6

CHAPTER

LINES AND ANGLES

CONTENTS

➢ SOME BASIC TERMS

◈ **Line segment:**

A part (or portion) of a line with two end points

◈ **Ray:**

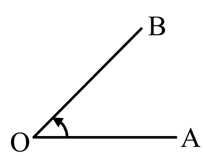
A part of a line with one end point.

◈ **Collinear points:**

If three or more points lie on the same line other wise these are called non collinear points.

◈ **Angle:**

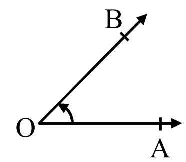
Two rays with a common end point form an angle.



OA, OB are rays & O is end point.

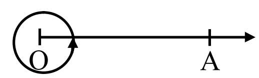
◈ **Measure of an angle:**

The amount of turning from OA to OB is called the measure of ∠AOB, written as m∠AOB. An angle is measured in degrees denoted by '°'.



◈ **An angle of 360°:**

If a ray OA starting from its original position OA, rotates about O, in the anticlockwise direction and after making a complete revolution it comes back to its original position, we say that it has rotated through 360 degrees, written as 360°.



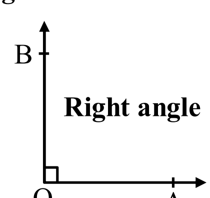
This complete rotation is divided into 360 equal parts. Each part measures 1°.

1° = 60 minutes, written as 60'.

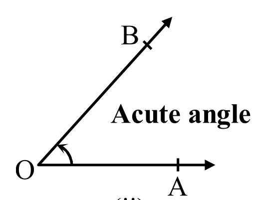
1' = 60 seconds, written as 60".

We use a protractor to measure an angle.

◈ **Types of angles:**

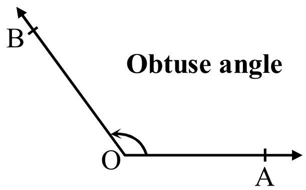


(i)



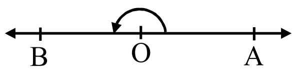
(ii)

Obtuse angle



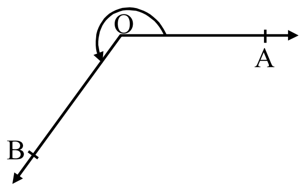
(iii)

Straight angle



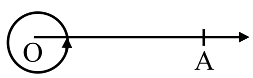
(iv)

Reflex angle



(v)

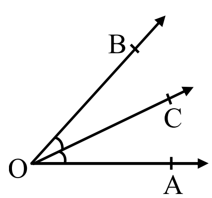
Complete angle



(vi)

◈ Bisector of an angle:

A ray OC is called the bisector of $\angle AOB$, if $m\angle AOC = m\angle BOC$.



In this case, $\angle AOC = \angle BOC = \frac{1}{2}\angle AOB$.

◈ Complementary angles:

Two angles are said to be complementary, if the sum of their measures is 90°.

Two complementary angles are called the complement of each other.

Ex. : Angles measuring 20° and 70° are complementary angles.

◈ Supplementary angles:

Two angles are said to be supplementary, if the sum of their measures is 180°.

Two supplementary angles are called the supplement of each other.

Ex. : Angles measuring 60° and 120° are supplementary angles.

❖ EXAMPLES ❖

Ex.1 Find the measure of an angle which is 20° more than its complement.

Sol. Let the measure of the required angle be x°.

Then, measure of its complement = $(90 - x)°$.

$\therefore\ x - (90 - x) = 20 \Leftrightarrow 2x = 110 \Leftrightarrow x = 55$

Hence, the measure of the required angle is 55°.

Ex.2 Find the measure of an angle which is 40° less than its supplement.

Sol. Let the measure of the required angle be x°.

Then, measure of its supplement = $(180 - x)°$.

$\therefore\ (180 - x) - x = 40 \Leftrightarrow 2x = 140 \Leftrightarrow x = 70$

Hence, the measure of the required angle is 70°.

Ex.3 Find the measure of an angle, if six times its complement is 12° less than twice its supplement.

Sol. Let the measure of the required angle be x°.

Then, measure of its complement = $(90 - x)°$.

Measure of its supplement = $(180 - x)°$.

$\therefore\ 6(90 - x) = 2(180 - x) - 12$

$\Leftrightarrow\ 540 - 6x = 360 - 2x - 12$

$\Leftrightarrow\ 4x = 192 \Leftrightarrow x = 48$.

Hence the measure of the required angle is 48°.

Ex.4 Convert 180° in degree, minute & second.

Sol. $180° = 179°\ 59'\ 60''$.

Ex.5 Find the measure of the supplement of an angle of $87°28'43''$.

Sol. We may write, $180° = 179°59'60''$.

$\therefore$ supplement of an angle of $(87°28'43'')$

$= \text{an angle of } [180° - (87°28'43'')]$

$= \text{an angle of } [179°59'60'' - 87°28'43'']$

$= \text{an angle of } (92°31'17'')$.

Hence, the measure of the required angle

$= (92°31'17'')$.

Ex.6 If $\angle A = 36°27'46''$ and $\angle B = 28°43'39''$, find $\angle A + \angle B$.

Sol. $\angle A + \angle B = 36°27'46'' + 28°43'39''$

$= 64°70'85'' = 64°71'25'' = 65°11'25''$

Ex.7 Find the complement of each of the following angles :

(i) 58° (ii) 16°

(iii) $\frac{2}{3}$ of a right angle (iv) 46° 30'

Sol. (i) $90° - 58° = 32°$

(ii) $90° - 16° = 74°$

(iii) $90° - \frac{2}{3}(90°) = 90° - 60° = 30°$

(iv) $90° - 46° 30'$

$= 89° 60' - 46° 30'$

$= 43° 30'$

Ex.8 Find the measure of an angle which is complement of itself.

Sol. Let the measure of the angle be x^o, Then,

Then, the measure of its complement is given to be x^o.

Since, the sum of the measures of an angle and its complement is 90^o

$\therefore \quad x^o + x^o = 90^o \Rightarrow 2x^o = 90^o \Rightarrow x^o = 45^o$

Ex.9 Find the measure of an angle which forms a pair of supplementary angles with itself.

Sol. Let the measure of the angle be x^o. Then,

$x^o + x^o = 180^o \Rightarrow 2x^o = 180^o \Rightarrow x^o = 90^o$

Ex.10 An angle is equal to five times its complement. Determine its measure.

Sol. Let the measure of the given angle be x degrees. Then, its complement is $(90 - x)^o$.

It is given that :

Angle = 5 × Its complement

$\Rightarrow x = 5(90 - x) \quad \Rightarrow x = 450 - 5x$

$\Rightarrow 6x = 450 \quad \Rightarrow x = 75$

Thus, the measure of the given angles is 75^o.

Ex.11 An angle is equal to one-third of its supplement. Find its measure.

Sol. Let the measure of the required angle be x degrees. Then,

Its supplement = $180^o - x$. It is given that :

Angle = $\frac{1}{3}$ (Its supplement)

$\Rightarrow x = \frac{1}{3}(180^o - x) \quad \Rightarrow 3x = 180^o - x$

$\Rightarrow 4x = 180^o \quad \Rightarrow x = 45^o$

Thus, the measure of the given angle is 45^o.

Ex.12 Two supplementary angles are in the ratio 2 : 3. Find the angles.

Sol. Let the two angles be 2x and 3x in degrees. Then,

$2x + 3x = 180^o$

$\Rightarrow 5x = 180^o \quad \Rightarrow x = 36^o$

Thus, the two angles are $2x = 2 \times 36^o = 72^o$

and $3x = 3 \times 36^o = 108^o$

Ex.13 Write the complement of the following angles: $30^o 20'$

Sol. Complement of

$30^o20' = 90^o - 30^o 20'$

$= 90^o - (30^o + 20')$

$= (89^o - 30^o) + (1^o - 20')$

$= 59^o + (60' - 20') \quad [\Theta\ 1^o = 60']$

$= 59^o + 40' = 59^o 40'$

Ex.14 Find the supplement of the following angles : $134^o 30' 26''$

Sol. Supplement of an angle of $134^o 30' 26''$

$= 180^o - (134^o 30' 26'')$

$= (179^o - 134^o) + (1^o - 30' 26'')$

$= 45^o + (60' - (30' + 26'')) \quad [\Theta\ 1^o = 60']$

$= 45^o + (59' - 30') + (1' - 26'')$

$= 45^o + 29' + 34'' \quad = 45^o29'34''$

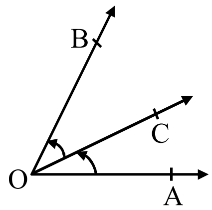
◈ Adjacent angles:

Two angles are called adjacent angles, if

(i) they have the same vertex,

(ii) they have a common arm and

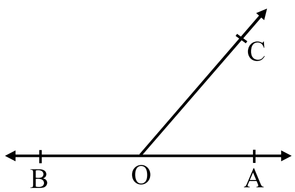
(iii) their non-common arms are on either side of the common arm.



In the given figure, ∠AOC and ∠BOC are adjacent angles having the same vertex O, a common arm OC and their non-common arms OA and OB on either side of OC.

◈ **Linear pair of angles:**

Two adjacent angles are said to form a linear pair of angles, if their non-common arms are two opposite rays.



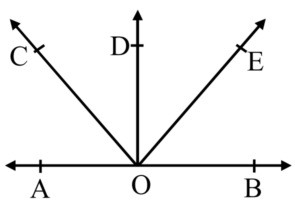
In the adjoining figure, ∠AOC and ∠BOC are two adjacent angles whose non-common arms OA and OB are two opposite rays, i.e., BOA is a line

∴ ∠AOC and ∠BOC form a linear pair of angles.

Theorem 1 :

Prove that the sum of all the angles formed on the same side of a line at a given point on the line is 180°.

Given : AOB is a straight line and rays OC, OD and OE stand on it, forming ∠AOC, ∠COD, ∠DOE and ∠EOB.



To prove : ∠AOC + ∠COD + ∠DOE + ∠EOB = 180°.

Proof : Ray OC stands on line AB.

∴ ∠AOC + ∠COB = 180°

⇒ ∠AOC + (∠COD + ∠DOE + ∠EOB) = 180°

[Θ∠COB = ∠COD + ∠DOE + ∠EOB]

⇒ ∠AOC + ∠COD + ∠DOE + ∠EOB = 180°.

Hence, the sum of all the angles formed on the same side of line AB at a point O on it is 180°.

Theorem 2 :

Prove that the sum of all the angles around a point is 360°.

Given : A point O and the rays OA, OB, OC, OD and OE make angles around O.

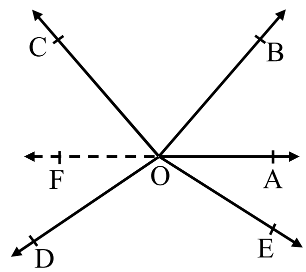
To prove : ∠AOB + ∠BOC + ∠COD + ∠DOE + ∠EOA = 360°

Construction : Draw a ray OF opposite to ray OA.

Proof : Since ray OB stands on line FA, we have : ∠AOB + ∠BOF = 180° [linear pair]

∴ ∠AOB + ∠BOC + ∠COF = 180° ….(i)

[Θ∠BOF = ∠BOC + ∠COF]



Again, ray OD stands on line FA.

∴ ∠FOD + ∠DOA = 180° [linear pair]

or ∠FOD + ∠DOE + ∠EOA = 180° …(ii)

[Θ∠DOA = ∠DOE + ∠EOA]

Adding (i) and (ii), we get :

∠AOB + ∠BOC + ∠COF + ∠FOD + ∠DOE + ∠EOA = 360°

∴ ∠AOB + ∠BOC + ∠COD + ∠DOE + ∠EOA = 360°

[Θ ∠COF + ∠FOD = ∠COD]

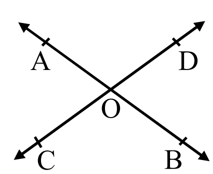
Hence, the sum of all the angles around a point O is 360°.

◈ **Vertically opposite angles:**

Two angles are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.

Let two lines AB and CD intersect at a point O. Then, two pairs of vertically opposite angles are formed :

(i) ∠AOC and ∠BOD (ii) ∠AOD and ∠BOC



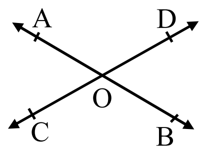
Theorem 3 :

If two lines intersect then the vertically opposite angles are equal.

Given : Two lines AB and CD intersect at a point O.

To prove : (i) ∠AOC = ∠BOD, (ii) ∠AOD = ∠BOC

Proof : Since ray OA stands on line CD, we have:



∠AOC + ∠AOD = 180° [linear pair].

Again, ray OD stands on line AB.

∴ ∠AOD + ∠BOD = 180° [linear pair]

∴ ∠AOC + ∠AOD = ∠AOD + ∠BOD

[each equal to 180°]

∴ ∠AOC = ∠BOD

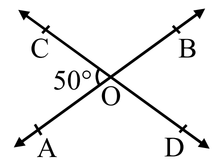
Similarly, ∠AOD = ∠BOC

➤ IMPORTANT POINTS

- Two angles are called adjacent angles if –
  - (i) they have the same vertex,
  - (ii) they have a common arm, and
  - (iii) uncommon arms are on either side of the common arm.
- Two adjacent angles are said to form a linear pair of angles, if the non-common arms are two opposite rays.
- If a ray stands on a line, then the sum of the adjacent angles so formed is 180º.
- If the sum of two adjacent angles is 180º, then their non-common arms are two opposite rays.
- The sum of all the angles round a point is equal to 360º.
- Two angle are called a pair of vertically opposite angles, if their arms form two pairs of opposite rays.
- If two lines intersect, then the vertically opposite angles are equal.

❖ EXAMPLES ❖

Ex.15 Two lines AB and CD intersect at O. If ∠AOC = 50°, find ∠AOD, ∠BOD and ∠BOC.



Sol. ∠AOD + ∠AOC = 180° (linear pair)

∠AOD + 50° = 180°

∠AOD = 130°

Also ∠BOD = ∠AOC

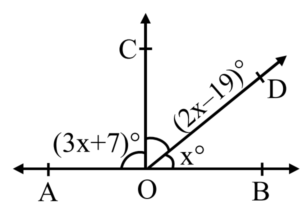
(vertically opposite angles)

& ∠BOC = ∠AOD = 130°

(vertically opposite angles)

∴ 130°, 50°, 130°.

Ex.16 In the adjoining figure, AOB is a straight line. Find the value of x. Hence, find ∠AOC, ∠COD and ∠BOD.



Sol. (3x + 7)° + (2x – 19)° + x° = 180' (linear pair)

⇒ (6x – 12) = 180°

⇒ 6x = 192°

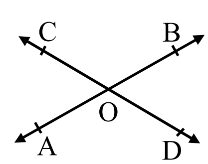
⇒ x = 32°

∴ ∠AOC = 3x + 7 = 3(32) + 7 = 96 + 7 = 103°

∠COD = 2x – 19 = 2(32) – 19 = 64 – 19 = 45°

∠BOD = x° = 32°.

Ex.17 Two lines AB and CD intersect at a point O such that ∠BOC + ∠AOD = 280°, as shown in the figure. Find all the four angles.



Sol. Θ $\angle AOC = \angle BOD = x$(Let)

(vertically opposite angles)

$\therefore \angle AOC + (\angle AOD + \angle BOC) + \angle BOD = 360°$

$\Rightarrow x + 280° + x = 360°$

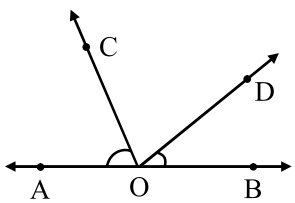
$\Rightarrow 2x = 80°$

$\Rightarrow x = 40°$

$\therefore \angle AOC = \angle BOD = x° = 40°$.

and $\angle BOC = \angle AOD = \dfrac{280}{2} = 140°$.

Ex.18 In figure, OA, OB are opposite rays and $\angle AOC + \angle BOD = 90°$. Find $\angle COD$.



Sol. Since OA and OB are opposite rays. Therefore, AB is a line. Since ray OC stands on line AB.

$\therefore \angle AOC + \angle COB = 180°$

$\Rightarrow \angle AOC + \angle COD + \angle BOD = 180°$

[Θ $\angle COB = \angle COD + \angle BOD$]

$\Rightarrow (\angle AOC + \angle BOD) + \angle COD = 180°$

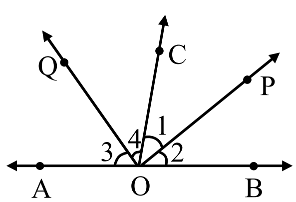
$\Rightarrow 90° + \angle COD = 180°$

[Θ $\angle AOC + \angle BOD = 90°$ (Given)]

$\Rightarrow \angle COD = 180° - 90° = 90°$

Ex.19 In figure, OP bisects $\angle BOC$ and OQ, $\angle AOC$. Show that $\angle POQ = 90°$.

Sol.



According to question, OP is bisector of $\angle BOC$

$\therefore \angle 1 = \angle 2$

or $\angle 1 = \dfrac{\angle BOC}{2}$

and OQ is bisector of $\angle AOC$

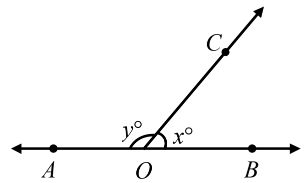
$\therefore \angle 3 = \angle 4$

or $\angle 4 = \dfrac{\angle AOC}{2}$

$\therefore \angle 1 + \angle 4 = \dfrac{\angle BOC}{2} + \dfrac{\angle AOC}{2}$

$= \dfrac{\angle BOC + \angle AOC}{2} = \dfrac{180}{2} = 90°$

Ex.20 In figure OA and OB are opposite rays :



(i) If x = 75, what is the value of y ?

(ii) If y = 110, what is the value of x ?

Sol. Since $\angle AOC$ and $\angle BOC$ form a linear pair.

Therefore, $\angle AOC + \angle BOC = 180°$

$\Rightarrow x + y = 180°$ ...(1)

(i) If x = 75, then from (i)

$75 + y = 180°$

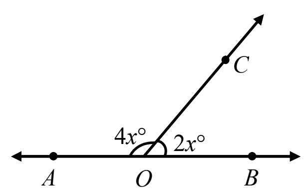
$y = 105°$.

(ii) If y = 110 then from (i)

$x + 110 = 180$

$\Rightarrow x = 180 - 110 = 70$.

Ex.21 In figure $\angle AOC$ and $\angle BOC$ form a linear pair. Determine the value of x.



Sol. Since $\angle AOC$ and $\angle BOC$ form a linear pair.

$\therefore \angle AOC + \angle BOC = 180°$

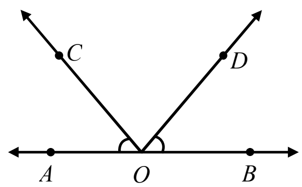
$\Rightarrow 4x + 2x = 180°$

$\Rightarrow 6x = 180°$

$\Rightarrow x = \dfrac{180°}{6} = 30°$

Thus, x = 30°

Ex.22 In figure OA, OB are opposite rays and $\angle AOC + \angle BOD = 90°$. Find $\angle COD$.



Sol. Since OA and OB are opposite rays. Therefore, AB is a line. Since ray OC stands on line AB. Therefore,

$\angle AOC + \angle COB = 180^\circ$ [Linear Pairs]

$\Rightarrow \angle AOC + \angle COD + \angle BOD = 180^\circ$

$[\Theta \angle COB = \angle COD + \angle BOD]$

$\Rightarrow (\angle AOC + \angle BOD) + \angle COD = 180^\circ$

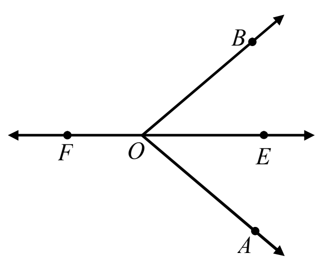
$\Rightarrow 90^\circ + \angle COD = 180^\circ$

$[\Theta \angle AOC + \angle BOD = 90^\circ$ (Given)]

$\Rightarrow \angle COD = 180^\circ - 90^\circ$

$\Rightarrow \angle COD = 90^\circ$

Ex.23 In figure ray OE bisects angle $\angle AOB$ and OF is a ray opposite to OE. Show that $\angle FOB = \angle FOA$.



Sol. Since ray OE bisects angle AOB. Therefore,

$\angle EOB = \angle EOA$ ....(i)

Now, ray OB stands on the line EF.

$\therefore \angle EOB + \angle FOB = 180^\circ$ ...(ii) {linear pair]

Again, ray OA stands on the line EF.

$\therefore \angle EOA + \angle FOA = 180^\circ$ ....(iii)

Form (ii) and (iii), we get

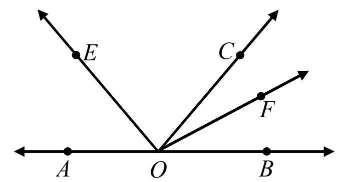
$\angle EOB + \angle FOB = \angle EOA + \angle FOA$

$\Rightarrow \angle EOA + \angle FOB = \angle EOA + \angle FOA$

$[\Theta \angle EOB = \angle EOA$ (from (i)]

$\Rightarrow \angle FOB = \angle FOA$.

Ex.24 In figure OE bisects $\angle AOC$, OF bisects $\angle COB$ and $OE \perp OF$. Show that A, O, B are collinear.



Sol. Since OE and OF bisect angles AOC and COB respectively. Therefore,

$\angle AOC = 2\angle EOC$ ....(i)

and $\angle COB = 2\angle COF$ ....(ii)

Adding (i) and (ii), we get

$\angle AOC + \angle COB = 2\angle EOC + 2\angle COF$

$\Rightarrow \angle AOC + \angle COB = 2(\angle EOC + \angle COF)$

$\Rightarrow \angle AOC + \angle COB = 2(\angle EOF)$

$\Rightarrow \angle AOC + \angle COB = 2 \times 90^\circ$

$[\Theta OE \perp OF \therefore \angle EOF = 90^\circ]$

$\Rightarrow \angle AOC + \angle COB = 180^\circ$

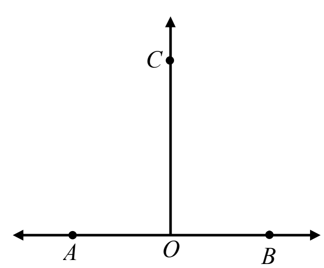
But $\angle AOC$ and $\angle COB$ are adjacent angles.

Thus, $\angle AOC$ and $\angle COB$ are adjacent supplementary angles. So, $\angle AOC$ and $\angle COB$ form a linear pair. Consequently OA and OB are two opposite rays. Hence, A, O, B are collinear.

Ex.25 If ray OC stands on line AB such that $\angle AOC = \angle COB$, then show that $\angle AOC = 90^\circ$.

Sol. Since ray OC stands on line AB. Therefore,

$\angle AOC + \angle COB = 180^\circ$ [Linear pair] ...(i)

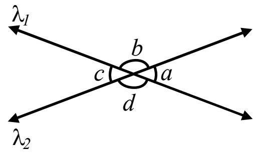


But $\angle AOC = \angle COB$ (Given)

$\therefore \angle AOC + \angle AOC = 180^\circ$

$\Rightarrow 2\angle AOC = 180^\circ \quad \Rightarrow \angle AOC = 90^\circ$

Ex.26 In Fig., lines λ_1 and λ_2 intesect at O, forming angles as shown in the figure. If a = 35º, find the values of b, c, and d.



Sol. Since lines λ_1 and λ_2 intersect at O. Therefore,

$\angle a = \angle c$ [Vertically opposite angles]

$\Rightarrow \angle c = 35^\circ$ $[\therefore \angle a = 35^\circ]$

Clearly, $\angle a + \angle b = 180^\circ$

[Since $\angle a$ and $\angle b$ are angles of a linear pair]

$\Rightarrow 35^\circ + \angle b = 180^\circ$

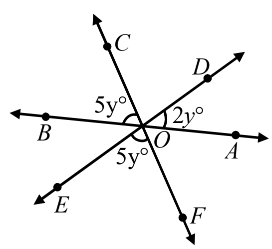
$\Rightarrow \angle b = 180^\circ - 35^\circ$

$\Rightarrow \angle b = 145^\circ$

Since $\angle b$ and $\angle d$ are vertically opposite angles. Therefore,

$\angle d = \angle b \Rightarrow \angle d = 145^\circ$ $[\therefore \angle b = 145^\circ]$

Ex.27 In Fig., determine the the value of y.



Sol. Since $\angle COD$ and $\angle EOF$ are vertically opposite angles. Therefore,

$\angle COD = \angle EOF \Rightarrow \angle COD = 5y^\circ$

$[\therefore \angle EOF = 5y^\circ$ (Given)]

Now, OA and OB are opposite rays.

$\therefore \angle AOD + \angle DOC + \angle COB = 180^\circ$

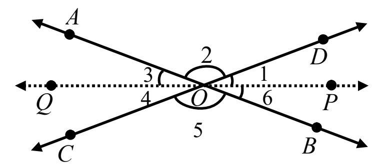
$\Rightarrow 2y^\circ + 5y^\circ + 5y^\circ = 180^\circ$

$\Rightarrow 12y^\circ = 180^\circ$

$\Rightarrow y^\circ = \frac{180^\circ}{12} = 15.$

Thus, $y^\circ = 15.$

Ex.28 In Fig., AB and CD are straight lines and OP and OQ are respectively the bisectors of angles BOD and AOC. Show that the rays OP and OQ are in the same line.



Sol. In order to prove that OP and OQ are in the same line, it is sufficient to prove that $\angle POQ = 180^\circ$.

Now, OP is the bisector of $\angle AOC$

$\Rightarrow \angle 1 = \angle 6$ ...(i)

and, OQ is the bisector of $\angle AOC$

$\Rightarrow \angle 3 = \angle 4$ ....(ii)

Clearly, $\angle 2$ and $\angle 5$ are vertically opposite angles.

$\therefore \angle 2 = \angle 5$ ....(iii)

We know that the sum of the angles formed at a point is 360º.

Therefore,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 360^\circ$

$\Rightarrow (\angle 1 + \angle 6) + (\angle 3 + \angle 4) + (\angle 2 + \angle 5) = 360^\circ$

$\Rightarrow 2\angle 1 + 2\angle 3 + 2\angle 2 = 360^\circ$

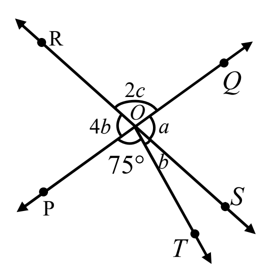
[Using (i), (ii) and (iii)]

$\Rightarrow 2(\angle 1 + \angle 3 + \angle 2) = 360^\circ$

$\Rightarrow \angle 1 + \angle 2 + \angle 3 = 180^\circ$ $\Rightarrow \angle POQ = 180^\circ$

Hence, OP and OQ are in the same straight line.

Ex.29 In Fig., two staright lines PQ and RS intersect each other at O. If $\angle POT = 75^\circ$, find the values of a, b and c.



Sol. Since OR and OS are in the same line. Therefore,

$$\angle ROP + \angle POT + \angle TOS = 180^\circ$$

$\Rightarrow$ $4b^\circ + 75^\circ + b^\circ = 180^\circ \Rightarrow 5b^\circ + 75^\circ = 180^\circ$

$\Rightarrow$ $5b^\circ = 105^\circ \Rightarrow$ $b^\circ = 21$

Since PQ and RS intersect at O. Therefore,

$\angle QOS = \angle POR$

[Vertically oppsostie angles]

$\Rightarrow$ $a = 4b$

$\Rightarrow$ $a = 4 \times 21 = 84$ $[\because\ b = 21]$

Now, OR and OS are in the same line. Therefore.

$\angle ROQ + \angle QOS = 180^\circ$ [Linear pair]

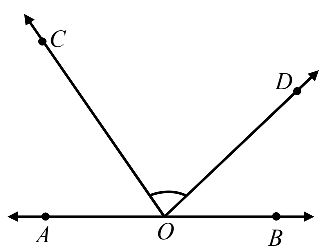
$\Rightarrow$ $2c + a = 180$

$\Rightarrow$ $2c + 84 = 180$ $[\because\ b = 84]$

$\Rightarrow$ $2c = 96 \Rightarrow$ $c = 48$

Hence, a = 84, b = 21 and c = 48

Ex.30 In fig if $\angle AOC + \angle BOD = 70^\circ$, find $\angle COD$.



Sol. $\angle AOC + \angle COD + \angle BOD = 180^\circ$

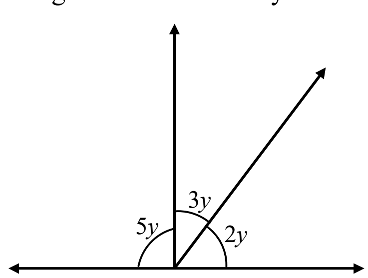
or $(\angle AOC + \angle BOD) + \angle COD = 180^\circ$

or $70^\circ + \angle COD = 180^\circ$

or $\angle COD = 180^\circ - 70^\circ$

or $\angle COD = 110^\circ$

Ex.31 In fig. find the value of y.

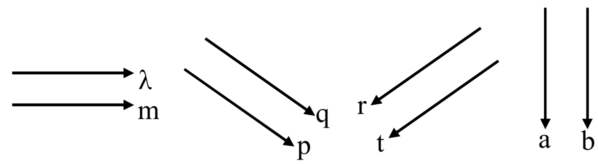


Sol. $2y + 3y + 5y = 180^\circ$

$\Rightarrow 10y = 180^\circ \Rightarrow y = \dfrac{180^\circ}{10^\circ} = 18^\circ$

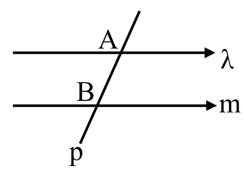
◈ **Parallel lines:**

The lines which are in same plane and do not intersect each other an y where, i.e. distance between parallel lines is same anywhere



◈ **Transversal line :**

A line which intersect two or more given lines at distinct points, is called a transversal of the given lines.



Here λ || m and p is transversal line.

* Part of transversal which is between the two lines is called intercept (AB).

➤ **IMPORTANT POINTS**

◈ Two angles on the same side of a transversal are known as the corresponding angles if both lie either above the two lines or below the two lines.

◈ The pairs of interior angles on the same side of the transversal are called pairs of consecutive interior angles.

◈ If a transversal intersect two parallel lines, then each pair of corresponding angles are equal.

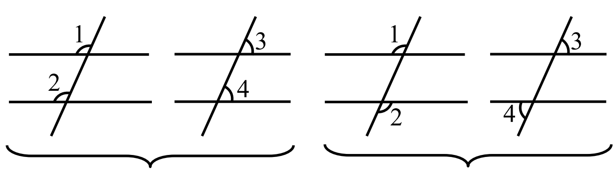
◈ If a transversal intersects two parallel lines, then each pair of alternate interior angles are equal .

◈ If a transversal intersects two lines in such a way that a pair of alternate interior angles are equal, then the two lines are parallel.

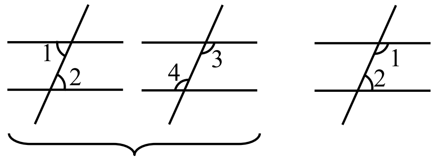
◈ If a transversal intersects two parallel lines, then each pair of consecutive interior angles are supplementary.

- ◈ If a transversal intersects two lines in such a way that a pair of consecutive interior angles are supplementary, then the two lines are parallel.
- ◈ If two parallel lines are intersected by a transversal, the bisectors of any pair of alternate interior angles are parallel.
- ◈ If two parallel lines are intersected by a transversal, then bisectors of any two corresponding angles are parallel.
- ◈ If the bisectors of a pair of corresponding angles formed by a transversal with two given lines are parallel, prove that the given lines are parallel.
- ◈ If a line is perpendicular to one of two given parallel lines, then it is also perpendicular to the other line.
- ◈ If a side of a triangle is produced, the exterior angle so formed is equal to the sum of two interior opposite angles.
- ◈ If all sides of a polygon are equal it is called a regular polygon.
- ◈ **Sum of all the interior angles of a polygon of n-sides = $(n-2) \times 180^o$ $(n \geq 3)$**
- ◈ **Each interior angle of a regular polygon of n-sides =** $\frac{(n-2)\times 180^o}{n}$
- ◈ **Sum of all the exterior angles formed by producing the sides of polygon = 360^o.**
- ◈ **No. of sides of polygon**

$$= \frac{360^o}{180^o - \text{each interior angle}}$$



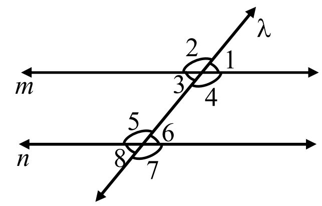
Corresponding angles   Exterior   Alternate angles



Interior alternate angles   Cointerior angles

❖ EXAMPLES ❖

Ex.32 In figure m || n and ∠1 = 65°. Find ∠5 and ∠8.



Sol. We have,

∠1 = ∠3 [Vertically opposite angles]

and, ∠3 = ∠8 [Corresponding angles]

∴ ∠1 = ∠8

⇒ ∠8 = 65° [Θ ∠1 = 65° (Given)]

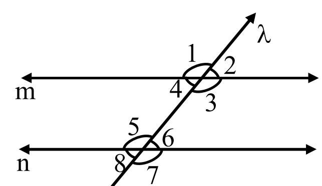
Now, ∠5 + ∠8 = 180°

⇒ ∠5 + 65° = 180°

⇒ ∠5 = 180° – 65° = 115°

Thus, ∠5 = 115° and ∠8 = 65°.

Ex.33 In figure m || n and angles 1 and 2 are in the raito 3 : 2. Determine all the angles from 1 to 8.



Sol. It is given that ∠1 : ∠2 = 3 : 2. So, let

∠1 = 3x° and ∠2 = 2x°

But ∠1 and ∠2 form a linear pair.

∴ ∠1 + ∠2 = 180°

⇒ 3x° + 2x° = 180° ⇒ 5x° = 180°

⇒ $x = \frac{180^o}{5} = 36^o$

∴ ∠1 = 3x° = (3 × 36)° = 108°

and, ∠2 = 2x° = (2 × 36)° = 72°

Now, ∠1 = ∠4 and ∠2 = ∠3

[Vertically opposite angles]

∴ ∠4 = 72° and ∠3 = 108°

Now, $\angle 6 = \angle 2^o$ and $\angle 3 = \angle 7$

[Corresponding angles]

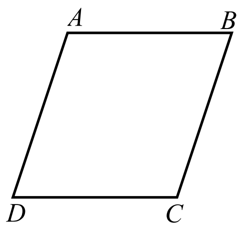
$\Rightarrow$ $\angle 6 = 72^o$ and $\angle 7 = 108^o$ [Θ $\angle 2 = 72^o$]

Again, $\angle 5 = \angle 7$ and $\angle 8 = \angle 6$

$\therefore$ $\angle 5 = 108^o$ and $\angle 8 = 72^o$

Hence, $\angle 1 = 108^o$, $\angle 2 = 72^o$, $\angle 3 = 108^o$, $\angle 4 = 72^o$, $\angle 5 = 108^o$ $\angle 6 = 72^o$, $\angle 7 = 108^o$ and $\angle 8 = 72^o$.

Ex.34 In figure AB || DC and AD || BC. Prove that $\angle DAB = \angle DCB$.



Sol. Since AD || BC and AB is a transversal intersecting them at A and B respectively. Therefore

$\angle DAB + \angle ABC = 180^o$

[Consecutive interior angles] ...(i)

Again, AB || CD and BC is a transversal interacting them at B and C respectively. Therefore,

$\angle ABC + \angle DCB = 180^o$

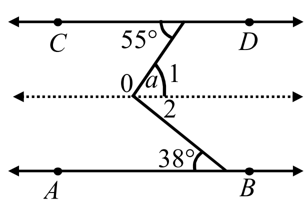
[Consecutive interior angles] ....(ii)

From (i) and (ii), we get

$\angle DAB + \angle ABC = \angle ABC + \angle DCB$

$\Rightarrow$ $\angle DA B = \angle DCB$

Ex.35 In figure AB || CD. Determine $\angle a$



Sol. Through O draw a line λ parallel to both AB and CD.

Clearly, $\angle a = \angle 1 + \angle 2$ ....(ii)

Now, $\angle 1 = 55^o$ [Alternate $\angle$s]

and $\angle 2 = 38^o$ [Alternate $\angle$s]

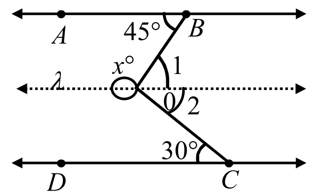
$\therefore$ $\angle a = 55^o + 38^o$ [Using (i)]

$\Rightarrow$ $\angle a = 93^o$.

Thus, $\angle a = 93^o$

Ex.36 In figure AB || CD. Determine X.

Sol. Through O, draw a line λ parallel to both AB and CD. Then,



$\angle 1 = 45^o$ [Alternate $\angle$s]

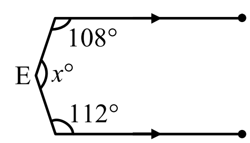
and $\angle 2 = 30^o$ [Alternate $\angle$s]

$\therefore$ $\angle BOC = \angle 1 + \angle 2$ $= 45^o + 30^o = 75^o$

So, $x = 360 - \angle BOC$ $= 360 - 75 = 285^o$

Hence, $x = 285^o$

Ex.37 In figure AB || CD. Find the value of x.



Sol. Draw EF parallel to both AB and CD.

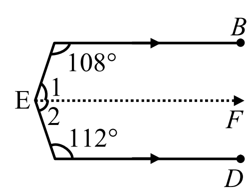
Now, AB || EF and transversal AE cuts them at A and E respectively.

$\angle BAE + \angle FEA = 180^o$

$\Rightarrow 108^o + \angle 1 = 180^o \Rightarrow \angle 1 = 180^o - 108^o = 72^o$

Again, EF || CD and transversal CE cuts them at E and F respectively.

$\therefore$ $\angle FEC + \angle ECD = 180^o$



$\Rightarrow$ $\angle 2 + 112^o = 180^o$

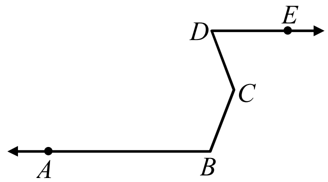
$\Rightarrow$ $\angle 2 = 180^o - 112^o$

$\Rightarrow$ $\angle 2 = 68^o$

Now, $x = \angle 1 + \angle 2$

$\Rightarrow$ $x = 72^o + 68^o = 140^o$

Ex.38 In Figure AB || DE. Prove that $\angle ABC + \angle BCD = 180^\circ + \angle CDE$.



Sol. Through C, draw CF parallel to both AB and DE. Since AB || CF and the transversal BC cuts them at B and C respectively. Therefore,

$\angle ABC + \angle 1 = 180^\circ$ ....(i)

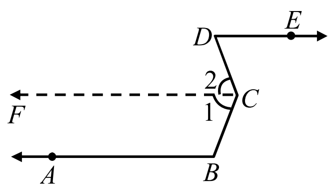
[Θ conseu. interior angles are supplementary]

Similarly, DE || CF and tranversal CD intersects them at C and D respectively. Therefore,

$\angle CDE = \angle 2$ [Alternate angles] ....(ii)

Adding (i) and (ii), we get

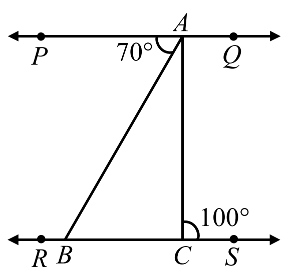
$\angle ABC + \angle 1 + \angle 2 = 180^\circ + \angle CDE$



$\Rightarrow \angle ABC + \angle BCD = 180^\circ + \angle CDE$

[Θ $\angle 1 + \angle 2 = \angle BCD$]

Ex.39 In Figure PQ || RS, $\angle PAB = 70^\circ$ and $\angle ACS = 100^\circ$. Determine $\angle ABC$, $\angle BAC$ and $\angle CAQ$.



Sol. Since PQ || RS and transversal AB cuts them at A and B respectively.

$\therefore \angle ABC = \angle PAB$ [Alternate angles]

$\Rightarrow \angle ABC = 70^\circ$ [Θ $\angle PAB = 70^\circ$ (Given)]

Now, PQ || RS and transversal AC cuts them at A and C respectively.

$\therefore \angle PAC = \angle ACS$ [Alternate angles]

$\Rightarrow \angle PAC = 100^\circ$ [Θ $\angle ACS = 100^\circ$]

$\Rightarrow \angle PAB + \angle BAC = 100^\circ$

[Θ $\angle PAC = \angle PAB + \angle BAC$]

$\Rightarrow 70^\circ + \angle BAC = 100^\circ$

$\Rightarrow \angle BAC = 30^\circ$

Now, ray AB stands at A on PQ.

$\therefore \angle PAC + \angle CAQ = 180^\circ$

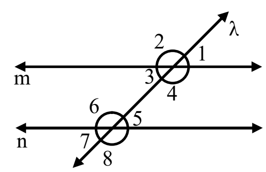
$\Rightarrow 100^\circ + \angle CAQ = 180^\circ$

$\Rightarrow \angle CAQ = 80^\circ$

Hence, $\angle ABC = 70^\circ$,

$\angle BAC = 30^\circ$ and $\angle CAQ = 80^\circ$.

Ex.40 In Figure if $\angle 2 = 120^\circ$ and $\angle 5 = 60^\circ$, show that m || n.



Sol. We have

$\angle 2 = 120^\circ$ and $\angle 5 = 60^\circ$

But $\angle 2 = \angle 4$ [Vertically opposite angles]

$\therefore$ $\angle 4 = 120^\circ$, $\angle 5 = 60^\circ$

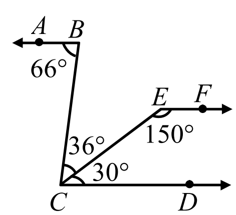
$\Rightarrow$ $\angle 4 + \angle 5 = 120^\circ + 60^\circ = 180^\circ$

$\Rightarrow$ $\angle 4$ and $\angle 5$ are supplementary angles.

$\Rightarrow$ Consecutive interior angles are supplementary.

$\Rightarrow$ m || n.

Ex.41 In figure show that AB || EF.



Sol. We have,

$\angle BCD = \angle BCE + \angle ECD$

$= 36^\circ + 30^\circ = 66^\circ$

$\therefore$ $\angle ABC = \angle BCD$

Thus, lines AB and CD are intersected by the line BC such that $\angle ABC = \angle BCD$ i.e. the alternate angles are equal. Therefore,

AB || CD ....(i)

Now, $\angle ECD + \angle CEF = 30^o + 150^o = 180^o$

This shows that the sum of the interior angles on the same side of the transversal CE is 180^o i.e. they are supplementary.

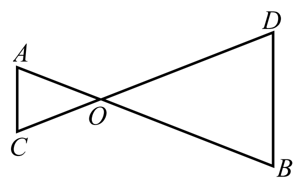
$\therefore$ EF || CD ....(ii)

From (i) and (ii), we have

AB || CD and CD || EF $\Rightarrow$ AB || EF.

Hence, AB || EF

Ex.42 In figure given that $\angle AOC = \angle ACO$ and $\angle BOD = \angle BDO$. Prove that AC || DB.



Sol. We have,

$\angle AOC = \angle ACO$ and $\angle BOD = \angle BDO$

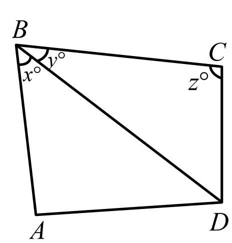
But $\angle AOC = \angle BOD$ [Vertically opp. $\angle$s]

$\therefore$ $\angle ACO = \angle BOD$ and $\angle BOD = \angle BDO$

$\Rightarrow$ $\angle ACO = \angle BDO$

Thus, AC and BD are two lines intersected by transversal CD such that $\angle ACO = \angle BDO$ i.e. alternate angles are equal. Therefore, AC || DB.

Ex.43 In figure AB || DC if $x = \frac{4}{3}y$ and $y = \frac{3}{8}z$, find the values of x, y and z.



Sol. Since AB || DC and transversal BD intersects them at B and D respectively. Therefore,

$\angle ABD = \angle CDB \quad \Rightarrow \angle CDB = x^o$

In ΔBCD, we have

$y^o + z^o + x^o = 180^o$

$\Rightarrow \frac{3}{8}z^o + z^o + \frac{4}{3} \times \frac{3}{8}z^o = 180$

$\Rightarrow \frac{3}{8}z^o + z^o + \frac{1}{2}z^o = 180$

$\Rightarrow \frac{15}{8}z^o = 180^o$

[Θ $x = \frac{4}{3}y$ and $y = \frac{3}{8}z$

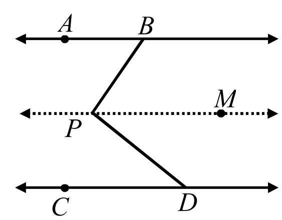
$\therefore x = \frac{4}{3} \times \frac{3}{8}z = \frac{z}{2}$]

$\Rightarrow z^o = 180^o \times \frac{8}{15} = 96^o$

Now, $y = \frac{3}{8}z \Rightarrow y = \frac{3}{8} \times 96^o = 36^o$

and $x = \frac{4}{3}y \Rightarrow x = \frac{4}{3} \times 36^o = 48$

Ex.44 In figure lines AB and CD are parallel and P is any point between the two lines. Prove that $\angle ABP + \angle CDP = \angle DPB$.



Sol. Through point P draw a line PM parallel to AB or CD.

Now,

PM || AB [By construction]

$\Rightarrow \angle ABP = \angle MPB$ [Alternate angles] ....(i)

It is given that CD || AB and PM || AB by construction. Therefore,

PM || CD

[Θ Lines parallel to the same line are parallel to each other]

$\Rightarrow \angle CDP = \angle MPD$ [Alternate angles] ....(ii)

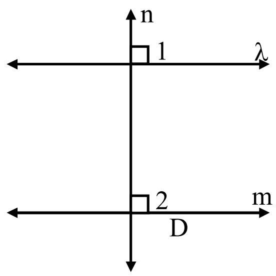
Adding (i) and (ii), we get

$\angle ABP + \angle CDP = \angle MPB + \angle MPD = \angle DPB$

Ex.45 Prove that two lines perpendicular to the same line are parallel to each other.

Sol. Let lines λ, m, n be such that $\lambda \perp n$ and $m \perp n$ as shown in figure.

We have to prove that $\lambda \parallel m$



Now,

$\lambda \perp n$ and $m \perp n$

$\Rightarrow \angle 1 = 90^\circ$ and $\angle 2 = 90^\circ$

$\Rightarrow \angle 1 = \angle 2$

Thus, the corresponding angles made by the transversal n with lines λ and m are equal.

Hence, $\lambda \parallel m$.

Ex.46 Prove that two angles which have their arms parallel are either equal or supplementary.

Sol. **Given :** Two angles ∠ABC and ∠DEF such that BA || ED and BC || EF.

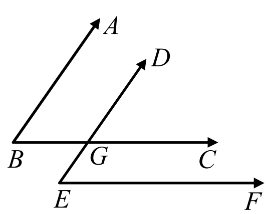
To prove : ∠ABC = ∠DEF

or $\angle ABC + \angle DEF = 180^\circ$

Proof : We have the following three cases:

Case I : When both pairs of arms are parallel in the same sense fig. in this case,

AB || DE and transversal BC cuts them at B and G respectively



$\therefore \angle ABC = \angle DGC$ ...(i)

[Corresponding angles]

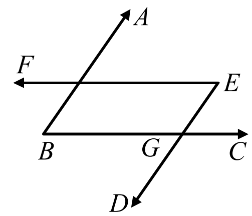
Again, BC || EF and transversal DE cuts them at G and E respectively.

$\therefore \angle DGC = \angle DEF$ ...(ii)

From (i) and (ii), we get

∠ABC = ∠DEF

Case II : When both pairs of arms are parallel in opposite sense in this case,



AB || DE and transversal BC cuts them at B and G respectively.

$\therefore \angle ABC = \angle EGC$ ....(iii)

[Corresponding angles]

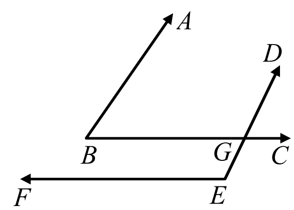
Again, BC || EF and transversal DE cuts them at G and E respectively.

$\therefore \angle DEF = \angle EGC$ [Alternate angles] ....(iv)

From (iii) and (iv), we get

∠ABC = ∠DEF.

Case III : When one pair of arms is parallel in the same sense and the other in opposite sense. In this case,



AB || DE and transversal BC cuts them

$\therefore \angle ABC = \angle BGE$ [Alternate angles] ....(v)

Again, BC || FE and transversal DE cuts them

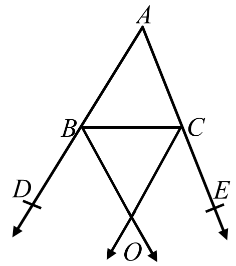
$\therefore \angle DEF + \angle BGE = 180^\circ$ ....(vi)

[Θ Consecutive interior angles are supplementary]

From (v) and (vi), we get

$\angle ABC + \angle DEF = 180^\circ$

Ex.47 In figure bisectors of the exterior angles B and C formed by producing sides AB and AC of ΔABC intersect each other at the point O. Prove that $\angle BOC = 90^\circ - \frac{1}{2}\angle A$.



Sol. $\angle DBC = 180^o - \angle B$

$\therefore \quad \frac{1}{2}\angle DBC = \angle OBC = 90^o - \angle B/2$

Similarly, $\angle OCB = 90^o - \angle C/2$

In $\angle OBC$, we have,

$\angle OBC + \angle OCB + \angle BOC = 180^o$

or $(90^o - \angle B/2) + (90^o - \angle C/2) + \angle BOC = 180^o$

$\angle BOC = 180^o - (180^o - \angle B/2 + \angle C/2)$

$= \angle B/2 + \angle C/2 \quad$ ...(i)

But $\angle A + \angle B + \angle C = 180^o$

or $\quad \angle B + \angle C = 180^o - \angle A$

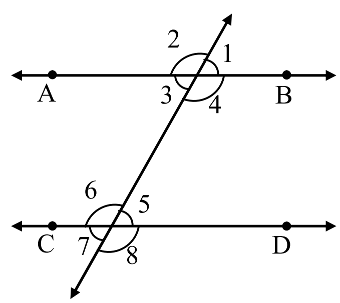
$\therefore \quad \frac{1}{2}(\angle B + \angle C) = \frac{1}{2}(180^o - \angle A)$

$\therefore \quad \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90^o - \angle A/2$

Hence, from (i), we have,

$\angle BOC = 90^o - \angle A/2$

Ex.48 In fig, given that AB || CD.



(i) If $\angle 1 = (120 - x)^o$ and $\angle 5 = 5x^o$, find the measures of $\angle 1$ and $\angle 5$.

(ii) If $\angle 4 = (x + 20)^o$ and $\angle 5 = (x + 8)^o$, find the measure of $\angle 4$ and $\angle 5$.

(iii) If $\angle 2 = (3x - 10)^o$ and $\angle 8 = (5x - 30)^o$, determine the measures of $\angle 2$ and $\angle 8$.

(iv) If $\angle 1 = (2x + y)^o$ and $\angle 6 = (3x - y)^o$, determine the measures of $\angle 2$ in terms of y.

(v) If $\angle 2 = (2x + 30)^o$, $\angle 4 = (x + 2y)^o$ and $\angle 6 = (3y + 10)^o$, find the measure of $\angle 5$.

(vi) If $\angle 2 = 2\,(\angle 1)$, determine $\angle 7$.

(vii) If the ratio of the measures of $\angle 3$ and $\angle 8$ is 4 **:** 5, find the measure $\angle 3$ and $\angle 8$

(viii) If the complement of $\angle 5$ equals the supplement of $\angle 4$, find the measures of $\angle 4$ and $\angle 5$.

Sol. (i) since $\angle 1$ and $\angle 5$ are the corresponding angles and corresponding angles are equal.

$\therefore \quad \angle 1 = \angle 5 \quad \Rightarrow \quad (120 - x)^o = 5x^o$

$\Rightarrow \quad 120^o = 6x \quad \Rightarrow \quad x = \frac{120}{6} = 20^o$

$\therefore \quad \angle 1 = (120 - x)^o = (120 - 20)^o = 100^o$

and, $\angle 5 = 5x^o = (5 \times 20)^o = 100^o$

(ii) Since $\angle 4$ and $\angle 5$ are consecutive interior angles. Therefore,

$\angle 4 + \angle 5 = 180^o$

[$\therefore$ Consecutive interior angles are supplementary]

$\Rightarrow \quad (x + 20)^o + (x + 8)^o = 180^o$

$\Rightarrow \quad 2x^o + 28^o = 180^o \quad \Rightarrow 2x^o = 180^o - 28^o$

$\Rightarrow 2x = 152^o \quad \Rightarrow x = 76^o$

$\therefore \quad \angle 4 = (x + 20)^o = (76 + 20)^o = 96^o$

and, $\angle 5 = (x + 8)^o = (76 + 8^o) = 84^o$

(iii) We have,

$\angle 2 = \angle 4 \quad$ [Vertically opposite angles]

and $\angle 4 = \angle 8 \quad$ [Corresponding angles]

$\therefore \quad \angle 2 = \angle 8$

$\Rightarrow \quad (3x - 10)^o = (5x - 30)^o \Rightarrow 3x - 10 = 5x - 30$

$\Rightarrow \quad 3x - 5x = -30 + 10 \quad \Rightarrow -2x = -20$

$\Rightarrow x = 10$

$\therefore \quad \angle 2 = (3x - 10)^o = (3 \times 10 - 10)^o = 20^o$

and $\angle 8 = (5x - 30)^o = (5 \times 10 - 30)^o = 20^o$

(iv) Since $\angle 3$ and $\angle 6$ are consecutive interior angles. Therefore

$\angle 3 + \angle 6 = 180^o$

But $\angle 1 = \angle 3 \quad \therefore \angle 1 + \angle 6 = 180^o$

$\Rightarrow \quad (2x + y)^o + (3x - y)^o = 180^o$

$\Rightarrow$ 5x = 180º $\Rightarrow$ x = 36.

$\therefore$ ∠1 = (2x + y)º = (72 + y)º [$\therefore$ x = 36]

But ∠1 + ∠2 = 180º

$\therefore$ (72 + y)º + ∠2 = 180º

$\Rightarrow$ ∠2 = (180 – (72 + y))º $\Rightarrow$ ∠2 = (108 – y)º.

(v) We have,

∠2 = ∠4 [Vertically opposite angles]

and ∠4 = ∠6 [Alternate angles]

$\therefore$ ∠2 = ∠4 = ∠6

Now, ∠2 = ∠4

$\Rightarrow$ 2x + 30 = x + 2y $\Rightarrow$ 2x – x – 2y + 30 = 0

$\Rightarrow$ x – 2y + 30 = 0 ....(1)

And, ∠4 = ∠6 $\Rightarrow$ (x + 2y) = (3y + 10)

$\Rightarrow$ x – y – 10 = 0 ....(2)

Subtracting (2) from (1), we get

(x – 2y + 30) – (x – y – 10) = 0

$\Rightarrow$ – y + 40 = 0 $\Rightarrow$ y = 40.

Putting y = 40 in (2), we get x = 50.

$\therefore$ ∠4 = (x + 2y)º = (50 + 2 × 40)º = 130º

But ∠4 + ∠5 = 180º

$\therefore$ 130º + ∠5 = 180º $\Rightarrow$ ∠5 = 50º

(vi) We have,

∠1 + ∠2 = 180º [Linear pairs]

$\therefore$ ∠2 = 2 ∠1 $\Rightarrow$ ∠1 + 2 ∠1 = 180º

$\Rightarrow$ 3 ∠1 = 180º $\Rightarrow$ ∠1 = 60º

But ∠1 = ∠3 [Vertically opposite angles]

$\therefore$ ∠3 = 60º

But ∠3 = ∠5 [Alternate angles]

and ∠5 = ∠7 [Vertically opposite angles]

$\therefore$ ∠3 = ∠7 $\Rightarrow$ ∠7 = 60º [Θ ∠3 = 60º]

(vii) We have, ∠3 : ∠8 = 4 : 5. So, let

∠3 = 4x and ∠8 = 5x.

$\Rightarrow$ ∠5 = 4x and ∠8 = 5x

[Θ ∠3 = ∠5 (Alternate angles)]

$\Rightarrow$ ∠5 + ∠8 = 4x + 5x

$\Rightarrow$ 180º = 9x $\Rightarrow$ x = 20º

$\therefore$ ∠3 = 4x = 4 × 20º = 80º

and ∠8 = 5x = 5 × 20º = 100º

(viii) We have, Complement of ∠5 = Supplement of ∠4

$\Rightarrow$ 90º – ∠5 = 180º – ∠4

$\Rightarrow$ 90º – ∠5 = 180º – (180º – ∠5)

$$\begin{bmatrix} \Theta\ \angle 4 + \angle 5 = 180^\circ \\ \Theta\ \angle 4 = 180^\circ - \angle 5 \end{bmatrix}$$

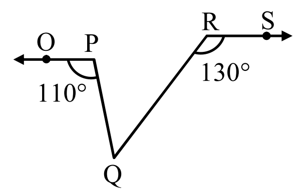
$\Rightarrow$ 90º – ∠5 = ∠5

$\Rightarrow$ 2 ∠5 = 90º $\Rightarrow$ ∠5 = 45º

$\therefore$ ∠4 + ∠5 = 180º

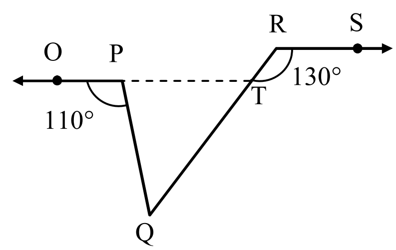
$\Rightarrow$ ∠4 + 45º = 180º $\Rightarrow$ ∠4 = 135º

Ex.49 In fig, OP || RS. Determine ∠PQR.



Sol. Produce OP to intersect RQ in a point T.

Now, OT || RS and transversal RT intersect them at T and R respectively.



$\therefore$ ∠RTP = ∠SRT [Alternate angles]

$\Rightarrow$ ∠RTP = 130º

$\Rightarrow$ ∠PTQ = 180º – 130º = 50º

$$\begin{bmatrix} \Theta\ \angle RTP + \angle PTQ = 180^\circ \\ \text{Linear Pairs} \end{bmatrix}$$

Since, ray QP stands at P on OT.

$\therefore$ ∠OPQ + ∠QPT = 180º

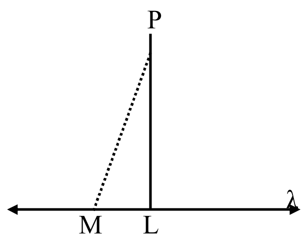
$\Rightarrow$ 110º + ∠QPT = 180º $\Rightarrow$ ∠QPT = 70º

$\therefore$ ∠PQR = 180º – (70º + 50º) = 60º

[Θ Sum of the angles of a triangle is 180º]

Ex.50 Prove that through a given point we can draw only one perpendicular to a given line.

Sol. If possible, let PL and PM be two perpendicular from a point P on a line λ as shown in fig.



We know that two lines perpendicular to the same line are parallel to each other. Therefore,

PL || PM

But there cannot be two parallel lines passing through the same point. Therefore, through a given point we can draw only one line perpendicular to a given line.

IMPORTANT POINTS TO BE REMEMBERED

1. An angle is the union of two non-collinear rays with a common initial point.
2. An angle whose measure is 90º is called a right angle.
3. An angle whose measure is less than 90º is called an acute angle.
4. An angle whose measure is more than 90º but less than 180 is called an obtuse angle.
5. An angle whose measure is 180º is called a straight angle.
6. An angle whose measure is more than 180º is called a reflex angle.
7. Two angles are complementary if their sum is 90º.
8. Two angles are supplementary if their sum is 180º.
9. Two angles having a common vertex and a common arm are called adjacent angles if their uncommon arms are on either side of the common arm.
10. Two adjacent angles are said to form a linear pair of angles, if their non-common arms are two opposite rays.
11. Two angles are pair of vertically opposite angles if their arms form two pairs of opposite rays.
12. If two lines intersect, then vertically opposite angles are equal.
13. If a transversal intersects two parallel lines, then each pair of -
    (i) corresponding angles are equal
    (ii) alternate interior angles are equal
    (iii) interior angles on the same of the same side of the transversal are supplementary.
14. If a transversal intersects two lines such that, either -
    (i) any one pair of corresponding angles are equal, or
    (ii) any one pair of alternate interior angles are equal, or
    (iii) any one pair of interior angles on the same side of the transversal are supplementary then the lines are parallel.
15. Lines which are parallel to a given line are parallel to each other.