CBSE Board
Class X Mathematics

Time: 3 hrs **Total Marks: 80**

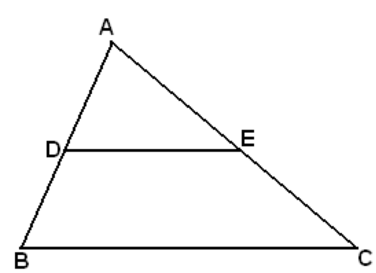
General Instructions:

1. **All** questions are **compulsory**.
2. The question paper consists of **30** questions divided into **four sections** A, B, C, and D. **Section A** comprises of **6** questions of 1 mark each, **Section B** comprises of **6** questions of 2 marks each, **Section C** comprises of **10** questions of 3 marks each and **Section D** comprises of **8** questions of 4 marks each.
3. Question numbers **1 to 6** in **Section A** are multiple choice questions where you are to select **one** correct option out of the given four.
4. Use of calculator is **not** permitted.

Section A
(Questions 1 to 6 carry 1 mark each)

1. If α and β are the zeroes of the quadratic polynomial f(x) = x^2 + 2x + 1, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$.

2. In $\triangle$ ABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $\frac{AD}{DB}=\frac{2}{3}$ and EC = 4 cm, then find AE.



3. Is number '7 × 11 × 13 + 13 + 13 × 2' a composite number?

4. The radius of a cylindrical tank is 28 m. If its capacity is equal to that of a rectangular tank of size 28 m × 16 m × 11 m then. Find the depth of the cylindrical tank.

5. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has its circumference equal to the sum of the circumferences of the two circles.

6. If the probability of winning a game is 0.3, then what is the probability of losing it?

Section B
(Questions 7 to 12 carry 2 marks each)

7. Show that (a – b), a and (a + b) form consecutive terms of an A.P.

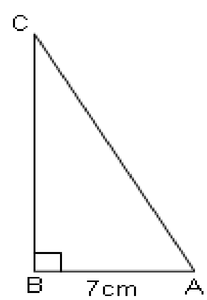
8. Determine the ratio in which the line $3x + y - 9 = 0$ divides the segment joining the points (1, 3) and (2, 7).

9. Determine the set of values of p for which the quadratic equation $px^2 + 6x + 1 = 0$ has real roots.

10. Find the mean of the following data:

Classes	Frequency
0-10	7
10-20	3
20-30	15
30-40	5

11. In ΔABC, $m\angle B = 90°$, AB = 7 cm and AC – BC = 1 cm. Determine the values of sin C and cos C.



12. Find the zeroes of the quadratic polynomial $x^2 + 7x + 12$ and verify the relationship between the zeroes and its coefficients.

Section C
(Questions 13 to 22 carry 3 marks each)

13. The point P divides the join of (2, 1) and (−3, 6) in the ratio 2 : 3. Does P lie on the line $x - 5y + 15 = 0$?

14. The cost of fencing a circular field at the rate of Rs. 24 per metre is Rs. 5280. The field is to be ploughed at the rate of Rs. 0.50 per m^2. Find the cost of ploughing the field. $\left(\text{Take } \pi = \frac{22}{7}\right)$

15. Solve for x:

$$\left(\frac{4x-3}{2x+1}\right)-10\left(\frac{2x+1}{4x-3}\right)=3,\ x\neq-\frac{1}{2},\frac{3}{4}$$

16. Construct a pair of tangents to a circle of radius 4 cm inclined at an angle of 45^{o}..

17. If in a rectangle, the length is increased and breadth is reduced each by 2 metres, then the area is reduced by 28 sq metres. If the length is reduced by 1 metre and breadth is increased by 2 metres, then the area is increased by 33 sq metres. Find the length and breadth of the rectangle.

18. Find the mode for the following data which gives the literacy rate (in %) in 40 cities of India.

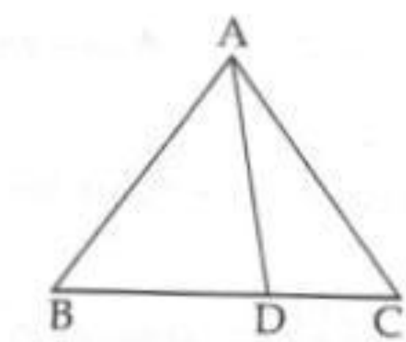
Literacy rate (%)	45-55	55-65	65-75	75-85	85-95
No. of cities	4	11	12	9	4

19. The sum of the numerator and denominator of a fraction is 8. If 3 is added to both the numerator and the denominator, the fraction becomes $\frac{3}{4}$. Find the fraction.

20. If one solution of the equation $3x^2 = 8x + 2k + 1$ is seven times the other. Find the solutions and the value of k.

21. If the zeros of the polynomial $f(x) = x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b.

22. In the figure, ΔABC is such that ∠ADC = ∠BAC. Prove that $CA^2 = CB \times CD$.

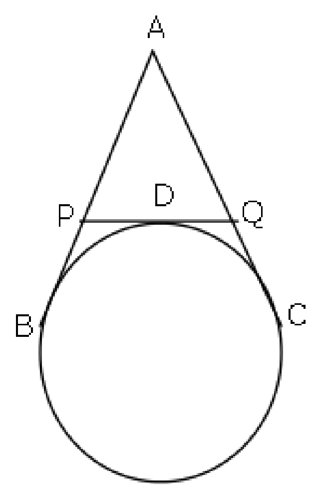


Section D
(Questions 23 to 30 carry 4 marks each)

23. A cylindrical container whose diameter is 12 cm and height is 15 cm is filled with ice-cream. Ice-cream is distributed to ten children in equal cones having hemispherical tops. If the height of conical portion is twice the diameter of its base, find the diameter of the ice-cream cone.

24. From a window of a house in a street, h metres above the ground, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are α and β respectively. Show that the height of the opposite house is $h(1+ \tan \alpha . \cot \beta)$ metres.

25. For a Science exhibition Samy presented a diagrammatic representation of 'Rain Water Harvesting' as his project. AB and AC are 5 m long pipes bringing water from the terrace of a building (as shown in the given figure). The triangular space is developed as a garden.



What is the perimeter of the triangular garden? What qualities do you think are encouraged by such exhibitions?

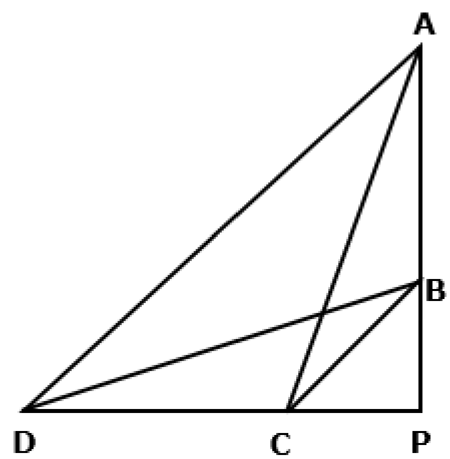
26. Two dice are thrown simultaneously. Find the probability that the sum of the two numbers appearing on their top is less than or equal to 10.

27. If m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then show that the sum of the m and terms is $\frac{1}{2}(mn + 1)$.

28. From a solid cylinder of height 7 cm and base diameter 12 cm, a conical cavity of same height and same base diameter is hollowed out. Find the total surface area of the remaining solid. [Use $\pi = 22/7$]

29. Solve the equations $2x - y + 6 = 0$ and $4x + 5y - 16 = 0$ graphically. Also determine the coordinate of the vertices of the triangle formed by these lines and the x-axis.

30. In a quadrilateral ABCD, given that $\angle A + \angle D = 90°$. Prove that $AC^2 + BD^2 = AD^2 + BC^2$.



CBSE Board

Class X Mathematics

Solution

Time: 3 hrs **Total Marks: 80**

Section A

1. It is given that α and β are the zeros of the quadratic polynomial $f(x) = x^2 + 2x + 1$

$\therefore \alpha+\beta=-\frac{2}{1}=-2$ and and $\alpha\beta=\frac{1}{1}=1$

Now, $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=\frac{-2}{1}=-2$

2. In ΔABC, DE ∥ BC.

Then, by Basic Proportionality Theorem,

$\frac{AD}{DB}=\frac{AE}{EC}$

$\Rightarrow \frac{2}{3}=\frac{AE}{4}$

$\Rightarrow AE=\frac{2\times 4}{3}=\frac{8}{3}=2.67$ cm

$\Rightarrow x^2-x=x^2-4$

$\Rightarrow x=4$

3. 7 × 11 × 13 + 13 + 13 × 2

= 13(7 × 11 + 1 + 2)

= 13(80)

Thus, the given number is a composite number.

4. Let d be the depth of the cylindrical tank.

According to the given information,

Volume of cylindrical tank = Volume of rectangular tank

$\Rightarrow \pi(28)^2 \times d = 28 \times 16 \times 11$

$\Rightarrow d = 2$

Thus, the depth of the cylindrical tank is 2 m.

5. Circumference of first circle = $C_1 = 2\pi \times 19 = 38\pi$

Circumference of second circle = $C_2 = 2\pi \times 9 = 18\pi$

$C_1 + C_2 = 56\pi$

Let R be the radius of the new circle.

$\therefore\ 2\pi R = 56\pi$

$\therefore\ 2R = 56$

$\therefore\ R = 28$ cm

6. P(winning) + P(losing) = 1

$\Rightarrow$ 0.3 + P(losing) = 1

$\Rightarrow$ P(losing) = 1 – 0.3 = 0.7

Section B

7. Three terms p, q and r are in A.P. if 2q = p + r.

Now, first term + third term = (a – b) + (a + b)

= 2a

= 2 × (second term)

Thus, (a – b), a and (a + b) are consecutive terms of an A.P.

8. Suppose the line 3x + y − 9 = 0 divides the segment joining the points A(1, 3) and B(2, 7) in the ratio k : 1 at point C.

Then, the co-ordinates of C $\equiv \left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

But, C lies on 3x + y − 9 = 0.

Therefore,

$$\left[3\left(\frac{2k+1}{k+1}\right)\right] + \left[\frac{7k+3}{k+1}\right] - 9 = 0$$

$\Rightarrow$ 6k + 3 + 7k + 3 − 9k − 9 = 0

$\Rightarrow k = \frac{3}{4}$

So, the required ratio is 3 : 4.

9. Given equation is $px^2 + 6x + 1 = 0$

Here, $a = p$, $b = 6$ and $c = 1$

The given equation will have real roots, if $b^2 - 4ac \geq 0$.

$\Rightarrow (6)^2 - 4(p)(1) \geq 0$

$\Rightarrow 36 - 4p \geq 0$

$\Rightarrow 36 \geq 4p$

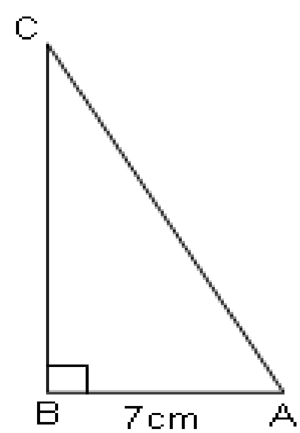
$\Rightarrow p \leq 9$

10.

Classes	Class Mark (x_i)	Frequency (f_i)	f_ix_i
0-10	5	7	35
10-20	15	3	45
20-30	25	15	375
30-40	35	5	175
		$\Sigma f_i = 30$	$\Sigma f_ix_i = 630$

Mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{630}{30} = 21$

11. In ABC, we have



$AC^2 = BC^2 + AB^2$

$(1 + BC)^2 = BC^2 + AB^2$

$\Rightarrow 1 + BC^2 + 2BC = BC^2 + AB^2$

$\Rightarrow 1 + 2BC = 7^2$

$\Rightarrow 2BC = 48$

$\Rightarrow BC = 24$ cm

$\Rightarrow AC = 1 + BC = 1 + 24 = 25$ cm

Hence, $\sin C = \frac{AB}{AC} = \frac{7}{25}$ and $\cos C = \frac{BC}{AC} = \frac{24}{25}$

12. $x^2 + 7x + 12$

$= x^2 + 3x + 4x + 12$

$= x(x + 3) + 4(x + 3)$

$= (x + 3)(x + 4)$

$\therefore$ −3 and −4 are the zeroes of the given polynomial.

Sum of zeroes = $-3 - 4 = -7 = \frac{-7}{1} = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

Product of zeroes = $(-3)(-4) = 12 = \frac{12}{1} = \frac{\text{constant term}}{\text{coefficient of } x^2}$

Section C

13. Given, the point P divides the join of (2, 1) and (−3, 6) in the ratio 2 : 3.

Co-ordinates of the point P $\equiv \left(\frac{2\times(-3)+3\times2}{2+3}, \frac{2\times6+3\times1}{2+3}\right) \equiv \left(\frac{-6+6}{5}, \frac{12+3}{5}\right) \equiv (0,3)$

Now, the given equation is $x - 5y + 15 = 0$.

Substituting $x = 0$ and $y = 3$ in this equation, we have

L.H.S. = $0 - 5(3) + 15 = -15 + 15 = 0$ = R.H.S.

Hence, the point P lies on the line $x - 5y + 15 = 0$.

14. 1 m of fencing costs Rs. 24.

Hence for Rs. 5280, the length of fencing = $\frac{5280}{24}$ = 220 m

$\Rightarrow$ Circumference of the field = 220 m

$\therefore 2\pi r = 220$

$\Rightarrow 2 \times \frac{22}{7} \times r = 220$

$\Rightarrow r = \frac{220\times7}{44} = 35$ m

Area of the field = $\pi r^2 = \pi(35)^2 = 1225\pi$ m^2

Cost of ploughing = Rs. 0.50 per m^2

Total cost of ploughing the field = Rs. $1225\,\pi \times 0.50 = \frac{1225\times22\times1}{7\times2}$ = Rs. 1925

15. $\left(\frac{4x-3}{2x+1}\right)-10\left(\frac{2x+1}{4x-3}\right)=3$

Let $\frac{4x-3}{2x+1}=y$

Then, the equation becomes

$y-\frac{10}{y}=3$

$\Rightarrow y^2-10=3y$

$\Rightarrow y^2-3y-10=0$

$\Rightarrow y^2-5y+2y-10=0$

$\Rightarrow y(y-5)+2(y-5)=0$

$\Rightarrow (y-5)(y+2)=0$

$\Rightarrow y = 5,\ -2$

$\therefore \frac{4x-3}{2x+1}=5$ or $\frac{4x-3}{2x+1}=-2$

$\Rightarrow 10x+5=4x-3$ or $4x-3=-4x-2$

$\Rightarrow 6x=-8$ or $8x=1$

$\Rightarrow x=-\frac{4}{3}$ or $x=\frac{1}{8}$

16. Steps of construction:

1) Draw a circle of radius 4 cm with centre O.

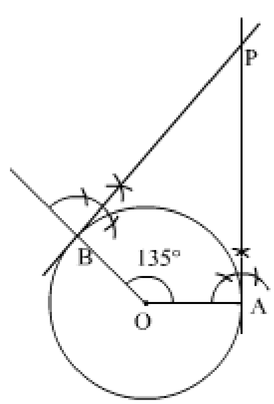
2) Take a point A on the circle. Join OA.

3) Draw perpendicular to OA at A.

4) Draw radius OB, making an angle of 135° (180° – 45°) with OA.

4) Draw perpendicular to OB at point B. Let these perpendiculars intersect at P.

PA and PB are the required tangents inclined at angle of 45°.



17. Let the length and breadth of the rectangle be x and y respectively.

So the original area of the rectangle = xy

According to question,

$(x + 2)(y - 2) = xy - 28$

$\Rightarrow xy - 2x + 2y - 4 = xy - 28$

$\Rightarrow 2x - 2y = 24$ .....(i)

And, $(x - 1)(y + 2) = xy + 33$

$\Rightarrow xy + 2x - y - 2 = xy + 33$

$\Rightarrow 2x - y = 35$ ....(ii)

Subtracting (i) from (ii), we get

$y = 11$

Substituting this value in (ii), we get

$2x - 11 = 35$

$\Rightarrow 2x = 46$

$\Rightarrow x = 23$

Thus, the length and breadth of the rectangle are 23 metres and 11 metres, respectively.

18. From the data given as above we may observe that maximum class frequency is 12 belonging to class interval 65 – 75.

So, modal class = 65 – 75

Lower class limit (l) of modal class = 65

Frequency (f_1) of modal class = 12

Frequency (f_0) of class preceding the modal class = 11

Frequency (f_2) of class succeeding the modal class = 9

Class size (h) = 10

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 65 + \left(\frac{12 - 11}{2(12) - 11 - 9}\right) \times 10$$

$$= 65 + \frac{1}{4} \times 10$$

$$= 65 + 2.5$$

$$= 67.5$$

19. Let the fraction be $\frac{x}{y}$.

According to the question,

x + y = 8 ....(1)

$\frac{x+3}{y+3}=\frac{3}{4}$

$\Rightarrow 4x+12=3y+9$

$\Rightarrow 4x-3y=-3$ ....(2)

Multiplying (1) by 3, we get

3x + 3y = 24 ....(3)

Adding (2) and (3), we get

7x = 21

$\Rightarrow$ x = 3

$\Rightarrow$ y = 8 – x = 8 – 3 = 5

Thus, the fraction is $\frac{3}{5}$.

20. Given equation is $3x^2 = 8x + 2k + 1$

$\Rightarrow 3x^2 - 8x - (2k + 1) = 0$

Let α be one zero and β = 7α be the other zero of the given equation.

Then, $\alpha + 7\alpha = \frac{8}{3} \Rightarrow 8\alpha = \frac{8}{3} \Rightarrow \alpha = \frac{1}{3}$

$\Rightarrow \beta = \frac{7}{3}$

Thus, the solutions of the given equations are $\frac{1}{3}$ and $\frac{7}{3}$.

Now,

$\alpha\beta=-\frac{(2k+1)}{3}$

$\Rightarrow \frac{1}{3}\times\frac{7}{3}=-\frac{(2k+1)}{3}$

$\Rightarrow \frac{7}{9}\times 3=-2k-1$

$\Rightarrow \frac{7}{3}+1=-2k$

$\Rightarrow -2k=\frac{10}{3}$

$\Rightarrow k=-\frac{5}{3}$

21. Since a – b, a and a + b are the zeros of $f(x) = x^3 - 3x^2 + x + 1$.

$$\therefore\ (a-b)+a+(a+b) = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\Rightarrow 3a = -\frac{-3}{1}$$

$$\Rightarrow 3a = 3$$

$$\Rightarrow a = 1$$

$$\text{And, } (a-b)\times a\times(a+b) = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$$

$$\Rightarrow a(a^2-b^2) = -\frac{1}{1}$$

$$\Rightarrow 1(1-b^2) = -1$$

$$\Rightarrow b^2 = 2$$

$$\Rightarrow b = \pm\sqrt{2}$$

22. In ΔABC and ΔDAC,

∠BAC = ∠ADC

∠C = ∠C

$\therefore$ ΔABC ~ ΔDAC (By AA similarity criterion)

$$\therefore \frac{CB}{CA} = \frac{CA}{CD}$$

$\Rightarrow CA^2 = CB \times CD$

Section D

23. Let the radius of the base of the cone be x cm.

So, diameter = 2x cm

$\Rightarrow$ Height of the cone = 2(diameter) = 2(2x) = 4x

Volume of one ice cream

= volume of conical portion + volume of hemispherical portion

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 [h + 2r]$

$= 2\pi x^3$

Diameter of the cylindrical container = 12 cm

Radius of cylindrical container = 6 cm

Height of cylindrical container = 15 cm

Volume of cylindrical container $= \pi r^2 h = \pi(6)^2(15)$

We have:

Number of children $= \frac{\text{volume of cylindrical container}}{\text{volume of one ice cream cone}}$

$\Rightarrow 10 = \frac{\pi(6)^2(15)}{2\pi x^3}$

$\Rightarrow x^3 = 27$

$\Rightarrow$ x = 3 cm

Thus, diameter of the base of the cone is 2x = 6 cm.

24. Let B be the window of a house AB and let CD be the other house.

Then, AB = EC = h metres.

Let CD = H metres.

Then, ED = (H − h) m

In ΔBED,

$\cot\alpha = \dfrac{BE}{ED}$

BE = (H − h)cot α ... (a)

In ΔACB,

$\dfrac{AC}{AB} = \cot\beta$

AC = h.cot β .... (b)

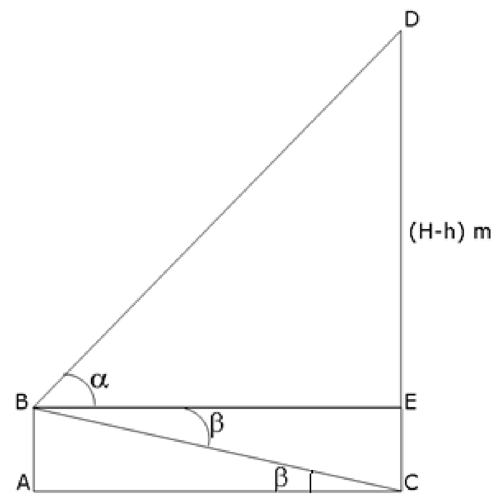
But BE = AC

$\therefore (H - h)\cot\alpha = h\cot\beta$

$H = h\dfrac{(\cot\alpha + \cot\beta)}{\cot\alpha}$

H = h(1 + tanα cotβ)

Thus, the height of the opposite house is h(1 + tanα.cotβ) meters.



25. It is known that the tangents from an external point to the circle are equal in length.

$\therefore$ AB = AC, PB = PD and QC = QD

$\therefore$ Perimeter of Δ APQ = AP + PQ + AQ

= (AB - BP) + (PD + DQ) + (AC - QC)

= (AB - BP) + (BP + QC) + (AB - QC)

= 2AB

= 10 m

Thus, the perimeter of the triangular garden is 10 m.

Qualities encouraged: Creativity, Social Consciousness, Environmental Friendliness, Problem Solving, Team work

26. Elementary events associated to the random experiment of throwing two dice are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

$\therefore$ Total number of elementary outcomes = 36

Favourable outcomes with the sum as 11 are (5, 6) and (6, 5).

Favourable outcome with the sum as 12 is (6, 6).

P(sum 11) = $\frac{2}{36}$

P(sum 12) = $\frac{1}{36}$

P(sum $\leq$ 10) = 1 − [P(sum 11) + P(sum 12)]

$$= 1 - \left[\frac{2}{36} + \frac{1}{36}\right]$$

$$= 1 - \frac{3}{36}$$

$$= 1 - \frac{1}{12}$$

$$= \frac{11}{12}$$

27. Let a and d respectively be the first term and the common difference of the A.P.

$a + (m - 1)d = \frac{1}{n}$ ....(i)

$a + (n - 1)d = \frac{1}{m}$ ....(ii)

On solving (i) and (ii) we get

$$(m - n)d = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow (m - n)d = \frac{m - n}{mn}$$

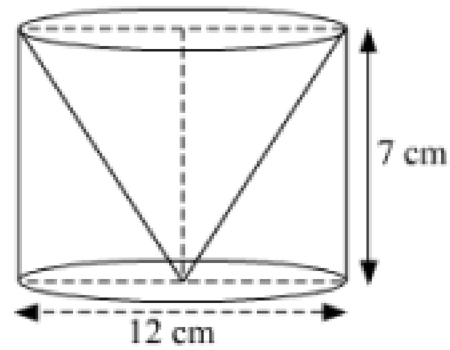
$$\Rightarrow d = \frac{1}{mn}$$

Therefore, $a = \frac{1}{n} - \frac{m-1}{mn} = \frac{m - m + 1}{mn} = \frac{1}{mn}$

$$S_{mn} = \frac{mn}{2}\left[2 \times \frac{1}{mn} + (mn - 1)\left(\frac{1}{mn}\right)\right] = \frac{1}{2}[2 + (mn - 1)] = \frac{1}{2}(mn + 1)$$

28. It is given that, height (h) of cylindrical part = height (h) of the conical part = 7 cm

Diameter of the cylindrical part = 12 cm



$\therefore$Radius (r) of the cylindrical part = $\frac{12}{2} = 6\text{cm}$

$\therefore$ Radius of conical part = 6 cm

Slant height (l) of conical part

$= \sqrt{r^2 + h^2}$ cm
$= \sqrt{6^2 + 7^2}$ cm
$= \sqrt{85}$ cm
$= 9.22$ cm (approx.)

Total surface area of the remaining solid

= CSA of cylindrical part + CSA of conical part + Base area of the circular part

$= 2\pi rh + \pi rl + \pi r^2$
$= 2 \times \frac{22}{7} \times 6 \times 7 + \frac{22}{7} \times 6 \times 9.22 + \frac{22}{7} \times 6 \times 6$
$= 264 + 173.86 + 113.14$
$= 551\ \text{cm}^2$

29. To solve the equations, make the table corresponding to each equation.

$2x - y + 6 = 0$

$\Rightarrow y = 2x + 6$

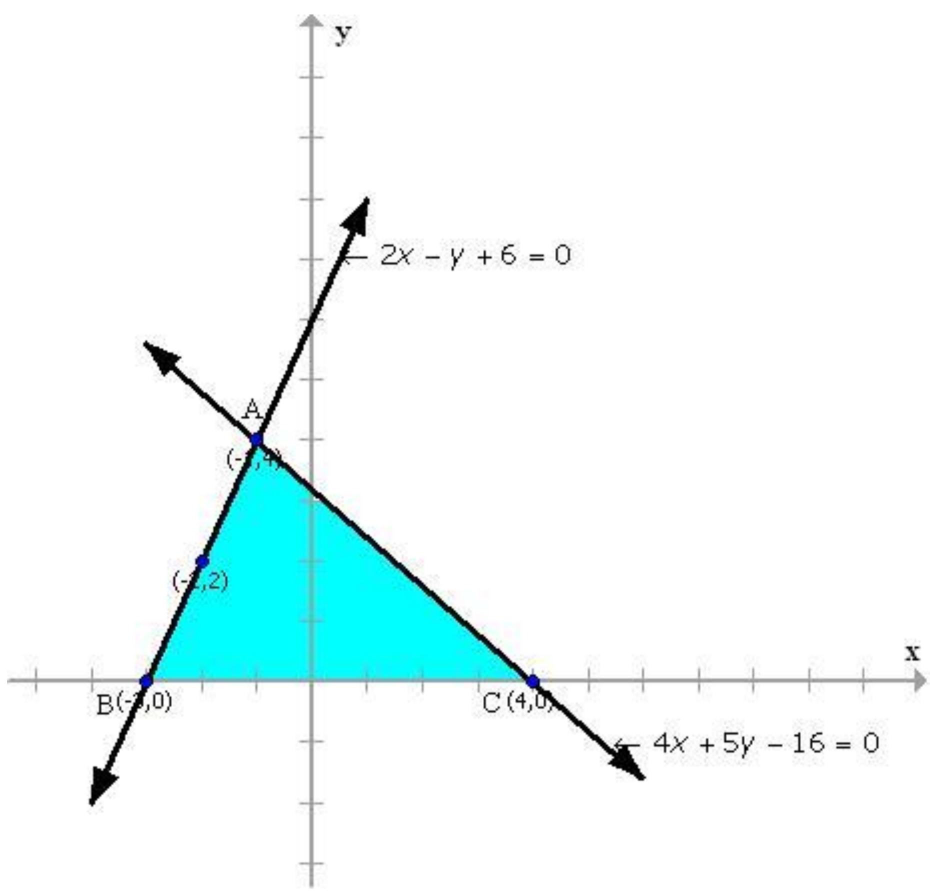
x	-1	-2	-3
y	4	2	0

$4x + 5y - 16 = 0$

$\Rightarrow y = \frac{16-4x}{5}$

x	4	-1
y	0	4

Now plot the points and draw the graph.



Because the lines intersect at the point (−1, 4), x = −1 and y = 4 is the solution.

Also, by observation, vertices of triangle formed by lines and x-axis are A(−1, 4), B(−3, 0) and C(4, 0).

30. We have, $\angle A + \angle D = 90°$

In ΔAPD, by angle sum property,

$\angle A + \angle D + \angle P = 180°$

$\Rightarrow 90° + \angle P = 180°$

$\Rightarrow \angle P = 180° - 90° = 90°$

In ΔAPC, by Pythagoras theorem,

$AC^2 = AP^2 + PC^2$ ....(1)

In ΔBPD, by Pythagoras theorem,

$BD^2 = BP^2 + DP^2$ ....(2)

Adding equations (1) and (2),

$AC^2 + BD^2 = AP^2 + PC^2 + BP^2 + DP^2$

$\Rightarrow AC^2 + BD^2 = (AP^2 + DP^2) + (PC^2 + BP^2) = AD^2 + BC^2$