Uniform plane waves

* Homogeneous Medium $\longrightarrow \mu, \epsilon, \sigma$ are not f^n of x, y, z
* Isotropic medium [iso – same, tropo – dirn] $\longrightarrow \mu, \epsilon, \sigma$ ~~which~~ values do not change with dirn

non-isotropic medium $\longrightarrow \mu, \epsilon, \sigma$ values change with dirn

* ~~Linear~~ medium

$$\begin{cases} \bar{J} = \sigma \bar{E} \\ \bar{D} = \epsilon \bar{E} \\ \bar{B} = \mu \bar{H} \end{cases}$$

* Wave eqn (or) HelmHoltz eqn $\rightarrow$

let the medium is homogeneous, isotropic, linear, $\rho_v = 0$ in the medium

for these condn maxwell's eqn becomes

$$* \quad \bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = \frac{-\partial \mu \bar{H}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t} \quad \text{—— (i)}$$

$$* \quad \nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$= \sigma \bar{E} + \frac{\partial \epsilon \bar{E}}{\partial t}$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t} \quad \text{—— (ii)}$$

$$* \quad \bar{\nabla} \cdot \bar{D} = \rho_v \Rightarrow \nabla \cdot (\epsilon \bar{E}) = 0$$

$$\epsilon (\nabla \cdot \bar{E}) = 0$$

$$\nabla \cdot \bar{E} = 0 \quad \text{—— (iii)}$$

$$* \quad \nabla \cdot \bar{B} = 0, \quad \nabla \cdot (\mu \bar{H}) = 0$$

$$\mu (\nabla \bar{H}) = 0$$

$$(\nabla \cdot \bar{H}) = 0 \quad \text{—— (iv)}$$

~~Electric~~ wave eqn $\rightarrow$

Take curl of eqn (i) on both sides.

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{E}) = -\mu \frac{\partial}{\partial t} (\bar{\nabla} \times \bar{H})$$

$$(\bar{\nabla} \cdot \bar{E}) \bar{\nabla} - (\nabla \cdot \bar{\nabla}) \bar{E} = -\mu \frac{\partial}{\partial t} (\bar{\nabla} \times \bar{H}) \quad \text{—— (v)}$$

From put ②, ③ in (v)

$$0 - \nabla^2 E = -\mu \frac{\partial}{\partial t}\left(\sigma E + \epsilon \frac{\partial E}{\partial t}\right)$$

$$\boxed{\nabla^2 E = \mu\sigma \frac{\partial E}{\partial t} + \mu\epsilon \frac{\partial^2 E}{\partial t^2}} \quad \text{—— (vi)}$$

Eqⁿ (6) is called electric ~~wave eqⁿ~~. The solⁿ is

$$\underbrace{\bar{E}(x, y, z, t)}_{\to \text{ 4 independant variables (difficult to solve)}}$$

Magnetic wave eqⁿ →

Take the curl of eqⁿ (2)

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{H}) = \sigma \bar{\nabla} \times \bar{E} + \epsilon \frac{\partial}{\partial t}(\bar{\nabla} \times \bar{E}) \quad \text{—— (7)}$$

$$(\nabla \cdot \bar{H})\bar{\nabla} - (\nabla \cdot \bar{\nabla})\bar{H} = \sigma \nabla \times \bar{E} + \epsilon \frac{\partial}{\partial t}(\nabla \times \bar{E}) \quad \text{—— (8)}$$

from ①, ④ in (8)

$$0 - \nabla^2 \bar{H} = \sigma\left(-\mu \frac{\partial \bar{H}}{\partial t}\right) + \epsilon \frac{\partial}{\partial t}\left(-\mu \frac{\partial \bar{H}}{\partial t}\right)$$

$$\boxed{\nabla^2 \bar{H} = \mu\sigma \frac{\partial \bar{H}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}} \quad \text{—— (9)}$$

eqⁿ (9) is called magnetic wave eqⁿ.

So eqⁿ (6) & (9) are called wave eqⁿ (or) Helmholtz eqⁿ

* Removing time term in eqⁿ (6), (9) leads to ~~phasor domain~~ (or) freq. domain (or) steady state domain.

1* To convert time domain to freq. domain supress Re, $e^{j\omega t}$

Example - write phasor form of $v(t) = V_m \cos(\omega t + 30°)$

$$v(t) = Re\left[V_m e^{j(\omega t + 30°)}\right] \qquad \left\{\begin{array}{l} e^{j\theta} = \cos\theta + j\sin\theta \\ e^{-j\theta} = \cos\theta - j\sin\theta \end{array}\right\}$$

$$v(t) = Re\left[V_m e^{j\omega t} e^{j30}\right]$$

$$V_s = V_m e^{j30} = V_m \angle 30°$$

(2) To get time domain eqⁿ $v(t)$ from phasor domain V_s use

$$v(t) = Re\left[V_s e^{j\omega t}\right]$$

time domain $\qquad$ phasor domain

$$\frac{\partial}{\partial t} \longrightarrow j\omega$$

time domain	phasor domain
* $\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$	$\vec{\nabla}\times\vec{E_s} = -(j\omega)\mu\vec{H_s}$
* $\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}$	$\vec{\nabla}\times\vec{H_s} = (\sigma + j\omega\epsilon)\vec{E_s}$
* $\vec{\nabla}\cdot\vec{E} = 0$	$\vec{\nabla}\cdot\vec{E_s} = 0$
* $\vec{\nabla}\cdot\vec{H} = 0$	$\vec{\nabla}\cdot\vec{H_s} = 0$
* $\nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$	$\nabla^2\vec{E_s} = \mu\sigma(j\omega)\vec{E_s} + \mu\epsilon(j\omega)^2\vec{E_s}$ $\nabla^2\vec{E_s} = \underbrace{j\omega\mu(\sigma + j\omega\epsilon)}_{\gamma^2}\vec{E_s}$ $\nabla^2\vec{E_s} = \gamma^2\vec{E_s}$
* $\nabla^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}$	$\nabla^2\vec{H_s} = \gamma^2\vec{H_s}$

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$x=0$

$x<0$	$x>0$
$\epsilon_1 = 1.5\epsilon_0$	$\epsilon_2 = 2.5\epsilon_0$ F/m
$E_1 = (\underbrace{2u_x}_{E_{N1}} - \underbrace{3u_y + 1.0u_z}_{E_{t1}})$ V/m	$E_2 = E_{t2} + E_{N2}$

Given $x=0$ constant ($x=0$) boundary, so x componant is normal to the boundary

$E_{N1} = 2u_x$, $E_t = -3u_y + 1.0u_z$

(i) $E_{t1} = E_{t2}$

$E_{t2} = -3u_y + 1.0u_z$

(ii) $D_{N1} = D_{N2} + \{\rho_s = 0\}$

$\overline{D_{N1}} = \overline{D_{N2}}$

$\epsilon_1\overline{E_{N1}} = \epsilon_2 E_{N2}$

$E_{N2} = \frac{\epsilon_1}{\epsilon_2}\overline{E_{N1}}$

$E_{N2} = 1.2u_x$

$E_2 = E_{t2} + E_{N2} = 1.2u_x - 3u_y + 1.0u_z$

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$D_{N1} = D_{N2} + \rho_s$

$E_{N1} = a\hat{x}$ & $E_{N2} = 2a\hat{x}$

& $D_{N1} - D_{N2} = \rho_s$

$\epsilon E_{N1} - \epsilon E_{N2} = \rho_s$

take air to dielectric medium
(m1) (m2)

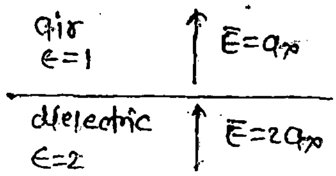
$\epsilon_0 \epsilon_r E_{N1} - \epsilon_0 \epsilon_r E_{N2} = \rho_s$

$\epsilon_0 (1)(1) - \epsilon_0 (2)(2) = \rho_s$

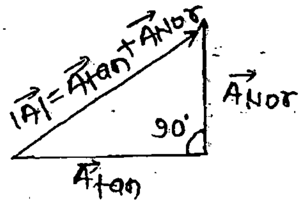
$\rho_s = -3\epsilon_0$

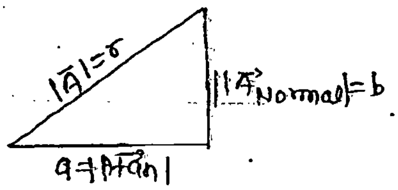
$\vec{D} = \epsilon \vec{E}$

$|E_{N1}| = \sqrt{1^2 + 0 + 0} = 1$



(32/57)

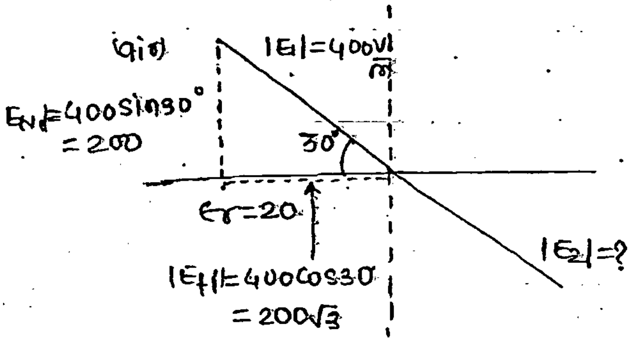




$r = \sqrt{a^2 + b^2}$

$|\vec{A}|^2 = \sqrt{|A_{tan}|^2 + |A_{Nor}|^2}$

Now in the question:-



From boundary cod^n

(i) $|E_{t1}| = |E_{t2}|$

$|E_{t2}| = 200\sqrt{3}$

(ii) $|D_{N1}| = |D_{N2}| + (\rho_s = 0)$

$\epsilon_1 E_{N1} = \epsilon_2 E_{N2}$

$E_{N2} = \frac{\epsilon_1}{\epsilon_2} E_{N1}$

$= \frac{\epsilon_0(1)}{\epsilon_0(2)} E_{N1}(200)$

$= \cancel{20}\ 10$

$|E_2| = \sqrt{|E_{t2}|^2 + |E_{N2}|^2}$

$= \sqrt{(200\sqrt{3})^2 + (10)^2}$

$|E_2| = 200\sqrt{3}$

(33/57)

$z<0$	$z=0$	$z>0$
$\mu_{r_1}=2$		$\mu_{r_2}=1$
$B_1=1.2\bar{a}_x+0.8\bar{a}_y+0.4\bar{a}_z$		$\bar{H_2}=?$

$\bar{B}_{N1}=0.4a_z$, $\bar{B}_{t1}=1.2\bar{a}_x+0.8\bar{a}_y$

Given $z=0$, so z componant is normal componants.

(1.) $\vec{H}_{t_1}=\vec{H}_{t_2}+(K=0)$

$$\frac{B_{t1}}{\mu_1}=H_{t2}$$

$$\frac{1.2a_x+0.8a_y}{2\mu_0}=\bar{H}_{t2}$$

$$\frac{(0.6a_x+0.4a_y)}{\mu_0}=\bar{H}_{t2}$$

(2.) $\bar{B}_{N1}=\bar{B}_{N2}$

$$0.4a_z=\mu_2\bar{H}_{N2}$$

$$\bar{H}_{N_2}=\frac{0.4a_z}{1\times\mu_0}$$

$$H_{N2}=\frac{0.4a_z}{\mu_0}$$

$$\vec{H_2}=\frac{0.6a_x+0.4a_y+0.4a_z}{\mu_0}$$

(34/57) $\bar{D}=2(\hat{a}_x-\sqrt{3}\hat{a}_z)\ c/m^2$

$\rho_s=?$

dielectric & cond^r boundary.

$$|D_{N_1}|=\rho_s$$

$$\sqrt{2^2+(2\sqrt{3})^2}=\rho_s$$

$$\rho_s=+4 \text{ (going away)}$$

★ Given $\bar{D}$ is pointing away from the surface

$$\rho_s=+4\frac{c}{m^2}$$

★ if its pointing towards the surface $\rho_s=-4\,c/m^2$

(35/57) On the surface of cond^r the tangential componant is 0. only normal componant is there.

dielectric cond^r boundary

$D_{N_1} = \rho_s$

$\epsilon_1 E_{N_1} = \rho_s$

$80(8.554\times10^{-12})\,2 = \rho_s$

$\rho_s = 1.41\times10^{-9}\,c/m^2$

Chapter (2) →

(1/58) Given; $\phi = \frac{1}{3}\lambda t^3$, $emf = 9V$ (at $t=3s$)

$emf = -\frac{d\phi}{dt}$

$emf = -\frac{1}{3}\lambda(3t^2)$

$emf = -\lambda t^2$

$9V = -\lambda(9)$

$\boxed{\lambda = -1\left(\frac{wb}{s^3}\right)}$

(2/58) $V_{ab} = emf$, $B = 10\cos(120\pi t)\ wb/m^2$, $mF = 0$, $\rho = 10cm = 0.1m$

$\cancel{emf =} -\frac{d\phi}{dt}$, $area = \pi\rho^2 = \pi(0.1)^2$

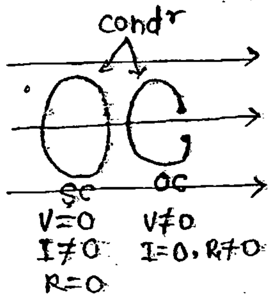
$= -\frac{\partial}{\partial t}(B \times area)$

$= -\frac{\partial}{\partial t}(10\cos 120\pi t \times \pi\, 0.1^2)$

$emf = +\frac{10\times\pi}{1}\times(0.1)^2\left[\sin 120\pi t\right]120\pi$

$\boxed{emf = 118.43\sin(\pi t)}$

(3/58) Let the cond^r coils perfect cond^r $\left(\begin{matrix}\sigma\to\infty\\ R\to 0\end{matrix}\right)$



Joule heating $= \frac{1}{2}I^2R$ | $1/2 I^2R$

$= 0$ | $= 0$

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$A = 10^{-4} m^2$, $d = 10^{-3} m$, $V = 0.5$, $f = 3.6\,GHz$

Because of freq. is given steady state domain

$\overline{Jd} = \frac{\partial D}{\partial t}$ (vector form)

$Jd = \frac{\partial D}{\partial t}$ (scalar form because Id is scalar)

$Jds = (j\omega) Ds$

$\frac{Ids}{area} = (j2\pi f) \epsilon E_s$

$Ids = (j\pi 2f) \epsilon \times \left(\frac{V}{d}\right) \times Area$

$= j2\pi \times (3.6 \times 10^{9}) \times \left(\frac{1}{36\pi} \times 10^{-9}\right) \times \left(\frac{0.5}{10^{-3}}\right) \times 10^{-4}$

$Ids = j10mA$

$\boxed{|Jds| = 10mA}$

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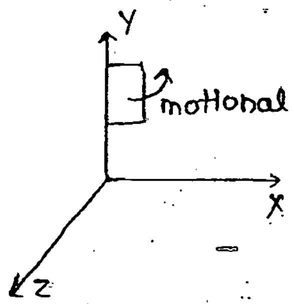

$\overline{B} = B_0 \cos(\omega t + \phi)\, \vec{a_x}$

↓

time varying (xmer)

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Radiation ⟶ EM wave generation

↓

E produces H. $(\nabla \times H = \frac{\partial D}{\partial t})$

H produces E $(\nabla \times E = -\frac{\partial B}{\partial t})$

* Sol^n of electric wave equation in phasor domain →

$$\nabla^2 \vec{E_s} = \gamma^2 \vec{E_s} \quad \text{——(i)}$$

$$\left\{ \begin{array}{l} \gamma^2 = j\omega\mu(\sigma + j\omega\epsilon) \\ \boxed{\gamma = \sqrt{j\omega\mu(\sigma+j\omega\epsilon)} = \text{Real} + j\,\text{imag.} = \alpha + j\beta\left(\frac{1}{m}\right)} \end{array} \right.$$

Let us solve in rectangular system;

$$\frac{\partial^2 \vec{E_s}}{\partial x^2} + \frac{\partial^2}{\partial y^2}\vec{E_s} + \frac{\partial^2 \vec{E_s}}{\partial z^2} = \gamma^2 \vec{E_s} \quad \text{——(ii)}$$

* Let the wave is travelling in z-dir^n; in this case, $\vec{E}$, $\vec{H}$, power are functions of z only.

$$\frac{\partial^2 \vec{E_s}}{\partial z^2} = \gamma^2 \vec{E_s} \quad \text{——(iii)}$$

in general in rectangular system vector has x, y, z componants

~~$\vec{E_s} = E_x a_x + E_y a_{ys} + E_{yz}$~~

$$\vec{E_s} = E_{xs} a_x + E_{ys} a_y + E_{zs} a_z \quad \text{——(iv)}$$

From eq^n (4.) in (3.)

$$\Rightarrow \frac{\partial^2}{\partial z^2}\left(E_{xs} a_x + E_{ys} a_y + E_{zs} a_z\right) = \gamma^2 \left(E_{xs} a_x + E_{ys} a_y + E_{zs} a_z\right)$$

$$\Rightarrow \frac{\partial^2}{\partial z^2} E_{xs} a_x + \frac{\partial^2}{\partial z^2} E_{ys} a_y + \frac{\partial^2}{\partial z^2} E_{zs} a_z = \gamma^2 E_{xs} a_x + \gamma^2 E_{ys} a_y + \gamma^2 E_{zs} a_z$$

Compare both sides;

$$\frac{\partial^2 E_{xs}}{\partial z^2} = \gamma^2 E_{xs} \quad \text{——(v)}$$

$$\frac{\partial^2 E_{ys}}{\partial z^2} = \gamma^2 E_{ys} \quad \text{——(vi)}$$

$$\frac{\partial^2 E_{zs}}{\partial z^2} = \gamma^2 E_{zs} \quad \text{——(vii)}$$

Observation →

$$\nabla \cdot \vec{E_s} = 0 \quad \text{——(8.)}$$

$$\frac{\partial E_{xs}}{\partial x} + \frac{\partial E_{ys}}{\partial y} + \frac{\partial E_{zs}}{\partial z} = 0 \quad \text{——(9.)}$$

For wave travelling in z-dir^n E_x, E_y, E_z are f^n of z only

so eq^n (9.) $\Rightarrow 0 + 0 + \frac{\partial E_{zs}}{\partial z} = 0$

$$\frac{\partial^2 E_{zs}}{\partial z^2} = 0 \quad \text{——(10)}$$

Note → (1.) for wave travelling in the z-dir^n

$$\frac{\partial^2 E_{zs}}{\partial z^2}=0 \quad \text{---(10)} \qquad \frac{\partial^2 H_{zs}}{\partial z^2}=0$$

from (10) in (7.)

$$0=\gamma^2 E_{zs}$$

$$\boxed{E_{zs}=0} \qquad \boxed{H_{zs}=0}$$

* In our case eq^n (7.) is not present;
eq^n(5.), (6.) are similar eq^n; so solving one is sufficient.

TEM → transverse = 90°

from eq^n (5.) $\frac{\partial^2 E_{xs}}{\partial z^2}=\gamma^2 E_{xs} \xrightarrow{\text{solve}} E_{xs} \xrightarrow[\text{produces}]{E\perp \bar{H}} H_{ys}$, $H_{zs}=0$

from eq^n (6.) $\frac{\partial^2 E_{ys}}{\partial z^2}=\gamma^2 E_{ys} \xrightarrow{\text{solve}} E_{ys} \xrightarrow[\text{produces}]{} H_{xs}$, $H_{zs}=0$

for wave travelling in z-dir^n $\vec{E},\vec{H}$ has only x, y componants so $\vec{E},\vec{H}$ are transeverse (90°) to the wave propogation dir^n (z dir^n). It is called as transeverse electro magnetic (TEM) wave.

solution →

$$\frac{\partial^2 E_{xs}}{\partial z^2}=\gamma^2 E_{xs} \quad \text{---(5.)}$$

$$D^2 E_{xs}=\gamma^2 E_{xs} \qquad * D=\frac{\partial}{\partial z}$$

$$(D^2-\gamma^2)E_{xs}=0$$

roots ⇒ $D^2-\gamma^2=0$

$$(D-\gamma)(D+\gamma)=0$$

$$D_1=+\gamma,\ D_2=-\gamma$$

$$E_{xs}=K_1 e^{+\gamma z}+K_2 e^{-\gamma z}$$

$$E_{xs}=\underbrace{E_{r0}e^{\gamma z}}_{\substack{\text{Reflected wave}\\ \text{(or)}\\ \text{travelling wave}\\ \text{in (-z) dir}^n}}+\underbrace{E_{i0}e^{-\gamma z}}_{\substack{\text{Incident wave}\\ \text{(or)}\\ \text{wave travelling}\\ \text{in (+z) dir}^n}}$$

If the medium impedances are matched then there is no reflected wave,

so; - $\boxed{E_{xs} = E_{i0}\, e^{-\gamma z} = E_{i0}\, e^{-(\alpha+j\beta)z} = E_{i0}\, e^{-\alpha z} e^{-j\beta z}}$ ——(11)

E_x in time domain

$$(E_x)_{z,t} = Re\{E_{xs} e^{j\omega t}\}$$

$$= Re\{E_{i0}\, e^{-\alpha z} . e^{-j\beta z} . e^{j\omega t}\}$$

$$= E_{i0}\, e^{-\alpha z} Re\{e^{j(\omega t - \beta z)}\}$$

$$\boxed{E_x(z,t) = \underbrace{E_{i0}\, e^{-\alpha z}}_{\text{magnitude}} \cos\underbrace{(\omega t - \beta z)}_{\text{phase}}}$$

Note →

(1) where α = attenuation constant $\left(\frac{\text{nepere}}{m}\right)$

β = ~~Phase shift~~ Phase constant $\left(\frac{\text{rad}}{m}\right)$

$\gamma = \alpha + j\beta$ = propogation constant $\left(\frac{1}{m}\right)$

(2) * As it is a f^n of time, distance (z), so it is called as travelling wave.

(3) For this wave on z = constant (eg:- z = 1) plane the elec. field $[E_{i0}\, e^{-\alpha t}]$ is uniform (same) at all points, ~~so the~~ so it is called uniform plane wave.

(4) Constant phase velocity

$$\boxed{U_p = \frac{\omega}{\beta}\left(\frac{\text{meter}}{\text{sec}}\right)}$$

$\omega t - \beta z$ = phase

if phase is constant then $\omega t - \beta z$ = constant

diff wrt t

$$\omega(1) - \beta\frac{dz}{dt} = 0$$

$$\frac{dz}{dt}\left(\frac{m}{s}\right) = \frac{\omega}{\beta}$$

5.) ~~If wave~~ $\underbrace{\omega t}_{\text{time}} - \underbrace{\beta z}_{\text{distance phase}}$ = phase

Time period (T) :- when wave travels t = T (sec) then time phase is changed by 2π rad.

ωt = time phase

$\omega T = 2\pi$, $\boxed{T = \frac{2\pi}{\omega}}$

$$\omega = 2\pi\left(\frac{1}{T}\right)$$

$$\omega = 2\pi f$$

distance period(λ) $\rightarrow$ when wave travels $z=\lambda$ meters then distance phase is changed by 2π rad.

$$\beta z = \text{distance phase}$$

$$\beta\lambda = 2\pi$$

$$\boxed{\lambda = \frac{2\pi}{\beta}}$$

* Magnetic Field Equation $\rightarrow$ ($\vec{H}$ due to E_{xs} only)

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$$

$$\vec{\nabla}\times\vec{E_s} = -j\omega\mu\vec{H_s}$$

$$\vec{\nabla}\times\vec{E_s} = -j\omega\mu\vec{H_s} \quad \Big| \quad E_{xs} = E_{io}e^{-\gamma z}$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{xs} & 0 & 0 \end{vmatrix} = -j\omega\mu\vec{H_s}$$

$$\Rightarrow a(0-0) - a_y\left(0 - \frac{\partial}{\partial z}E_{xs}\right) + a_z\left(0 - \underbrace{\frac{\partial}{\partial y}E_{xs}}_{0}\right) = -j\omega\mu\vec{H_s}$$

$$\Rightarrow \quad \frac{\partial}{\partial z}(E_{xs})\,a_y = -j\omega\mu\vec{H_s}$$

$$\frac{\partial}{\partial z}\left(E_{io}e^{-\gamma z}\right)a_y = -j\omega\mu\vec{H_s}$$

$$E_{io}e^{-\gamma z}(-\gamma)\,a_y = -j\omega\mu\vec{H_s}$$

$$E_{xs}(-\gamma)\,a_y = -j\omega\mu\left(H_{xs}a_x + H_{ys}a_y + H_{zs}a_z\right)$$

Compare both sides $\Rightarrow E_{xs}(-\gamma) = -j\omega\mu H_{ys}$

$$\frac{(V/m)\,E_{xs}(V)}{(A/m)\,H_{ys}(A)} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma+j\omega\epsilon)}} = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \eta$$

$$\boxed{\frac{E_{xs}}{H_{ys}}\left(\frac{V}{A}\right) = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \eta = \eta_{real} + j\eta_{img} = |\eta| e^{j\theta_\eta} (\Omega)}$$

$$\boxed{\frac{V_s}{I_s} = \frac{V_m e^{j\theta_V}}{I_m e^{j\theta_I}} = \frac{V_m}{I_m} e^{j(\theta_V - \theta_I)} = Z = R + jX = |Z| e^{j\theta} \\ = |Z| e^{j\theta}}$$

$$\frac{E_{xs}}{H_{ys}} = |\eta| e^{j\theta_\eta} \Rightarrow H_{ys} = \frac{E_{xs}}{|\eta| e^{j\theta_\eta}} = \frac{E_{i0} e^{-\alpha z} e^{-j\beta z}}{|\eta| e^{j\theta_\eta}}$$

$$H_{ys} = \frac{E_{i0}}{|\eta|} e^{-\alpha z} e^{-j\beta z} e^{-j\theta_\eta}$$

Time domain →

$$H_y(z,t) = Re\{H_{ys} e^{j\omega t}\} = Re\left\{\frac{E_{i0}}{|\eta|} e^{-\alpha z} e^{-j\beta z} e^{-j\theta_\eta} e^{j\omega t}\right\}$$

$$H_y(z,t) = \frac{E_{i0}}{|\eta|} e^{-\alpha z} Re\{e^{j(\omega t - \beta z - \theta_\eta)}\}$$

$$\boxed{H_y(z,t) = \frac{E_{i0}}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta) \\ = H_{i0} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)}$$

where $\frac{E_{i0}}{H_{i0}} = |\eta|$

Network theory

$\frac{V_m}{I_m} = |Z|$

$$= \cancel{H_{i0} \cos(\omega t - \theta_\eta)}$$

Pointing vector → Instantaneous poynting vector is defined as

$$\vec{P}(t) = \underset{\frac{V}{m}}{\vec{E}(t)} \times \underset{\frac{A}{m}}{\vec{H}(t)} \left(\frac{watt}{m^2}\right)$$

avg. pointing vector is defined as

$$P_{avg} = \frac{1}{2} Re\{\vec{E_s} \times \vec{H_s}^*\} = \frac{1}{2} Re\{\vec{E_s}^* \times \vec{H_s}\} \left(\frac{watt}{m^2}\right)$$

$$\left[\text{avg power} = \frac{1}{2} Re[V_s I_s^*] = \frac{1}{2} Re[V_s^* I_s] (watt)\right]$$

$$\vec{E} = E_{xs}\, a_x \; ; \quad E_{xs} = E_{i0}\, e^{-\alpha z} e^{-j\beta z}$$

$$\vec{H} = H_{ys}\, a_y \; ; \quad H_{ys} = \frac{E_{i0}}{|\eta|} e^{-\alpha z} e^{-j\beta z} e^{-j\theta_\eta}$$

$$\vec{P}_{avg} = \frac{1}{2} Re \left\{ \vec{E_s} \times \vec{H_s}^* \right\}$$

35/61 (i) $\bar{E} = 25\sin(\omega t + 4x)(a_y + 6a_z)$ → wave is travelling in x-dirⁿ (so E field can have y, z componants)

$(1 \cdot e^{j0} a_y + 6 e^{j0} a_z)$

Diff magnitudes (1, 6) & phase diff · 0 → linear.

(ii) $\bar{E} = 25\sin(\omega t + 4x)(a_y + j a_z)$

$(1 \cdot e^{j0} + 1 e^{j90} a_z)$

same magnitude & phase diff $90°$ → circular.

(iii) $\bar{E} = 25\sin(\omega t + 4x + 60°)(a_y + j a_z)$ → circular.

(iv) $\bar{E} = 25\sin(\omega t + 4x)\,[a_y + (1+j)a_z]$

$(1e^{j0} a_y + \sqrt{2} e^{j45} a_y)$

diff magnitudes & $45°$ ϕ diff. → elliptical

(v) $\vec{E} = 25\sin(\omega t + 4x)\,[(1-j)a_y + (1+j)a_z]$

$(\sqrt{2}e^{-j45°} a_y + \sqrt{2} e^{j45°} a_z)$

Same magnitude $(\sqrt{2}, \sqrt{2})$ & $90°$ ϕ ⇒ circular

36/62 $\vec{E} = 2\cos(\omega t + 30°)\, a_x + 2\cos(\omega t - 90°)\, a_y$

equal magnitudes & $60°$ ϕ → elliptical.

Ans (a)

37/62 Diff magnitude (E_1, E_2) & $90°$ ϕ diff.

ans (b)

38/62

Free space	dielectric
$\epsilon_r = 1, \mu_r = 1$	$\epsilon_r = 4, \mu_r = 1$

Fraction of power xmitted $= \dfrac{P_{xmitted}}{P_{incident}} = \dfrac{\frac{1}{2}\left(\frac{E_{t0}^2}{\eta_2}\right)}{\frac{1}{2}\left(\frac{E_{i0}^2}{\eta_1}\right)} = \dfrac{(E_{t0})^2 \eta_1}{(E_{i0})^2 \eta_2}$

$$= T^2 \frac{\frac{\eta_0}{\sqrt{\epsilon_{r1}}}}{\frac{\eta_0}{\sqrt{\epsilon_{r2}}}}$$

$$= T^2 \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}}$$

$$= \left(\frac{2}{3}\right)^2 \cdot \sqrt{\frac{4}{1}}$$

$$= \left(\frac{4}{9}\right) \cdot 2 = \frac{8}{9}$$

$$T = \frac{2\eta_2}{\eta_1+\eta_2} = \frac{2\sqrt{\epsilon_{r1}}}{\sqrt{\epsilon_{r1}}+\sqrt{\epsilon_{r2}}}$$

$$T = \frac{2\sqrt{1}}{\sqrt{1}+\sqrt{4}} = \frac{2}{3}$$

(40/62)

$\mu_0 \epsilon_1$ medium (1) | $\mu_0 \epsilon_2$ medium (2)

given $E_{t0} = -2E_{r0}$ find $\frac{\epsilon_2}{\epsilon_1}$

divide with E_{i0}

$$\frac{E_{t0}}{E_{i0}} = -2\frac{E_{t0}}{E_{i0}}$$

$$T = -2\Gamma$$

$$\frac{2\eta_2}{\eta_1+\eta_2} = -2\left(\frac{\eta_2-\eta_1}{\eta_1+\eta_2}\right)$$

$$\eta_2 = \eta_1 - \eta_2$$

$$2\eta_2 = \eta_1$$

$$2 = \frac{\eta_1}{\eta_2}$$

$$2 = \frac{\sqrt{\frac{\mu_0}{\epsilon_1}}}{\sqrt{\frac{\mu_1}{\epsilon_2}}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\frac{\epsilon_2}{\epsilon_1} = 4$$

Std que.

(41/62) On a cond^r surface

$E_{t1} = 0$ $\left(P_{avg} = \frac{1}{2}\frac{E_{t0}^2}{\eta} = 0\right)$

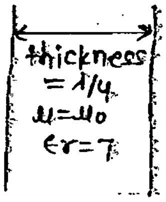
$E_{t1} = 0 \longrightarrow$ E field is min^m

$\downarrow E \perp H$

H field is max^m

ans (c)

45/63 $f = 10\times10^{9}\,\text{Hz}$



$$\text{thickness} = \frac{\lambda}{4} = \frac{\frac{v}{f}}{4} = \frac{\frac{c}{\sqrt{\mu_r\epsilon_r}}\cdot\frac{1}{f}}{4}$$

$$= \frac{\frac{3\times10^{8}}{\sqrt{(1)(7)}}\cdot\frac{1}{10^{10}}}{4} = 0.283\,\text{cm}$$

22/60 $\delta = 4\,\mu m,\ f = 400\times10^{3}$

$v = \omega\delta = 5\,m/s$

ans (c)

23/60 $v = \omega\delta$

$$= 2\pi f\,\frac{1}{\sqrt{\pi f\mu\sigma}} \propto \sqrt{f}$$

$$v \propto \sqrt{f} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{f_2}{f_1}}$$

$$\frac{v_2}{5} = \sqrt{\frac{4f_1}{f_1}}\ ;\ v_2 = 5(2)\ ;\ v_2 = 10\,m/s$$

ans (b)

24/60 Given, $\tan\theta = 1.73$

$\theta = \tan^{-1}(1.73) = 60^\circ$

$2\theta_n = \theta$

$\theta_n = \frac{\theta}{2} = 30^\circ$ ans (a)

25/60 Given, $\sigma = 10^{-2}\,\frac{\mho}{m}$; $\epsilon_r = 4$

$$\tan\theta = \frac{|J_{cs}|}{|J_{ds}|} = \frac{\sigma}{\omega\epsilon}$$

$$1 = \frac{\sigma}{\omega\epsilon}$$

$$\sigma = \omega\epsilon = 2\pi f\epsilon$$

$$f = \frac{\sigma}{2\pi 4\epsilon_0} = \frac{10^{-2}}{4\pi(4)\frac{1}{36\pi}\times10^{-9}} = 45\,\text{MHz}$$

26/60 $I_c = 1A$, $f = 50Hz$, $\epsilon = \epsilon_0$, $\mu = \mu_0$, $\sigma = 58\ \mho/m$

$$\frac{I_c}{I_d} = \frac{\sigma}{\omega\epsilon}$$

$$\frac{\frac{I_c}{area}}{\frac{I_d}{area}} = \frac{\sigma}{\omega\epsilon} \quad ; \quad \frac{I_c}{I_d} = \frac{\sigma}{\omega\epsilon}$$

$$\frac{1}{I_d} = \frac{50}{2\pi(50)\left(\frac{1}{36\pi}\times10^{-9}\right)}$$

$$I_d = 4.8\times10^{-11}A$$

27/61 $\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1-j\frac{\sigma}{\omega\epsilon}}} = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1-0}} = \sqrt{\frac{\mu}{\epsilon}}$ (real)

ans (a)

28/61 substitue in $\eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}} = 0.02e^{j\pi/4}$.

ans (a)

29/61 ans (a)

30/61 $\vec{H} = 0.1\sin(10^8t+\beta y)\,a_x$ ← wave is travelling in (-y) dire

free space

$10^8t+\beta y = 0$

$y = -\frac{10^8 t}{\beta}$

$$\vec{P}_{avg} = \frac{1}{2}\frac{|\vec{E}_0|^2}{\eta_0}a_\rho$$

$$= \frac{1}{2}|H_0|^2\eta_0 a_\rho \qquad \because \frac{E_0}{H_0} = |\eta| = \eta_0$$

$$= \frac{1}{2}(0.1)^2(120\pi)(-a_y)$$

ans:(b) $= -0.6\pi a_y$

31/61 .ans (b) in free space

$$\vec{E} = 60\cos(\omega t - 2z)\,a_y$$

$$\vec{P}_{avg} = \frac{1}{2}\cdot\frac{|E_0|^2}{\eta_0}a_\rho$$

$$= \frac{1}{2}\frac{(60)^2}{120\pi}a_x\left(\frac{Watt}{m^2}\right)$$

$\text{Power} = \vec{P}_{avg}\,(\text{area})$

$$= \frac{1}{2}\cdot\frac{60^2}{120\pi}\pi(4)^2 = 240\,W$$

(32/61) ans (c) ; $P(t) = \bar{E}(t)\times\bar{H}(t)$

(38/61) (i) $\vec{H} = 50\sin(\omega t-\beta z)a_x + 150\sin(\omega t-\beta z)a_y$

$$\vec{P}_{avg} = \frac{1}{2}\frac{|E_{i0}|^2}{\eta_0}a_\rho = \frac{1}{2}|H_{i0}|^2\eta_0 a_\rho$$

$$= \frac{1}{2}\left(\sqrt{50^2+150^2}\right)^2(120\pi)a_z$$

(39/62)

m-1

$\vec{E}_i = E_0\cos(\omega t-\beta z)\hat{a}_y$

$\beta_1 = \beta = 10\pi,\ \omega = 3\times10^9\pi$

$v_p = \frac{\omega}{\beta} = \frac{3\times10^9\pi}{10\pi} = 3\times10^8 = c$

free space ; $\epsilon_{r_1}=1$; $\mu_{r_1}=1$

given $\beta = \omega\sqrt{\epsilon_0\mu_0} = \beta = 10\pi$

m-2

$\epsilon_r = 4$

$\epsilon_t = ?$

$\vec{E}_t = E_{t0}\cos(\omega t-\beta_2 z)a_y$

$$T = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_1+\eta_2} = \frac{2\sqrt{\epsilon_{r_1}}}{\sqrt{\epsilon_{r_1}}+\sqrt{\epsilon_{r_2}}} = \frac{2\sqrt{1}}{\sqrt{1}+\sqrt{4}} = \frac{2}{3}$$

$E_{t0} = \frac{2}{3}E_{i0} = \frac{2}{3}E_0$

$\beta_2 = \omega\sqrt{\mu_2\epsilon_2}$

$\beta_2 = \omega\sqrt{\mu_0(1)\epsilon_0(4)}$

$= \omega\sqrt{\mu_0\epsilon_0}\sqrt{4}$

$\beta_2 = 2\beta$

ans (b)

(7/59) $E_x(z,t) = E_{i0}e^{-\alpha z}\cos(\omega t-\beta z)$

Initial magnitude $(z=0) = E_{i0}$

at a distance z magnitudes $= E_{i0}e^{-\alpha z}$

when em wave travels $z = 20m$ then magnitude $= \frac{1}{e}E_{i0} = e^{-1}E_{i0}$

$$\underbrace{\cancel{E_{i0}}e^{-\alpha(20)}}_{\text{Formula}} = \underbrace{\cancel{E_{i0}}e^{-1}}_{\text{given}} \Rightarrow \alpha(20) = 1$$

$$\alpha = \frac{1}{20}$$

distance phase $= \beta z$

Given for $z = 20m$ distance phase $= \frac{\pi}{6}$ rad.

$$\beta(20) = \pi/6$$

(formula) (given)

$$\beta = \frac{\pi}{120}$$

$$p = \alpha + j\beta = \frac{1}{20} + j\frac{\pi}{120}$$

8/59 Given for $z = 5m$. magnitudes decays 20%. E_{io}

$$E_{io} e^{-\alpha(5)} = \frac{20}{100} E_{io}$$

$$\alpha = 0.321$$

after what distance magnitudes decays 40%. E_{io}

$$E_{io} \cdot e^{-(0.321)z} = \frac{40}{100} E_{io}$$

$$z = 2.85m$$

9/59

$$E_{(x,t)} = \underset{\substack{\downarrow \\ E_{io}}}{25} \sin(\omega t + \underset{\substack{\downarrow \\ \beta = 4}}{4x})\, \underset{\substack{\downarrow \\ a_E = a_y}}{a_y}$$

$\alpha = 0$

$\rightarrow$ wave is travelling in $-x$ dirn.

$a_p = -a_x$

* propogation $\rightarrow -x$ dirn
* $\boxed{v_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{\mu_r \epsilon_r}}}$ $\boxed{\lambda = \frac{2\pi}{\beta} = \frac{\lambda_0}{\sqrt{\mu_r \epsilon_r}}}$

$$c = \frac{\omega}{\beta}\ ; \ \omega = c\beta = 3\times10^8(4) = 12\times10^8$$

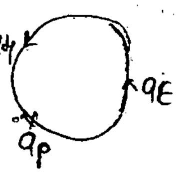
$$f = \frac{12\times10^8}{2\pi}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{4}$$

* $H(x,t) = H_{io} \sin(\omega t + 4x)\, a_H \rightarrow a_p \times a_E$

$-a_x \times a_y$ a_H

$-a_z$



$$\frac{E_{io}}{H_{io}} = |\eta| = \eta_0 = 120\pi$$

$$H_{io} = \frac{E_{io}}{120\pi} = \frac{25}{120\pi}$$

$$H(x,t) = \frac{25}{120\pi} \sin(\omega t + 4x)(-a_z)$$

(11/59) $\vec{H} = 0.5e^{-0.1x}\cos(10^6t - 2x)a_z$

$$a_E = a_H \times a_\beta$$
$$= a_z \times a_x$$
$$a_E = +a_y$$ = wave is polarised in +y dirn.

ans. (d)

(13/59) ans.(a)

(12/59) ans.(b) Free space $\vec{E} = \eta_0 \sin(10^7 t + kz)a_y$

$$\lambda = \frac{v}{f} = \frac{c}{f} = \frac{3\times10^8}{(10^7/2\pi)} = 188.5\,m$$

(14/59) $\mu = \mu_0;\ \epsilon = 81\epsilon_0$

~~$\vec{E} = 0.5\sin(10^8t$~~ $\vec{E} = 10\cos(2\pi\times10^8 t - \beta x)a_y$

$$v_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

$$\frac{6\pi\times10^8}{\beta} = \frac{3\times10^8}{\sqrt{(1)(81)}}$$

$$\beta = 18\pi$$

ans.(c)

(15/59) $\epsilon_r = 8,\ \mu_r = 2;\ \sigma = 0$

$$\eta = \eta_0\sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0}}\sqrt{\frac{\mu_r}{\epsilon_r}}$$

ans.(d)

$$= 120\pi\sqrt{\frac{2}{8}} = 60\pi$$

(16/59) ans.(d)

(17/60) $\lambda_0 = 2cm$

ans(c)

$\lambda = 1cm$ (dielectric)

$$\lambda = \frac{\lambda_0}{\sqrt{\mu_r\epsilon_r}}$$

$$1cm = \frac{2cm}{\sqrt{(1)\epsilon_r}} \qquad \epsilon_r = 4$$

18/60 ans·(c)

19/60 $f = 10\times10^9$

ans·(d)

3mm

$\beta(3\times10^{-3}) = \frac{\pi}{2}$

$\beta = \frac{\pi}{2(3\times10^{-3})}$

$$v_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

$$\frac{2\pi(10)^{10}}{\frac{\pi}{6\times10^{-3}}} = \frac{3\times10^8}{\sqrt{(1)\epsilon_r}} \qquad \epsilon_r = 6.25$$

20/60 +z dir^n; $E = E_0 e^{-j\beta z}$

$= E_0 e^{-jkz}$

ans·(c)

21/60

$f = 14\times10^9$ Hz

ϵ_r; $\mu_r = 1$

$E = E_0 e^{j(\omega t - 280\pi y)} a_z$

$\vec{H} = 3 e^{j(\omega t - 280\pi y)} a_x$

$\beta = 280\pi$

$$v_p = \frac{\omega}{\beta} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

$$\frac{2\pi(14\times10^9)}{280\pi} = \frac{3\times10^8}{\sqrt{\epsilon_r(1)}}$$

$\epsilon_r = 9$

$$\frac{E_{i0}}{H_{i0}} = 120\pi\sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$= 120\pi\sqrt{\frac{1}{9}}$$

$$\frac{E_{i0}}{3} = \frac{120\pi}{\sqrt{9}}$$

$E_{i0} = 120\pi$