr} 4 & 1 & -8 & 4 & -32 \\ & & 4 & -16 & -48 \\ \hline & 1 & -4 & -12 & -80 \end{array}$$

Since the remainder is not 0, 4 is not a zero of the given polynomial.

Now, test the zero 8 using synthetic division.

$$\begin{array}{r|rrrr} 8 & 1 & -8 & 4 & -32 \\ & & 8 & 0 & 32 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

The remainder is 0. Thus, 8 is a zero of the given polynomial.
The new quotient is $(x-8)(x^2+4)$.

STEP3 Use the factor theorem.
Set each factor to 0 and solve for x.

$$\begin{aligned} x-8 &= 0 \quad \text{and} \quad & x^2+4 &= 0 \\ x &= 8 & x^2 &= -4 \\ & & \sqrt{x} &= \sqrt{-4} \\ & & x &= \pm 2i \end{aligned}$$

Thus, the rational zeros are 8, $2i$, and $-2i$.

Answer 72e.

Let us consider the expression $f(x)=x^3+4x^2+25x+100$

$\therefore \quad f(x)=x^3+4x^2+25x+100$ [Write the original function]

$f(-4)=(-4)^3+4(-4)^2+25(-4)+100$ [Replace x with -4]

$\Rightarrow \quad f(-4)=-64+4(16)-100+100$

$\Rightarrow \quad f(-4)=-64+64-100+100$ [Multiplicaiton]

$\Rightarrow \quad f(-4)=0$ [add]

By factorization theorem

"if $f(c)=0$ then $(x-c)$ is a factor of the given polynomial"

$\therefore (x+4)$ is a factor of $f(x)$

$$\begin{array}{r|rrrr} \therefore -4 & 1 & 4 & 25 & 100 \\ & 0 & -4 & 0 & -100 \\ \hline & 1 & 0 & 25 & \underline{|0} \end{array}$$

$\therefore f(x)=(x+4)(x^2+0x+25)$

<u>Suppose</u> $\quad f(x)=0$

i.e. $(x+4)(x^2+0x+25)=0$

$\Rightarrow \quad x+4=0 \quad$ or $\quad x^2+25=0$ $\left[\begin{array}{l}\text{Use the rule } a\cdot b=0 \\ \text{either } a=0 \text{ or } b=0\end{array}\right]$

$\Rightarrow \quad x=-4 \quad$ or $\quad x^2=-25$

$\Rightarrow \quad x=-4 \quad$ or $\quad x=\pm\sqrt{-25}$ $\left[\begin{array}{l}\text{Use the rule if } a^n=b \\ \text{then } a=\sqrt[n]{b}\end{array}\right]$

$\Rightarrow \quad x=-4 \quad$ or $\quad x=\pm 5i$ $\left[\sqrt{-25}=5i\right]$

$\therefore \boxed{x=-4,-5i,+5i}$ are the zero's of the given function $f(x)$

Answer 73e.

According to rational zero theorem, if $f(x)=a_nx^n+\ldots\ldots+a_1x+a_0$ the every rational zero of f will be of the form $\dfrac{p}{q}=\dfrac{\text{factor of constant term } a_0}{\text{factor of leadingcoeffecient } a_n}$.

STEP1 **Find** the rational zeros of the given function. We know that the leading coefficient of $f(x)$ is 1, and the constant term is 90. The factors of 90 are $\pm1, \pm2, \pm3, \pm5, \pm6, \pm9, \pm10, \pm15, \pm18, \pm30, \pm45, \pm90$.

The possible rational zeros are

$\dfrac{\pm1}{1}, \dfrac{\pm2}{1}, \dfrac{\pm3}{1}, \dfrac{\pm5}{1}, \dfrac{\pm6}{1}, \dfrac{\pm9}{1}, \dfrac{\pm10}{1}, \dfrac{\pm15}{1}, \dfrac{\pm18}{1}, \dfrac{\pm30}{1}, \dfrac{\pm45}{1}, \dfrac{\pm90}{1}$

or $\pm1, \pm2, \pm3, \pm5, \pm6, \pm9, \pm10, \pm15, \pm18, \pm30, \pm45, \pm90$.

Test the zeros using synthetic division.

First, test the zero 1.
Write the terms in the dividend without the variables.
Bring down the first coefficient, 1. Multiply 1 by 1 to get 1 and then place this number under the next coefficient –3. Add –3 and 1.

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & -31 & 63 & 90 \\ & & 1 & & & \\ \hline & 1 & & & & \end{array}$$

Similarly, multiply and add till we get the remainder.

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & -31 & 63 & 90 \\ & & 1 & -2 & -33 & 30 \\ \hline & 1 & -2 & -33 & 30 & 120 \end{array}$$

Since the remainder is not 0, 1 is not a zero of the given polynomial.

Test the zero –1 using synthetic division.

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & -31 & 63 & 90 \\ & & -1 & 4 & 27 & -90 \\ \hline & 1 & -4 & -27 & 90 & 0 \end{array}$$

The remainder is 0. Thus, –1 is a zero of the given polynomial.

STEP2 **Write** $f(x)$ in factored form.

$f(x) = (x + 1)(x^3 - 4x^2 - 27x + 90)$

Repeat the steps above for $g(x) = x^3 - 4x^2 - 27x + 90$.
The possible rational zeros of $g(x)$ are
$\pm1, \pm2, \pm3, \pm5, \pm6, \pm9, \pm10, \pm15, \pm18, \pm30, \pm45, \pm90$

Test the zeros using the synthetic division.
First, test the zero 1 using synthetic division.

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -27 & 90 \\ & & 1 & -3 & -30 \\ \hline & 1 & -3 & -30 & 60 \end{array}$$

Since the remainder is not 0, 1 is not a zero of the polynomial.

Test the zero 3 using synthetic division.

$$\begin{array}{r|rrrr} 3 & 1 & -4 & -27 & 90 \\ & & 3 & -3 & -90 \\ \hline & 1 & -1 & -30 & 0 \end{array}$$

The remainder is 0. Thus, 3 is a zero of the polynomial.

Write $g(x)$ in factored form.
$g(x) = (x-3)(x^2 - x - 30)$

Factorize $x^2 - x - 30$.
$x^2 - x - 30 = (x-6)(x+5)$

Use the factor theorem.
Set each factor to 0 and solve for x.

$$x - 3 = 0 \qquad x - 6 = 0 \qquad x + 5 = 0$$
$$x = 3 \qquad x = 6 \qquad x = -5$$

Thus, the zeros of the function are $-5, -1, 3$, and 6.

Answer 74e.

Let us consider the given equation $x^4 + 10x^3 + 25x^2 - 36$

$$f(1) = 1^4 + 10(1)^3 + 25(1)^2 - 36 = 0$$

By factorization theorem

If $f(0) = 0$ then $(x-a)$ is a factor of the given polynomial"

$\therefore (x-1)$ is a factor of $f(x)$.

$$\begin{array}{r|rrrrr} x=1 & 1 & 10 & 25 & 0 & -36 \\ & 0 & 1 & 11 & 36 & 36 \\ \hline x=-2 & 1 & 11 & 36 & 36 & \lfloor 0 \\ & 0 & -2 & -18 & -36 & \\ \hline x=-3 & 1 & 9 & 18 & \lfloor 0 & \\ & 0 & -3 & -18 & & \\ \hline & 1 & 6 & \lfloor 0 & & \end{array}$$

Factor of $f(x) = (x-1)(x+2)(x+3)(x+6)$

Suppose $f(x) = 0$

$\therefore \quad (x-1)(x+2)(x+3)(x+6) = 0$

$\Rightarrow \quad x-1=0 \text{ (or) } x+2=0 \text{ (or) } x+3=0 \text{ (or) } x+6=0$

$\Rightarrow \quad x=1 \text{ (or) } x=-2 \text{ (or) } x=-3 \text{ (or) } x=-6$

$\therefore \quad x = 1, -2, -3, -6$

$\boxed{x = -6, -3, -2, 1}$

The zero of the function $f(x)$ is $-6, -3, -2, 1$

Answer 75e.

According to the quotient of powers property, $\frac{a^m}{a^n} = a^{m-n}$, where a and b are real numbers, m and n are integers.

$$\frac{x^{-4}}{x^3} = x^{-4-3}$$
$$= x^{-7}$$

Use the negative exponent property $a^{-m} = \frac{1}{a^m}$.

$$x^{-7} = \frac{1}{x^7}$$

Thus, the given expression simplifies to $\frac{1}{x^7}$.

Answer 76e.

Let us consider the expression $\left(x^4\right)^{-3}$

$\left(x^4\right)^{-3} = x^{4\cdot(-3)}$ $\quad \left[\text{Use the rule } \left(a^m\right)^n = a^{m\cdot n}\right]$

$\left(x^4\right)^{-3} = x^{-12}$

$\left(x^4\right)^{-3} = \frac{1}{x^{12}}$ $\quad \left[\text{Use the rule } a^{-n} = \frac{1}{a^n}\right]$

$$\boxed{\left(x^4\right)^{-3} = \frac{1}{x^{12}}}$$

Answer 77e.

According to the negative exponent property, $a^{-m} = \frac{1}{a^m}$, where a and b are real numbers, m and n are integers.

$$\left(3x^2y\right)^{-3} = \frac{1}{\left(3x^2y\right)^3}$$

Use the power of a product property $(ab)^m = a^m b^n$.

$$\frac{1}{\left(3x^2y\right)^3} = \frac{1}{\left(3^3\right)\left(x^2\right)^3\left(y^3\right)}$$

Apply power of a power property, $\left(a^m\right)^n = a^{mn}$, and simplify.

$$\frac{1}{\left(3^3\right)\left(x^2\right)^3\left(y^3\right)} = \frac{1}{27x^6y^3}$$

Thus, the given expression simplifies to $\frac{1}{27x^6y^3}$.

Answer 78e.

Let us consider the expression $4x^0y^{-4}$

$$4x^0y^{-4} = 4\cdot 1\cdot y^{-4} \qquad \left[\text{Use the rule } \left(a^m\right)^n = a^{m\cdot n}\right]$$

$$\Rightarrow \quad 4x^0y^{-4} = 4\frac{1}{y^4} \qquad \left[\text{Use the rule } a^{-n} = \frac{1}{a^n}\right]$$

$$\Rightarrow \quad 4x^0y^{-4} = \frac{4}{y^4}$$

$$\therefore \boxed{4x^0y^{-4} = \frac{4}{y^4}}$$

Answer 79e.

According to the product of powers property, $a^m \cdot a^n = a^{m+n}$, where a and b are real numbers, m and n are integers.

$$x^6 \cdot x^{-2} = x^{6+(-2)}$$
$$= x^{6-2}$$

Simplify, and evaluate the power.

$$x^{6-2} = x^4$$

Thus, the given expression simplifies to x^4.

Answer 80e.

Let us consider the expression $\left(\frac{x^3}{y^{-2}}\right)^2$

$$\left(\frac{x^3}{y^{-2}}\right)^2 = \left(x^3y^2\right)^2 \qquad \left[\text{Use the rule } a^{-n} = \frac{1}{a^n}\right]$$

$$\Rightarrow \quad \left(\frac{x^3}{y^{-2}}\right)^2 = \left(x^3\right)^2\cdot\left(y^2\right)^2 \qquad \left[\text{Use the rule } \left(a\cdot b\right)^m = a^m\cdot b^n\right]$$

$$\Rightarrow \quad \left(\frac{x^3}{y^{-2}}\right)^2 = x^6y^4$$

$$\therefore \boxed{\left(\frac{x^3}{y^{-2}}\right)^2 = x^6y^4}$$

Answer 81e.

Rewrite the given expression as a product such that exponential expressions with the same bases are separated.

$$\frac{4x^3y^6}{10x^5y^{-3}} = \frac{4}{10}\cdot\frac{x^3}{x^5}\cdot\frac{y^6}{y^{-3}}$$

According to the quotient of powers property, $\frac{a^m}{a^n} = a^{m-n}$, where a and b are real numbers, m and n are integers.

$$\begin{aligned}\frac{4}{10}\cdot\frac{x^3}{x^5}\cdot\frac{y^6}{y^{-3}} &= \frac{4}{10}\cdot x^{3+5}\cdot y^{6-3}\\ &= \frac{2x^8y^3}{5}\end{aligned}$$

Thus, the given expression simplifies to $\frac{2x^8y^3}{5}$.

Answer 82e.

Let us consider the expression $\frac{3x}{x^3y^2}\cdot\frac{y^4}{9x^{-2}}$

$$\begin{aligned}\frac{3x}{x^3y^2}\cdot\frac{y^4}{9x^{-2}} &= \frac{3}{9}\cdot x\cdot\frac{1}{x^3}\cdot\frac{1}{x^{-2}}\cdot y^4\cdot\frac{1}{y^2}\\
&= \frac{3}{9}\cdot x\cdot\left(x^3\right)^{-1}\cdot\left(x^{-2}\right)^{-1}\cdot y^4\left(y^2\right)^{-1} && \left[\text{Use the rule } \frac{1}{a^n} = \left(a^n\right)^{-1}\right]\\
&= \frac{3}{9}\cdot x\cdot x^{3(-1)}\cdot x^{-2(-1)}\cdot y^4\cdot y^{2(-1)} && \left[\text{Use the rule } \left(a^m\right)^n = a^{m\cdot n}\right]\\
&= \frac{3}{9}\cdot x\cdot x^{-3}\cdot x^2\cdot y^4\cdot y^{-2}\\
&= \frac{3}{9}\cdot x^{1-3+2}\cdot y^{4-2} && \left[\text{Use the rule } a^m\cdot a^n = a^{m+n}\right]\\
&= \frac{3}{9}\cdot x^{3-3}\cdot y^{4-2}\\
&= \frac{3}{9}\cdot x^0\cdot y^2\\
&= \frac{3}{9}\cdot 1\cdot y^2 && \left[\text{Use the rule } a^0 = 1\right]\\
&= \frac{1}{3}\cdot y^2 && [\text{Simplify}]\end{aligned}$$

$$\therefore \boxed{\frac{3x}{x^3y^2}\cdot\frac{y^4}{9x^{-2}} = \frac{1}{3}\cdot y^2}$$