Chapter 1: Sets and Relations

EXERCISE 1.1 [PAGES 9 - 10]

Exercise 1.1 | Q 1.1 | Page 9

Describe the following set in the Roster Form.
{x/x is a letter of the word 'MARRIAGE'}

SOLUTION

Let A = {x/x is a letter of the word 'MARRIAGE'}

∴ A = {M, A, R, I, G, E}

Exercise 1.1 | Q 1.2 | Page 9

Describe the following set in the Roster Form.

$\{x/x \text{ is an integer}, -\frac{1}{2} < x < \frac{9}{2}\}$

SOLUTION

Let B = $\{x/x \text{ is an integer}, -\frac{1}{2} < x < \frac{9}{2}\}$

∴ B = {0, 1, 2, 3, 4}

Exercise 1.1 | Q 1.3 | Page 9

Describe the following set in the Roster Form.
{x/x = 2n, n ∈ N}

SOLUTION

Let C = {x/x = 2n, n ∈ N}

∴ C = {2, 4, 6, 8, ….}

Exercise 1.1 | Q 2.1 | Page 9

Describe the following set in the Set-Builder form.
{0}

SOLUTION

Let A = {0}

0 is a whole number but it is not a natural number

∴ A = {x/x ∈ W, x ∉ N}

Exercise 1.1 | Q 2.2 | Page 9

Describe the following set in the Set-Builder form.

{0, ±1, ±2, ±3}

SOLUTION

Let B = {0, ±1, ±2, ±3}

B is the set of elements that belongs to Z from -3 to 3.

∴ B = {x/x ∈ Z, -3 ≤ x ≤ 3}

Exercise 1.1 | Q 2.3 | Page 9

Describe the following set in the Set-Builder form.

$$\left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$$

SOLUTION

$$\text{Let C} = \left\{\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \frac{4}{17}, \frac{5}{26}, \frac{6}{37}, \frac{7}{50}\right\}$$

$$\therefore \text{C} = \left\{x/x = \frac{\text{n}}{\text{n}^2 + 1}, \text{n} \in \text{N}, \text{n} \leq 7\right\}$$

Exercise 1.1 | Q 3 | Page 9

If A = {x/6x^2 + x - 15 = 0}
B = {x/2x^2 - 5x - 3 = 0}
C = {x/2x^2 - x - 3 = 0} then
find
i) (A ∪ B ∪ C)
ii) (A ∩ B ∩ C)

SOLUTION

A = {x/6x^2 + x - 15 = 0}

∴ 6x^2 + x - 15 = 0

∴ 6x^2 + 10x - 9x - 15 = 0

∴ 2x(3x + 5) - 3(3x + 5) = 0

∴ (3x + 5)(2x - 3) = 0

∴ 3x + 5 = 0 or 2x - 3 = 0

$$\therefore x = \frac{-5}{3} \text{ or } x = \frac{3}{2}$$

$$\therefore A = \left\{\frac{-5}{3}, \frac{3}{2}\right\}$$

B = {x/$2x^2$ - 5x - 3 = 0}

∴ $2x^2$ - 5x - 3 = 0

∴ $2x^2$ - 6x + x - 3 = 0

∴ 2x(x - 3) + 1(x - 3) = 0

∴ (x - 3)(2x + 1) = 0

∴ x - 3 = 0 or 2x + 1 = 0

$$\therefore x = 3 \text{ or } x = \frac{-1}{2}$$

$$\therefore B = \left\{\frac{-1}{2}, 3\right\}$$

C = {x/$2x^2$ - x - 3 = 0}

∴ $2x^2$ - x - 3 = 0

∴ $2x^2$ - 3x + 2x - 3 = 0

∴ x(2x - 3) + 1(2x - 3) = 0

∴ (2x - 3)(x + 1) = 0

∴ 2x - 3 = 0 or x + 1 = 0

$$\therefore x = \frac{3}{2} \text{ or } x = -1$$

$$\therefore c = \left\{-1, \frac{3}{2}\right\}$$

i. $A \cup B \cup C = \left\{-\frac{5}{3}, \frac{3}{2}\right\} \cup \left\{\frac{-1}{2}, 3\right\} \cup \left\{-1, \frac{3}{2}\right\}$

$= \left\{\frac{-5}{3}, -1, \frac{-1}{2}, \frac{3}{2}, 3\right\}$

ii. $A \cap B \cap C = \{\}$

Exercise 1.1 | Q 4 | Page 9

If A, B, C are the sets for the letters in the words 'college', 'marriage' and 'luggage' respectively, then verify that

$A - (B \cup C) = (A - B) \cap (A - C)$

SOLUTION

A = {c, o, l, g, e}

B = {m, a, r, i, g, e}

C = {l, u, g, a, e}

$B \cup C$ = {m, a, r, i, g, e, l, u}

$A - (B \cup C)$ = {c, o}

A - B = {c, o, l}

A - C = {c , o}

$\therefore [(A - B) \cap (A - C)]$ = {c , o} = $A - (B \cup C)$

$\therefore [A - (B \cup C)] = [(A - B) \cap (A - C)]$

Exercise 1.1 | Q 5.1 | Page 10

If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

SOLUTION

A = {1, 2, 3, 4}, B = {3, 4, 5, 6},
C = {4, 5, 6, 7, 8}
X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$B \cap C$ = {4,5,6}

$\therefore A \cup (B \cap C)$ = {1, 2, 3, 4, 5, 6} .....(i)

$A \cup B$ = {1, 2, 3, 4, 5, 6}

$(A \cup C)$ = {1, 2, 3, 4, 5, 6, 7, 8}

$\therefore (A \cup B) \cap (A \cup C)$ = {1, 2, 3, 4, 5, 6} …….....(ii)

From (i) and (ii), we get

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Exercise 1.1 | Q 5.2 | Page 10

If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

A∩(B∪C) = (A∩B) ∪ (A∩C)

SOLUTION

A = {1, 2, 3, 4}, B = {3, 4, 5, 6},
C = {4, 5, 6, 7, 8},
X ={1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

B ∪ C = {3, 4, 5, 6, 7, 8}

∴ A ∩ (B ∪ C) = {3, 4} .....(i)

A ∩ B = {3, 4}

A ∩ C = {4}

∴ (A ∩ B) ∪ (A ∩ C) = {3, 4} .....(ii)

From (i) and (ii), we get

A∩(B∪C) = (A∩B) ∪ (A∩C)

Exercise 1.1 | Q 5.3 | Page 10

If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

(A∪B)' = (A'∩B')

SOLUTION

A = {1, 2, 3, 4}, B = {3, 4, 5, 6},
C = {4, 5, 6, 7, 8}
X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A ∪ B = {1, 2, 3, 4, 5, 6}

∴ (A∪B)' = ={7, 8, 9, 10} .....(i)

A' = {5, 6, 7, 8, 9, 10},

B' = {1, 2, 7, 8, 9, 10}

∴ A' ∩ B' = {7, 8, 9, 10} .....(ii)

From (i) and (ii), we get

(A∪B)' = (A'∩B')

Exercise 1.1 | Q 5.4 | Page 10

If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:

(A∩B)' = A'∪B'

SOLUTION

A = {1, 2, 3, 4}, B = {3, 4, 5, 6},
C = {4, 5, 6, 7, 8}
X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10},

A ∩ B = {3, 4}

∴ (A ∩ B)' = {1, 2, 5, 6, 7, 8, 9, 10} …(i)

A' = {5, 6, 7, 8, 9, 10}

B' = {1, 2, 7, 8, 9, 10}

∴ A'∪B' = {1, 2, 5, 6, 7, 8, 9, 10} .....(ii)

From (i) and (ii), we get

(A∩B)' = A'∪B'

Exercise 1.1 | Q 5.5 | Page 10
If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:
A = (A∩B) ∪ (A∩B')

SOLUTION

A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8}
X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A = {1, 2, 3, 4} ….....(i)

A ∩ B = {3, 4}

B' = {1, 2, 7, 8, 9, 10}

A ∩ B' = {1, 2}

∴ (A ∩ B) ∪ (A ∩ B') = {1, 2, 3, 4} ….....(ii)

From (i) and (ii), we get

A = (A ∩ B) ∪ (A ∩ B')

Exercise 1.1 | Q 5.6 | Page 10
If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:
B = (A∩B) ∪ (A'∩B)

SOLUTION

A = {1, 2, 3, 4}, B = {3, 4, 5, 6},
C = {4, 5, 6, 7, 8}
X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

B = {3, 4, 5, 6} .....(i)

(A∩B) = {3,4}

A' = {5, 6, 7, 8, 9, 10}

A' ∩ B = {5, 6}

∴ (A ∩ B) ∪ (A' ∩ B) = {3, 4, 5, 6} .....(ii)

From (i) and (ii), we get

B = (A ∩ B) ∪ (A' ∩ B)

Exercise 1.1 | Q 5.7 | Page 10

If A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following:
n (A∪B) = n(A) + n(B) - n(A∩B)

SOLUTION

A = {1, 2, 3, 4}, B = {3, 4, 5, 6},
C = {4, 5, 6, 7, 8}
X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A ∩ B = {3, 4}, A ∪ B = {1, 2, 3, 4, 5, 6}

∴ n(A) = 4, n(B) = 4,

n(A ∩ B) = 2,

n(A ∪ B) = 6 .....(i)

∴ n(A) + n(B) - n(A ∩ B) = 4 + 4 - 2

∴ n(A) + n(B) - n(A ∩ B) = 6 .....(ii)

From (i) and (ii), we get

n(A ∪ B) = n(A) + n(B) - n(A ∩ B)

Exercise 1.1 | Q 6.1 | Page 10

If A and B are subsets of the universal set X and n(X) = 50, n(A) = 35, n(B) = 20, n(A'∩B') = 5,

find: n (A∪B)

n(X) = 50, n(A) = 35, n(B) = 20, n(A'∩B') = 5

n (A∪B) = n(X) - [n(A ∪ B)']

= n(X) - n(A' ∩ B')

= 50 - 5

= 45

Exercise 1.1 | Q 6.2 | Page 10

If A and B are subsets of the universal set X and n(X) = 50, n(A) = 35, n(B) = 20, $n(A' \cap B') = 5$,
find: $n(A \cap B)$

SOLUTION

$n(X) = 50, n(A) = 35, n(B) = 20, n(A' \cap B') = 5$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

= 35 + 20 - 45

= 10

Exercise 1.1 | Q 6.3 | Page 10

If A and B are subsets of the universal set X and n(X) = 50, n(A) = 35, n(B) = 20, $n(A' \cap B') = 5$,
find: $n(A' \cap B)$

SOLUTION

$n(X) = 50, n(A) = 35, n(B) = 20, n(A' \cap B') = 5$

$n(A' \cap B) = n(B) - n(A \cap B)$

= 20 - 10

= 10

Exercise 1.1 | Q 6.4 | Page 10

If A and B are subsets of the universal set X and n(X) = 50, n(A) = 35, n(B) = 20, $n(A' \cap B') = 5$,
find: $n(A \cap B')$

SOLUTION

$n(X) = 50, n(A) = 35, n(B) = 20, n(A' \cap B') = 5$

$n(A \cap B') = n(A) - n(A \cap B)$

= 35 - 10

= 25

Exercise 1.1 | Q 7 | Page 10

Out of 200 students; 35 students failed in MHT-CET, 40 in AIEEE and 40 in IIT entrance, 20 failed in MHT-CET and AIEEE, 17 in AIEEE and IIT entrance, 15 in MHT-CET and IIT entrance, and 5 failed in all three examinations. Find how many students.

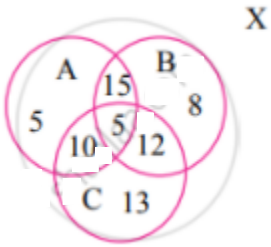
i) did not fail in any examination.

ii) failed in AIEEE or IIT entrance.

SOLUTION

Let A = set of students who failed in MHT-CET
B = set of students who failed in AIEEE
C = set of students who failed in IIT entrance
X = set of all students

∴ n(X) = 200, n(A) = 35, n(B) = 40, n(C) = 40,
n(A ∩ B)= 20, n(B ∩ C) = 17, n(A ∩ C) = 15, n(A ∩ B ∩ C) = 5



i. n(A ∪ B ∪ C)

= n(A) + n(B) + n(C) - n(A ∩ B) - n(B ∩ C) - n(A ∩ C) + n(A ∩ B ∩ C)

= 35 + 40 + 40 - 20 - 17 - 15 + 5

= 68

∴ No. of students who did not fail in any exam

= n(X) - n(A ∪ B ∪ C)

= 200 - 68 = 132

ii. No. of students who failed in AIEEE or IIT entrance

= n(B ∪ C)

= n(B) + n(C) - n(B ∩ C)

= 40 + 40 - 17

= 63

Exercise 1.1 | Q 8 | Page 10

From amongst 2000 literate individuals of a town, 70% read Marathi newspapers, 50% read English newspapers and 32.5% read both Marathi and English newspapers. Find the number of individuals who read.

i) at least one of the newspapers.
ii) neither Marathi nor English newspaper.
iii) Only one of the newspapers.

SOLUTION

Let M = set of individuals who read Marathi newspapers

E = set of individuals who read English newspapers

X = set of all literate individuals

∴ n(X) = 2000,

$$n(M) = \frac{70}{100} \times 2000$$

$$= 1400$$

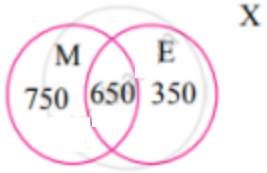
$$n(E) = \frac{50}{100} \times 2000 = 1000$$

$$n(M \cap E) = \frac{32.5}{100} \times 2000 = 650$$

$$n(M \cup E) = n(M) + n(E) - n(M \cap E)$$

$$= 1400 + 1000 - 650$$

$$= 1750$$



i. No. of individuals who read at least one of the newspapers = $n(M \cup E) = 1750$.

ii. No. of individuals who read neither Marathi nor English newspaper

$= n(M' \cap E')$

$= n(M \cup E)'$

$= n(X) - n(M \cup E)$

$= 2000 - 1750$

$= 250$

iii. No. of individuals who read only one of the newspapers = $n(M \cap E') + n(M' \cap E)$

$= n(M \cup E) – n(M \cap E)$

$= 1750 – 650$

= 1100

Exercise 1.1 | Q 9 | Page 10

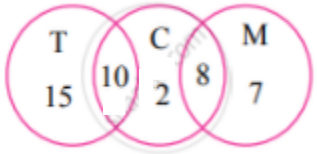
In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 students take both tea and coffee, 8 students take both milk and coffee. None of them take tea and milk both and everyone takes at least one beverage, find the number of students in the hostel.

SOLUTION

Let T = set of students who take tea
C = set of students who take coffee
M = set of students who take milk

∴ n(T) = 25, n(C) = 20, n(M) = 15,

n(T ∩ C) = 10, n(M ∩ C) = 8, n(T ∩ M) = 0, n(T ∩ M ∩ C) = 0



∴ Number of students in the hostel

= n(T ∪ C ∪ M)

= n(T) + n(C) + n(M) - n(T ∩ C) - n(M ∩ C) - n(T ∩ M) + n(T ∩ M ∩ C)

= 25 + 20 + 15 - 10 - 8 - 0 + 0

= 42

Exercise 1.1 | Q 10.1 | Page 10

There are 260 persons with skin disorders. If 150 had been exposed to the chemical A, 74 to the chemical B, and 36 to both chemicals A and B, find the number of persons exposed to Chemical A but not Chemical B.

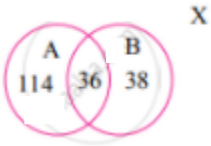
SOLUTION

Let A = set of persons exposed to chemical A

B = set of persons exposed to chemical B

X = set of all persons

∴ n(X) = 260, n(A) = 150, n(B) = 74, n(A ∩ B) = 36



No. of persons exposed to chemical A but not to chemical B

= n(A ∩ B')

= n(A) - n(A ∩ B)

= 150 - 36

= 114

Exercise 1.1 | Q 10.2 | Page 10

There are 260 persons with skin disorders. If 150 had been exposed to the chemical A, 74 to the chemical B, and 36 to both chemicals A and B, find the number of persons exposed to Chemical B but not Chemical A.

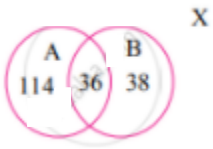
SOLUTION

Let A = set of persons exposed to chemical A

B = set of persons exposed to chemical B

X = set of all persons

∴ n(X) = 260, n(A) = 150, n(B) = 74, n(A ∩ B) = 36



No. of persons exposed to chemical B but not to chemical A

= n(A' ∩ B)

= n(B) - n(A ∩ B)

= 74 - 36

= 38

Exercise 1.1 | Q 10.3 | Page 10

There are 260 persons with skin disorders. If 150 had been exposed to the chemical A, 74 to the chemical B, and 36 to both chemicals A and B, find the number of persons exposed to Chemical A or Chemical B.

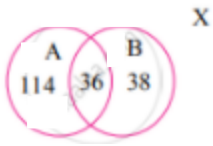
SOLUTION

Let A = set of persons exposed to chemical A

B = set of persons exposed to chemical B

X = set of all persons

∴ n(X) = 260, n(A) = 150, n(B) = 74, n(A ∩ B) = 36



No. of persons exposed to chemical A or chemical B

= n(A ∪ B)

= n(A) + n(B) - n(A ∩ B)

= 150 + 74 - 36

= 188

Exercise 1.1 | Q 11 | Page 10

If A = {1,2,3} write the set of all possible subsets of A.

SOLUTION

A = {1, 2, 3}

∴ { }, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3} and {1, 2, 3} are all the possible subsets of A.

Exercise 1.1 | Q 12.1 | Page 10

Write the following interval in the set-builder form.

(-3, 0)

SOLUTION

(–3, 0) = {x/x ∈ R, –3 < x < 0}

Exercise 1.1 | Q 12.2 | Page 10

Write the following interval in the set-builder form.

[6,12]

SOLUTION

[6, 12] = {x/x ∈ R, 6 ≤ x ≤ 12}

Exercise 1.1 | Q 12.3 | Page 10

Write the following interval in the set-builder form.

(6, 12)

SOLUTION

(6, 12) = {x/x ∈ R, 6 < x < 12}

Exercise 1.1 | Q 12.4 | Page 10

Write the following interval in the set-builder form.

(-23, 5)

SOLUTION

(–23, 5) = {x/x ∈ R, –23 < x < 5}

EXERCISE 1.2 [PAGES 15 - 16]

Exercise 1.2 | Q 1 | Page 15

If (x - 1, y + 4) = (1,2) find the values of x and y.

SOLUTION

(x - 1, y + 4) = (1,2)

By the definition of equality of ordered pairs,

we have

x - 1 = 1 and y + 4 = 2

∴ x = 2 and y = - 2

Exercise 1.2 | Q 2 | Page 15

If $\left(x+\frac{1}{3}, \frac{y}{3}-1\right)=\left(\frac{1}{3}, \frac{3}{2}\right)$, find x and y.

SOLUTION

$$\left(x+\frac{1}{3}, \frac{y}{3}-1\right)=\left(\frac{1}{3}, \frac{3}{2}\right)$$

By the definition of equality of ordered pairs, we have

$$x+\frac{1}{3}=\frac{1}{3} \text{ and } \frac{y}{3}-1=\frac{3}{2}$$

$$\therefore x=\frac{1}{3}-\frac{1}{3} \text{ and } \frac{y}{3}=\frac{3}{2}+1=\frac{5}{2}$$

$$\therefore x = 0 \text{ and } y = \frac{15}{2}$$

Exercise 1.2 | Q 3.1 | Page 15

If A = {a, b, c}, B = (x , y} find A × B.

SOLUTION

A = {a, b, c}, B = (x , y}

A × B = {(a, x), (a, y), (b, x), (b, y), (c, x), (c, y)}

Exercise 1.2 | Q 3.2 | Page 15
If A = {a, b, c}, B = (x , y} find B × A.

SOLUTION

A = {a, b, c}, B = (x , y}

B × A = {(x, a), (x, b), (x, c), (y, a), (y, b), (y, c)}

Exercise 1.2 | Q 3.3 | Page 15
If A = {a, b, c}, B = (x , y} find A × A.

SOLUTION

A = {a, b, c}, B = (x , y}

A × A = {(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)}

Exercise 1.2 | Q 3.4 | Page 15
If A = {a, b, c}, B = (x , y} find B × B.

SOLUTION

A = {a, b, c}, B = (x , y}

B × B = {(x, x), (x, y), (y, x), (y, y)}

Exercise 1.2 | Q 4 | Page 15
If P = {1, 2, 3} and Q = {6, 4}, find the sets P × Q and Q × P.

SOLUTION

P = {1, 2, 3} and Q = {6, 4}

P × Q = {(1, 6), (1, 4), (2, 6), (2, 4), (3, 6), (3, 4)}

Q × P = {(6, 1), (6, 2), (6, 3), (4, 1), (4, 2), (4, 3)}

Exercise 1.2 | Q 5.1 | Page 16
Let A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}. Find A × (B ∩ C).

SOLUTION

A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}

(B ∩ C) = {5, 6}

∴ A × (B ∩ C) = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}

Exercise 1.2 | Q 5.2 | Page 16
Let A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}. Find (A × B) ∩ (A × C).

SOLUTION

A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}

A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}

A × C = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}

∴ (A × B) ∩ (A × C)

= {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}

Exercise 1.2 | Q 5.3 | Page 16

Let A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}. Find A × (B ∪ C).

SOLUTION

A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}

(B ∪ C) = {4, 5, 6}

∴ A × (B ∪ C)

= {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}

Exercise 1.2 | Q 5.4 | Page 16

Let A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}. Find (A × B) ∪ (A × C).

SOLUTION

A = {1, 2, 3, 4}, B = {4, 5, 6}, C = {5, 6}

A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}

A × C = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6)}

∴ (A × B) ∪ (A × C)

= {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}

Exercise 1.2 | Q 6 | Page 16

Express {(x, y) / $x^2 + y^2 = 100$ where x, y ∈ W} as a set of ordered pairs.

SOLUTION

{(x, y) / $x^2 + y^2 = 100$ where x, y ∈ W}

We have, $x^2 + y^2 = 100$

When x = 0 and y = 10,

$x^2 + y^2 = 0^2 + 10^2 = 100$

When x = 6 and y = 8,

$x^2 + y^2 = 6^2 + 8^2 = 100$

When x = 8 and y = 6,

$x^2 + y^2 = 8^2 + 6^2 = 100$

When x = 10 and y = 0,

$x^2 + y^2 = 10^2 + 0^2 = 100$

∴ Set of ordered pairs = {(0, 10), (6, 8), (8, 6), (10, 0)}

Exercise 1.2 | Q 7.1 | Page 16

Write the domain and range of the following relation.
{(a, b) / a ∈ N, a<6 and b = 4}

SOLUTION

Let R_1 = {(a, b)/ a ∈ N, a < 6 and b = 4}

Set of values of 'a' are domain and set of values of 'b' are range.

a ∈ N and a < 6

∴ a = 1, 2, 3, 4, 5 and b = 4

Domain (R_1) = {1, 2, 3, 4, 5}

Range (R_1) = {4}

Exercise 1.2 | Q 7.2 | Page 16

Write the domain and range of the following relation.
{(a, b) / a, b ∈ N, a+b = 12}

SOLUTION

Let R_2 = {(a, b)/a, b ∈ N and a + b = 12}

Now, a, b ∈ N and a + b = 12

When a = 1, b = 11

When a = 2, b = 10

When a = 3, b = 9

When a = 4, b = 8

When a = 5, b = 7

When a = 6, b = 6

When a = 7, b = 5

When a = 8, b = 4

When a = 9, b = 3

When a = 10, b = 2

When a = 11, b = 1

∴ Domain (R_2) = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

Range (R_2) = {11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1}

Exercise 1.2 | Q 7.3 | Page 16

Write the domain and range of the following relation.
(2, 4), (2, 5), (2,6), (2, 7)}

SOLUTION

Let R_3 = {(2, 4), (2, 5), (2, 6), (2, 7)}

Domain (R_3) = {2}

Range (R_3) = {4, 5, 6, 7}

Exercise 1.2 | Q 8 | Page 16

Let A = {6, 8} and B = {1, 3, 5}
Let R = {(a, b)/a ∈ A, b ∈ B, a - b is an even number}, Show that R is an empty relation from A to B.

SOLUTION

A = {6, 8} and B = {1, 3, 5}

R = {(a, b)/a ∈ A, b ∈ B, a - b is an even number}

a ∈ A

∴ a = 6, 8

b ∈ B

∴ b = 1, 3, 5

When a = 6 and b = 1, a - b = 5 which is odd

When a = 6 and b = 3, a - b = 3 which is odd

When a = 6 and b = 5, a - b = 1 which is odd

When a = 8 and b = 1, a - b = 7 which is odd

When a = 8 and b = 3, a - b = 5 which is odd

When a = 8 and b = 5, a - b = 3 which is odd

Thus, no set of values of a and b gives a - b even

∴ R is an empty relation from A to B.

Exercise 1.2 | Q 9.1 | Page 16

Write the relation in the Roster form and hence find its domain and range.
$R_1 = \{(a, a^2) / a \text{ is prime number less than } 15\}$

SOLUTION

$R_1 = \{(a, a^2) / a \text{ is prime number less than } 15\}$

$\therefore a = 2, 3, 5, 7, 11, 13$

$\therefore a^2 = 4, 9, 25, 49, 121, 169$

$\therefore R_1 = \{(2, 4), (3, 9), (5, 25), (7, 49), (11, 121), (13, 169)\}$

$\therefore$ Domain (R_1)

$= \{a/a \text{ is a prime number less than } 15\}$

$= \{2, 3, 5, 7, 11, 13\}$

Range (R_1)

$= \{a^2 / a \text{ is a prime number less than } 15\}$

$= \{4, 9, 25, 49, 121, 169\}$

Exercise 1.2 | Q 9.2 | Page 16
Write the relation in the Roster form and hence find its domain and range.

$R_2 = \{(a, \frac{1}{a}) / 0 < a \leq 5, a \in N\}$

SOLUTION

$R_2 = \{(a, \frac{1}{a}) / 0 < a \leq 5, a \in N\}$

$\therefore a = 1, 2, 3, 4, 5$

$\therefore \frac{1}{a} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$

$\therefore R_2 = \left\{(1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right), \left(4, \frac{1}{4}\right), \left(5, \frac{1}{5}\right)\right\}$

$\therefore$ Domain $(R_2) = \{a / 0 < a \leq 5, a \in N\}$

$= \{1, 2, 3, 4, 5\}$

Range $(R_2) = \{\frac{1}{a} / 0 < a \leq 5, a \in N\}$

$= \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

Exercise 1.2 | Q 10 | Page 16

R = {(a, b) / b = a + 1, a ∈ z, 0 < a < 5} Find the Range of R.

SOLUTION

R = {(a, b) / b = a + 1, a ∈ z, 0 < a < 5}

∴ a = 1, 2, 3, 4

∴ b = 2, 3, 4, 5

∴ Range (R) = {2, 3, 4, 5}

Exercise 1.2 | Q 11.1 | Page 16

Find the following relation as sets of ordered pairs.
{(x, y) / y = 3x, x ∈ {1,2,3}, y ∈ {3,6,9,12}}

SOLUTION

{(x, y) / y = 3x, x ∈ {1,2,3}, y ∈ {3, 6, 9, 12}}

Here y = 3x

When x = 1, y = 3(1) = 3

When x = 2, y = 3(2) = 6

When x = 3, y = 3(3) = 9

∴ Ordered pairs are {(1, 3), (2, 6), (3, 9)}

Exercise 1.2 | Q 11.2 | Page 16

Find the following relation as sets of ordered pairs.
{(x, y) / y > x + 1, x ∈ {1, 2} and y ∈ {2, 4, 6}}

SOLUTION

{(x, y) / y > x + 1, x ∈ {1, 2} and y ∈ {2, 4, 6}}

Here, y > x + 1

When x = 1 and y = 2, 2 ≯ 1 + 1

When x = 1 and y = 4, 4 > 1 + 1

When x = 1 and y = 6, 6 > 1 + 1

When x = 2 and y = 2, 2 ≯ 2 + 1

When x = 2 and y = 4, 4 > 2 + 1

When x = 2 and y = 6, 6 > 2 + 1

∴ Ordered pairs are {(1, 4), (1, 6), (2, 4), (2, 6)}

Exercise 1.2 | Q 11.3 | Page 16

Find the following relation as sets of ordered pairs.
{(x, y) / x+y = 3, x, y ∈ {0, 1, 2, 3}}

SOLUTION

{(x, y) / x+y = 3, x, y ∈ {0, 1, 2, 3}}

Here, x + y = 3

When x = 0, y = 3

When x = 1, y = 2

When x = 2, y = 1

When x = 3, y = 0

∴ Ordered pairs are {(0, 3), (1, 2), (2, 1), (3, 0)}

MISCELLANEOUS EXERCISE 1 [PAGES 16 - 17]

MISCELLANEOUS EXERCISE 1 | Q 1.1 | Page 16

Write the following set in the set-builder form.
{10, 20, 30, 40, 50}

SOLUTION

Let A = {10, 20, 30, 40, 50}

∴ A = {x/x = 10n, n ∈ N and n ≤ 5}

MISCELLANEOUS EXERCISE 1 | Q 1.2 | Page 16

Write the following set in the set-builder form.
{a, e, i, o, u)

SOLUTION

Let B = {a, e, i, o, u}

∴ B = {x/x is a vowel of English alphabets}

MISCELLANEOUS EXERCISE 1 | Q 1.3 | Page 16

Write the following set in the set-builder form.
{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

Let C = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

∴ C = {x/x represents days of a week}

MISCELLANEOUS EXERCISE 1 | Q 2.1 | Page 17

If U = {x/x ∈ N, 1 ≤ x ≤ 12}
A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11} C = {3, 5, 8, 9, 12}
Write the set: A ∪ B.

SOLUTION

U = {x / x ∈ N, 1 ≤ x ≤ 12} = {1, 2, 3, …., 12}

A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11}

C = {3, 5, 8, 9, 12}

A ∪ B = {1, 2, 4, 6, 7, 10, 11}

MISCELLANEOUS EXERCISE 1 | Q 2.2 | Page 17

If U = {x/x ∈ N, 1 ≤ x ≤ 12}
A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11} C = {3, 5, 8, 9, 12}
Write the set: B ∩ C.

SOLUTION

U = {x / x ∈ N, 1 ≤ x ≤ 12} = {1, 2, 3, …., 12}

A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11}

C = {3, 5, 8, 9, 12}

B ∩ C = { }

MISCELLANEOUS EXERCISE 1 | Q 2.3 | Page 17

If U = {x/x ∈ N, 1 ≤ x ≤ 12}
A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11} C = {3, 5, 8, 9, 12}
Write the set: A - B

SOLUTION

U = {x / x ∈ N, 1 ≤ x ≤ 12} = {1, 2, 3, …., 12}

A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11}

C = {3, 5, 8, 9, 12}

A - B = {1, 10}

MISCELLANEOUS EXERCISE 1 | Q 2.4 | Page 17

If U = {x/x ∈ N, 1 ≤ x ≤ 12}
A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11} C = {3, 5, 8, 9, 12}
Write the set: B - C

SOLUTION

U = {x / x ∈ N, 1 ≤ x ≤ 12} = {1, 2, 3, …., 12}

A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11}

C = {3, 5, 8, 9, 12}

B – C = {2, 4, 6, 7, 11}

MISCELLANEOUS EXERCISE 1 | Q 2.5 | Page 17

If U = {x/x ∈ N, 1 ≤ x ≤ 12}
A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11} C = {3, 5, 8, 9, 12}
Write the set: A ∪ B ∪ C.

SOLUTION

U = {x / x ∈ N, 1 ≤ x ≤ 12} = {1, 2, 3, …., 12}

A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11}

C = {3, 5, 8, 9, 12}

A ∪ B ∪ C ={1, 2, 3, 4, 5, 6, 7, 8, 9,10, 11, 12}

MISCELLANEOUS EXERCISE 1 | Q 2.6 | Page 17

If U = {x/x ∈ N, 1 ≤ x ≤ 12}
A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11} C = {3, 5, 8, 9, 12}
Write the set: A ∩ (B ∪ C).

SOLUTION

U = {x / x ∈ N, 1 ≤ x ≤ 12} = {1, 2, 3, …., 12}

A = {1, 4, 7, 10} B = {2, 4, 6, 7, 11}

C = {3, 5, 8, 9, 12}

B ∪ C = {2, 3, 4, 5, 6, 7, 8, 9, 11, 12}

∴ A ∩ (B ∪ C) = {4, 7}

MISCELLANEOUS EXERCISE 1 | Q 3 | Page 17

In a survey of 425 students in a school, it was found that 115 drink apple juice, 160 drink orange juice, and 80 drink both apple as well as orange juice. How many drinks neither apple juice nor orange juice?

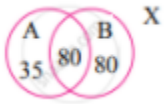
SOLUTION

Let A = set of students who drink apple juice

B = set of students who drink orange juice

X = set of all students

∴ n(X) = 425, n(A) = 115, n(B) = 160, n(A ∩ B) = 80



No. of students who neither drink apple juice

nor orange juice = n(A' ∩ B') = n(A ∪ B)'

= n(X) - n(A ∪ B)

= 425 - [n(A) + n(B) - n(A ∩ B)]

= 425 - (115 + 160 - 80)

= 230

MISCELLANEOUS EXERCISE 1 | Q 4 | Page 17

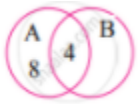
In a school there are 20 teachers who teach Mathematics or Physics, of these, 12 teach Mathematics and 4 teach both Physics and Mathematics. How many teachers teach Physics?

SOLUTION

Let A = set of teachers who teach Mathematics

B = set of teachers who teach Physics

∴ n(A ∪ B) = 20, n(A) = 12, n(A ∩ B) = 4



Since, n(A ∪ B) = n(A) + n(B) - n(A ∩ B)

∴ 20 = 12 + n(B) - 4

∴ n(B) = 12

∴ Number of teachers who teach physics = 12

MISCELLANEOUS EXERCISE 1 | Q 5.1 | Page 17

If A = {1, 2, 3} and B = {2, 4}, state the elements of A × A, A × B, B × A, B × B, (A × B)∩(B × A).

SOLUTION

A = {1, 2, 3} and B = {2, 4}

A × A = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

A × B = {(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)}

B × A = {(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)}

B × B = {(2, 2), (2, 4), (4, 2), (4, 4)}

(A × B)∩(B × A) = {(2, 2)}

MISCELLANEOUS EXERCISE 1 | Q 5.2 | Page 17
If A = {-1, 1} , find A × A × A.

SOLUTION

A = {-1, 1}

∴ A × A × A

= {(-1, -1, -1), (-1, -1, 1), (-1 , 1, -1), (-1, 1, 1), (1 , -1 , -1), (1, -1 , 1), (1 , 1 , -1), (1 , 1 , 1)}

MISCELLANEOUS EXERCISE 1 | Q 6.1 | Page 17
If A = {1, 2, 3}, B = {4, 5, 6} which of following are relation from A to B
R_1 = {(1, 4), (1, 5), (1, 6)}

SOLUTION

A = {1, 2, 3}, B = {4, 5, 6}

∴ A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

R_1 = {(1, 4), (1, 5), (1, 6)}

Since $R_1 \subseteq A \times B$

∴ R_1 is a relation from A to B.

MISCELLANEOUS EXERCISE 1 | Q 6.2 | Page 17
If A = {1, 2, 3}, B = {4, 5, 6} which of following are relation from A to B
R_2 = {(1, 5), (2, 4), (3, 6)}

SOLUTION

A = {1, 2, 3}, B = {4, 5, 6}

∴ A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

R_2 = {(1, 5), (2, 4), (3, 6)}

Since $R_2 \subseteq A \times B$

$\therefore R_2$ is a relation from A to B.

MISCELLANEOUS EXERCISE 1 | Q 6.3 | Page 17

If A = {1, 2, 3}, B = {4, 5, 6} which of following are relation from A to B

$R_3 = \{(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)\}$

SOLUTION

A = {1, 2, 3}, B = {4, 5, 6}

$\therefore A \times B = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

$R_3 = \{(1, 4), (1, 5), (3, 6), (2, 6), (3, 4)\}$

Since, $R_3 \subseteq A \times B$

$\therefore R_3$ is a relation from A to B.

MISCELLANEOUS EXERCISE 1 | Q 6.4 | Page 17

If A = {1, 2, 3}, B = {4, 5, 6} which of following are relation from A to B

$R_4 = \{(4, 2), (2, 6), (5, 1), (2, 4)\}$

SOLUTION

A = {1, 2, 3}, B = {4, 5, 6}

$\therefore A \times B = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

$R_4 = \{(4, 2), (2, 6), (5, 1), (2, 4)\}$

Since, $(4, 2) \in R_4$, but $(4, 2) \notin A \times B$

$\therefore R_4 \subseteq A \times B$

$\therefore R_4$ is not a relation from A to B.

MISCELLANEOUS EXERCISE 1 | Q 7 | Page 17

Determine the Domain and Range of the following relations.

$R = \{(a, b) / a \in N, a < 5, b = 4\}$

SOLUTION

$R = \{(a, b) / a \in N, a < 5, b = 4\}$

$\therefore$ Domain (R) $= \{a / a \in N, a < 5\} = \{1, 2, 3, 4\}$

Range (R) $= \{b / b = 4\} = \{4\}$