CBSE Test Paper 04
Chapter 2 Inverse Trigonometric Functions

1. The solution of the equation $\cos^{-1}(\sqrt{3}x) + \cos^{-1}x = \frac{\pi}{2}$ is given by

   a. $-\frac{1}{2}$
   b. None of these
   c. $\pm\frac{1}{2}$
   d. $\frac{1}{2}$

2. If $\theta = \tan^{-1}x$, then sin 2θ is equal to

   a. None of these
   b. $\frac{2x}{1+x^2}$
   c. $\frac{2x}{1-x^2}$
   d. $\frac{1-x^2}{1+x^2}$

3. $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + \cot^{-1}(3) =.$

   a. $\frac{\pi}{4}$
   b. $\frac{\pi}{2}$
   c. $\frac{\pi}{3}$
   d. $\frac{\pi}{6}$

4. $\frac{\cos 8^0 - \sin 8^0}{\cos 8^0 + \sin 8^0}$ is equal to

   a. $\tan 37^0$
   b. $\tan 53^0$
   c. $\tan 82^0$
   d. None of these

5. Which of the following is different from $2\tan^{-1}x$?

   a. $\tan^{-1}\left(\frac{2x}{1-x^2}\right), |x| < 1$
   b. $\sin^{-1}\left(\frac{2x}{1-x^2}\right), |x| \leqslant 1$

c. None of these

d. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), |x| \geqslant 0$

6. The value of sin (2tan^{-1})(0.75)) is equal to ________.

7. The domain of the function cos^{-1}(2x - 1) is ________.

8. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is ________.

9. Write the value of sin $\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$.

10. Find the value of $\sec\left(\tan^{-1}\frac{y}{2}\right)$.

11. $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)=$. **(1)**

12. Find the value of $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$.

13. If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the value of x.

14. Evaluate $\cos\left[\sin^{-1}\frac{1}{4}+\sec^{-1}\frac{4}{3}\right]$.

15. If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$ find x.

16. Prove that: $\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{12}{13}=\cos^{-1}\frac{33}{65}$

17. Write the following function in the simplest form: $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}, |x| < a$.

18. Find the value of $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$.

Solution

1. d. $\frac{1}{2}$

   Explanation: $\cos^{-1}(\sqrt{3}x) + \cos^{-1}x = \frac{\pi}{2}$

   $\Rightarrow \cos^{-1}(\sqrt{3}x) = \frac{\pi}{2} - \cos^{-1}x = (\sin^{-1}x)$

   $\Rightarrow \sqrt{3}x = \cos(\sin^{-1}x) = \sqrt{1-x^2}$

   $\Rightarrow 3x^2 = 1 - x^2 \Rightarrow x = \pm\frac{1}{2}$

   x= -$\frac{1}{2}$ does not satisfy the given equation.

   Consider $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

   $\Rightarrow \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$, which does not satisfy given equation

2. b. $\frac{2x}{1+x^2}$

   Explanation: $\theta = \tan^{-1}x.$

   $\Rightarrow x = \tan\theta$

   We know that

   $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2x}{1+x^2}$

3. a. $\frac{\pi}{4}$

   Explanation: $sin^{-1}\left(\frac{1}{\sqrt{5}}\right) + cot^{-1}(3)$

   $\Rightarrow \tan^{-1}\left(\frac{1}{2}\right) + tan^{-1}\left(\frac{1}{3}\right)$ because $\frac{1}{2}.\frac{1}{3} < 1$

   $\Rightarrow \tan^{-1}\left(\frac{\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}\right)$

   $\Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$

4. a. $\tan 37^0$

   Explanation: $\frac{\cos 8^0 - \sin 8^0}{\cos 8^0 + \sin 8^0}$

   $= \frac{\frac{\cos 8^0}{\cos 8^0} - \frac{\sin 8^0}{\cos 8^0}}{\frac{\cos 8^0}{\cos 8^0} + \frac{\sin 8^0}{\cos 8^0}}$

   $= \frac{1-\tan 8^0}{1+\tan 8^0} = \tan(45^0 - 8^0) = \tan 37^0$

5. c. None of these

Explanation: we know that,

$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$, $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ $and\ \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

6. 0.96
7. [0, 1]
8. $\frac{2\pi}{3}$
9. Given, $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$
   $\left[\because \sin^{-1}(-x) = -\sin^{-1}x; \forall x \in [-1,1]\right]$
   $= \sin\left[\frac{\pi}{3} + \sin^{-1}\left(\sin\frac{\pi}{6}\right)\right]\ \left[\because \sin\frac{\pi}{6} = \frac{1}{2}\right]$
   $= \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] = \sin\frac{\pi}{2} = 1\ \left[\because \sin^{-1}(\sin\theta) = \theta; \forall\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right]$
10. Let $\tan^{-1}\frac{y}{2} = \theta$, where $\theta \in \left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)$. So, $\tan\theta = \frac{y}{2}$, which gives $\sec\theta = \frac{\sqrt{4+y^2}}{2}$.
    Therefore, $\sec\left(\tan^{-1}\frac{y}{2}\right) = \sec\theta = \frac{\sqrt{4+y^2}}{2}$.
11. $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$
    Put $x = a\tan\theta$
    $= \tan^{-1}\left(\frac{3a^3\tan\theta - a^3 tan^3\theta}{a^3-3a^3\tan^2\theta}\right)$
    $= \tan^{-1}\left[\frac{a^3(3\tan\theta-\tan^3\theta)}{a^3(1-3\tan^2\theta)}\right]$
    $= \tan^{-1}(\tan 3\theta)$
    $= 3\theta$
    $= 3\tan^{-1}\frac{x}{a}$
12. $\tan^{-1}\left(\tan\frac{9\pi}{8}\right)$
    $= \tan^{-1}\tan\left(\pi + \frac{\pi}{8}\right)$
    $= \tan^{-1}\left(\tan\left(\frac{\pi}{8}\right)\right) = \frac{\pi}{8}$
13. Given: $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$
    $\Rightarrow \tan^{-1}\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)} = \frac{\pi}{4}\ \left[\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}\right]$
    $\Rightarrow \tan^{-1}\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)-(x-1)(x+1)} = \frac{\pi}{4}$
    $\Rightarrow \tan^{-1}\frac{x^2+2x-x-2+x^2-2x+x-2}{x^2-4-(x^2-1)} = \frac{\pi}{4}$
    $\Rightarrow \frac{2x^2-4}{x^2-4-x^2+1} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{2x^2-4}{-3} = 1$

$\Rightarrow$ 2x^2 - 4x = -3

$\Rightarrow$ 2x^2 = 1

$\Rightarrow x^2 = \frac{1}{2}$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$

14. $= \cos\left[\sin^{-1}\frac{1}{4} + \cos^{-1}\frac{3}{4}\right]$

$\cos\left(\sin^{-1}\frac{1}{4}\right)\cos\left(\cos^{-1}\frac{3}{4}\right) - \sin\left(\sin^{-1}\frac{1}{4}\right)\sin\left(\cos^{-1}\frac{3}{4}\right)$

$= \frac{3}{4}\sqrt{1-(\frac{1}{4})^2} - \frac{1}{4}\sqrt{1-(\frac{3}{4})^2}$

$= \frac{3}{4}\frac{\sqrt{15}}{4} - \frac{1}{4}\frac{\sqrt{7}}{4} = \frac{3\sqrt{15}-\sqrt{7}}{16}$

15. $\tan^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$

$\Rightarrow \frac{\frac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)}}{\frac{(x-2)(x+2)-(x-1)(x+1)}{(x-2)(x+2)}} = \tan\frac{\pi}{4}$

$\Rightarrow \frac{x^2+2x-x-2+x^2-2x+x-2}{(x^2+2x-2x-4)-(x^2-1)} = 1$

$\Rightarrow \frac{2x^2-4}{x^2-4-x^2+1} = 1$

$\Rightarrow \frac{2x^2-4}{-3} = \frac{1}{1}$

$\Rightarrow$ 2x^2 - 4 = - 3

$\Rightarrow$ 2x^2 = -3 + 4

$\Rightarrow$ 2x^2 = 1

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$

16. Let $\cos^{-1}\frac{4}{5} = \theta$ so that $\cos\theta = \frac{4}{5}$

$\therefore \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

Again, Let $\cos^{-1}\frac{12}{13} = \phi$ so that $\cos\phi = \frac{12}{13}$

$\therefore \sin\phi = \sqrt{1-\cos^2\phi} = \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$

Since $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi = \frac{4}{5}\times\frac{12}{13} - \frac{3}{5}\times\frac{5}{13}$

$= \frac{48-15}{65} = \frac{33}{65}$

$\Rightarrow \theta+\phi = \cos^{-1}\frac{33}{65}$

$\Rightarrow \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

17. Putting $x = a\sin\theta$ so that $\theta = \sin^{-1}\frac{x}{a}$

$$\Rightarrow \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2(1-\sin^2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2\cos^2\theta}}\right)$$
$$= \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$
$$= \tan^{-1}\tan\theta$$
$$= \theta = \sin^{-1}\frac{x}{a}$$

18. We have, $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$

$$= 2.2\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$$
$$= 2\left[\tan^{-1}\frac{\frac{2}{5}}{1-\left(\frac{1}{5}\right)^2}\right] - \tan^{-1}\frac{1}{239}\ \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$
$$= 2.\left[\tan^{-1}\left(\frac{\frac{2}{5}}{1-\frac{1}{25}}\right)\right] - \tan^{-1}\frac{1}{239}$$
$$= 2.\left[\tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right)\right] - \tan^{-1}\frac{1}{239}$$
$$= 2\tan^{-1}\frac{5}{12} - tan^{-1}\frac{1}{239}$$
$$= \tan^{-1}\frac{2.\frac{5}{12}}{1-\left(\frac{5}{12}\right)^2} - \tan^{-1}\frac{1}{239}\ \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$$
$$= \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{25}{144}}\right) - \tan^{-1}\frac{1}{239}$$
$$= \tan^{-1}\left(\frac{144\times 5}{119\times 6}\right) - \tan^{-1}\frac{1}{239}$$
$$= \tan^{-1}\left(\frac{120}{199}\right) - \tan^{-1}\frac{1}{239}$$
$$= \tan^{-1}\left(\frac{\frac{120}{119} - \frac{1}{239}}{1+\frac{120}{119}\cdot\frac{1}{239}}\right)\left[\because\ \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$
$$= \tan^{-1}\left(\frac{120\times 239 - 119}{119\times 239\times 120}\right)$$
$$= \tan^{-1}\left[\frac{28680-119}{28441+120}\right] = \tan^{-1}\frac{28561}{28561}$$
$$= \tan^{-1}(1) = \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{\pi}{4}$$