

Chapter—1

SQUARE & CUBE

Square Numbers

In the illustration given below each row (horizontal) and column (vertical) show equal number of dots. Each row and column has five dots, can you count the total number of dots?

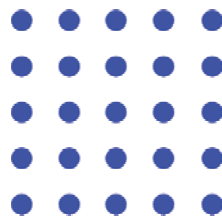


Fig.1

Now use two-two, three- three, four –four , eight-eight dots in rows and columns(equal numbers)and make some more square patterns and then complete the table given below -

Table 1.1

S.No.	No. of dots in each row or column	Total number of dots in the pattern made
1.	5	25
2.	2	-----
3.	-----	9
4.	4	-----
5.	-----	-----
6.	-----	-----

Look at the last column in the above table. All the numbers indicate that they can be obtained by multiplying a number with the same number e.g. $25 = 5 \times 5$, $4 = 2 \times 2$, $9 = 3 \times 3$. So, all these number like 1, 4, 9, 16, 25etc. are (Perfect Square Numbers) Now can you write five more perfect Square Numbers?

We have ourselves made these numbers Perfect Square Numbers. If any number is given to you, how will you find out whether it is a Perfect Square Number or not?

Recognizing a Perfect Square Number

You have seen that you can arrange 9 dots in 3 rows of 3-3 dots each, 16 dots in 4 rows of 4-4 dots each, but when we have 10, 11 or 12 dots, we won't be able to arrange them in such a manner where the number of dots in each row and the number of rows be equal. You can verify such situations for small number of dots. If the number of dots are 109 or 784 or even larger, it would certainly be difficult to verify them using the above pattern. So, the Perfect Square Numbers can be verified by another method, which is the method of prime factorisation.

The Prime factor and the identification of the Perfect Square Number

For a Perfect Square Number, the number of dots in each row and the number of rows are equal e.g. the Perfect Square Number like 6×6 , 5×5 , 3×3 , 7×7 , etc.

For any number, where the multiple factors are made of complete pairs, would be Perfect Square Numbers. To get this, we would first break the number into its multiple factors and then make pairs

The Prime Factorisation Method

In this method, first the prime multiple factors of the given number is obtained and then pairs are made. The numbers in which all the prime multiple factors get in pairs, is recognized as a Perfect Square Number.

For example.

- (1) Take a number 144

The prime multiple factors of 144 are

$$\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$$

Here, pair of prime multiples of 144 are formed.

- (2) Take a number 252.

The prime multiple factors of 252 are $\underline{2 \times 2} \times \underline{3 \times 3} \times 7$

For this number, the factor 7 does not have any pair,

Therefore, 252 is not a Perfect Square Number.

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	252
2	126
3	63
3	21
7	7
	1

Activity 1.

Now find out the prime multiple factors for the numbers given in the Table and fill in the blanks given below.

Table 1.2

S.No.	Number	Prime factor	Are all the prime multiple factors in pairs	Whether the number is a Perfect Square or not
1.	16	$\underline{2} \times \underline{2} \times \underline{2} \times \underline{2}$	Yes	Perfect Square
2.	32	$\underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times 2$	No	No
3.	36			
4.	48			
5.	40			
6.	49			
7.	56			
8.	64			

Now verify, whether the numbers given below are Perfect Square Number or not.

- (i) 164 (ii) 256 (iii) 81 (iv) 120 (v) 576
 (vi) 205 (vii) 625 (viii) 324 (ix) 216 (x) 196

Some characteristics of Perfect Square Numbers

Identifying a Perfect Square by the unit number -

Carefully observe all the Perfect Square Numbers like 4,9,16, 25, 81, 100, 169, 324,256,625 etc. Is there any Perfect Square Number that has the digit 2,3,7 or 8 in the unit's place? Take some more numbers and check the unit's place. What conclusion do you draw? Let's observe the table below.

Table 1.3

Number	Perfect Square	Numbers	Perfect Square Numbers
1	1	2	4
3	9	4	16
5	25	6	36
7	49	8	64
9	81	10	100
11	121	12	144
.....
.....

You can still take the table forward. What do you notice about the perfect squares of the odd and even numbers in the table?
Put a $\sqrt{}$ mark in the statements that follow:

- (1) The Perfect Squares of odd numbers are : Odd/ Even
- (2) The Perfect Squares of even numbers are : Odd/ Even

Another Interesting Observation

Look at the figure given below-

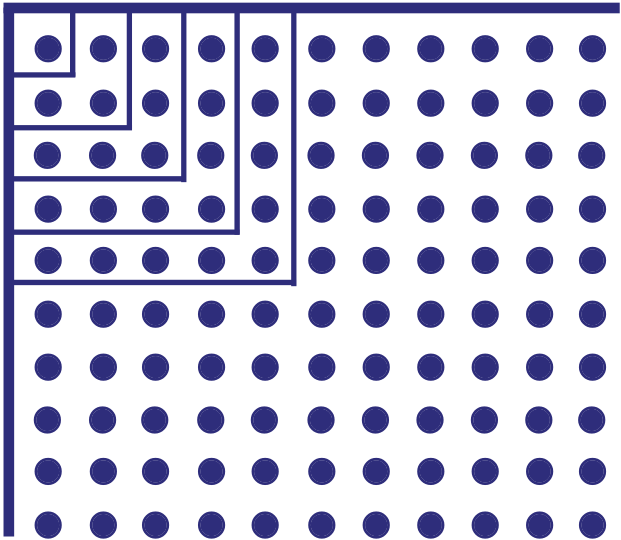


Figure 2

In the figure, starting from one end several squares have been drawn. Some parts of these squares have automatically been included in the new squares. If we add the total number of dots included in each part separately, we will see that the number of dots in the squares would be as follows.

1 st square	=	1	=	1 ²
2 nd square	=	1 + 3	=	2 ²
3 rd square	=	1 + 3 + 5	=	3 ²
4 th square	=	1 + 3 + 5 + 7	=	4 ²
5 th square	=	1 + 3 + 5 + 7 + 9	=	5 ²
6 th square	=	1 + 3 +	=	6 ²
7 th square	=	=
8 th square	=	=

What do you notice? We can see that whichever square is taken into consideration, the total number of dots in that square is a perfect square number. Can you tell what will be the number of dots in the eight & tenth squares?

We will notice that the dots included in the first, second, third and other squares are in the following manner :

1 st square	=	first odd number	=	1 ²
2 nd square	=	sum of first two odd number	=	2 ²
3 rd square	=	sum of first three odd number	=	3 ²

This continues, for example: the sum of the first 8 odd numbers is equal to 8². This holds true for as many numbers as you would work to check.

Hence we conclude that ‘The square of any natural number ‘n’ is equal to the sum of the first ‘n’ consecutive odd number.

Some more interesting patterns:

Observe the square numbers of 1, 11, 111

1 ²	=	1
11 ²	=	1 2 1
111 ²	=	1 2 3 2 1
1111 ²	=	1 2 3 4 3 2 1
11111 ²	=
111111 ²	=

Activity 2.

Ask your friend to say any two consecutive numbers. Add those numbers orally and write the sum in your notebook. Now ask your friend to find out the squares of the two numbers and subtract the smaller square number from the greater square number. After this, show the number you wrote in your notebook to your friend. Aren't the numbers the same?

How did that happen?

Can you guess how this will work out. Look at the following patterns:

$$4^2 - 3^2 = 16 - 9 = 7 = 4 + 3,$$

$$9^2 - 8^2 = 81 - 64 = 17 = 9 + 8,$$

$$13^2 - 12^2 = 169 - 144 = 25 = 13 + 12$$

Now look at the examples below:

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2, \quad 6^2 + 8^2 = 36 + 64 = 100 = 10^2$$

$$5^2 + 12^2 = 25 + 144 = 169 = 13^2 \text{ i.e. } 5^2 + 12^2 = 13^2$$

Can you find out more such examples? You will find that in every example there is a triplet of numbers and each triplet has some special numbers. The square of the larger number is the sum of the squares of remaining two numbers. Such numbers are known as Pythagoral Triplets. Triplets means a set of three numbers, like

(3, 4, 5), (6, 8, 10), or (5, 12, 13) are Pythagoral triplets.

Example 1. Verify whether (9, 40, 41) is a pythagoral triplet?

Solution : Here $9^2 + 40^2 = 81 + 1600 = 1681$

And $41^2 = 1681$

Therefore $9^2 + 40^2 = 41^2$. So, (9, 40, 41) is a Pythagoral Triplet.

PRACTICE - 1

Verify whether the sets given below are Pythagoral Triplet or not?

(i) (5, 12, 13) (ii) (8, 15, 17)

(iii) (10, 15, 25) (iv) (4, 7, 11)

NOTE : Our country knew about this triplet relationship much earlier. It is believed that in 600 BC an Indian mathematician, Bodhayan expressed it in an expanded form and explained it through multi digit numbers .

Making Perfect Squares From Numbers

As you have seen in the example on page 2 that in the multiple factors of 252, 2 and 3 had pairs but 7 didn't have any pair. If we divide or multiply the number by 7, we would be able to get pairs of all the multiple factors. This means 7 is the least number which when divided or multiplied by the number 252 will make the product or quotient, a perfect square. Now let us understand this by some more examples:

Example 2. Find out the least number which when multiplied by 720 will make a perfect square number.

Solution : $720 = \underline{2} \times \underline{2} \times \underline{2} \times \underline{2} \times \underline{3} \times \underline{3} \times 5$

Out of the prime multiple factors of 720, 5 did not make any pair. Therefore the perfect square number would be that in which 5 will also get a pair and for this we will have to multiply 720 by 5. Therefore 5 is the least number that will give a perfect square when multiplied with 720.

Example 3. Find out the least number which when divided by 140 will make a quotient a perfect square.

Solution : $140 = \underline{2} \times \underline{2} \times 5 \times 7$

Out of the prime multiple factors of 140, 5 and 7 did not have pairs. If 140 is divided by 35 (5×7), then the quotient will be a perfect square.

Example 4. Find out the lowest perfect square that is completely divisible by 6 and 8.

Solution : The lowest common multiple of 6 and 8 = 24

The prime factor of $24 = \underline{2} \times \underline{2} \times 2 \times 3$

Now we see that 2 and 3 are the multiple factors of 24 that do not have pairs. If we multiply 24 by $(2 \times 3) = 6$, it will be a perfect square. That would get divided by both 6 and 8. Therefore, the desired number is $24 \times 6 = 144$.

Try These Also

$$(10)^2 = 100, (300)^2 = 90000, (5000)^2 = 25000000$$

The square of 10 has two zeroes, the square of 300 has four zeroes, and that of 5000 has six zeroes. Can you think of a number the square of which has zero in the unit's place or has zeroes in all three places – units, tens and hundreds.

PRACTICE - 2

- (i) Find out the smallest number which when multiplied by 200, so that the product becomes a perfect square.
- (ii) Find out the smallest number which when multiplied by 180, so that the product becomes a perfect square.
- (iii) Find out the smallest number which when divided by 2352 make a quotient a perfect square.

EXERCISE - 1.1

Q.1 Pairs the multiple factors of the following numbers and say whether they are perfect squares or not?

- | | | |
|----------|----------|-----------|
| (i) 164 | (ii) 121 | (iii) 289 |
| (iv) 729 | (v) 1100 | |

Q.2 Give reasons why the given numbers are not perfect squares.

- | | | |
|-----------|------------|-----------------------------------|
| (i) 12000 | (ii) 1227 | (iii) 790 |
| (iv) 1482 | (v) 165000 | (vi) 15050 (vii) 1078 (viii) 8123 |

Q.3 Find out the numbers whose squares are even numbers and whose squares are odd numbers?

- | | | |
|------------|-------------|------------|
| (i) 14 | (ii) 277 | (iii) 179 |
| (iv) 205 | (v) 608 | (vi) 11288 |
| (vii) 1079 | (viii) 4010 | (ix) 1225 |

Q.4 Look at the given pattern and fill in the blanks:-

11^2	=	121
101^2	=	10201
1001^2	=	1002001
10001^2	=	-----
100001^2	=	-----
-----	=	1000002000001

Cube Numbers

Till now we have thought about square numbers. When a number is multiplied by the same number, the product obtained is known as the square of that number. Now if the product is again multiplied by the original number, the number obtained would be the cube of that number.

$$\begin{array}{ll} \text{e.g. } 2 \times 2 \times 2 = 8 & \text{or } 2^3 = 8 \\ 7 \times 7 \times 7 = 343 & \text{or } 7^3 = 343 \end{array}$$

Here 8 and 343 are the cubes of 2 and 7 respectively.
Observe the table below and fill in the blanks:-

Table 1.4

Number	Multiplied thrice	Exponential form	Cube number
1	$1 \times 1 \times 1$	1^3	1
2	$2 \times 2 \times 2$	2^3	8
3	$3 \times 3 \times 3$	-----	-----
4	-----	-----	-----
5	-----	-----	-----
6	-----	-----	-----
7	-----	-----	-----
8	-----	-----	-----
9	-----	-----	-----
10	-----	-----	-----

In the above table, the numbers obtained i.e. 1, 8, 27.....etc are cube numbers of integer numbers, 1, 2, 3 etc.

such numbers are known as **Perfect cubes**.

Think about the even numbers and their cubes given in the tables, what do you conclude ? Are cubes of odd numbers also odd? Are cubes of even number also even?

How will you recognise whether a given number is cube or not? To recognise Square numbers ,we had made pairs of the prime multiple factors .Those numbers that made perfect squares had complete pairs made.

We can extend the same method for cubes. We can write 8 as $2 \times 2 \times 2$. On finding the prime multiple factors, we notice that 2 has been multiplied thrice and after a triplet there is no other multiple factor left. Let us now take a look at the number 27. Here 3 is multiplied thrice and can make a triplet. If we take 24 as the number then we shall have $24 = 2 \times 2 \times 2 \times 3$ i.e. 2 come thrice and form a triplet but 3 remains without any pair. This means 24 is not a cube number.

Thus to identify a perfect cube number, we can conclude that if the prime multiple factors get arranged in the form of triplets of that number then the Number is perfect cube, otherwise not.

Let us take few more examples:

Example 5. Is 216 a perfect cube number?

Solution : $216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$ (on finding in divisible multiple factors)
 $= 2^3 \times 3^3$ (all the factors get arranged in triplets)
 $= (2 \times 3)^3$
 $= 6^3$

So here, 216 can be represented as (6^3) the cube of 6.

Example 6. Say whether the number 23625 is a perfect cube?

Solution : $23625 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} \times 7$ (on finding the prime multiple factors)
 Here 23625 get arranged into triplets of 3 and 5 but not so for the multiple factor 7. Therefore 23625 is not a perfect cube.

Example 7. Find the smallest number that when multiplied by 68600 given a perfect cube?

Solution : $68600 = \underline{2 \times 2 \times 2} \times 5 \times 5 \times \underline{7 \times 7 \times 7}$

Here in the multiple factors of 68600, 2 and 7 have made triplets but 5 is pair and the number 68600 will have to be multiplied by 5 to make it a perfect cube.

Example 8. Find the smallest number which when divided by 408375, gives the quotient as a perfect cube.

Solution : $408375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} \times 11 \times 11$

Here, 408375 has multiple factor that has triplets for 3 and 5 not for 11. Therefore, if 408375 is divided by $11 \times 11 = 121$, then the quotient would be a perfect cube.

EXERCISE - 1.2

Q 1. Identify the perfect cubes in the given numbers.

- | | | |
|-----------|-----------|-----------|
| (i) 125 | (ii) 800 | (iii) 729 |
| (iv) 2744 | (v) 22000 | (vi) 832 |

Q 2. Find out that smallest number which when multiplied by 256 will make the product a perfect cube.

Q 3. Find out the smallest number which when multiplied by 1352 will make the product a perfect cube.

Q 4. Find out that smallest number which when multiplied by 8019 will make the quotient a perfect cube.

Q 5. Find out the smallest number which when multiplied by the numbers which are not cube in Question-1 give cube numbers.

Square Root

In the beginning of this chapter, we have learnt about perfect square numbers. Let's revise it through an activity once again:

**Activity 3.**

Table 1.5

S.No.	Number	Prime multiple factor	The number for which it is a square
1.	16	$\underline{2} \times \underline{2} \times \underline{2} \times \underline{2}$	$2 \times 2 = 4$
2.	25	$\underline{5} \times \underline{5}$	5
3.	36		
4.	49		
5.	64		
6.	100		
7.	144		
8.	196		

In the above activity you have seen that the square of 4 is 16, the square of 5 is 25 and that of 8 is 64. This can be also stated as follows: the square root of 16 is 4, the square root of 25 is 5 and it is written as; the square root of $16 = \sqrt{16} = 4$

The square root of $25 = \sqrt{25} = 5$ (We indicate the square root by the symbol “ $\sqrt{\quad}$ ”).

You must have noticed that the square of any natural number “n” is equal to the sum of the “n” initial consecutive odd numbers.

For example: $5^2 = 1 + 3 + 5 + 7 + 9 = 25$

Just as five (initial) consecutive odd numbers have been added to obtain the square of 5 (25); can we get the square root of 25 by subtracting the consecutive odd numbers from 25? Let's find out :

$$\begin{aligned} 25 - 1 &= 24, & 24 - 3 &= 21, & 21 - 5 &= 16 \\ 16 - 7 &= 9, & 9 - 9 &= 0, \end{aligned}$$

Here on subtracting the first five odd numbers from 25, the remainder obtained is zero (0). This means, the square root of 25 is 5 i.e. $\sqrt{25} = 5$.

Try to verify this process for some more perfect squares. You will find that **the number of initial odd numbers that need to be subtracted from a perfect square number to obtain zero, that number itself is the square root of the perfect square number.**

Can we verify perfect square number by this process?

You will find if the remainder on subtraction is not zero, the number is not a perfect square.

PRACTICE - 3

Orally state the square root of the following numbers:

- (i) 25 (ii) 49 (iii) 64 (iv) 81 (v) 121 (vi) 144

We can find out the square root of some numbers orally but not for all. Let us discuss the method of finding out square roots.

Square root by prime multiple factors

To find out the square roots of numbers by this method we first, determine the prime multiple factors for the number given. Then we make pairs of these and taking one number from each of the pairs, multiply them to get the square root.

Example 9. Find out the square root of 441.

Solution : $441 = \underline{3 \times 3} \times \underline{7 \times 7}$

$$\begin{aligned} \therefore \sqrt{441} &= \sqrt{\underline{3 \times 3} \times \underline{7 \times 7}} \\ &= 3 \times 7 \quad (\text{taking one number from each pair}) \\ &= 21 \end{aligned}$$

3	441
3	147
7	49
7	7
	1

Example 10. Find out the square root of 1296 .

Solution : $1296 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

$$\begin{aligned} \therefore \sqrt{1296} &= \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}} \\ &= 2 \times 2 \times 3 \times 3 \\ &= 36 \end{aligned}$$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

PRACTICE - 4

Find out the square roots of the following numbers by the prime factorisation method.

- (i) 289 (ii) 625 (iii) 900 (iv) 361 (v) 1764

Example 11. If the area of a square figure is 2025 square cm, find out the length of one side of the figure.

Solution : The area of the square figure = (side)² = 2025 cm².

$$\begin{aligned} \therefore \text{One side of the figure} &= \sqrt{2025} \\ &= \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5}} \\ &= 3 \times 3 \times 5 \\ &= 45 \text{ cm.} \end{aligned}$$

Example 12. A person planted 11025 mango saplings in his garden in a way that the number of plants in each row is equal to number of rows. Find the number of rows.

Solution : Let the number of rows in the garden be 'X'
 Since the total number of plants = $X \times X = X^2$
 $X^2 = 11025$ or $X = \sqrt{11025}$
 $= \sqrt{\underline{3 \times 3} \times \underline{5 \times 5} \times \underline{7 \times 7}}$
 $= 3 \times 5 \times 7 = 105$

Therefore, the number of rows in the garden = 105

EXERCISE - 1.3

- Find out the square root of the given numbers by prime factorisation method.

(i) 361	(ii) 400	(iii) 784
(iv) 1024	(v) 2304	(vi) 7056
- A group of boys bought 256 mangoes and distributed it among themselves. If each boy got the number of mangoes equal to the number of boys in the group, find out the number of boys in the group

Finding Square Root through Division Method.

Till now we have learnt to find out square root of perfect square numbers. In mathematics there is an interesting method by which we can find the square root of perfect square numbers and also the square root of those numbers which are not perfect square.

You know that the square root of one digit and two digit perfect square numbers are one digit numbers. Using their tables you can easily understand this.

Such as : $1 \times 1 = 1$, So 1 is the square root of 1
 $3 \times 3 = 9$, So 3 is the square root of 9
 $9 \times 9 = 81$, So 9 is the square root of 81

100 is the perfect square number after 81 which is a three digit number. Its square root is 10 which is a two digit number. (Do you find any pattern in digits of a number and its square root?) How will you find the square root of largest three digit number? Let's understand this by an example.

Example 13. Find the square root of 625?

Step 1. Starting from the unit place of the number 625 make pairs of numbers. A small horizontal line can be drawn to make pairs above the numbers. In this case only one pair 25 will be formed and the number 6 will $\overline{6} \overline{25}$ remain alone.

Step 2. Put 625 inside the division sign. Now find the largest divisor whose square cannot be greater than 6.

Here it is 2,

($\because 2 \times 2 = 4 < 6$, $3 \times 3 = 9 > 6$)

	quotient
divisor	$\overline{6} \overline{25}$ dividend
$\sqrt{}$	

Step 3. Now placing the number 2 in divisor and quotient and subtracting their product 4 after placing it below the number 6. The Remainder 2 will come.

$$\begin{array}{r} 2 \\ 2 \overline{) 6 \ 25} \\ \underline{-4} \\ 2 \end{array}$$

Step 4. Addition of the same number 2 to the divisor will result to number 4. Now write it below. After that shift the pair 25 besides the result 2 obtained from the step3. Here the new dividend will become 225.

$$\begin{array}{r} 2 \\ 2 \overline{) 6 \ 25} \\ \underline{-4} \\ 2 \end{array}$$

Step 5. Now we have to put such number after 4 in the divisor and after 2 in the quotient that its product with the new divisor should not be exceed 225. if we put 3 in the divisor after 4 the new divisor will be 43.

$$\begin{array}{r} 2 \ 5 \\ 2 \overline{) 6 \ 25} \\ \underline{-4} \\ 2 \end{array}$$

$$\therefore 43 \times 3 = 129 < 225$$

Successively placing 4 and 5 in the divisor we see –

$$44 \times 4 = 176 < 225$$

$$\text{and } 45 \times 5 = 225 = 225$$

It is clear that taking 5. in the quotient is appropriate. Subtract the product 225 from the new dividend 225. 0 will be the Remainder. So the final quotient 25 will be the square root of 625.

$$\text{i.e. } \sqrt{625} = 25$$

Example 14. Find the square root of 9409.

Solution –

Step 1. We have given a number of 4 digit, Starting from the unit place of the number 9409 make pairs of two digit numbers. Two pairs 94 and 09 will be formed. Put them inside the division sign.

$$\sqrt{94 \ 09}$$

Step 2. Considering the second pair 94 as a dividend find the largest divisor whose square should not be greater than 94. It is obvious that the divisor is 9. Now placing the number 9 in divisor and quotient and subtracting their product 81 after placing it below the number 94. The Remainder 13 will come.

$$\begin{array}{r} 9 \\ 9 \overline{) 94 \ 09} \\ \underline{-81} \\ 13 \end{array}$$

Step 3. Adding the same number 9 to the divisor. Write the sum 18 below it. Now write down the pair number 09 beside the remainder 13. the new dividend will be 1309. Now as shown in the previous example, what should be written beside the divisor 18 so that it's product with the new divisor become 1309, nearer or lesser.

$$\begin{array}{r} 9 \\ 9 \overline{) 94 \ 09} \\ \underline{-81} \\ 13 \end{array}$$

Here we see that the divisor is a 3 digit number and the dividend is a 4 digit number. If we remove the unit place digit of both the divisor and the dividend then the divisor remains 18 and the dividend remains 130. Now it can be easily seen that $18 \times 7 = 126$ which is less than 130.

So we can put 7 in the divisor and the quotient.

$$187 \times 7 = 1309$$

Subtracting this product 1309 from the new dividend 1309. The Remainder 0 will come. The final quotient 97 will be the square root of 9409.

$$\begin{array}{r} 97 \\ 9 \overline{) 94 \ 09} \\ \underline{+ 9} \\ 187 \\ \underline{- 13 \ 09} \\ 0 \end{array}$$

From both the above examples we have got square roots of perfect square numbers. Now let's take an example which is not a perfect square. In such cases the square root gives digits after decimal point.

Example 15. Find the square root of 8772.

Solution – You know that 8772 can be written as 8772.0000. As in the last two examples we made pairs of 2 digit numbers starting from the unit place, the same procedure will be applied here. Digits in the unit and ten's place will make a pair and digits in the hundred and thousand's place will make a pair. In the right side of decimal points digits in the tenth and hundredth place put together to make a pair and so on.

We will write the number

8772.0000 as $\overline{87} \ \overline{72} . \overline{00} \ \overline{00}$ let's find out the square root of 8772 as before –

After the remainder 123 bring down the first pair of zeroes which are after the decimal points. Now put the decimal point before the number to be written in the quotient. Do the process of division in the similar way.

If we want square root upto two decimal places only then we can stop our procedure over here. If we have to proceed further, then put a pair of zero each time after remainder and proceed it and get new quotient. So the square root of 8772 will be approximately 93.65 i.e. approximate.

$$\begin{array}{r} 93.65 \\ 9 \overline{) 87 \ 72.00 \ 00} \\ \underline{+ 9} \\ 183 \\ \underline{+ 3} \\ 1866 \\ \underline{+ 6} \\ 18725 \\ \sqrt{} \\ 110400 \\ \underline{- 93625} \\ 16775 \end{array}$$

EXERCISE - 1.4

- Q. 1. Find out the square root of following by division method.
 (i) 529 (ii) 1369 (iii) 1024 (iv) 5776
 (v) 900 (vi) 7921 (vii) 50625 (viii) 363609
- Q. 2. In a cinema hall the owner wants to organize the chair in this way that the number of rows and columns of seats should be equal. If there are total 1849 seats then find out the number of rows and column.
- Q. 3. The area of a square garden is 1444 square meter, so find out the length and breadth of that garden.

Example 16. Determine the square root of 51.84

Solution –

$$\begin{array}{r}
 7.2 \\
 \hline
 7 \quad \overline{51.84} \\
 +7 \quad \underline{-49} \\
 142 \quad \overline{02.84} \\
 \quad \underline{-284} \\
 \quad \quad \overline{000} \\
 \quad \quad \underline{-000} \\
 \quad \quad \quad \sqrt{}
 \end{array}$$

$$\sqrt{51.84} = 7.2$$

Example 17. Determine the square root of 23.1 up to two places of decimal.

Solution –

$$\begin{array}{r}
 4.80 \\
 \hline
 4 \quad \overline{23.1000} \\
 +4 \quad \underline{-16} \\
 88 \quad \overline{07.10} \\
 8 \quad \underline{-704} \\
 960 \quad \overline{600} \\
 \quad \underline{-600} \\
 \quad \quad \overline{000} \\
 \quad \quad \underline{-000} \\
 \quad \quad \quad \sqrt{}
 \end{array}$$

$$\sqrt{23.1} = 4.80$$

Example 18. Determine the square root of 2 up to three places of decimal.

Solution –

$$\begin{array}{r}
 1.414 \\
 \sqrt{2.00\ 00\ 00} \\
 \underline{1} \\
 24 \\
 \underline{+ 4} \\
 281 \\
 \underline{+ 1} \\
 2824 \\
 \underline{- 1\ 12\ 96} \\
 00604
 \end{array}$$

EXERCISE - 1.5

Q. 1. Find out the squares root of following numbers.

(i) 7.29

(ii) 16.81

(iii) 9.3025

Q. 2. Find out the squares root of following up to two places of decimal.

(i) 0.9

(ii) 5

(iii) 7

Cube root

To find square root of a number we were making pair of two same number from the prime factors of that number. After that we were tarring one number from each pair and multiply them. To find cube root of any multiply, we processed this method. For cube root we trio the prime factors of that number and multiply one-one number from each trio. Obtained number will our required cube root.

Example 19. Determine cube root of 512.

Solution – $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$$\sqrt[3]{512} = 2 \times 2 \times 2$$

$$\sqrt[3]{512} = 8$$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Example 20. Determine cube root of 91125.

Hint - We see that the number has 5 in its unit place. So the number is completely divisible by 5.

Solution – $91,125 = 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$$\sqrt[3]{91,125} = 5 \times 3 \times 3$$

$$\sqrt[3]{91,125} = 45$$

5	91,125
5	18,225
5	3,645
3	729
3	243
3	81
3	27
3	9
3	3
	1

EXERCISE - 1.6

Determine the cube root of following.

- | | | |
|------------|--------------|-------------|
| (i) 125 | (ii) 343 | (iii) 1331 |
| (vi) 2197 | (v) 9261 | (vi) 166375 |
| (vii) 4913 | (viii) 42875 | |

WE HAVE LEARNT

1. If “n” is a number, then $n \times n$ or n^2 will be known as its square and $n \times n \times n$ or n^3 will be called its cube.
2. Those numbers whose unit place have numbers like 2,3,7 or 8 cannot be perfect square numbers.
3. If a perfect square number ends in an even number of zeroes, then they would also be perfect squares.
4. The squares and cubes of even numbers are always **even numbers** and squares and cubes of odd numbers are always **odd numbers**.
5. The square of any natural number “n” is the sum of the initial consecutive odd numbers .
6. If three numbers are in such a sequence that the square of the greater number is equal to the sum of the square of the remaining two numbers , then the numbers are known as Pythagoral Triplets e.g. $3^2 + 4^2 = 5^2$ therefore (3,4,5) make a Pythagoral Triplet.
7. Square root is represented by the symbol “ $\sqrt{\quad}$ ”. This is known as the symbol of under root or the square root of the number. The number written under this symbol is determined .

