Vector Algebra

Short Answer type Quesitons

1. Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

Sol. Let $\vec{c}$ denote the sum of $\vec{a}$ and $\vec{b}$. We have

$\vec{c} = (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} + 3\hat{k}) = \hat{i} + 5\hat{k}$

Now $|\vec{c}| = \sqrt{1^2 + 5^2} = \sqrt{26}$.

Thus, the required unit vector is $\hat{c} = \dfrac{\vec{c}}{|\vec{c}|} = \dfrac{1}{\sqrt{26}}(\hat{i} + 5\hat{k}) = \dfrac{1}{\sqrt{26}}\hat{i} + \dfrac{5}{\sqrt{26}}\hat{k}$.

2. Find a vector of magnitude 11 in the direction opposite to that of $\overrightarrow{PQ}$, where P and Q are the points $(1,\ 3,\ 2)$ and $(-1,\ 0,\ 8)$, respectively.

Sol. The vector with initial point P(1, 2, 3) and terminal point Q(-1, 0, 8) is given by

$\overrightarrow{PQ} = (-1-1)\hat{i} + (0-3)\hat{j} + (8-2)\hat{k} = -2\hat{i} - 3\hat{j} + 6\hat{k}$

Thus, $\overrightarrow{QP} = -\overrightarrow{PQ} = 2\hat{i} + 3\hat{j} - 6\hat{k}$

$\Rightarrow |\overrightarrow{QP}| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4+9+36} = \sqrt{49} = 7$

Therefore, unit vector in the direction of $\overrightarrow{QP}$ is given by

$$\widehat{QP} = \frac{\overrightarrow{QP}}{|\overrightarrow{QP}|} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$$

Hence, the required vector of magnitude 11 in direction of $\overrightarrow{PQ}$ is

$$11\ \widehat{QP} = 11\left(\frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}\right) = \frac{22}{7}\hat{i} + \frac{33}{7}\hat{j} - \frac{66}{7}\hat{k}.$$

3. Find the position vector of a point R which divides the line joining the two points P and Q with position vectors $\overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\overrightarrow{OQ} = \vec{a} - 2\vec{b}$, respectively, in the ratio $1:2$, (i) internally and (ii) externally.

Sol. (i) The position vector of the point R dividing the join of P and Q internally in the ratio 1:2 is given by

$$\overrightarrow{OR} = \frac{2(2\vec{a} + \vec{b}) + 1(\vec{a} - 2\vec{b})}{1+2} = \frac{5\vec{a}}{3}.$$

(ii) The position vector of the point R' dividing the join of P and Q in the ratio $1:2$ externally is given by

$$\overrightarrow{OR} = \frac{2(2\vec{a} + \vec{b}) - 1(\vec{a} - 2\vec{b})}{2-1} = 3\vec{a} + 4\vec{b}.$$

4. If the points (-1, -1, 2), (2, m, 5) and (3, 11, 6) are collinear, find the value of m.

Sol. Let the given points be $A(-1,-1,2)$, $B(2,m,5)$ and $C(3,11,6)$. Then

$\overrightarrow{AB} = (2+1)\hat{i} + (m+1)\hat{j} + (5-2)\hat{k} = 3\hat{i} + (m+1)\hat{j} + 3\hat{k}$

And $\overline{\mathrm{AC}} = (3+1)\hat{i} + (11+1)\hat{j} + (6-2)\hat{k} = 4\hat{i} + 12\hat{j} + 4\hat{k}.$

Since A, B, C, are collinear, we have $\overrightarrow{\mathrm{AB}} = \lambda \overrightarrow{\mathrm{AC}}$, i.e.,

$(3\hat{i} + (m+1)\hat{j} + 3\hat{k}) = \lambda(4\hat{i} + 12\hat{j} + 4\hat{k})$

$\Rightarrow 3 = 4\lambda \text{ and } m+1 = 12\lambda$

Therefore, $m = 8$.

5. Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with y and $z-axes$, respectively.

Sol. Here $m = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $n = \cos\frac{\pi}{2} = 0.$

Therefore, $l^2 + m^2 + n^2 = 1$ gives

$l^2 + \frac{1}{2} + 0 = 1$

$\Rightarrow l = \pm\frac{1}{\sqrt{2}}$

Hence, the required vector $\vec{r} = 3\sqrt{2}(l\hat{i} + m\hat{j} + n\hat{k})$ is given by

$\vec{r} = 3\sqrt{2}(\pm\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + 0\hat{k}) = \vec{r} = \pm 3\hat{i} + 3\hat{j}.$

6. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\hat{c} = \hat{i} + 3\hat{j} - \hat{k}$, find λ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$.

Sol. We have

$\lambda\vec{b} + \vec{c} = \lambda(\hat{i} + \hat{j} - 2\hat{k}) + (\hat{i} + 3\hat{j} - \hat{k})$

$= (\lambda+1)\hat{i} + (\lambda+3)\hat{j} - (2\lambda+1)\hat{k}$

Since $\vec{a} \perp (\lambda\vec{b} + \vec{c}), \vec{a}.(\lambda\vec{b} + \vec{c}) = 0$

$\Rightarrow 2(\hat{i} - \hat{j} + \hat{k}).[(\lambda+1)\hat{i} + (\lambda+3)\hat{j} - (2\lambda+1)\hat{k}] = 0$

$\Rightarrow 2(\lambda+1) - (\lambda+3) - (2\lambda+1) = 0$

$\Rightarrow \lambda = -2.$

7. Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{i} + 2\hat{j} + \hat{k}$ and $-\hat{i} + 3\hat{j} + 4\hat{k}$.

Sol. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$. Then

$$\vec{a}\times\vec{b}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{vmatrix} = \hat{i}(8-3) - \hat{j}(4+1) + \hat{k}(3+2) = 5\hat{i} - 5\hat{j} + 5\hat{k}$$

$\Rightarrow |\vec{a}\times\vec{b}|\sqrt{(5)^2 + (-5)^2 + (5)^2} = \sqrt{3(5)^2} = 5\sqrt{3}.$

Therefore, unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$ is given by

$$\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}=\frac{5\hat{i}-5\hat{j}+5\hat{k}}{5\sqrt{3}}$$

Hence, vectors of magnitude of $10\sqrt{3}$ that are perpendicular to plane of $\vec{a}$ and $\vec{b}$ are $\pm 10\sqrt{3}\left(\frac{5\hat{i}-5\hat{j}+5\hat{k}}{5\sqrt{3}}\right)$, *i.e.*, $\pm 10(\hat{i}-\hat{j}+\hat{k})$.

Long AnswerType Questions

8. Using vectors, prove that $cos(A-B)=cos\,A\;cos\,B+sin\,A\;sin\,B$**.**

Sol. Let $\widehat{OP}$ and $\widehat{OQ}$ be unit vectors making angles A and B, respectively, with positive direction of $x-axis$. Then $\angle QOP = A-B$ [Fig. 10.1]

We know $\widehat{OP}=\overrightarrow{OM}+\overrightarrow{MP}=\hat{i}\cos A+\hat{j}\sin A$ and $\widehat{OQ}=\overrightarrow{ON}+\overrightarrow{NQ}\;\hat{i}\cos B+\hat{j}\sin B$.

By definition $\widehat{OP}.\widehat{OQ}=|\widehat{OP}|\;|\widehat{OQ}|\cos A\text{-}B$

$= cos(A-B)$ ...(1) $\because (|\widehat{OP}|=1=|\widehat{OQ}|)$

In terms of components, we have

$\widehat{OP}.\widehat{OQ}=(\hat{i}\cos A+\hat{j}\sin A).(\hat{i}\cos B+\hat{j}\sin B)$

$= cos\,A\;cos\,B+sin\,A\;sin\,B$ ...(2)

From (1) and (2), we get

$cos(A-B)=cos\,A\;cos\,B+sin\,A\;sin\,B$.

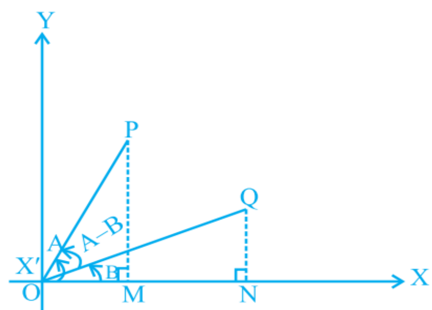


Fig. 10.1

9. Prove that in a $\Delta ABC, \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$, **where** $a,\ b,\ c$ **represents the magnitude of the sides opposite to vertices** $A, B, C,$ **respectively.**

Sol. Let the three sides of the triangle BC, CA and AB be represented by $\vec{a},\vec{b}$ and $\vec{c}$, respectively [Fig. 10.2]

We have $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. i.e., $\vec{a}+\vec{b}=-\vec{c}$

Which pre-cross multiplying by $\vec{a}$, and post cross multiplying by $\vec{b}$, gives

$\vec{a}\times\vec{b}=\vec{c}\times\vec{a}$

and $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}$

respectively. Therefore,

$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

$\Rightarrow |\vec{a}\times\vec{b}|\;|\vec{b}\times\vec{c}|\;|\vec{c}\times\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}|\sin(\pi-C)=|\vec{b}||\vec{c}|\sin(\pi-A)|\vec{c}||\vec{a}|\sin(\pi-B)$

$\Rightarrow ab\sin C=bc\sin A=ca\sin B$

Dividing by abc, we get

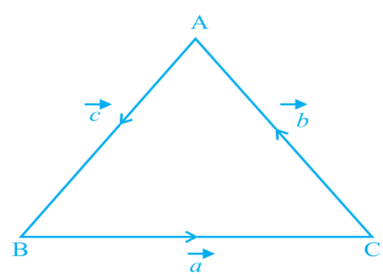


Fig. 10.2

$$\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b} \text{ i.e., } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Objective Type Questions

Choose the correct answer from the given the four options in each of the Examples 10 to 21.

10. **The magnitude of the vector** $6\hat{i}+2\hat{j}+3\hat{k}$ **is**

(A) 5

(B) 7

(C) 12

(D) 1

Sol. (B) is the correct answer.

11 **The position vector of the point which divides the join of points with position vectors** $\vec{a}+\vec{b}$ **and** $2\vec{a}-\vec{b}$ **in the ratio 1: 2 is**

(A) $\frac{3\vec{a}+2\vec{b}}{3}$

(B) $\vec{a}$

(C) $\frac{5\vec{a}-\vec{b}}{3}$

(D) $\frac{4\vec{a}+\vec{b}}{3}$

Sol. (D) is the correct answer. Applying section formula, the position vector of the required point is $\frac{2(\vec{a}+\vec{b})+1(2\vec{a}-\vec{b})}{2+1} = \frac{4\vec{a}+\vec{b}}{3}$

12. **The vector with initial point** $P(2,-3,5)$ **and terminal point** $Q(3,-4,7)$ **is**

(A) $\hat{i}-\hat{j}+2\hat{k}$

(B) $5\hat{i}-7\hat{j}+12\hat{k}$

(C) $\hat{i}+\hat{j}-2\hat{k}$

(D) None of these

Sol. (A) is the correct answer.

13. **The angle between the vectors** $\hat{i}-\hat{j}$ **and** $\hat{j}-\hat{k}$ **is**

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{-\pi}{3}$

(D) $\frac{5\pi}{6}$

Sol. (B) is the correct answer. Apply in formula $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$.

14. The value of λ for which the two vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \lambda\hat{j} + \hat{k}$ are perpendicular is

(A) 2

(B) 4

(C) 6

(D) 8

Sol. (D) is the correct answer.

15. The area of the parallelogram whose adjacent sides are $\hat{i} + \hat{k}$ and $2\hat{i} + \hat{j} + \hat{k}$ is

(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) 3

(D) 4

Sol. (B) is the correct answer. Area of the parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$ is $|\vec{a}\times\vec{b}|$.

16. If $|\vec{a}| = 8, |\vec{b}| = 3$ and $|\vec{a}\times\vec{b}| = 12,$ then value of $\vec{a}.\vec{b}$ is

(A) $6\sqrt{3}$

(B) $8\sqrt{3}$

(C) $12\sqrt{3}$

(D) None of these

Sol. (C) is the correct answer. Using the formula $|\vec{a}\times\vec{b}| = |\vec{a}|.|\vec{b}||\sin\theta|,$ we get

$\theta = \pm\frac{\pi}{6}.$

Therefore, $\vec{a}.\vec{b} = |\vec{a}|.|\vec{b}|\cos = 8\times3\times\frac{\sqrt{3}}{2} = 12\sqrt{3}.$

17. The 2 vectors $\hat{j} + \hat{k}$ and $3i - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC. The length of the Median through A is

(A) $\frac{\sqrt{34}}{2}$

(B) $\frac{\sqrt{48}}{2}$

(C) $\sqrt{18}$

(D) None of these

Sol. (A) is the correct answer. Median $\overrightarrow{\text{AD}}$ is given by

$|\overrightarrow{\text{AD}}| = \frac{1}{2}|3\hat{i} + \hat{j} + 5\hat{k}| = \frac{\sqrt{34}}{2}$

18. **The projection of vector $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ along $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is**

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) 2

(D) $\sqrt{6}$

Sol. (A) is the correct answer. Projection of a vector $\vec{a}$ on $\vec{b}$ is

$$\frac{\vec{a}.\vec{b}}{|\vec{b}|} = \frac{(2\hat{i} - \hat{j} + \hat{k}).(\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{1+4+4}} = \frac{2}{3}.$$

19. **If $\vec{a}$ and $\vec{b}$ are unit vector, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\vec{a} - \vec{b}$ be a unit vector?**

(A) 30^0

(B) 45^0

(C) 60^0

(D) 90^0

Sol. (A) is the correct answer. We have

$$(\sqrt{3}\vec{a} - \vec{b})^2 = 3\vec{a}^2 + \vec{b}^2 - 2\sqrt{3}\vec{a}.\vec{b}$$

$$\Rightarrow \vec{a}.\vec{b} = \frac{\sqrt{3}}{2} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^0$$

20. **The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ forming a right-handed system is**

(A) $\hat{k}$

(B) $-\hat{k}$

(C) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$

(D) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

Sol. (A) is the correct answer. Required unit vector is $\frac{(i-j)\times(i+j)}{|(i-j)\times(i+j)|} = \frac{2\hat{k}}{2}\hat{k}.$

21. **If $|\vec{a}|\ 3$ and $-1\ \ k\ \ 2,$ then $|k\vec{a}|$ lies in the interval**

(A) $[0,\ 6]$

(B) $[-3,\ 6]$

(C) $[3,\ 6]$

(D) $[1,\ 2]$

Vector Algebra
Objective Type Questions

Choose the correct answer from the given four options in each of the Exercises from 19 to 33 (M.C.Q)

19. The vector in the direction of the vector $\hat{i}-2\hat{j}+2\hat{k}$ that has magnitude 9 is

(A) $\hat{i}-2\hat{j}+2\hat{k}$

(B) $\dfrac{\hat{i}-2\hat{j}+2\hat{k}}{3}$

(C) $3\left(\hat{i}-2\hat{j}+2\hat{k}\right)$

(D) $9\left(\hat{i}-2\hat{j}+2\hat{k}\right)$

Sol. (C) Let $\vec{a}=\hat{i}-2\hat{j}+2\hat{k}$

Any vector in the direction of a vector $\vec{a}$ is given by $\dfrac{\vec{a}}{|\vec{a}|}$.

$$=\frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{1^2+2^2+2^2}}=\frac{\hat{i}-2\hat{j}+2\hat{k}}{3}$$

$\therefore$ Vector in the direction of $\vec{a}$ with magnitude $9=9\cdot\dfrac{\hat{i}-2\hat{j}+2\hat{k}}{3}$

$$=3(\vec{i}-2\vec{j}+2\vec{k})$$

20. The position vector of the point which divides the join of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio 3: 1 is

(A) $\dfrac{3\vec{a}-2\vec{b}}{2}$

(B) $\dfrac{7\vec{a}-8\vec{b}}{4}$

(C) $\dfrac{3\vec{a}}{4}$

(D) $\dfrac{5\vec{a}}{4}$

Sol. (D) Let the position vector of the point R divides the join of points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$.

$\therefore$ Position vector R $=\dfrac{3(\vec{a}+\vec{b})+1(2\vec{a}-3\vec{b})}{3+1}$

Since, the position vector of a point R dividing the line segment joining the points P and Q, whose position vectors are $\vec{p}$ and $\vec{q}$ in the ration *m: n* internally, is given by $\dfrac{m\vec{q}+n\vec{p}}{m+n}$.

$\therefore \quad R = \frac{5\vec{a}}{4}$

21. The vector having initial and terminal points as (2, 5, 0) and (–3, 7, 4), respectively is

(A) $-\hat{i} + 12\hat{j} + 4\hat{k}$

(B) $5\hat{i} + 2\hat{j} - 4\hat{k}$

(C) $-5\hat{i} + 2\hat{j} + 4\hat{k}$

(D) $\hat{i} + \hat{j} + \hat{k}$

Sol. (C) Required vector $= (-3-2)\hat{i} + (7-5)\hat{j} + (4-0)\hat{k}$

$= -5\hat{i} + 2\hat{j} + 4\hat{k}$

Similarly, we can say that for having initial and terminal points as

(i) $(4,\ 1,\ 1)$ *and* $(3,\ 13,\ 5)$, respectively

(ii) $(1,\ 1,\ 9)$ *and* $(6,\ 3,\ 5)$, respectively

(iii) $(1,\ 2,\ 3)$ *and* $(2,\ 3,\ 4)$, respectively, we shall get (a), (b) and (d) as its correct options.

22. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a}.\vec{b} = 2\sqrt{3}$ is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) $\frac{5\pi}{2}$

Sol. (B) Here, $|\vec{a}| = \sqrt{3}, |\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2\sqrt{3}$ [given]

We know that, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 2\sqrt{3} = \sqrt{3} \cdot 4 \cdot \cos\theta$

$\Rightarrow \cos\theta = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{3}$

23. Find the value of λ such that the vectors $\vec{a} = 2\vec{i} + \lambda\vec{j} + \vec{k}$ and $\vec{b} = \vec{i} + 2\vec{j} + 3\vec{k}$ are orthogonal

(A) 0

(B) 1

(C) $\frac{3}{2}$

(D) $-\frac{5}{2}$

Sol. (D) Since, two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal i.e., $\vec{a}\cdot\vec{b}=0$.

$\therefore (2\hat{i}+\lambda\hat{j}+\hat{k})\cdot(\hat{i}+2\hat{j}+3\hat{k})=0$

$\Rightarrow 2+2\lambda+3=0$

$\therefore \lambda=\frac{-5}{2}$

24. The value of λ for which the vectors $3\hat{i}-6\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+\lambda\hat{k}$ are parallel is

(A) $\frac{2}{3}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $\frac{2}{5}$

Sol. (A) Since, two vectors are parallel i.e., angle between them is zero.

$\therefore (3\hat{i}-6\hat{j}+\hat{k})\cdot(2\hat{i}-4\hat{j}+\lambda\hat{k})=|3\hat{i}-6\hat{j}+\hat{k}|\cdot|2\hat{i}-4\hat{j}+\lambda\hat{k}|$

$[\because \vec{a}\cdot\vec{b}=|a||b|\cos 0^o \Rightarrow \vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|]$

$\Rightarrow 6+24+\lambda=\sqrt{9+36+1}\sqrt{4+16+\lambda^2}$

$\Rightarrow 30+\lambda=\sqrt{46}\sqrt{20+\lambda^2}$

$\Rightarrow 900+\lambda^2+60\lambda=46(20+\lambda^2)$ [on squaring both sides]

$\Rightarrow \lambda^2+60\lambda-46\lambda^2=920-900$

$\Rightarrow -45\lambda^2+60\lambda-20=0$

$\Rightarrow -45\lambda^2+30\lambda+30\lambda-20=0$

$\Rightarrow -15\lambda(3\lambda-2)+10(3\lambda-2)=0$

$\Rightarrow (10-15\lambda)(3\lambda-2)=0$

$\therefore \lambda=\frac{2}{3},\frac{2}{3}$

Alternate method

Let $\vec{a}=3\hat{i}-6\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-4\hat{j}+\lambda\hat{k}$

Since, $\vec{a}\parallel\vec{b}$

$\Rightarrow \frac{3}{2}=\frac{-6}{-4}=\frac{1}{\lambda}\Rightarrow \lambda=\frac{2}{3}$

25. The vectors from origin to the points A and B are $\vec{a}=2\hat{i}-3\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}+\hat{k}$, respectively, then the area of the triangle OAB is

(A) **340**

(B) $\sqrt{25}$

(C) $\sqrt{229}$

(D) $\frac{1}{2}\sqrt{229}$

Sol. (D) $\therefore$ Area of $\Delta\text{OAB}=\frac{1}{2}|\overrightarrow{\text{OA}}\times\overrightarrow{\text{OB}}|$

$$=\frac{1}{2}|(2\hat{i}-3\hat{j}+2\hat{k})\times(2\hat{i}+3\hat{j}+\hat{k})|$$

$$=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -3 & 2\\ 2 & 3 & 1\end{vmatrix}$$

$$=\frac{1}{2}|[\vec{i}(-3-6)-\vec{j}(2-4)+\vec{k}(6+6)]|$$

$$=\frac{1}{2}|-9\hat{i}+2\hat{j}+12\hat{k}|$$

$$\therefore \text{Area of } \Delta\text{OAB}=\frac{1}{2}\sqrt{(81+4+144}=\frac{1}{2}\sqrt{229}$$

26. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to

(A) $\vec{a}^2$

(B) $3\vec{a}^2$

(C) $4\vec{a}^2$

(D) $2\vec{a}^2$

Sol. (D) Let $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore \vec{a}^2=x^2+y^2+z^2$

$$\therefore \vec{a}\times\hat{i}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ 1 & 0 & 0\end{vmatrix}$$

$=\hat{i}[0]-\hat{j}[-z]+\hat{k}[-y]$

$=z\hat{j}-y\hat{k}$

$\therefore (\vec{a}\times\hat{i})^2=(z\hat{j}-y\hat{k})(z\hat{j}-y\hat{k})$

$=y^2+z^2$

Similarly, $(\vec{a}\times\hat{j})^2=x^2+z^2$

And $(\vec{a}\times\hat{k})^2=x^2+y^2$

$\therefore (\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2 = y^2+z^2+x^2+z^2+x^2+y^2$

$= 2(x^2+y^2+z^2) = 2\vec{a}^2$

27. **If $|\vec{a}|=10, |\vec{b}|=2$ and $\vec{a}.\vec{b}=12,$ then value of $|\vec{a}\times\vec{b}|$ is**

(A) 5

(B) 10

(C) 14

(D) 16

Sol. (D) Here, $|\vec{a}|=10, |\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=12$ [given]

$\therefore \vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$

$12 = 10\times 2\cos\theta$

$\Rightarrow \cos\theta = \frac{12}{20} = \frac{3}{5}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{25}}$

$\sin\theta = \pm\frac{4}{5}$

$\therefore |\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}||\sin\theta|$

$= 10\times 2\times\frac{4}{5} = 16$

28. **The vectors $\lambda\hat{i}+\hat{j}+2\hat{k}, \hat{i}+\lambda\hat{j}-\hat{k}$ and $2\hat{i}-\hat{j}+\lambda\hat{k}$ are coplanar if**

(A) $\lambda=-2$

(B) $\lambda=0$

(C) $\lambda=1$

(D) $\lambda=-1$

Sol. (A) Let $\vec{a}=\lambda\hat{i}+\hat{j}+2\hat{k}, \vec{b}=\hat{i}+\lambda\hat{j}-\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+\lambda\hat{k}$

For $\vec{a}, \vec{b}$ and $\vec{c}$ to be coplanar,

$$\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \lambda(\lambda^2-1)-1(\lambda+2)+2(-1-2\lambda)=0$

$\Rightarrow \lambda^3-\lambda-\lambda-2-2-4\lambda=0$

$\Rightarrow \lambda^3-6\lambda-4=0$

$\Rightarrow (\lambda+2)(\lambda^2-2\lambda-2)=0$

$\Rightarrow \lambda=-2$ or $\lambda=\frac{2\pm\sqrt{12}}{2}$

$\Rightarrow \lambda=-2$ or $\lambda=\frac{2\pm 2\sqrt{3}}{2}=1\pm\sqrt{3}$

29. **If $\vec{a},\vec{b},\vec{c}$ are unit vector such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is**

(A) 1

(B) 3

(C) $-\frac{3}{2}$

(D) None of these

Sol. (C) We have, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, and $\vec{a}^2=1,\vec{b}^2=1,\vec{c}^2=1$

$\because (\vec{a}+\vec{b}+\vec{c})(\vec{a}+\vec{b}+\vec{c})=0$

$\Rightarrow \vec{a}^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}^2+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}^2=0$

$\Rightarrow \vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$[\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a},\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{b}$ and $\vec{c}\cdot\vec{a}=\vec{a}\cdot\vec{c}]$

$\Rightarrow 1+1+1+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=-\frac{3}{2}$

30. **Projection vector of $\vec{a}$ on $\vec{b}$ is**

(A) $\left(\frac{\vec{a}.\vec{b}}{|\vec{b}|^2}\right)\vec{b}$

(B) $\frac{\vec{a}.\vec{b}}{|\vec{b}|}$

(C) $\frac{\vec{a}.\vec{b}}{|\vec{a}|}$

(D) $\left(\frac{\vec{a}.\vec{b}}{|\vec{a}|^2}\right)\hat{b}$

Sol. (A) Projection vector of $\vec{a}$ on $\vec{b}$ is given by $=\vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\vec{b}=\left(\vec{a}\cdot\frac{\vec{b}}{|\vec{b}|}\right)\cdot\vec{b}$

31. **If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=2,|\vec{b}|=3,|\vec{c}|=5$, then value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ is**

(A) 0

(B) 1

(C) –19

(D) 38

Sol. (C) Here, $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $\vec{a}^2=4,\vec{b}^2=9,\vec{c}^2=25$

$\therefore \quad (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})=\vec{0}$

$\Rightarrow \quad \vec{a}^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{a}+\vec{b}^2+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}^2=\vec{0}$

$\Rightarrow \quad \vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0 \qquad [\because \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}]$

$\Rightarrow \quad 4+9+25+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$

$\Rightarrow \quad \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\dfrac{-38}{2}=-19$

32. If $|\vec{a}|=4$ and $-3\le\lambda\le 2$, then the range of $|\lambda\vec{a}|$ is

(A) [0, 8]

(B) [– 12, 8]

(C) [0, 12]

(D) [8, 12]

Sol. (C) We have, $|\vec{a}|=4$ and $-3\le\lambda\le 2$

$\therefore |\lambda\vec{a}|=|\lambda||\vec{a}|=\lambda|4|$

$\Rightarrow |\lambda\vec{a}|=|-3|4=12$, at $\lambda=-3$

$|\lambda\vec{a}|=|0|4=0$, at $\lambda=0$

And $|\lambda\vec{a}|=|2|4=8$, at $\lambda=2$

So, the range of $|\lambda\vec{a}|$ is (0, 12).

Alternate Method

Since, $-3\le\lambda\le 2$

$0\le|\lambda|\le 3$

$\Rightarrow 0\le 4|\lambda|\le 12$

$|\lambda\vec{a}|\in[0,12]$

33. The number of vectors of unit length perpendicular to the vectors $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$ is

(A) one

(B) two

(C) three

(D) infinite

Sol. (B) The number of vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is $\vec{c}$ (say)

i.e., $\vec{c}=\pm(\vec{a}\times\vec{b})$

So, there will be two vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$.

Fill in the blanks in each of the Exercises from 34 to 40.

34. The vector $\vec{a}+\vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$ if _________.

Sol. If vector $\vec{a}+\vec{b}$ bisects the angle between the non-collinear vectors, then

$\vec{a}\cdot(\vec{a}+\vec{b})=|\vec{a}|\cdot|\vec{a}+\vec{b}|\cos\theta$

$\vec{a}\cdot(\vec{a}+\vec{b})=a\sqrt{a^2+b^2}\cos\theta$

$\Rightarrow \cos\theta=\dfrac{\vec{a}\cdot(\vec{a}+\vec{b})}{a\sqrt{a^2+b^2}} \quad ...(i)$

And $\vec{b}\cdot(\vec{a}+\vec{b})=|\vec{b}|\cdot|\vec{a}+\vec{b}|\cos\theta$ -

$\vec{b}\cdot(\vec{a}+\vec{b}) = a\sqrt{a^2+b^2}\cos\theta$ [since, θ should be same]

$\Rightarrow \cos\theta = \dfrac{\vec{b}\cdot(\vec{a}+\vec{b})}{b\sqrt{a^2+b^2}}$

From Eqs. (i) and (ii),

$\dfrac{\vec{a}\cdot(\vec{a}+\vec{b})}{a\sqrt{a^2+b^2}} = \dfrac{\vec{b}\cdot(\vec{a}+\vec{b})}{b\sqrt{a^2+b^2}} \Rightarrow \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{\vec{b}}{|\vec{b}|}$

$\therefore \hat{a} = \hat{b} \Rightarrow \vec{a}$ and $\vec{b}$ are equal vector.

35. If $\vec{r}.\vec{a}=0, \vec{r}.\vec{b}=0,$ and $\vec{r}.\vec{c}=0$ for some non-zero vector $\vec{r}$, then the value of $\vec{a}(\vec{b}\times\vec{c})$ is ________.

Sol. Since, $\vec{r}$ is a non-zero vector, So, we can say that $\vec{a}, \vec{b}$ and $\vec{c}$ are in a same plane.

$\vec{a}(\vec{b}\times\vec{c}) = 0$

[since, angle between $\vec{a}, \vec{b}$ and $\vec{c}$ are zero i.e., $\theta = 0$]

36. The vectors $\vec{a} = 3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b} = -\hat{i}-2\hat{k}$ are the adjacent sides of a parallelogram. The acute angle between its diagonals is ________.

Sol. We have, $\vec{a} = 3\hat{i}-2\hat{j}+2\hat{k}$ and $\vec{b} = -\hat{i}-2\hat{k}$

$\therefore \vec{a}+\vec{b} = 2\hat{i}-2\hat{j}$ and $\vec{a}-\vec{b} = 4\hat{i}-2\hat{j}+4\hat{k}$

Now, let θ is the acute angle between the diagonals $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

$\therefore \cos\theta \dfrac{(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})}{|\vec{a}+\vec{b}||\vec{a}-\vec{b}|}$

$= \dfrac{(2\hat{i}-2\hat{j})\cdot(4\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{8}\sqrt{16+4+16}} = \dfrac{8+4}{2\sqrt{2}\cdot 6} = \dfrac{1}{\sqrt{2}}$

$\therefore \theta = \dfrac{\pi}{4} \left[\because \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}\right]$

37. The value of k for which $|k\vec{a}| < |\vec{a}|$ and $k\vec{a} = \frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true are ________.

Sol. We have, $|k\vec{a}| < |\vec{a}|$ and $k\vec{a} = \frac{1}{2}\vec{a}$ parallel to $\vec{a}$.

$\therefore |k\vec{a}| < |\vec{a}| \Rightarrow k|\vec{a}| < |\vec{a}|$

$\Rightarrow |k| < 1 \Rightarrow -1 < k < 1$

Also, since $k\vec{a}+\frac{1}{2}\vec{a}$ is parallel to $\vec{a}$, then we see that at $k = \frac{-1}{2}, k\vec{a}+\frac{1}{2}\vec{a}$ becomes a null vector and then it will not be parallel to $\vec{a}$.

So, $k\vec{a}+\frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true when $k \in]-1,1[k \neq \frac{-1}{2}$.

38. The value of the expression $|\vec{a}\times\vec{b}|^2 + (\vec{a}.\vec{b})^2$ is ________.

Sol. $|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\theta + (\vec{a}\cdot\vec{b})^2$

$=|\vec{a}|^2|\vec{b}|^2\ (1-\cos^2\theta)+(\vec{a}\cdot\vec{b})^2$

$=|\vec{a}|^2|\vec{b}|^2-|\vec{a}|^2|\vec{b}|^2\cos^2\theta+(\vec{a}\cdot\vec{b})^2$

$=|\vec{a}|^2|\vec{b}|^2-(\vec{a}\cdot\vec{b})^2+(\vec{a}\cdot\vec{b})^2$

$|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$

39. If $|\vec{a}\times\vec{b}|^2+|\vec{a}.\vec{b}|^2=144$ and $|\vec{a}|=4$, then $|\vec{b}|$ is equal to ________.

Sol. $\because |\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144\,|\vec{a}|^2\cdot|\vec{b}|^2$

$\Rightarrow |\vec{a}|^2|\vec{b}|^2=144$

$\Rightarrow |\vec{b}|^2=\dfrac{144}{|\vec{a}|^2}-=\dfrac{144}{16}=9$

$\therefore |\vec{b}|=3$

40. If $\vec{a}$ is any non-zero vector, then $(\vec{a}.\hat{i})\hat{i}+(\vec{a}.\hat{j})\hat{j}+(\vec{a}.\hat{k})\hat{k}$ equals __________.

Sol. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\therefore \vec{a}\cdot\hat{i}=a_1, \vec{a}\cdot\hat{j}=a_2$ and $\vec{a}\cdot\hat{k}=a_3$

$\therefore (\vec{a}\cdot\hat{i})\hat{i}+(\vec{a}\cdot\hat{j})\hat{j}+(\vec{a}\cdot\hat{k})\hat{k}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}=\vec{a}$

State True or False in each of the following Exercises.

41. If $|\vec{a}|=|\vec{b}|$, then necessarily it implies $\vec{a}=\pm\vec{b}$.

Sol. True

If $|\vec{a}|=|\vec{b}|\Rightarrow \vec{a}=\pm\vec{b}$

So, it is a true statement.

42. Position vector of a point P is a vector whose initial point is origin.

Sol. True

Since, $\vec{P}=\overrightarrow{OP}$ = displacement of vector $\vec{P}$ from origin.

43. If $\left|\vec{a}+\vec{b}\right|=\left|\vec{a}-\vec{b}\right|$, then the vectors $\vec{a}$ and $\vec{b}$ are orthogonal.

Sol. True

Since, $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$

$\Rightarrow |\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$

$\Rightarrow 2|\vec{a}||\vec{b}|=-2|\vec{a}||\vec{b}|$

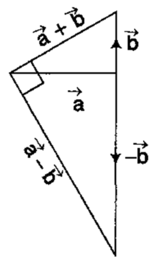
$\Rightarrow 4|\vec{a}||\vec{b}|=0$

$\Rightarrow |\vec{a}||\vec{b}|=0$

Hence, $\vec{a}$ and $\vec{b}$ are orthogonal. $[\because \vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|\cos 90^o=0]$

44. The formula $(\vec{a}+\vec{b})^2=\vec{a}^2+\vec{b}^2+2\vec{a}\times\vec{b}$ is valid for non-zero vectors $\vec{a}$ and $\vec{b}$.

Sol. False



$(\vec{a}+\vec{b}) = (\vec{a}+\vec{b})\cdot(\vec{a}+\vec{b})$

$= \vec{a}^2 + \vec{b}^2 + 2\vec{a}\cdot\vec{b}$

45. If $\vec{a}$ and $\vec{b}$ are adjacent sides of a rhombus, then $\vec{a}.\vec{b} = 0.$

Sol. False

If $\vec{a}\cdot\vec{b} = 0,$ then $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 90^o$

Hence, angle between $\vec{a}$ and $\vec{b}$ is 90^o, which is not possible in a rhombus.

Since, angle between adjacent sides in a rhombus is not equal to $90^o.$

Vector Algebra
Short Answer Type Quesitons

1. Find the unit vector in the direction of sum of vectors $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=2\hat{j}+\hat{k}$.

Sol. Let $\vec{c}$ denote the sum of $\vec{a}$ and $\vec{b}$.

We have, $\vec{c}=\vec{a}+\vec{b}$

$=2\hat{i}-\hat{j}+\hat{k}+2\hat{j}+\hat{k}=2\hat{i}+\hat{j}+2\hat{k}$

$\therefore$ Unit vector in the direction of $\vec{c}=\dfrac{\vec{c}}{|\vec{c}|}=\dfrac{2\hat{i}+\hat{j}+2\hat{k}}{\sqrt{2^2+1^2+2^2}}=\dfrac{2\hat{i}+\hat{j}+2\hat{k}}{\sqrt{9}}$

$\hat{c}=\dfrac{2\hat{i}+\hat{j}+2\hat{k}}{3}$

2. If $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}+2\hat{k}$, find the unit vector in the direction of

(i) $6\vec{b}$

(ii) $2\vec{a}-\vec{b}$

Sol. Here, $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ **and** $\vec{b}=2\hat{i}+\hat{j}+2\hat{k}$,

(i) since, $6\vec{b}=12\hat{i}+6\hat{j}-12\hat{k}$

$\therefore$ Unit vector in the direction of $6\vec{b}=\dfrac{6\vec{b}}{|6\vec{b}|}$

$=\dfrac{12\hat{i}+6\hat{j}-12\hat{k}}{\sqrt{12^2+6^2+12^2}}=\dfrac{6(2i+j-2k)}{\sqrt{324}}$

$=\dfrac{6(2\hat{i}+\hat{j}-2\hat{k})}{18}=\dfrac{2\hat{i}+\hat{j}-2\hat{k}}{3}$

(ii) Since, $2\vec{a}-\vec{b}=2(\hat{i}+\hat{j}+2\hat{k})-(2\hat{i}+\hat{j}-2\hat{k})$

$=2\hat{i}+2\hat{j}+4\hat{k}-2\hat{i}-\hat{j}+2\hat{k}=\hat{j}+6\hat{k}$

$\therefore$ Unit vector in the direction of $2\vec{a}-\vec{b}=\dfrac{2\vec{a}-\vec{b}}{|2\vec{a}-\vec{b}|}=\dfrac{\hat{j}+6\hat{k}}{\sqrt{1+36}}=\dfrac{1}{\sqrt{37}}(\hat{j}+6\hat{k})$

3. Find a unit vector in the direction of $\overrightarrow{PQ}$, where P *and* Q have co-ordinates $(5,0,8)$ *and* $(3,3,2)$, respectively.

Sol. Since, the coordinates of P *and* Q are $(5,0,8)$ *and* $(3,3,2)$, respectively.

$\therefore \overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}$

$=(3\hat{i}+3\hat{j}+2\hat{k})-(5\hat{i}+0\hat{j}+8\hat{k})$

$=-2\hat{i}+3\hat{j}-6\hat{k}$

$\therefore$ Unit vector in the direction of $\overrightarrow{PQ}=\dfrac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}=\dfrac{-2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{2^2+3^2+6^2}}$

$$= \frac{-2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{49}} = \frac{-2\hat{i}+3\hat{j}-6\hat{k}}{7}$$

4. **If $\vec{a}$ and $\vec{b}$ are the position vectors of A *and* B, respectively, find the position vector of a point C in BA produced such that $BC = 1.5\ BA$.**

Sol. Since, $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OB} = \vec{b}$

$\therefore\ \overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = \vec{a} - \vec{b}$

And $1.5\overrightarrow{BA} = 1.5(\vec{a} - \vec{b})$

Since, $\overrightarrow{BC} = 1.5\overrightarrow{BA} = 1.5(\vec{a} - \vec{b})$

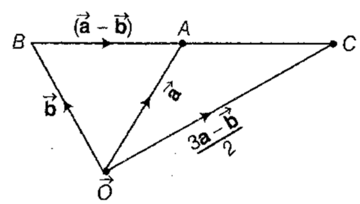
$\overrightarrow{OC} - \overrightarrow{OB} = 1.5\vec{a} - 1.5\vec{b}$

$\overrightarrow{OC} = 1.5\vec{a} - 1.5\vec{b} + \vec{b}\quad [\because \overrightarrow{OB} = \vec{b}]$

$= 1.5\vec{a} - 0.5\vec{b}$

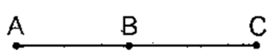
$= \dfrac{3\vec{a} - \vec{b}}{2}$

Graphically, explanation of the above solution is given below



5. **Using vectors, find the value of k such that the points (k, -10, 3), (1, -1, 3) and (3, 5, 3) are collinear.**

Sol. Let the points are $A(k,-10,3)$, $B(1,-1,3)$ *and* $C(3,5,3)$



So, $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (\hat{i} - \hat{j} + 3\hat{k}) - (k\hat{i} - 10\hat{j} + 3\hat{k})$

$= (1-k)\hat{i} + (-1+10)\hat{j} + (3-3)\hat{k}$

$= (1-k)\hat{i} + 9\hat{j} + 0\hat{k}$

$\therefore |\overrightarrow{AB}| = \sqrt{(1-k)^2 + (9)^2 + 0} = \sqrt{(1-k)^2 + 81}$

Similarly, $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= (3\hat{i} + 5\hat{j} + 3\hat{k}) - (\hat{i} - \hat{j} + 3\hat{k})$

$= 2\hat{i} + 6\hat{j} + 0\hat{k}$

$\therefore |\overrightarrow{BC}| = \sqrt{2^2 + 6^2 + 0} = 2\sqrt{10}$

And $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= (3\hat{i} + 5\hat{j} + 3\hat{k}) - (k\hat{i} - 10\hat{j} + 3\hat{k})$

$= (3-k)i + 15\hat{j} + 0\hat{k}$

$\therefore |\overrightarrow{AC}| = \sqrt{(3-k)^2 + 225}$

If A, B and C are collinear, then sum of modules of any two vectors will be equal to the modules of third vectors.

For $|\overrightarrow{AB}| + |\overrightarrow{BC}| = |\overrightarrow{AC}|$

$\sqrt{(1-k)^2 + 81} + 2\sqrt{10} = \sqrt{(3-k)^2 + 225}$

$\Rightarrow \sqrt{(3-k)^2 + 225} - \sqrt{(1-k)^2 + 81} = 2\sqrt{10}$

$\Rightarrow \sqrt{9 + k^2 - 6k + 225} - \sqrt{1 + k^2 - 2k + 81} = 2\sqrt{10}$

$\Rightarrow \sqrt{k^2 - 6k + 234} - 2\sqrt{10} = \sqrt{k^2 - 2k + 82}$

$\Rightarrow k^2 - 6k + 234 + 40 - 2\sqrt{k^2 - 6k + 234} \cdot 2\sqrt{10} = k^2 - 2k + 82$

$\Rightarrow k^2 - 6k + 234 + 40 - k^2 + 2k - 82 = 4\sqrt{10}\sqrt{k^2 + 234 - 6k}$

$\Rightarrow -4k + 192 = 4\sqrt{10}\sqrt{k^2 + 234 - 6k}$

$\Rightarrow -k + 48 = \sqrt{10}\sqrt{k^2 + 234 - 6k}$

On squaring both sides, we get

$48 \times 48 + k^2 - 96k = 10(k^2 + 234 - 6k)$

$\Rightarrow k^2 - 96k - 10k^2 + 60k = -48 \times 48 + 2340$

$\Rightarrow -9k^2 - 36k = -48 \times 48 + 2340$

$\Rightarrow (k^2 + 4k) = +16 \times 16 - 260$ [dividing by 9 in both sides]

$\Rightarrow k^2 + 4k = -4$

$\Rightarrow k^2 + 4k + 4 = 0$

$\Rightarrow (k+2)^2 = 0$

$\therefore k = -2$

6. A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, find $\vec{r}$.

Sol. We have, $|\vec{r}| = 2\sqrt{3}$

Since, $\vec{r}$ is equally inclined to the three axes, $\vec{r}$ so direction cosines of the unit vector $\vec{r}$ will be same i.e., $l = m = n$.

We know that,

$l^2 + m^2 + n^2 = 1$

$\Rightarrow l^2 + l^2 + l^2 = 1$

$\Rightarrow l^2 = \dfrac{1}{3}$

$\Rightarrow l = \pm\left(\dfrac{1}{\sqrt{3}}\right)$

So, $\hat{r} = \pm\dfrac{1}{\sqrt{3}}\hat{i} \pm \dfrac{1}{\sqrt{3}}\hat{j} \pm \dfrac{1}{\sqrt{3}}\hat{k}$

$\therefore \vec{r} = \hat{r}|\vec{r}| \quad \left[\because \hat{r} = \frac{\vec{r}}{|\vec{r}|}\right]$

$= \left[\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right]2\sqrt{3} \quad [\because |r| = 2\sqrt{3}]$

$= \pm 2\hat{i} + 2\hat{j} \pm 2\hat{k} = \pm 2(\hat{i} + \hat{j} + \hat{k})$

7. A vector $\vec{r}$ has magnitude 14 and direction ratios $2, 3, -6$. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with x-axis.

Sol. Here, $|\vec{r}| = 14, \vec{a} = 2k, \vec{b} = 3k$ and $\vec{c} = -6k$

$\therefore$ Direction cosines l, m *and* n are

$$l = \frac{\vec{a}}{|\vec{r}|} = \frac{2k}{14} = \frac{k}{7}$$

$$m = \frac{\vec{b}}{|\vec{r}|} = \frac{3k}{14}$$

And $n = \frac{\vec{c}}{|\vec{r}|} = \frac{-6k}{14} = \frac{-3k}{7}$

Also, we know that

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{k^2}{49} + \frac{9k^2}{196} + \frac{9k^2}{49} = 1$

$\Rightarrow \frac{4k^2 + 9k^2 + 36k^2}{196} = 1$

$\Rightarrow k^2 = \frac{196}{49} = 4$

$\Rightarrow k = \pm 2$

So, the direction cosines (l, m, n) are $\frac{2}{7}, \frac{3}{7}$ and $\frac{-6}{7}$

[since, $\vec{r}$ makes an acute angle with X-axis]

$\because \vec{r} = \hat{r} \cdot |\vec{r}|$

$\therefore r = (l\hat{i} + m\hat{j} + n\hat{k})|\vec{r}|$

$= \left(\frac{+2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) \cdot 14$

$= +4\hat{i} + 6\hat{j} - 12\hat{k}$

8. Find a vector of magnitude 6, which is perpendicular to both the vector $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$.

Sol. Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 4\hat{i} - \hat{j} + 3\hat{k}$

So, any vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$ is given by

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 2\\ 4 & -1 & 3\end{vmatrix}$$

$$=\hat{i}(-3+2)-\hat{j}(6-8)+\hat{k}(-2+4)$$

$$=-\hat{i}+2\hat{j}+2\hat{k}=\vec{r}$$

A vector of magnitude 6 in the direction of $\vec{r}$

$$=\frac{\vec{r}}{|\vec{r}|}\cdot 6\frac{-\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{1^2+2^2+2^2}}\cdot 6$$

$$=\frac{-6}{3}\hat{i}+\frac{12}{3}\hat{j}+\frac{12}{3}\hat{k}$$

$$=-2\hat{i}+4\hat{j}+4\hat{k}$$

9. Find the angle between the vector $2\hat{i}-\hat{j}+\hat{k}$ and $3\hat{i}+4\hat{j}-\hat{k}$.

Sol. Let $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=3\hat{i}+4\hat{j}-\hat{k}$

We know that, angle between two vectors $\vec{a}$ and $\vec{b}$ is given by

$$\cos\theta=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

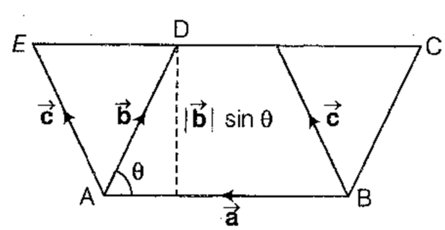
$$=\frac{(2i-\hat{j}+\hat{k})(3\hat{i}+4\hat{j}-\hat{k})}{\sqrt{4+1+1}\sqrt{9+16+1}}$$

$$=\frac{6-4-1}{\sqrt{6}\sqrt{26}}=\frac{1}{2\sqrt{39}}$$

$$\theta=\cos^{-1}\left(\frac{1}{2\sqrt{39}}\right)$$

10. If $\vec{a}+\vec{b}+\vec{c}=0$, show that $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$. Interpret the result geometrically?

Sol. Since, $\vec{a}+\vec{b}+\vec{c}=0$



$$\Rightarrow \vec{b}=-\vec{c}-\vec{a}$$

Now, $\vec{a}\times\vec{b}=\vec{a}\times(-\vec{c}-\vec{a})$

$$=\vec{a}\times(-\vec{c})+\vec{a}\times(-\vec{a})=-\vec{a}\times\vec{c}$$

$$\Rightarrow \vec{a}\times\vec{b}=\vec{c}\times\vec{a}\ \ ...(i)$$

Also, $\vec{b}\times\vec{c}=(-\vec{c}-\vec{a})\times\vec{c}$

$$=(-\vec{c}\times\vec{c})+(-\vec{a}\times\vec{c})=-\vec{a}\times\vec{c}$$

$$\Rightarrow \vec{b}\times\vec{c}=\vec{c}\times\vec{a}\ \ ...(ii)$$

From Eqs. (i) and (ii), $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

Geometrical interpretation of the result

If ABCD is parallelogram such that $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{AD}=\vec{b}$ and these adjacent sides are making angle θ between each other, then we say that

Area of parallelogram $ABCD=|\vec{a}||\vec{b}|\sin\theta|=|\vec{a}\times\vec{b}|$

Since, parallelogram on the same base and between the same parallels are equal in area.

We can say that, $|\vec{a}\times\vec{b}|=|\vec{a}\times\vec{c}|=|\vec{b}\times\vec{c}|$

This also implies that, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}=\vec{b}\times\vec{c}$

So, area of the parallelogram formed by taking any two sides represented by $\vec{a},\vec{b}$ and $\vec{c}$ as adjacent are equal.

11. Find the sine of the angle between the vectors $\vec{a}=3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}-2\hat{j}+4\hat{k}$.

Sol. Here, $a_1=3, a_2=1, a_3=2$ and $b_1=2, b_2=-2, b_3=4$

We know that,

$$\cos\theta=\frac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$$

$$=\frac{3\times2+1\times(-2)+2\times4}{\sqrt{3^2+1^2+2^2}\sqrt{2^2+(-2)^2+4^2}}$$

$$=\frac{6-2+8}{\sqrt{14}\sqrt{24}}=\frac{12}{2\sqrt{14}\sqrt{6}}=\frac{6}{\sqrt{84}}=\frac{6}{2\sqrt{21}}=\frac{3}{\sqrt{21}}$$

$$\therefore \sin\theta=\sqrt{1-\cos^2\theta}$$

$$=\sqrt{1-\frac{9}{21}}=\sqrt{\frac{12}{21}}=\frac{2\sqrt{3}}{\sqrt{3}\sqrt{7}}=\frac{2}{\sqrt{7}}$$

12. If A, B, C, D are the points with position vectors $\hat{i}-\hat{j}+\hat{k}, 2\hat{i}-\hat{j}+3\hat{k},\ 2\hat{i}-3\hat{k}$ and $3\hat{i}-2\hat{j}+\hat{k}$, respectively, find the projection of $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

Sol. Here, $\overrightarrow{OA}=\hat{i}+\hat{j}-\hat{k}, \overrightarrow{OB}=2\hat{i}-\hat{j}+3\hat{k}, \overrightarrow{OC}=2\hat{i}-3\hat{k}$ and $\overrightarrow{OD}=3i-2\hat{i}+\hat{k}$

$$\therefore \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(2-1)\hat{i}+(-1-1)\hat{j}+(3+1)\hat{k}$$

$$=\hat{i}-2\hat{j}+4\hat{k}$$

And $\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}=(3-2)\hat{i}+(-2-0)\hat{j}+(1+3)\hat{k}$

$$=\hat{i}-2\hat{j}+4\hat{k}$$

So, the projection of $\overrightarrow{AB}$ along $\overrightarrow{CD}=\overrightarrow{AB}\cdot\frac{\overrightarrow{CD}}{|\overrightarrow{CD}|}$

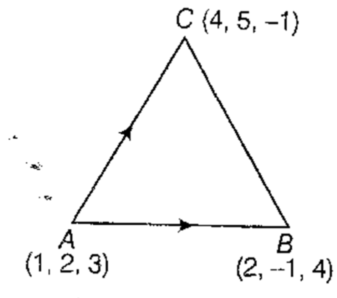
$$=\frac{(i-2j+4k)\cdot(\hat{i}-2\hat{j}+4\hat{k})}{\sqrt{1^2+2^2+4^2}}$$

$$=\frac{1+4+16}{\sqrt{21}}=\frac{21}{\sqrt{21}}=\sqrt{21}\text{ units}$$

13. Using vectors, find the area of the triangle ABC with vertices A(1, 2, 3), B(2, 1, 4) and C(4, 5, -1).

Sol. Here, $\overrightarrow{AB} = (2-1)\hat{i} + (-1-2)\hat{j} + (4-3)\hat{k}$

$= \hat{i} - 3\hat{j} + \hat{k}$



And $\overrightarrow{AC} = (4-1)\hat{i} + (5-2)\hat{j} + (-1-3)\hat{k}$

$= 3\hat{i} + 3\hat{j} - 4\hat{k}$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$$

$= \hat{i}(12-3) - \hat{j}(-4-3) + \hat{k}(3+9)$

$= 9\hat{i} + 7\hat{j} + 12\hat{k}$

And $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{9^2 + 7^2 + 12^2}$

$= \sqrt{81 + 49 + 144}$

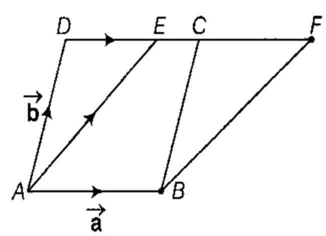
$= \sqrt{274}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2}\sqrt{274}$ sq units

14. Using vectors, prove that the parallelogram on the same base and between the same parallels are equal in area.

Sol. Let ABCD and ABFE are parallelograms on the same base AB and between the same parallel line AB and DF.



Let $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AD} = \vec{b}$

$\therefore$ Area of parallelogram $\overline{ABCD} = \vec{a} \times \vec{b}$

Now, area of parallelogram of $\overline{ABFE} = \overrightarrow{AB} \times \overrightarrow{AE}$

$= \overrightarrow{AB} \times (\overrightarrow{AD} + \overrightarrow{DE})$

$= \overrightarrow{AB} \times (\vec{b} + k\vec{a})$ [let $\overrightarrow{DE} = k\vec{a}$, where k is a scalar]

$= \vec{a} \times (\vec{b} + k\vec{a})$

$= (\vec{a} \times \vec{b}) + (\vec{a} \times k\vec{a})$

$= (\vec{a} \times \vec{b}) + k(\vec{a} \times \vec{a})$

$= (\vec{a} \times \vec{b})$ [$\because \vec{a} \times \vec{a} = 0$]

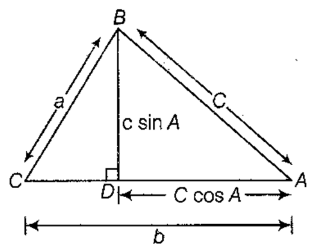
=Area of parallelogram ABCD.

Hence proved

Vector Algebra
Long Answer Type Questions

15. **Prove that in any triangle ABC,** $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$**, where** a, b, c **are the magnitudes of the sides opposite to the vertices** $A, B, C,$ **respectively.**

Sol. Here, components of C are $c \cos A$ *and* $c \sin$ A is drawn.



Since, $\overrightarrow{\text{CD}} = b - c\cos A$

In ΔBDC,

$a^2 = (b - c\cos A)^2 + (c\sin A)^2$

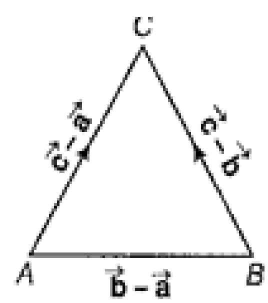
$\Rightarrow a^2 = b^2 + c^2\cos^2 A - 2bc\cos A + c^2\sin^2 A$

$\Rightarrow 2bc\cos A = b^2 - a^2 + c^2(\cos^2 A + \sin^2 A)$

$\therefore \cos A = \dfrac{b^2+c^2-a^2}{2bc}$

16. **If** $\vec{a}, \vec{b}, \vec{c}$ **determine the vertices of a triangle, show that** $\frac{1}{2}[\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}]$ **gives the vector area of the triangle. Hence deduce the condition that the three points** $\vec{a},\vec{b},\vec{c}$ **are collinear. Also, find the unit vector normal to the plane of the triangle.**

Sol. Since, $\vec{a},\vec{b}$ and $\vec{c}$ are the vertices of a ΔABC as shown.



$\therefore$ Area of $\Delta\text{ABC} = \frac{1}{2}|\overrightarrow{\text{AB}}\times\overrightarrow{\text{AC}}|$

Now, $\overrightarrow{\text{AB}} = \vec{b} - \vec{a}$ and $\overrightarrow{\text{AC}} = \vec{c} - \vec{a}$

$\therefore$ Area of $\Delta\text{ABC} = \frac{1}{2}|\vec{b} - \vec{a}\times\vec{c} - \vec{a}|$

$= \frac{1}{2}|\vec{b}\times\vec{c} - \vec{b}\times\vec{a} - \vec{a}\times\vec{c} + \vec{a}\times\vec{a}|$

$= \frac{1}{2}|\vec{b}\times\vec{c} + \vec{a}\times\vec{b} + \vec{c}\times\vec{a} + 0|$

$= \frac{1}{2}|\vec{b}\times\vec{c} + \vec{a}\times\vec{b} + \vec{c}\times\vec{a}|$ ...(*i*)

For three points to be collinear, area of the ΔABC should be equal to zero.

$$\Rightarrow \frac{1}{2}[\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}]=0$$

$$\Rightarrow \vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{a}\times\vec{b}=0 \;\; ...(ii)$$

This is the required condition for collinearity of three points $\vec{a},\vec{b}$ and $\vec{c}$.

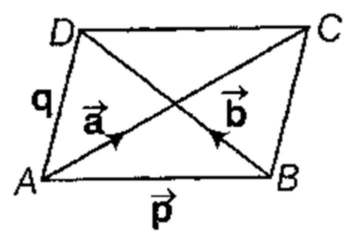
Let $\hat{n}$ be the unit vector normal to the plane of the ΔABC

$$\therefore \hat{n}=\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC}|}$$

$$=\frac{\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}}{|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|}$$

17. Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{a}\times\vec{b}|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

Sol. Let ABCD be a parallelogram such that



$\overrightarrow{AB}=\vec{p}, \overrightarrow{AD}=\vec{q}\Rightarrow\overrightarrow{BC}=\vec{q}$

By triangle law of addition, we get

$\overrightarrow{AC}=\vec{p}+\vec{q}=\vec{a}\;\; [say]\ldots(i)$

Similarly, $\overrightarrow{BD}=-\vec{p}+\vec{q}=\vec{b}\;\; [say]\ldots(ii)$

On adding Eqs. (i) and (ii), we get

$$\vec{a}+\vec{b}=2\vec{q}\Rightarrow\vec{q}=\frac{1}{2}(\vec{a}+\vec{b})$$

On subtracting Eq.(ii) from Eq.(i), we get

$$\vec{a}-\vec{b}=2\vec{p}\Rightarrow\vec{p}=\frac{1}{2}(\vec{a}-\vec{b})$$

Now, $\vec{p}\times\vec{q}=\frac{1}{4}(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$

$$=\frac{1}{4}(\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b})$$

$$=\frac{1}{4}[\vec{a}\times\vec{b}+\vec{a}\times\vec{b}] \;\; =\frac{1}{2}(\vec{a}\times\vec{b})$$

So, area of a parallelogram ABCD $=|\vec{p}\times\vec{q}|=\frac{1}{2}|\vec{a}\times\vec{b}|$

Now, area of a parallelogram, whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.

$$=\frac{1}{2}|(2\hat{i}-\hat{j}+\hat{k})\times(\hat{i}+3\hat{j}-\hat{k})$$

$$=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}$$

$=\frac{1}{2}|[\hat{i}(1-3)-\hat{j}(-2-1)+\hat{k}(6+1)]|$

$=\frac{1}{2}|-2\hat{i}+3\hat{j}+7\hat{k}|$

$=\frac{1}{2}\sqrt{4+9+49}\ =\frac{1}{2}\sqrt{62}$ sq units

18. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$.

Sol. Let $\vec{c}=x\hat{i}+y\hat{j}+z\hat{k}$

And $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$

For $\vec{a}\times\vec{c}=\vec{b}$,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix}=\hat{j}-\hat{k}$$

$\Rightarrow \hat{i}(z-y)-\hat{j}(z-x)+\hat{k}(y-x)=\hat{j}-\hat{k}$

$\therefore\ z-y=0\ \ldots(i)$

$x-z=1\ \ldots(ii)$

$x-y=1\ \ldots(iii)$

Also, $\vec{a}\cdot\vec{c}=3$

$(\hat{i}+\hat{j}+\hat{k})\cdot(x\hat{i}+y\hat{j}+z\hat{k})=3$

$\Rightarrow\ x+y+z=3\ \ldots(iv)$

On adding Eqs. (ii) and (iii), we get

$2x-y-z=2\ \ldots(v)$

On solving Eqs. (iv) and (v), we get

$x=\frac{5}{3}$

$\therefore\ y=\frac{5}{3}-1=\frac{2}{3}$ and $z=\frac{2}{3}$

Now, $\vec{c}=\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$

$=\frac{1}{3}(5\hat{i}+2\hat{j}+2\hat{k})$