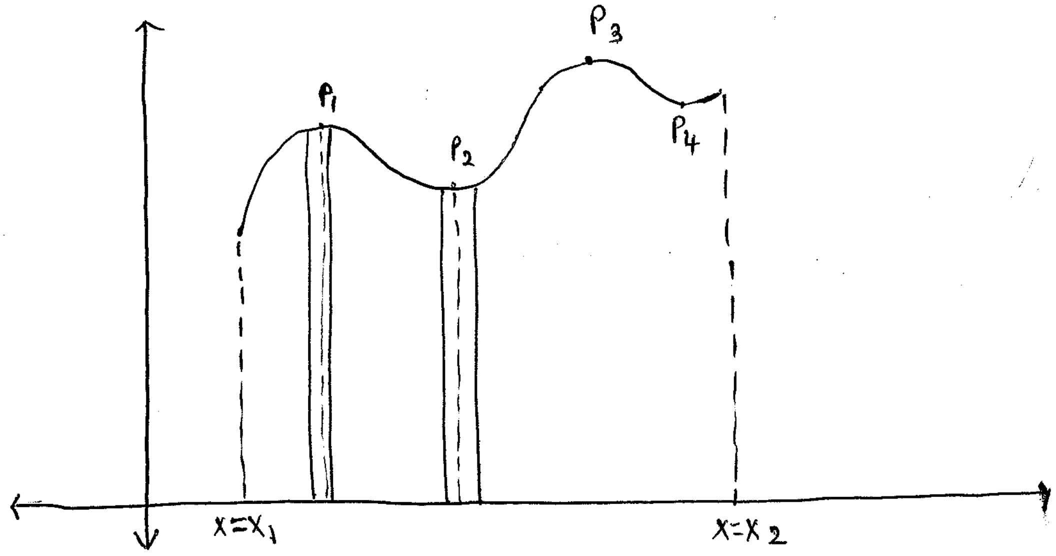
*Maxima and minima



~~Maximum Value~~

*Maxima

If the continuous function $f(x)$ increases to certain value then decreases then the value is called maxima

Minima

If the continuous function $f(x)$ decreases to certain value then increases then the value is called minima.

Maxima & Minima occurs alternatively.

NOTE: There ~~should~~ may be several maxima and minima.

The least minimum value is called global minima

The highest maximum value is called global maxima

Process for finding maxima & minima:-

Find $f(x), f'(x), f''(x), \ldots.$

① $f'(x)=0$, $x=\alpha, \beta, \gamma$

then put $x=\alpha$

$f''(\alpha)<0$, maxima at $x=\alpha$

maximum value is $f(\alpha)$

$x=\beta$

$f''(\beta)>0$, minima at $x=\beta$

minimum value is $f(\beta)$

$x=\gamma$

$f''(\gamma)=0$ point of inflection

further investigation is required.

Q:- $f(x) = x^x$. Find maxima, minima, and value (min OR max)

Sol^n:- $f(x) = x^x$

$y = x^x$

$\cancel{y = ex^x}$

$\log y = x \log x$

$\frac{1}{y}\frac{dy}{dx} = \log x + \cancel{x} \cdot 1/\cancel{x}$

$\frac{dy}{dx} = y(1+\log x)$

$y' = x^x(1+\log x) = 0$

$1+\log x = 0$ OR $x^x = 0$

$\log x = -1$ not possible

$x = e^{-1} = 1/e > 0$ minima

$y' = x^x + x^x \log x$ $\quad f(x) = (1/e)^{1/e}$

$y' = y + y \log x$ minimum value

$y'' = y' + y' \log x + y \cdot 1/x$

$y'' = x^x(1+\log x) + x^x(1+\log x)\log x + \frac{x^x}{x}$

$y'' = \left(\frac{1}{e}\right)^{1/e}(1+\log 1/e) + (1/e)^{1/e}(1+\log 1/e)\log 1/e + \frac{\left(\frac{1}{e}\right)^{1/e}}{1/e} > 0$

Q:- $y(x) = \frac{\log x}{x}$

In minima & maxima imaginary roots are neglected

$$y = \frac{\log x}{x}$$

$$y' = \frac{x(1/x) - \log x \cdot 1}{x^2}$$

$$y' = \frac{1 - \log x}{x^2}$$

$$y' = 0$$

$$\log x = 1$$

$$x = e^1$$

$$y'' = \frac{x^2(-1/x) - (1-\log x)(2x)}{x^4}$$

$$y'' = \frac{-x - 2x + 2x\log x}{x^4} < 0$$

$-1-2+2\cdot 1$

At $x = e$, ~~maxima~~ minima will be there

$y(x) = \frac{\log e}{e} = \frac{1}{e}$ ← minimum value.

Q:- $f(x) = x^3 + 3/x$

$$f'(x) = 3x^2 + -3/x^2$$

$$0 = 3x^2 - 3/x^2$$

$$0 = \frac{3x^4 - 3}{x^2}$$

$$3x^4 - 3 = 0$$

$$x^4 = 1$$

$$x = 1, -1$$

$f''(x) = 6x - \frac{3(-2)}{x^3} = 6x + \frac{6}{x^3}$

$f''(1) = 6 + 6 = 12 > 0$

$f''(-1) = -6 - 6 = -12 < 0$

At $x = 1$ ~~maximum~~ minimum

$f(1) = 1 + 3 = 4$

At $x = -1$ maximum

$f(-1) = -1 + 3/-1 = -4$

Q:- $f(x) = x^2 + \frac{250}{x}$

$$y = x^2 + \frac{250}{x}$$

$$y' = 2x + \frac{-250}{x^2}$$

$$0 = 2x - \frac{250}{x^2}$$

$$0 = 2x^3 - 250$$

$$2x^3 = 250$$

$$x^3 = 125$$

$$\boxed{x = 5}$$

$$y'' = 2 - \frac{250(-2)}{x^3}$$

$$y'' = 2 + \frac{500}{x^3}$$

$$y'' = 2 + \frac{500}{125} > 0$$

$\downarrow$
minima

$$f(x) = 25 + \frac{250}{5}$$

$$f(x) = 25 + 50 = 75$$

$\downarrow$
minimum value

Q:- $f(x) = x^2 e^{-x}$

$$y = x^2 \cdot e^{-x}$$

$$y' = 2x \cdot e^{-x} + x^2 \cdot e^{-x}$$

$$0 = 2x\, e^{-x} + x^2(-e^{-x})$$

$$2x\, e^{-x} - x^2 e^{-x} = 0$$

$$\cancel{e^{-x} x^2 [2 \quad -1] \neq 0}$$

$$\cancel{x^2 e^{-x} = 0}$$

$$x e^{-x} [2 - x] = 0$$

$$x = 2, 0$$

$$y'' = 2e^{-x} - 2xe^{-x} - 2xe^{-x} + x^2 e^{-x}$$

$$y''(2) = 2e^{-2} = 2e^{-2} < 0 \quad \text{maxima}$$

$$y''(2) = 2e^{-2} - 2(2)e^{-2} - 2 \cdot 2 e^{-2} + 4e^{-x}$$

$$\boxed{f(2) = 4e^{-2}}$$

$\downarrow$
maximum value

Q. Find the height of right circular cone of largest volume that can be enclosed by a sphere of 1 unit radius.
1m

Sol:-

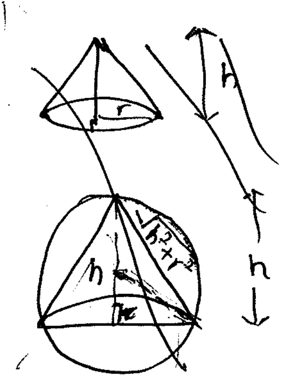
$\cancel{V = \frac{2}{3}\pi r^2 h}$

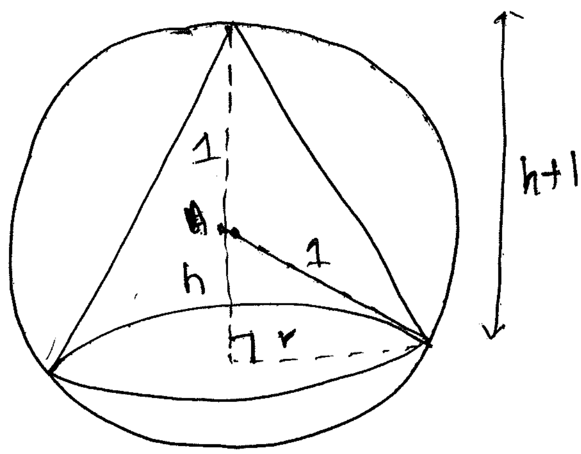
$\cancel{V' = \frac{2}{3}\pi \cdot \frac{dr}{dt} \cdot \frac{dh}{dt} \cdot 2r^2}$

$\cancel{V = \frac{2}{3}\pi r^2 h}$

$V = \frac{1}{3}\pi r^2 h$

$\cancel{\frac{dV}{dh} = \frac{1}{3}\pi r^2 \cdot dh}$





$V = \frac{1}{3}\pi (1-h^2)(h+1)$

$V = \frac{\pi}{3}(h - h^3 + 1 - h^2)$

$r_{sp.} = \sqrt{h^2+r^2}$

$1 = h^2 + r^2$

$r = \sqrt{1-h^2}$

$\cancel{V' = \frac{\pi}{3} \quad \frac{\pi}{3} 3h^2}$

$V' = \frac{\pi}{3}\left[1 - 3h^2 - 2h\right]$

$3h^2 + 2h - 1 = 0$

$h = \frac{-2 \pm \sqrt{4+12}}{2 \cdot 3}$

$h = \frac{-2 \pm 4}{6}$ $\quad h = 1/3$

But height of right circular cone is $1 + h$

$1 + 1/3 = 4/3$

Q:- Find height of cylinder of maximum volume that can be inscribed in a sphere of radius a unit.
(Note:- cylinder is symmetric)

Sol^n:-

$V = \pi r^2 h$

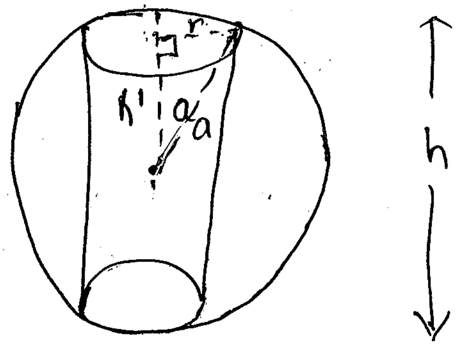
$V = \pi (a^2 - h^2) h$

$V' = \pi (a^2 - 3h^2)$

$V' = 0$

$a^2 = 3h^2$

$h = \frac{a}{\sqrt{3}}$



But height of cylinder $2h = \frac{2a}{\sqrt{3}}$

Q:- The maximum area of a rectangle whose vertices lies on ellipse $x^2+4y^2=1$ is ___. (EC-2015)

Sol^n:- Area $= f_1 = (2x)(2y) = 4xy$

$f = \text{AREA}^2 = 16x^2y^2$

$f = 4x^2(1-x^2)$

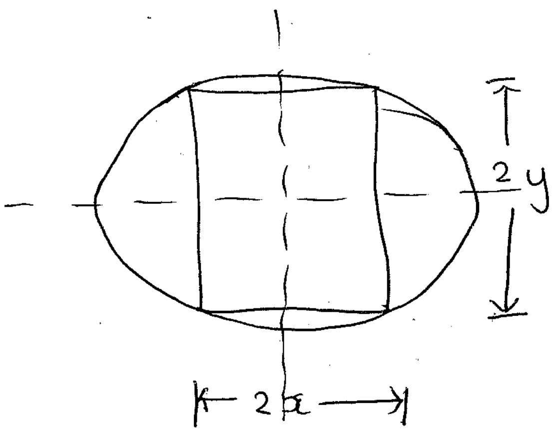
$f' = \frac{d}{dx}(4x^2 - 4x^4)$

$0 = 8x - 16x^3$

$x^2 = \frac{1}{2}$

$x = \pm 1/\sqrt{2}$

$y = \pm 1/\sqrt{8}$



AREA $= 4xy$
$= 4 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{8}} = 1$

Q:- $f(x) = a\cos x + b\sin x + c$

min. value $= c - \sqrt{a^2+b^2}$

max^m value $= c + \sqrt{a^2+b^2}$

~~max. value~~.

$f'(x) = -a\sin x + b\cos x$

$0 = -a\sin x + b\cos x$

$b\cos x = a\sin x$

$\tan x = b/a$

Q:- $f(x) = 3\cos x + 4\sin x + 2$. Find max^m & min^n value

max. value $= 2 + \sqrt{9+16} = 7$

min. value $= -3$

Q:- $f(x) = 5\cos x + 3\cos(x + \pi/2) + 2$

$5\cos x - 3\sin x + 2$

max. $= 2 + \sqrt{25+9} = 2 + \sqrt{34}$

min. $= 2 - \sqrt{25+9} = 2 - \sqrt{34}$

GATE-2017

Q①:- Two cars p and q are moving in a racing track continuously for two hours. Assume that no other vehicle are using the track during this time. The expression relating the distance travelled d in km and time t in hours for both the vehicles are given as:-

P: $d = 60t$

Q:- $d = 60t^2$

Within the first one hour, the maximum space headway would be.
(distance bet^n 2 vehicles)

(a) 15km at 30min.
(b) 15Km at 15min.

Sol^n:- For p,

$d = 60t$

$v = 60\,km/hr$

$v_1 = \frac{d_2 - d_1}{1} =$

$d_1 - d_2 = 60t - 60t^2$

$\frac{d(d_1 - d_2)}{dt} = -120t + 60$

$-120t + 60 = 0$

$t = \frac{1}{2}\,hr.$

$\frac{d^2(d_2 - d_1)}{d^2t} = 120 > 0$

For q,

$d = 60t^2$

$v = 120t$

$a = 120\,km/hr^2$

$|d_1 - d_2| = |60(\frac{1}{2})^2 - 60(\frac{1}{2})|$

$|d_1 - d_2| = |\frac{60}{4} - \frac{60}{2}| = |15 - 30| = 15\,Km$

Q:- Let $f(x) = e^{x+x^2}$ for real x. From among the following choose the Taylor's series approximation of $f(x)$ around $x = 0$, which includes all power of x less than or equal to 3.

$f(x) = e^{x+x^2}$, $f(0) = e^0 = 1$

$f'(x) = e^{x+x^2}(1+2x)$, $f'(0) = e^0(1) = 1$

$f''(x) = e^{x+x^2}(1+2x) + e^{x+x^2} \cdot 2$, $f''(0) = 2+1 = 3$

$f'''(x) = e^{x+x^2}(1+2x)(1+2x) + e^{x+x^2} \cdot 2 + e^{x+x^2} \cdot 2(1+2x)$

$= 1 + 4 + 2 = 7$

$= f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!}$

$= 1 + x \cdot 1 + \frac{x^2 \cdot 3}{2} + \frac{x^3 \cdot 7}{6}$

ESE-2017

Q:- The minimum value of $f(x) = \frac{x^3}{3} - x$ occurs at

$f'(x) = \frac{\cancel{3}x^2}{\cancel{3}} - 1 = 0$

$x = \pm 1$

$\boxed{x = 1}$ ← minimum value

$f''(x) = 2x$

$f''(1) = 2 > 0$

Q:- A cubic polynomial with real co-efficient

(a) can possibly have no extrema and no zero crossing (roots)

(b) ~~they~~ may have upto three extrema & upto 2 zero crossings

(c) cannot have more than two extrema and more than 3 zero crossings.

(d) will always have an equal no. of extrema and zero crossings.

Sol^n:- crossing = roots

NOTE: n^{th} polynomial can have $(n-1)$ extrema and n zero crossings (roots)

Q:-At $x=0$ the $f(x)=x^3+1$ as

(a) maximum value

(b) minimum value

(c) point of inflection

(d) N.O.T.A.

Sol^n:- $f(x)=x^3+1$

$f'(x)=3x^2=0$

$x=0$

$f''(x)=0 \longrightarrow$ point of inflection

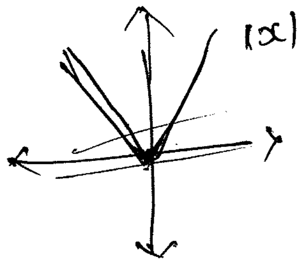
Q:- At $x=0$ the function $f(x)=|x|$ has

(a) a minimum ✓

(b) a maximum

(c) a point of inflextion

(d) neither max^m nor min^m



$f'(x) = -1 \quad , x<0$

$f'(x) = 1 \quad , x>0$

but at $x=0$ check neighbourhood point above the tangent i.e. we get <u>minimum</u>

Q:- The optimum value of $f(x)=x^2-4x+2$ is

$f'(x)=2x-4$

$x=2$

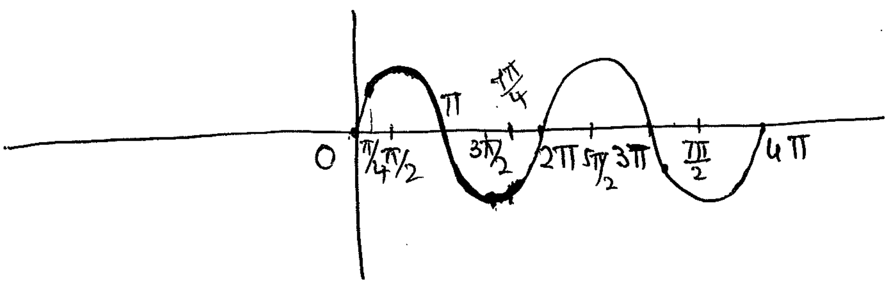
$f''(x)=2>0 \longrightarrow x=2$ minima

$= 4-8+2$

$\boxed{f(x) = -2}$ ⟵ optimum value minimum value.

Q:- Consider the function $f(x)=\sin x$ in interval $x \in [\pi/4, 7\pi/4]$. The no. of locations of local minima of this function are:-

$x \in [\pi/4, 7\pi/4]$



$\boxed{x = 3\pi/2}$

Q:- As x varies from -1 to $+3$ which one of following describe behaviour of the function.

$f(x) = x^3 - 3x^2 + 1$

(a) $f(x)$ decreases, then increases and decreases again

(b) $f(x)$ increases, " decreases " increases again.

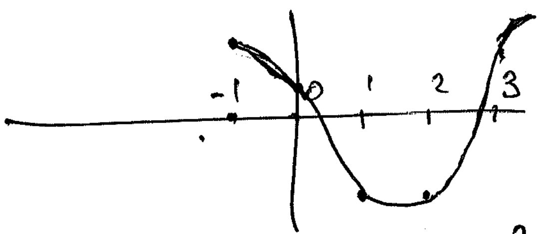
(c) $f(x)$ increases and then decreases

(d) NOTA

$f(-1) = -1 + 3 + 1 = 3$

$f(0) = 1$ $\qquad$ $f(1) = 1 - 3 + 1 = -3$

$f(3) = 27 - 27 + 1 = 1$ $\qquad$ $f(2) = 8 - 12 + 1 = -3$



$f'(x) = 3x^2 - 6x$

$f'(x) = 0$

$x = 0, 2$

$f''(x) = 6x - 6$

$f''(0) = -6 \leftarrow$ At $x = 0$ maxima increases

$f''(2) = 6 \leftarrow$ At $x = 2$ minima decreases

Q:- Find $\frac{dy}{dx}$ $\quad ax^2 + 2hxy + by^2 = 0$

$$\cancel{2}ax + \cancel{2}hy + \cancel{2}h\frac{dy}{dx}\cdot x + b\cdot\cancel{2}y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(xh + by) = -ax - hy$$

$$\frac{dy}{dx} = -\frac{(ax + hy)}{xh + by}$$

Explicit fun. $y = f(x)$

Implicit $\rightarrow$ To partial derivative

By other method,

$$-\frac{dy}{\partial x} = -(2ax + 2hy)$$

$$\frac{\partial y}{\partial y} = 2hx + 2by$$

$$\frac{dy}{dx} = \frac{\frac{\partial y}{\partial x}}{\frac{\partial y}{\partial y}} = -\frac{(ax+hy)}{hx+by}$$

If $z = f(x, y)$

$\qquad\qquad\downarrow\ \downarrow$

$\qquad\qquad t\ \ t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

EULER's equation

If $z = f(x,y)$, homogeneous function

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nZ$$

$\uparrow$ degree (algebraic)

$$x\frac{\partial^2 z}{\partial x^2} + y\partial \cdot \qquad + 2xy\partial$$

Q:- $z = \frac{x^2+y^2}{x+y}$

then $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial y} = 1z$

*Maxima and Minima of two variables:-

$z = f(x, y)$

$p = \frac{\partial z}{\partial x}$ $\quad s = \frac{\partial^2 z}{\partial x \partial y}$

$Q = \frac{\partial z}{\partial y}$ $\quad t = \frac{\partial^2 z}{\partial y^2}$

$r = \frac{\partial^2 z}{\partial x^2}$

- For maxima and minima let $P=0$ and $Q=0$ since P and Q are function in x and y. So we have two eq^n in x and y. Solve this eq^n and get relation bet^n x & y. Put these relation in either in $P=0$ or $Q=0$ then eq^n transforms to purely in x or purely in y.

- Solve this eq^n and get roots

$P=0$

$Q=0$

$x = x_0, x_1, x_2, \ldots\ldots$

$y = y_0, y_1, y_2, \ldots\ldots$

(x_0, y_0)

$\downarrow$

Stationary point or Critical point

Case:-1 $rt-s^2>0$, $r>0$ $\min^m. (x_0, y_0)$

$f(x_0, y_0) \leftarrow$ minimum value

Case:-2 $rt-s^2>0$, $r<0$ $\max^m. (x_0, y_0)$

$f(x_0, y_0) \leftarrow$ maximum value

Case:-3 $rt-s^2<0$, there is no. maxima & minima

Case:4 $rt-s^2=0$ further investigation is to be needed. (Doubtfull)

Q:- A function $(x,y) = x^2+y^2+6x+12$ has

(a) minimum at $(-3,0)$ ✓

(b) maximum at $(-3,0)$

(c) no extremum at $(-3,0)$

(d) NOTA

Sol:-

$P = 2x+6$

$Q = 2y$

$R = 2$

$S = 0$

$T = 2$

$Rt-s^2$, $r>0$

$2(2)-0=0$

$4<0$

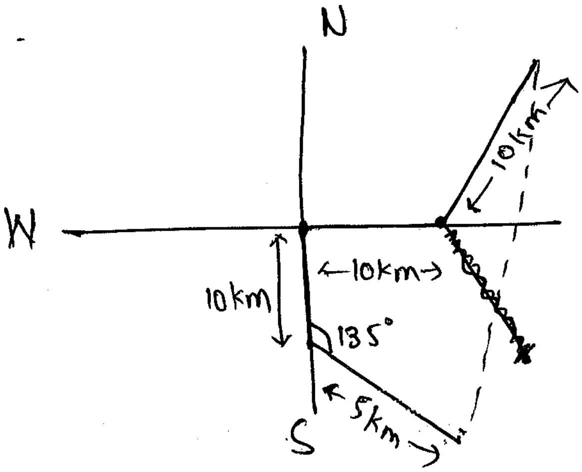
$P=0$ $\quad$ $Q=0$

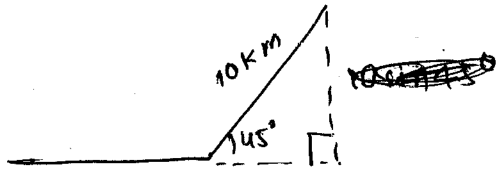
$\boxed{x=-3}$ $\quad$ $\boxed{y=0}$

$(-3,0)$

Both x and y travel from same point. X travels 10km East and again 10km North-East. Y travels 10km South and 5km South-East. The minimum distance betn x & y is _

Solⁿ:-

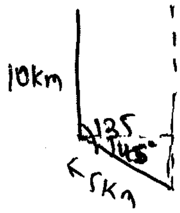




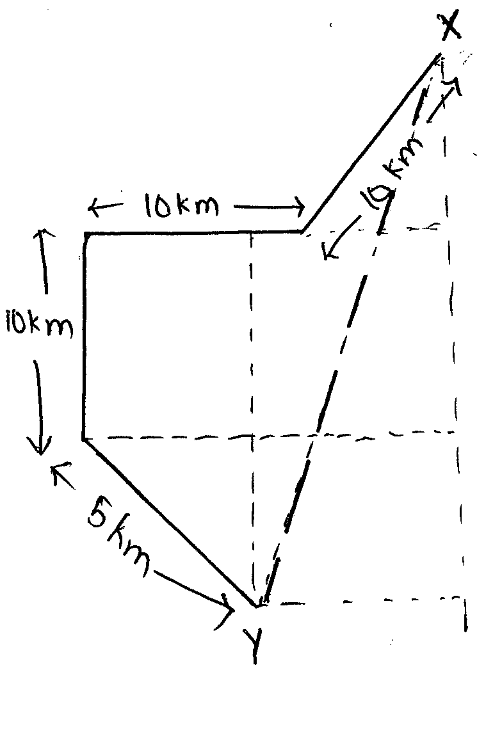
$10\cos 45^\circ$

$$\frac{10}{\sqrt{2}} = \frac{10}{1.414}$$

$= 7.07$



$$5\cos 45^\circ = \frac{5}{\sqrt{2}} = 3.53$$



Q:- $z = 1 - x^2 - y^2$

(a) minimum at (0,0)

(b) maximum at (0,0)

(c) no extremum at (0,0)

(d) NOTA

Sol^n:- $p = -2x$

$q = -2y$

$r = -2$

$s = 0$

$t = -2$

$rt - s^2$, $r < 0$

$(-2)(-2) - 0$

$4 > 0$

maxima

$P = 0$, $Q = -2y = 0$

$-2x = 0$

$x = 0$

$y = 0$

$(x, y) = (0, 0)$

max^m value at (0,0).

Q:- $f(x,y) = x^3 + y^3 - 3xy$ has (a) minimum at $(-1,-1)$

(b) maximum at $(1,-1)$ and (c) minimum at $(1,1)$

& (d) maximum at $(1,1)$

Sol^n:- $p = 3x^2 - 3y$

$q = 3y^2 - 3x$

$r = 6x$

$s = -3$

$t = 6y$

$P = 0$, $Q = 0$

$3x^2 - 3y = 0$ $\quad$ $3y^2 - 3x = 0$

$x^2 = y$ $\quad$ $y^2 = x$

$(1,1)$

$rt - s^2$ $\quad$ $r > 0$

$6 \cdot 6 - 9 = 36 - 9 = +25 > 0$

minima at $(1,1)$.

Q:- $f(x,y) = x^2 + y^2 + xy + x - 4y + 5$ has

(a) minimum at $(2,-3)$

(b) maximum at $(2,-3)$

(c) minimum at $(-2,3)$

(d) maximum at $(-2,3)$

Sol^n:- $p = 2x + y + 1$

$q = 2y + x - 4$

$r = 2$

$s = 1$

$t = 2$

$P = 0$

$2x = -1 - y$

$2x + y = -1$

$Q = 0$

$2y + x = 4$

$\boxed{y = 3}$

$$
\begin{array}{rcl}
4x + 2y &=& -2 \\
2y + x &=& 4 \\
- \quad - & & - \\
\hline
3x &=& -6
\end{array}
$$

$\boxed{x = -2}$

$rt - s^2$, $r = 2 > 0$

$2(2) - 1$

$4 - 1 = 3 > 0 \rightarrow$ minima at $(-2,3)$

Q:- $f(x,y) = 4x^2 + 6y^2 - 8x - 4y + 8$ has

(a) minimum $(8/3)$

(b) maximum $(8/3)$

(c) minimum $(10/3)$

(d) maximum $(10/3)$

Sol^n:- $p = 8x - 8$

$q = 12y - 4$

$r = 8$

$s = 0$

$t = 12$

$P=0 \quad , \quad Q=0$

$x=1 \qquad y=\frac{1}{3}$

$rt-s^2$

$8\cdot 12 - 0$

$96 > 0$

$f(1, 1/3) = 4 + \overset{2}{\cancel{6}}\cdot 1/\underset{3}{\cancel{9}} - 8\cdot 1 - 4\cdot 1/3 + 8$

$= 4 + \frac{2}{3} - \cancel{8} - 4/3 + \cancel{8}$

$= 4 - 2/3$

$\boxed{f(1, 1/3) = 10/3}$

Q:- $f(x,y) = \sin x + \sin y + \sin(x+y)$ has

(a) minimum $(\sqrt{3}/2)$

(b) maximum $(\sqrt{3}/2)$

(c) minimum $(3\cdot\sqrt{3}/2)$

(d) maximum $(3\cdot\sqrt{3}/2)$ ✓

Solⁿ:- $p = \cos x + \cos(x+y) \; (1)$

$q = \cos y + \cos(x+y)$

$r = -\sin x - \sin(x+y)$

$s = -\sin(x+y)$

$t = -\sin y - \sin(x+y)$

$P=0$

$\cos x + \cos(x+y) = 0$

$\cos x + \cos x \cos y - \sin x \sin y = 0$

$Q=0$

$\cos y + \cos(x+y) = 0$

$\cos y + \cos x \cos y - \sin x \sin y = 0$

$\cos x - \cos y = 0$

$\cos x = \cos y$

$x = y$

$P = 0$

$\cos x + \cos(x + x) = 0$

$\cos x + \cos 2x = 0$

$\cancel{\cos x + 2\cos^2 x - 1}$

$\cos 2x = -\cos x$

$\cos 2x = \cos(\pi - x)$

$rt - s^2$

$\left(-\frac{1}{\sqrt{2}} - 1\right)\left(-\frac{1}{\sqrt{2}} - 1\right) + 1$

$\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 1 + 1$

$\sqrt{2} + 2 + \frac{1}{2} > 0$, $r = -\frac{1}{\sqrt{2}} - 1 < 0$

$\sqrt{2} + \frac{5}{2} > 0$

$f(x, y) = \sin \frac{\pi}{4} + \cos \frac{\pi}{4} + \sin \frac{\pi}{2}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

$2x = \pi - x$

$3x = \pi$

$x = \pi/3$

$y = \pi/3$

$rt - s^2$

$\left(-\sin \frac{\pi}{3} - \sin\left(\overset{\pi - \pi/3}{\frac{2\pi}{3}}\right)\right)^2 - \left(-\sin\left(\frac{2\pi}{3}\right)\right)$

$\left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)^2 + \frac{\sqrt{3}}{2}$

$\left(-\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right)^2$

$3 + \frac{\sqrt{3}}{2} > 0$, $r < 0$

max^m at $(\pi/3, \pi/3)$

Q:- $f(x,y) = x^3 + 3xy^2 - 15x^2 + 72x - 15y^2$

$p = 3x^2 + 3y^2 - 30x + 72$

$q = 3x \cdot 2y - 30y$

$r = 6x - 30$

$s = 6\cancel{y}x$ ~~-30~~

$t = 6x - 30$

$p = 0$

$3x^2 + 3y^2 - 30x + 72 = 0$

$3 \cdot 25 + 3y^2 - 150 + 72 = 0$

$75 + 3y^2 - 150 + 72 = 0$

$3y^2 - 150 + 147 = 0$

$3y^2 = 3$

$y^2 = 1$

$y = \pm 1$

$q = 0$

$6xy - 30y = 0$

~~Suppose~~

$y(6x - 30) = 0$

$x = 5, \; y = 0$

At $x = 5$

$y = \pm 1$

At $y = 0$

$x = 6, 4$

① (5,0) ④ (6,0)

② (5,1) ⑤ (4,0)

③ (5,-1)

① (5,0)

$rt - s^2$ $\quad r = 0$

$0 - 30 < 0$

② (5,1) $\quad r = 0$

$0 - 6 < 0$

③ (5,-1) $\quad r = 0$

$0 + 6 > 0$

④ (4,0) $\quad r < 0$

$(-6) \cdot (-6) - 24$

$36 - 24 = 12 > 0$

⑤ (6,0)

$(6)(6) - 36 = 0.$

further investigation is to be needed