Chapter - 13
Exponent and Power

A2S2F6

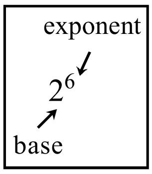
13.1 Can you guess the distance of the sun from the earth?

The distance of the sun from the earth is 149,600,000,000 meter. Similarly the distance of the sun from the Saturn is 1,433,500,000,000 meter and distance of the Uranus from the Saturn is 1,439,000,000,000 meter. Can you read these numbers? Can you tell which one is less among the three distances ? These are large numbers. These very large numbers are difficult to read. Such large numbers are difficult to compare also. If we can write these large numbers in concise form then it will be easier to compare. Perhaps you are eager to know how to write the large number in short. To express the large number in short we use Exponents. In this chapter we shall learn the preliminary concept of exponents.

13.2 Exponent :

Can you tell what is the product of $2\times2\times2\times2\times2\times2$? The product is 64.

Here 2 is multiplied by itself 6 times.



$\therefore$ $2\times2\times2\times2\times2\times2 = 2^6$

Here 2 is called base and 6 is called exponent.

The number 2^6 is read as '2 to the power 6'or simply as 6^{th} power of 2.

Do notice that 2^6 is the exponential form of 64.

Example 1 : Let us write in exponential form –

$3\times3\times3\times3\times3 = 3^5$ [5 times 3 is multipied]

If base is 3 and exponent is 5 then the number

$3^5 = 3\times3\times3\times3\times3 = 243$

The exponential form of 243 is 3^5

$5\times5\times5\times5\times5\times5\times5 = 5^7$ [7 times 5 is multiplied]

the number $5^7 = 5\times5\times5\times5\times5\times5\times5 = 15625$

The exponential from of 15625 is 5^7

$10\times10\times10\times10 = 10^4$

If 10 is base and exponent is 4 then the number will be $10^4 = 10\times10\times10\times10 = 10000$

10^4 is the exponential form of 10000

$(-3)\times(-3)\times(-3)\times(-3) = (-3)^4$

We can also take the negative integers as base. For example in $(-3)^4$, -3 is base and 4 is exponent.

Here 4 times (-3) is multiplied by itself. Here base is negative integer.

What will be the value of $(-3)^4$?

$$(-3)^4 = (-3)\times(-3)\times(-3)\times(-3)$$
$$= 81$$

Again $3\times3\times3\times3 = 3^4$

Now find, if $(-3)^4 = 3^4$

Similarly find if $(-3)^5 = 3^5$

If '*a*' is an integer, then taking '*a*' as base let us write the exponential form –

$a\times a = a^2$ [read as '*a*' square]

$a\times a\times a = a^3$ [read as '*a*' cubed]

$a\times a\times a\times a = a^4$

$a\times a\times a\times a\times a\times a\times a\times a = a^8$

Let us see the exponential form of different integer

$2\times2\times2\times3\times3\times3\times3 = 2^3\times3^4$

Here 2 is multiplied 3 times by itself and 4 times 3 is multiplied by itself

Similarly $(-3)\times(-3)\times(-3)\times(-3)\times5\times5\times5 = (-3)^4\times5^3$

$(-2)\times(-2)\times(-2)\times(-5)\times(-5)\times(-5) = (-2)^3\times(-5)^3$

$a\times a\times a\times a\times b\times b\times b = a^4\times b^3$

$p\times p\times q\times q\times q\times q\times r\times r\times r = p^2\times q^4\times r^3$

We have already mentioned earlier that the large numbers can be written in concise form using exponents.

$1000 = 10\times10\times10 = 10^3$

10^3 is the exponential form of 1000

Similarly $10000 = 10\times10\times10\times10 = 10^4$

10^4 is the exponential form of 10000

$100000 = 10\times10\times10\times10\times10 = 10^5$

$\therefore$ 1 Lakh $= 10^5$

Observe the exponential form of a few large numbers

1 million = 10^6

1 billion = 10^9

1 Crore = 10^7

Googol = 10^{100} etc.

You have already learnt about the expanded form of numbers in Class V.

What will be the expanded form of 32574 ?

$32574 = 3\times10000+2\times1000+5\times100+7\times10+4$

Which can be written as $3\times10^4+2\times10^3+5\times10^2+7\times10+4$. Here hundred, thousand and ten thousand are written in exponential form.

Expand 54729 in exponential form –

$54729 = 5\times10^4+4\times10^3+7\times10^2+2\times10+9$

In the same way, try to expand 125, 7632, 576 in exponential form. You will learn the exponential forms of large numbers in the later part of this chapter.

Suppose we have to express 81 in exponential form.

Lina expressed the number in exponential form as follows –

$81= 3\times3\times3\times3 = 3^4$

Bina expressed the number in exponential form as follows –

$81= 9\times9 = 9^2$

What do you observe from both the exponential forms ? The base and exponent are different in both the exponential forms one is 3 and the other is 9. Like wise the exponents are also different. One of the exponent is 4 and other is 2. Thus same number can be expressed in different bases and exponents.

Let us see the exponential form of 256 –

$256 = 2\times2\times2\times2\times2\times2\times2\times2 = 2^8$

$256 = 4\times4\times4\times4 = 4^4$

$256 = 16\times16 = 16^2$

Thus we get three different exponential forms of 256.

Example 2 : Express 729 as the power of 3.

Solution : Since we have to express 729 as the power of 3. So we need to divide 729 by 3 repeatedly.

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

Example 3: Express 8000 as the power of the factor of prime numbers.

Solution :

$$\begin{array}{r|l} 2 & 8000 \\ \hline 2 & 4000 \\ \hline 2 & 2000 \\ \hline 2 & 1000 \\ \hline 2 & 500 \\ \hline 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$\therefore\ 8000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^6 \times 5^3$

Let us try in other way –

$$\begin{aligned} 8000 &= 8 \times 1000 \\ &= 2 \times 2 \times 2 \times 10 \times 10 \times 10 \\ &= 2 \times 2 \times 2 \times 2 \times 5 \times 2 \times 5 \times 2 \times 5 \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^6 \times 5^3 \end{aligned}$$

$8 = 2 \times 2 \times 2$

$1000 = 10 \times 10 \times 10$

Example 4 : Find the value of $5^2 \times 3^4$

Solution : $5^2 \times 3^4$

$= 5 \times 5 \times 3 \times 3 \times 3 \times 3 = 2025$

Example 5 : Find the value of

$(-1)^3, (-1)^4, (-1)^5, (-1)^8, (-1)^9, (-1)^9$

Solution :

$$\begin{aligned} (-1)^3 &= \underbrace{(-1) \times (-1)} \times (-1) \\ &= 1 \times (-1) \\ &= -1 \end{aligned}$$

$$(-1)^4 = \underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}$$
$$= 1\times 1$$
$$= 1$$

$$(-1)^5 = \underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}\times(-1)$$
$$= 1\times 1\times(-1)$$
$$= 1\times(-1) = -1$$

or $(-1)^5 = (-1)^4\times(-1)$
$$= 1\times(-1) = -1$$

$$(-1)^8 = \underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}$$
$$= 1\times 1\times 1\times 1 = 1$$

$$(-1)^7 = \underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}\times(-1)$$
$$= 1\times 1\times 1\times(-1) = (-1)$$

$$(-1)^9 = \underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}\times\underbrace{(-1)\times(-1)}\times(-1)$$
$$= 1\times 1\times 1\times(-1)$$
$$= 1\times(-1) = -1$$

If you observe the above examples, you will notice that if the exponent of (-1) is even number then the value will be 1 and if the exponent of (-1) is odd number then the value will be (-1)

$\therefore$ $(-1)^{\text{even number}} = 1$

$(-1)^{\text{odd number}} = -1$

So $(-1)^{48} = 1$ [$\because$ exponent is even number]

$(-1)^{70} = 1$ [$\because$ exponent is even number]

$(-1)^{65} = -1$ [$\because$ exponent is odd number]

$(-1)^{73} = -1$ [$\because$ exponent is odd number]

Exercise -13.1

1. Find out the correct answer

(i) The value of $(-1)^5$ is

(a) -1 (b) 1 (c) 5 (d) -5

(ii) The value of $(-5)^4$ is

(a) -625 (b) 625 (c) 256 (d) -256

2. Express in exponential form

(i) $5\times5\times5\times5\times5$ (ii) $3\times3\times2\times2\times2$

(iii) $(-2)\times(-2)\times(-2)\times3\times3\times3\times3$ (iv) $b\times b\times b\times b\times b\times c\times c\times c$

(v) $a\times a\times a\times b\times b\times c\times c\times c\times c\times c$

3. Find the value of

(i) 2^7 (ii) $(-2)^7$ (iii) 3^6 (iv) $(-3)^6$

(v) $2^5\times4^4$ (vi) $5^2\times3^3$ (vii) $(-3)^2\times(-5)^3$

4. Express in exponential form

(i) 343 (ii) 729 (iii) 2187

(iv) –2187 (v) 3125 (vi) – 3125

5. Express each of the following numbers as product of the powers of their prime factors :

(i) 100 (ii) 300 (iii) 1000

(iv) 2700 (v) 405 (vi) 1600

6. Fill in the blanks with appropriate sign (>, < or =)

(i) $(-5)^3 \square 5^3$ (ii) $(-5)^2 \square 5^2$ (iii) $(-7)^4 \square 7^4$

(iv) $(-1)^{15} \square (-1)^{10}$ (v) $(-1)^{11} \square 1^{11}$ (vi) $2^7 \square 2^6$

7. If $2592 = 2^m\times3^n$, then find the value of m and n.

8. If $16875 = 3^m\times5^n$, then find the value of m and n.

13. 3 Laws of Exponents :

There are some laws of exponents. By applying these laws of multiplication and division of numbers can be done easily. Let us discuss the laws of exponents–

13.3.1 Multiplying powers with the same base

(i) Let us calculate $3^2\times3^5$

$3^2\times3^5$

$= (3\times3)\times(3\times3\times3\times3\times3)$

$= 3\times3\times3\times3\times3\times3\times3$

$= 3^7$ [again expressed in exponential form]

Here in 3^2 and 3^5, the base is same (base 3), the exponents are 2 and 5.

3 is multiplied by itself 2 times in 3^2.

3 is multiplied by itself by 5 times in 3^5.

So in $3^2 \times 3^5$ there are total 2+5 = 7 times 3 are multiplied.

Hence $3^2 \times 3^5 = 3^7$

You have noticed that $3^2 \times 3^5 = 3^{2+5}$

$= 3^7$

Here in above example, base is same in both right and left sides. The value of the exponent in the right side is equal to the sum of the exponents in the left side.

(ii) Let us calculate $(-3)^4 \times (-3)^5 = (-3)^4 \times (-3)^5$

$= (-3)^{4+5}$

$= (-3)^9$

Here the base is same in both right and left side. The value of the exponent of right side is equal to the sum of the exponent of the left side.

(iii) Suppose that *'a'* is an integer other then zero.

$$a^3 \times a^4 = a^{3+4}$$
$$= a^7$$

Here the base is same and sum of the exponent is 7

$$\therefore \quad a^3 \times a^4 = a^{3+4} = a^7$$

(iv) Like the above examples, let us see

$$a^m \times a^n = \underbrace{(a \times a \times .... \times a)}_{m \text{ times } a} \times \underbrace{(a \times a \times .... \times a)}_{n \text{ times } a}$$
$$= a \times a \times a \times .... \times a$$
$$(m + n \text{ times } a)$$
$$= a^{m+n}$$

If *'a'* is an integer ($a \neq 0$) and m and n are whole numbers,

then $a^m \times a^n = a^{m+n}$

This is a law of exponent

Let us simplify, with the help of the above rule –

(i) $5^3 \times 5^7 = 5^{3+7} = 5^{10}$

(ii) $(-2)^{10} \times (-2)^8 = (-2)^{10+8} = (-2)^{18}$

This law can be applied in case of multiplication of more numbers of same base.

(i) $7^2 \times 7^4 \times 7^5 = 7^{2+4+5} = 7^{11}$

(ii) $(-4)^3 \times (-4)^7 \times (-4)^2 \times (-4)^5$

$= (-4)^{3+7+2+5} = (-4)^{17}$

Now can we apply the above law in case of $3^4 \times 2^5$? The base of 3^4 is 3 and 2^5 is 2. This means both the bases are different. So we can not apply the above law.

13. 3.2 Dividing powers with the same base

(i) Let us calculate $3^6 \div 3^2$

Here both bases are same (base 3). The exponents are 6 and 2.

$$3^6 \div 3^2 = \frac{3^6}{3^2}$$

$$= \frac{3\times3\times3\times3\times3\times3}{3\times3} \quad \text{[dividing both numerator and denominator by } 3\times3\text{]}$$

$$= 3\times3\times3\times3$$

$$= 3^4$$

In the above, we see that 4 is obtained by subtracting the exponent of numerator and denominator. Thus 4 is the exponent of the quotient.

$\therefore \quad 3^6 \div 3^2 = 3^4 = 3^{6-2}$

(ii) Let us calculate $(-2)^5 \div (-2)^2$.

Here also both the bases are same i.e. (-2)

$$(-2)^5 \div (-2)^2 = \frac{(-2)^5}{(-2)^2}$$

$$= \frac{(-2)\times(-2)\times(-2)\times(-2)\times(-2)}{(-2)\times(-2)}$$

$$= (-2)\times(-2)\times(-2)$$

$$= (-2)^3$$

$$= (-2)^{5-2}$$

(iii) Let us calculate $a^7 \div a^4$, where 'a' is a non zero integer –

$$a^7 \div a^4 = \frac{a^7}{a^4}$$
$$= \frac{a \times a \times a \times a \times a \times a \times a}{a \times a \times a \times a}$$
$$= a \times a \times a$$
$$= a^3$$
$$= a^{7-4}$$

(iv) As in the above example, let us see

$$a^m \div a^n = \frac{a^m}{a^n}, \text{ here } m>n$$
$$= \frac{a \times a \times a \times \ldots\ldots\ldots\ldots \times a \quad (m \text{ times } a)}{a \times a \times a \times \ldots\ldots\ldots\ldots \times a \quad (n \text{ times } a)}$$
$$= a^{m-n}$$

If 'a' is a non zero integer and 'm' and 'n' are whole numbers where $m>n$, then $a^m \div a^n = a^{m-n}$, this is also a law of exponent.

Let us simplify using the above law –

(i) $5^{20} \div 5^{12} = 5^{20-12} = 5^8$ (ii) $(-3)^{12} \div (-3)^7 = (-3)^{12-7} = (-3)^5$

Law of exponent $a^m \div a^n = a^{m-n}$ when $\boldsymbol{m>n}$. Now if $m<n$, then what happens ? $(n-m)$ times 'a' will remain after dividing by equal numbers of 'a'. Now the exponent of 'a' will be negative integer. You will learn more about the exponent of negative integer in Class VIII.

Let us take an example

Example : Find the value of $(3^2)^4$

Solution : Meaning of $(3^2)^4$ is that (3^2) is multiplied by itself four times.

$$\therefore (3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$$
$$= (3 \times 3) \times (3 \times 3) \times (3 \times 3) \times (3 \times 3)$$
$$= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$
$$= 3^8$$

Let us try to find out the value of $(3^2)^4$ in another way

$\therefore\ (3^2)^4 = 3^2\times3^2\times3^2\times3^2$ [Product of the power of same base]

$= 3^{2+2+2+2}$

$= 3^8$

Observe that 2+2+2+2 = 8, means 2 is added 4 times.

$\therefore 2+2+2+2 = 2\times4 = 8$

In $(3^2)^4$, the exponent of 3 is 2.

Again exponent of 3^2 is 4, so there is two exponents of 3. One is 2 and other is 4.

The product of the exponent is $2\times4 = 8$

$\therefore \quad (3^2)^4 = 3^{2\times4}=3^8$

Similarly $(3^3)^5 = 3^{3\times5} = 3^{15}$

$(3^6)^4 = 3^{6\times4} = 3^{24}$

$(7^2)^8 = 7^{2\times8} = 7^{16}$

The same rule is applied in negative bases

$\{(-2)^5\}^3 = (-2)^{5\times3} = (-2)^{15}$

$\{(-3)^4\}^3 = (-3)^{4\times3} = (-3)^{12}$

On the basis of above examples we can write

For any non zero integer 'a', $(a^m)^n = a^{m\times n}$, where m and n are two whole numbers.

Now $(a^m)^n$

$= a^m\times a^m\times a^m\times a^m........\times a^m$ [n numbers]

$= a^{(m+m+m+............+m)}$ [Sum of n numbers of m]

$= a^{m\times n}$

This is a law of exponent.

13.5. Multiplying different bases of same exponent :

You have already learnt that

(i) $4^3\times4^5 = 4^{3+5} = 4^8$

(ii) $5^4\times5^4 = 5^{4+4} = 5^8$

Do notice that the bases are same in the above multiplications. But the exponent may differ.

Let us take another example –

Can you simplify $3^4\times5^4$

The bases are different in both terms. But the exponent is same. Can you simplify with the help of laws of exponent ?

$3^4 \times 5^4 = (3\times3\times3\times3)\times(5\times5\times5\times5)$

$= (3\times5)\times(3\times5)\times(3\times5)\times(3\times5)$ [Think why ?]

$= 15\times15\times15\times15$

$= 15^4$

$\therefore\ 3^4\times5^4 = 15^4$.

The product of the bases of the terms to be multiplied is 15. Are the exponents of the two numbers to be multiplied some with the exponent of the product ?

$\therefore\ 3^4\times5^4 = (3\times5)^4 = 15^4$

Let us take another example –

Simplify $(-2)^5\times3^5$

In this case also the bases are different but the exponent is same.

$(-2)^5\times3^5$

$= (-2)\times(-2)\times(-2)\times(-2)\times(-2)\times3\times3\times3\times3\times3$

$= \{(-2)\times3\}\times\{(-2)\times3\}\times\{(-2)\times3\}\times\{(-2)\times3\}\times\{(-2)\times3\}$

$= (-6)\times(-6)\times(-6)\times(-6)\times(-6)$

$= (-6)^5$ [Notice that (–6) is the product of (–2) and 3]

$\therefore\ (-2)^5\times3^5$

$= (-2\times3)^5$

$= (-6)^5$

Similarly $5^3\times7^3 = (5\times7)^3 = 35^3$

$(-2)^7\times2^7 = (-2\times2)^7 = (-4)^7$

$(-3)^4\times(-5)^4 = \{(-3)\times(-5)\}^4 = 15^4$

If *a* and *b* are two non zero integers
Then $a^m\times b^m = (a\times b)^m$, where *m* is a whole number
This is also a law of exponent

Let us examine the law–

$$a^m \times b^m = (\underbrace{a\times a\times ......\times a}_{(m \text{ times } a)})\times(\underbrace{b\times b\times .....\times b}_{(m \text{ times } b)})$$

$$= (a\times b)\times(a\times b)\times.....\times(a\times b) \qquad [m \text{ times } (a\times b)]$$

$$= (a\times b)^m$$

$$\therefore\ a^m\times b^m = (a\times b)^m$$

13.6 Dividing powers with the same base :

Observe the following examples

(i) Simplify $\frac{3^4}{5^4}$

Here the power are same but the bases are different (exponent is 4)

$$\frac{3^4}{5^4} = \frac{3\times3\times3\times3}{5\times5\times5\times5}$$

$$= \frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}$$

$$= \left(\frac{3}{5}\right)^4 \quad [\text{exponent is } 4]$$

(ii) $\frac{a^5}{b^5} = \frac{a\times a\times a\times a\times a}{b\times b\times b\times b\times b}$

$$= \frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}$$

$$= \left(\frac{a}{b}\right)^5$$

Similarly $\frac{(-2)^4}{3^4} = \left(\frac{-2}{3}\right)^4$

$$\frac{5^6}{(-3)^6} = \left(\frac{5}{-3}\right)^6$$

$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$, Here a and b are two non zero integers and n is a whole number

This is also a law of exponent

This law can also be proved by the following example

$$\frac{a^n}{b^n} = \frac{a \times a \times .... \times a}{b \times b \times .... \times b}$$

$$= \frac{a}{b} \times \frac{a}{b} \times .... \times \frac{a}{b} \qquad [n \text{ times } \frac{a}{b}]$$

$$= \left(\frac{a}{b}\right)^n$$

13.7 Numbers with exponent Zero

Can you tell what 5^0 is equal to ?

Can 5 be multiplied 0 times ?

$$\frac{5^3}{5^3} = \frac{5 \times 5 \times 5}{5 \times 5 \times 5}$$

$$= 1$$

$$\therefore \frac{5^3}{5^3} = 1$$

In case of $\frac{5^3}{5^3}$ if you apply the law of exponent $\frac{a^m}{a^n} = a^{m-n}$, what will you get?

$$\frac{5^3}{5^3} = 5^{3-3} = 5^0$$

$$\therefore \quad 5^0 = 5^{3-3} = \frac{5^3}{5^3} = 1$$

$\therefore \quad 5^0 = 1$

Similarly $7^0 = 1$

$(-2)^0 = 1$

You have already learnt that

$a^m \div a^n = a^{m-n}$

if $m = n$ then

$$a^m \div a^m = a^{m-m}$$

$$\frac{a^m}{a^m} = a^0$$

$\therefore\ a^0 = 1$, where a is a non zero integer $(a \neq 0)$

The laws of exponent are

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^n \times b^n = (a \times b)^n$

(v) $\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n$

(vi) $a^0 = 1$

Exemple 8 :

Simplify (i) $(3^5 \times 3^2) \div 3^3$ (ii) $\dfrac{2^3 \times 3^5 \times 4^2}{9 \times 8}$

Solution : (i) $(3^5 \times 3^2) \div 3^3$

$= 3^{5+2} \div 3^3$

$= 3^7 \div 3^3$

$= 3^{7-3}$

$= 3^4$

(ii) $\dfrac{2^3 \times 3^5 \times 4^2}{9 \times 8}$

$$= \frac{2^3 \times 3^5 \times (2^2)^2}{3^2 \times 2^3}$$

$$= \frac{2^3 \times 3^5 \times 2^4}{3^2 \times 2^3}$$

$$= \frac{2^{3+4} \times 3^5}{2^3 \times 3^2}$$

$$= \frac{2^7 \times 3^5}{2^3 \times 3^2}$$

$$= 2^{7-3} \times 3^{5-2}$$

$$= 2^4 \times 3^3$$

Exemple 9 : Simplify (using laws of exponent)

$$\frac{(2a^3b^2)^3 \times (3ab)^4}{2 \times 9 \times (a^2b^2)^3}$$

Solution : $\dfrac{(2a^3b^2)^3 \times (3ab)^4}{2 \times 9 \times (a^2b^2)^3}$

$$= \frac{2^3 (a^3)^3 (b^2)^3 . 3^4 . a^4 b^4}{2 \times 3^2 \times (a^2)^3 (b^2)^3}$$

$$\frac{2^3 \times 3^4 . a^{3\times3} b^{2\times3} . a^4 . b^4}{2 \times 3^2 . a^{2\times3} . b^{2\times3}}$$

$$= \frac{2^3 \times 3^4 . a^9 . b^6 . a^4 . b^4}{2 \times 3^2 . a^6 . b^6}$$

$$= \frac{2^3 \times 3^4 . a^{9+4} . b^{6+4}}{2 \times 3^2 . a^6 . b^6}$$

$$= \frac{2^3 \times 3^4 . a^{13} . b^{10}}{2 \times 3^2 . a^6 . b^6}$$

$$= \frac{2^3}{2} \times \frac{3^4}{3^2} . \frac{a^{13}}{a^6} . \frac{b^{10}}{b^6}$$

$$= 2^{3-1} \times 3^{4-2} . a^{13-6} . b^{10-6}$$

$$= 2^2 \times 3^2 . a^7 . b^4$$

Exercise - 13 .2

1. Simplify using laws of exponents (Write the answer in the exponential form)

(i) $3^5 \times 3^7 \times 3^{10}$ (ii) $(2^7 \times 2^6) \div 2^5$

(iii) $(2^0 \times 2^5 \times 2^8) \div (2^0 \times 2^6 \times 2^7)$ (iv) $(3^4)^2 \times (3^2)^3$

(v) $(16^2 \times 8^3) \div (2^5)^2$

(vi) $\frac{3 \times 7^3 \times 5^8}{21 \times 15}$ (vii) $\frac{25 \times 5^0 \times 9^4}{10 \times 9^3}$

(viii) $\frac{(2^5)^3 \times 16 \times 3^0}{(2^3)^5 \times 5^0}$ (ix) $\frac{2^3 \times 3^3}{6^2}$

(x) $\frac{5^3 \times 7^3 \times 2^3}{70^2}$ (xi) $\frac{7^5 \times 3^2 \times 6^4 \times 4}{(21)^2 \times 343 \times 2^6 \times 81}$

2. Express in terms of prime factors and write in exponential form

(i) 768 (ii) 729 (iii) 128×625

(iv) 64×729 (v) 1000

3. Simplify

(i) $\frac{(2a^2b^3)^3 \times (3ab^2)^4}{6^2 \times (ab)^5}$ (ii) $\frac{(a^m \times b^n)^p \times (a^p \times b^m)^n}{(a \times b)^p}$

(iii) $\frac{(ab^2)^3 \times (a^2b^3)^4 \times (a^3c^2)^3}{(a^2b^2c^2)^2}$

4. If $3^m = 81$, then find the value of *m*.

5. Check whether true or false

(i) $3a^0 = (3a)^0$ (ii) $2^3 > 3^2$

(iii) $(5^0)^4 = (5^4)^0$ (iv) $2^3 \times 3^3 = 6^5$

(v) $\frac{2^5}{3^5} = (\frac{2}{3})^{5-5}$ (vi) $2^5 = 5^2$

13.8 Expressing large number in the standard form.

In the begining of this chapter we discussed how to express large numbers in a very concise form by using exponents. Now we shall try to express large numbers in standard form.

Let us learn what standard form is ? Before going to express large numbers in standard form let us consider the following numbers : 81, 810, 8100, 81000, 810000, 8125, 81751, 792657, 7456 etc.

$$81 = \frac{81}{10} \times 10 = 8{\cdot}1 \times 10$$

$$810 = 81 \times 10 = \frac{81}{10} \times 100 = 8{\cdot}1 \times 100 = 8{\cdot}1 \times 10^2$$

$$8100 = 81 \times 100 = \frac{81}{10} \times 1000 = 8{\cdot}1 \times 1000 = 8{\cdot}1 \times 10^3$$

$$81000 = 81 \times 1000 = \frac{81}{10} \times 10000 = 8{\cdot}1 \times 10000 = 8{\cdot}1 \times 10^4$$

$$810000 = 81 \times 10000 = \frac{81}{10} \times 100000 = 8{\cdot}1 \times 100000 = 8{\cdot}1 \times 10^5$$

Again,

$$8125 = \frac{8125}{1000} \times 1000 = 8{\cdot}125 \times 10^3$$

$$81751 = \frac{81751}{10000} \times 10000 = 8{\cdot}1751 \times 10^4$$

$$792657 = \frac{792657}{100000} \times 100000 = 7{\cdot}92657 \times 10^5$$

$$745{\cdot}6 = \frac{7456}{10} = \frac{7456}{100} \times 100 = 7{\cdot}456 \times 10^2$$

Every number in above given expressed as the product of the power of ten. Observe that in these numbers there is one integer before the decimal point in all the numbers.

Notice that though we can express the number 4786 in different ways –

For example — $4786 = 4{\cdot}786 \times 10^3$

$4786 = 47{\cdot}86 \times 10^2$

$4786 = 478{\cdot}6 \times 10^1$

But remember $47{\cdot}86 \times 10^2$ or $478{\cdot}6 \times 10^1$; is not the standard in form of 4786.

The standard form of 4786 is $\mathbf{4{\cdot}786 \times 10^3}$.

Observe that if we want to express any number in standard form then one less than the digit counted (number of digits) to the left of the decimal point in a given number is the exponent of 10 in the standard form. For example in 4786 there is no decimal in the number. So, we will consider that the decimal in $4786 = 4786{\cdot}0$ will be at the right and here the number of digits to the left of decimal is 4. The exponent of 10 in the standard form is $4 - 1 = 3$

$\therefore\ 4786 = 4{\cdot}786 \times 10^3$

In $745{\cdot}6$ there are 3 digits to the left of decimal part. Hence the exponent of 10 in the standard form is

$\therefore\ 745{\cdot}6 = 7{\cdot}456 \times 10^2$

Example 10 : Express the following numbers in standard form

(i) $45{\cdot}26$ (ii) $4326{\cdot}3$ (iii) 6,450,000 (iv) 5265300

(v) 80,100,000,000 (vi) $7467{\cdot}293$ (vii) $5{\cdot}256$

Solution : (i) $45{\cdot}26 = \dfrac{4526}{100}$

$$= \frac{4526}{1000} \times 10 = 4{\cdot}526 \times 10^1$$

[Notice that, $45{\cdot}26$ there are 2 digits to the left of decimal point, hence the exponent of 10 in the standard form is $2 - 1 = 1$]

(ii) $4326{\cdot}3 = 4{\cdot}3263 \times 10^3$

[There are 4 digits to the left of decimal point, so the exponent of the 10 will be $4 - 1 = 3$]

(iii) $6{,}450{,}000 = \dfrac{6450000}{10^6} \times 10^6$

$= 6{\cdot}45 \times 10^6$

[6450000 = 6450000·0 there are 7 digits to the left of decimal point. So the exponent of 10 is 6]

(iv) $5265300 = 5{\cdot}2653 \times 10^6$

(v) $80{,}100{,}000{,}000 = 8{\cdot}01 \times 10^{10}$

(vi) $7467{\cdot}293 = 7{\cdot}467293 \times 10^3$

(vii) $5{\cdot}256 = 5{\cdot}256 \times 10^0$

Example 11 : Compare which one is bigger

(i) 71270000 and 695690000

Solution : $71270000 = 7{\cdot}127 \times 10^7$

$695690000 = 6{\cdot}9569 \times 10^8$

By comparing the powers of 10 we can say that 95690000 is greater than 71270000.

When we express the large numbers in standard form it becomes easy to remember the numbers. This helps us in fields of science and technology and also in other fields of application of large numbers. We shall conclude this chapter with a few examples –

Let us begin with some data from our Galaxy. Do you know what is the mass of sun. The mass of sun is –

1,989,100,000,000,000,000,000,000,000,000 kg.

Mass of Mercury is 330,220,000,000,000,000,000,000 kg.

The average distance of Mercury from sun is = 57,910,000 km.

Mass of Venus = 4867000,000,000,000,000,000,000 kg.

Which planet between Mercury or Venus, has a greater mass ?

If we express the above large numbers in standard form then it becomes easier to read, remember and compare them–

Mass of Sun = 1,989,100,000,000,000,000,000,000,000,000 kg.

$= 19891 \times 10^{26}$ kg.

$= \dfrac{1{\cdot}9891}{10000} \times 10^{26} \times 10000$ kg.

$= 1{\cdot}9891 \times 10^{26} \times 10^{4}$ kg.

$= 1{\cdot}9891 \times 10^{30}$ kg.

Has the number formed by 31 numerals become easier to read and understand ?

Mass of Mercury $= 330{,}220{,}000{,}000{,}000{,}000{,}000{,}000$ kg.

$= 33022 \times 10^{19}$ kg.

$= \dfrac{33022}{10000} \times 10^{19} \times 10000$ kg.

$= 3{\cdot}3022 \times 10^{23}$ kg.

Mass of Venus $= 4867000{,}000{,}000{,}000{,}000{,}000{,}000$ kg.

$= 4867 \times 10^{21}$ kg.

$= \dfrac{4867}{1000} \times 10^{21} \times 10^{3}$ kg.

$= 4{\cdot}867 \times 10^{24}$ kg.

After expressing the mass of the two planets in standard form you can easily compare and tell that the mass of Venus is greater than that of Mercury.

So when we express two numbers in standard form they become to compare.

The overage distance from Sun to Mercury $= 57{,}910{,}000$ km.

$= 57910000$ km.

$= 5791 \times 10^{4}$ km.

$= \dfrac{5791}{1000} \times 10^{4} \times 10^{3}$ km.

$= 5{\cdot}791 \times 10^{4} \times 10^{3}$ km.

$= 5{\cdot}791 \times 10^{7}$ km.

For comparing two numbers sometimes it becomes easier if we express them in standard form and equate in the power of 10.

Distance from Sun to Earth $= 149600000$ km.

Distance from Sun to Venus $= 108200000$ km.

Look, the distance from Sun to Earth $= 149600000$ km.

$= 1496 \times 10^{5}$ km.

$= \dfrac{1496}{100} \times 10^{4} \times 10^{3}$ km.

$= 14{\cdot}96 \times 10^7$ km.

Distance from Sun to Venus $= 108200000$ km.

$= 1082 \times 10^5$ km.

$$= \frac{1082}{100} \times 10^5 \times 10^2 \text{ km.}$$

$= 10{\cdot}82 \times 10^7$ km.

Since $14{\cdot}96$ is larger than $10{\cdot}82$.

$\therefore$ $14{\cdot}96 \times 10^7 > 10{\cdot}82 \times 10^7$ [While expressing in standard form, the power of 10 have been equaliser]

$\therefore$ The distance of earth from the sun is greater than the distance of Venus from the sun.

According to 2011 census the population of India was 1,220,000,000; If we express this in standard form then it will be $1{\cdot}22 \times 10^9$.

Exercise - 13.3

1. Express one following numbers in standard form

(i) 5,273,294 (ii) 7,10,021 (iii) 6,400,000 (iv) 18,129

(v) 23961,32 (vi) 75,000,000,000 (vii) 70,010,000,000

(viii) 45026·9 (ix) 3206·19 (x) 475000000000

2. Express the numbers in following statment in standard form–

(i) Radius of Moon 1737·1 km.

(ii) Radius of Earth 6771000 m.

(iii) Distance between Mercury and Venus 50,290,000 km.

(iv) Distance between Mercury and Jupiter 720,420,000 km.

(v) 1 light year = 9,460,700,000,000 km.

(vi) 1 Nautical Unit (AU)=149,600,000 km.

(vii) Mass of Moon 73490,000,000,000,000,000,000 km.

(viii) Radius of Sun 695510 km.

(ix) There is 1,386,000,000 cubic Kilometer sea water on Earth.

(x) Spead of light in Vaccum 299,792,458 meter / second (approx 300,000,000 meter/second).

3. Compare (which one is greater)

(i) 5761000000000000; 57600000000000000

(ii) $343{\cdot}6\times10^{19}$; $\cdot03436\times10^{17}$

What we have to learnt

1. Very large numbers which are difficult to read, understand, compare and operate upon, can be made easier by converting them into exponential form.
2. Examples of Exponential form
   $1024 = 2^{10}$
   $729 = 3^{6}$
   $625 = 5^{4}$
   $100{,}000 = 10^{5}$
3. **Laws of Exponents :** For any non Zero integer a and b; and for whole numbers m, and n

(i) $a^m \times a^n = a^{m+n}$ (ii) $a^m \div a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$ (iv) $a^m \times b^m = (ab)^m$

(iv) $a^m \div b^m = \left(\frac{a}{b}\right)^m$ (v) $a^0 = 1$

(vi) $(-1)^{\text{even number}} = 1$ (vii) $(-1)^{\text{odd number}} = -1$

(viii) $a^{-m} = \frac{1}{a^m}$, (where m is whole number)

(ix) $a^m = \frac{1}{a^{-m}}$, where m is whole number

Let us know

Srinivas Ramanujan (1887-1920) was a world famous Mathematician. The number 1729 as known as Ramanujan Number. He showed that the number 1729 is the smallest number which can be expressed as the sum of two cubes in two different ways.

$$1729 = 1^3+12^3 = 9^3+10^3$$