

Topics : Solution of Triangle, Vector, Sequence & Series, Continuity & Derivability

Type of Questions		M.M., Min.
Comprehension (no negative marking) Q.1 to Q.4	(3 marks, 3 min.)	[12, 12]
Single choice Objective (no negative marking) Q.5,6,7	(3 marks, 3 min.)	[9, 9]
Subjective Questions (no negative marking) Q.8,9,10	(4 marks, 5 min.)	[12, 15]

**COMPREHENSION (FOR Q. 1 TO 4)**

A regular heptagon (seven sides) is inscribed in a circle of radius 1.  $A_1, A_2, \dots, A_7$  be its vertices,  $G_1$  is centroid of  $\Delta A_1 A_2 A_5$  and  $G_2$  be centroid of  $\Delta A_3 A_6 A_7$ . P is centroid of  $\Delta O G_1 G_2$ , where O (origin) is centre of circumscribing circle.

1. Angle  $\angle POA_1$  is equal to

- (A)  $\frac{\pi}{7}$                       (B)  $\frac{2\pi}{7}$                       (C)  $\frac{5\pi}{7}$                       (D)  $\frac{6\pi}{7}$

2. OP is equal to

- (A)  $\frac{10}{9}$                       (B)  $\frac{8}{9}$                       (C)  $\frac{1}{9}$                       (D) 1

3.  $G_3$  is such that centroid of triangle  $G_1 G_2 G_3$  is O, then

- (A)  $3OG_3 = OA_2$               (B)  $3OG_2 = A_2 G_3$               (C)  $2OG_3 = OA_2$               (D)  $OG_3 = G_3 A_2$

4.  $PA_1$  is equal to

- (A)  $\frac{1}{9} \sqrt{\left(82 - 18 \cos \frac{\pi}{7}\right)}$                       (B)  $\frac{1}{9} \sqrt{\left(82 + 18 \cos \frac{2\pi}{7}\right)}$   
 (C)  $\frac{1}{9} \sqrt{\left(82 - 18 \sin \frac{2\pi}{7}\right)}$                       (D) None of these

5. If sine of the acute angle between two vectors  $-3\hat{i} + 4\hat{j} + \hat{k}$  and  $-2\hat{i} - \hat{j} - \hat{k}$  be  $1 + \frac{1}{2}x - \frac{1}{8}x^2$  ..... to  $\infty$  then x is equal to

- (A)  $\frac{155}{156}$                       (B)  $\frac{1}{156}$                       (C)  $-\frac{1}{156}$                       (D) None of these

6. Let  $\vec{a}, \vec{b}, \vec{c}$  be three unit vectors such that  $|\vec{a} + \vec{b} + \vec{c}| = 1$  and  $\vec{a} \perp \vec{b}$ . If  $\vec{c}$  makes angles  $\alpha, \beta$  with  $\vec{a}, \vec{b}$  respectively then  $\cos\alpha + \cos\beta$  is equal to  
 (A)  $3/2$  (B) 1 (C)  $-1$  (D) none of these
7. If  $\vec{a} = (1, -1, 2), \vec{b} = (-2, 3, 5), \vec{c} = (2, -2, 4)$  and  $\hat{i}$  is the unit vector in positive x-direction, then  $(\vec{a} - 2\vec{b} + 3\vec{c}) \cdot \hat{i}$  is equal to  
 (A) 11 (B) 15 (C) 18 (D) 10
8. Find the sum of infinite terms of the series :  $\frac{3}{2.4} + \frac{5}{2.4.6} + \frac{7}{2.4.6.8} + \frac{9}{2.4.6.8.10} + \dots$
9. Let  $f : [a, b] \rightarrow \mathbb{R}$  a continuous positive function, differentiable on  $[a, b]$ . Prove that there exists  $c \in (a, b)$  such that  $\frac{f(b)}{f(a)} = e^{(b-a)\frac{f'(c)}{f(c)}}$
10. If  $\vec{a}$  and  $\vec{b}$  are two vectors such that  $|\vec{a} + \vec{b}| = |\vec{a}|$  then prove that  $2\vec{a} + \vec{b}$  is a perpendicular to  $\vec{b}$ .

## Answers Key

1. (A)      2. (C)      3. (A)      4. (A)  
 5. (C)      6. (C)      7. (A)      8.  $1/2$