14 PRACTICE PAPER

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: *As given in Practice Paper – 1.*

Section–A

Choose the correct option:

1. **Which of the given values of x and y make the following pair of matrix equal**

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}?$$

(*a*) $x = -\frac{1}{3}, y = 7$ (*b*) x cannot be determined and $y = 7$

(*c*) $x = -\frac{2}{3}, y = 7$ (*d*) $x = -\frac{1}{3}, y = -\frac{2}{3}$

2. **For any 2 × 2 matrix A if $A(\text{adj } A) = \begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix}$, then $|A|^3$ equals**

(*a*) 12^2 (*b*) 12^3 (*c*) 12^4 (*d*) None of these

3. **Let A and B be two invertible matrices of order 3 × 3. If $\det(ABA') = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B')$ is equal to**

(*a*) 1 (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{16}$ (*d*) 16

4. **If $f(x) = \log(\sin x)$ then $f''\left(\frac{\pi}{4}\right)$ is equal to**

(*a*) 2 (*b*) –2 (*c*) 1 (*d*) –1

5. **Let $f(x) = 2\sin^3 x - 3\sin^2 x + 12\sin x + 5,\ 0 \le x \le \frac{\pi}{2}$. Then $f(x)$ is**

(*a*) decreasing in $\left[0, \frac{\pi}{2}\right]$ (*b*) increasing in $\left[0, \frac{\pi}{2}\right]$

(*c*) increasing in $\left[0, \frac{\pi}{4}\right]$ and decreasing in $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (*d*) none of these

6. $\int \frac{e^{2x}\,dx}{\sqrt[4]{e^x - 1}}$ **equals**

(*a*) $\frac{4}{21}(e^x - 1)^{3/4}(3e^x + 4) + C$ (*b*) $\frac{(e^x - 1)^{1/4}(3e^x + 4)}{21} + C$

(*c*) $\frac{4}{21}(e^x + 1)(3e^x + 4) + C$ (*d*) None of these

7. $\int \frac{x^{1/2}}{x^3+a^3}dx$ equals

(a) $\frac{1}{3}\tan^{-1}\left(\frac{x}{a}\right)^{3/2}+C$

(b) $\frac{2}{a^{3/2}}\tan^{-1}\left(\frac{x}{a}\right)^{3/2}+C$

(c) $\frac{2}{3}\frac{1}{a^{3/2}}\tan^{-1}\left(\frac{x}{a}\right)^{3/2}+C$

(d) None of these

8. The values of a and b, if the equations $\int(\sin 2x-\cos 2x)\,dx=\frac{1}{\sqrt{2}}\sin(2x-a)+b$ hold good are

(a) $a=-\frac{5\pi}{4}, b\in\mathbb{R}$

(b) $a=\frac{5\pi}{4}, b\in\mathbb{R}$

(c) $a\in\mathbb{R}, b=\frac{5\pi}{4}$

(d) $b=-\frac{5\pi}{4}, a\in\mathbb{R}$

9. *Read the following statements.*

Statement I : If f is an even function, then $\int_{-a}^{a} f(x)\,dx=2\int_{0}^{a} f(x)\,dx$.

Statement II : $\int_{1}^{\sqrt{3}}\frac{dx}{1+x^2}=-\frac{\pi}{12}$

Choose the correct option:

(a) Statement I is correct but statement II is not correct.

(b) Statement II is correct but statement I is not correct.

(c) Both statements I and II are correct.

(d) None of these

10. The area bounded by the curve $y=x^3$, the x-axis and the ordinates $x=-1$ and $x=1$ is

(a) $\frac{1}{2}$ sq. unit (b) 4 sq. units (c) $\frac{14}{3}$ sq. units (d) None of these

11. The order and degree of differential equation whose solution is $Ax^2+By^2=1$, where A and B are arbitrary constants is

(a) 1, 2 (b) 2, 3 (c) 3, 2 (d) 2, 1

12. The solution of the equation $\frac{d^2y}{dx^2}=e^{-3x}$ is

(a) $y=\frac{e^{-3x}}{9}+Cx$ (b) $y=\frac{e^{-3x}}{9}+C$ (c) $y=\frac{e^{-3x}}{9}+C_1x+C_2$ (d) None of these

13. Objective function of a LPP is

(a) a quadratic function

(b) a constant

(c) a linear function to be optimised

(d) None of these

14. The probability distribution of X is given by

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	k	$\frac{1}{8}$

then k is equal to

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$ (d) None of these

15. A die is thrown six times, the chance that exactly 3 times an odd number turn up is

(a) $\frac{1}{16}$ (b) $\frac{3}{16}$ (c) $\frac{5}{16}$ (d) none of these

Section–B (B1)

16. Let R be the relation defined on the set N of natural numbers by the rule $x\ R\ y$ iff $x + 2y = 8$, then domain of R is

(*a*) {2,4,8} (*b*) {2,4,6}

(*c*) {2,4,6,8} (*d*) {1,2,3,4}

17. If $f:[1,\infty) \to [3,\infty)$ is given by $f(x) = x + \frac{2}{x}$, then $f^{-1}(x)$ is equal to

(*a*) $\frac{x-\sqrt{x^2-8}}{2}$ (*b*) $\frac{x}{2+x^2}$ (*c*) $\frac{x+\sqrt{x^2-8}}{2}$ (*d*) None of these

18. Let $f: R \to R$ be a function such that $f(x) = \frac{1}{1+x}$, then $fofof(x)$ is

(*a*) $\frac{2+x}{3+2x}$ (*b*) 0 (*c*) does not exist (*d*) None of these

19. Which of the following functions from Z into Z are bijections?

(*a*) $f(x) = x + 3$ (*b*) $f(x) = x^3$ (*c*) $f(x) = 2x + 1$ (*d*) $f(x) = 2x$

20. Let * be binary operation on Z (set of integers) as $a * b = 2 + ab$ then $(2 * 3) * 4$ is equal to

(*a*) 8 (*b*) 14 (*c*) 34 (*d*) None of these

21. The value of $\sin(2\sin^{-1}(0.8))$ is

(*a*) sin 1.6 (*b*) 1.6 (*c*) 0.96 (*d*) 4.8

22. The greatest and least value of $(\sin^{-1}x)^2 + (\cos^{-1}x)^2$ are respectively

(*a*) $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ (*b*) $\frac{5\pi^2}{4}$ and $\frac{\pi^2}{8}$ (*c*) $\frac{\pi^2}{2}$ and $-\frac{\pi^2}{2}$ (*d*) $\frac{\pi}{2}$ and 0

23. If $\tan^{-1} y = 4\tan^{-1} x$, then y is not finite if

(*a*) $x^2 = 3 \pm 2\sqrt{2}$ (*b*) $x^2 = 2 - 2\sqrt{2}$ (*c*) $x^4 = 6x^2 - 1$ (*d*) $x^4 = 6x^2 + 1$

24. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ and $f(1) = 1$, $f(p+q) = f(p).f(q)$, $\forall\ p, q \in R$, then

$x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$ is equal to

(*a*) 2 (*b*) 1 (*c*) 0 (*d*) 3

25. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is

(*a*) 27 (*b*) 18 (*c*) 81 (*d*) 512

26. Matrix A and B will be inverse of each other only if

(*a*) $AB = BA$ (*b*) $AB = BA = 0$ (*c*) $AB = 0, BA = I$ (*d*) $AB = BA = I$

27. The value of $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ where a, b and c are in AP is

(*a*) 1 (*b*) −1 (*c*) 0 (*d*) None of these

28. The determinant $\begin{vmatrix} \sin A & \cos A & \sin A + \cos B \\ \sin B & \cos A & \sin B + \cos B \\ \sin C & \cos A & \sin C + \cos B \end{vmatrix}$ is equal to

(*a*) cos A (*b*) −1 (*c*) 10 (*d*) 0

29. *Read the following statements.*

Statement I : If $x^y = y^x$, then $\frac{dy}{dx} = \frac{y(x \log y + y)}{x(y \log x + x)}$

Statement II : If $y = e^x$ then $\frac{d^2y}{dx^2} = y$

Choose the correct option:

(*a*) Statement I is correct but statement II is not correct.

(*b*) Statement II is correct but statement I is not correct.

(*c*) Both statements I and II are correct.

(*d*) None of these

30. **The function** $f(x) = \begin{cases} \sin \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ **at** $x = 0$

(*a*) is continuous.

(*b*) has removable discontinuity.

(*c*) has jump discontinuity.

(*d*) has oscillating discontinuity.

31. **The function** $f(x) = \begin{cases} 2 - x, & \text{if } x < 2 \\ 2 + x, & \text{if } x \geq 2 \end{cases}$ **at** $x = 2$

(*a*) is continuous.

(*b*) has removable discontinuity.

(*c*) has jump discontinuity.

(*d*) has oscillating discontinuity.

32. **Let** y **be a function of** x **such that** $\log(x + y) - 2xy = 0$, **then** $y'(0)$ **is**

(*a*) 0 (*b*) 1 (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{2}$

33. **Let** $f(x)$ **be a function such that** $f'(a) \neq 0$. **Then at** $x = a$, $f(x)$

(*a*) cannot have a maximum

(*b*) cannot have a minimum

(*c*) must have neither a maximum nor a minimum

(*d*) None of these

34. ***Read the following statements.***

Statement I : $\int_0^1 \frac{e^x}{1 + e^{2x}} dx = \tan^{-1} e + \frac{\pi}{4}$

Statement II : $\int_a^b f(x)\,dx = \int_b^a f(t)\,dt$

Choose the correct option:

(*a*) Statement I is correct but statement II is not correct.

(*b*) Statement II is correct but statement I is not correct.

(*c*) Both statements I and II are correct.

(*d*) None of these

35. **If** $\int \frac{dx}{4 \sin^2 x + 4 \sin x \cos x + 5 \cos^2 x} = A \tan^{-1}(B \tan x + C)$, **then**

(*a*) $A = \frac{1}{4}; B = \frac{1}{2}, C = 1$ (*b*) $A = \frac{1}{2}, B = \frac{1}{4}, C = 1$ (*c*) $A = 1, B = \frac{1}{2}, C = \frac{1}{4}$ (*d*) $A = \frac{1}{4}, B = 1, C = \frac{1}{2}$

36. **If** $\int \frac{(\sqrt{x})^3}{(\sqrt{x})^5 + x^4} dx = A \log\left(\frac{x^k}{x^k + 1}\right) + C$, **then the value of** A **and** k **respectively are**

(*a*) $\frac{3}{2}$ and $\frac{2}{3}$ (*b*) $\frac{3}{2}, 2$ (*c*) $\frac{2}{3}$ and $\frac{3}{2}$ (*d*) does not exists

37. The area of the region bounded by $y = \sqrt{x}$ and $y = x$ is

(a) $\frac{1}{2}$ sq. unit (b) $\frac{1}{4}$ sq. unit (c) $\frac{1}{6}$ sq. unit (d) $\frac{1}{16}$ sq. unit

38. Let $y(x)$ be the solution of the differential equation $(x \log x)\frac{dy}{dx} + y = 2x \log x, x \geq 1$. Then the value of $y(e)$ is

(a) 3 (b) 2 (c) $2e$ (d) 0

39. The order and degree of differential equation $x = 1 + \frac{dy}{dx} + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots\ldots\ldots\ldots$ is

(a) 1, 2 (b) 2, 3 (c) 3, 2 (d) 1, 1

40. $(\vec{a}\cdot\hat{i})^2 + (\vec{a}\cdot\hat{j})^2 + (\vec{a}\cdot\hat{k})^2$ equals

(a) 1 (b) $\vec{a}$ (c) $-\vec{a}$ (d) $\vec{a}^2$

41. If $ABCDEF$ is regular hexagon and $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{BC} = \vec{b}$, $\overrightarrow{CD} = \vec{c}$, then $\overrightarrow{AE}$ is

(a) $\vec{a} + \vec{b} + \vec{c}$ (b) $\vec{b} + \vec{c}$ (c) $\vec{a} + \vec{b}$ (d) $\vec{a} + \vec{c}$

42. Unit vectors $\vec{\alpha}$ and $\vec{\beta}$ are inclined at an angle ϕ and $|\vec{\alpha} - \vec{\beta}| < 1$, if $0 \leq \phi \leq \pi$, then ϕ may belong to

(a) $\left[0, \frac{\pi}{3}\right]$ (b) $\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$ (c) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d) None of these

43. If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following is incorrect?

(a) $\vec{b} = \lambda\vec{a}$, for some scalar λ.

(b) $\vec{a} = \pm\vec{b}$

(c) The respective components of $\vec{a}$ and $\vec{b}$ are proportional.

(d) Both the vectors $\vec{a}$ and $\vec{b}$ will always have same direction, but different magnitudes.

44. If the projections of a line segment AB on x, y and z axis are respectively 3, 4 and 5, then the length of the line segment is

(a) $7\sqrt{2}$ (b) $\sqrt{2}$ (c) $3\sqrt{2}$ (d) $5\sqrt{2}$

45. The two lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ will be perpendicular, if and only if

(a) $aa' + bb' + cc' = 1$ (b) $aa' + cc' + 1 = 0$

(c) $(a + a')(b + b') + (c + c') = 0$ (d) $a' + b' + c' = 0$

46. Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a', b', c' respectively from the origin, such that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = k\left(\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}\right)$. Then the value of k is

(a) 4 (b) 1 (c) 5 (d) 7

47. If the projection of a line segment OA on the axes (where O is origin) are 6, 2, 3, then direction cosines are

(a) $\frac{1}{3}, \frac{2}{3}, \frac{1}{3}$ (b) $\frac{2}{7}, \frac{5}{7}, \frac{3}{7}$ (c) $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$ (d) $\frac{6}{7}, \frac{2}{7}, \frac{6}{7}$

48. If A and B are two events such that $P(A) = P\left(\frac{A}{B}\right) = \frac{1}{4}$ and $P\left(\frac{B}{A}\right) = \frac{1}{2}$, then $P\left(\frac{A'}{B'}\right)$ equals

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) none of these

49. If A and B are independent events. The probability that the both A and B occurs is $\frac{1}{12}$ and probability neither A nor B occurs is $\frac{1}{2}$, then

(*a*) $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$ (*b*) $P(A)=\frac{1}{2}, P(B)=\frac{1}{6}$ (*c*) $P(A)=\frac{1}{6}, P(B)=\frac{1}{2}$ (*d*) $P(A)=\frac{1}{4}, P(B)=\frac{1}{6}$

50. A mapping is selected at random from set $A = \{1, 2, ..., 10\}$ into itself. The probability that mapping selected is an injective, is

(*a*) $\frac{10}{10^9}$ (*b*) $\frac{9!}{10^9}$ (*c*) $\frac{9}{|\underline{10}}$ (*d*) none of these