Test Booklet Set No.
05

SUBJECT : MATHEMATICS

GUJARAT COMMON ENTRANCE TEST (GUJCET) 2019

Date: 26 April, 2019 | Duration: 1 Hours | Max. Marks: 40

:: IMPORTANT INSTRUCTIONS ::

1. The Mathematics test consists of 40 questions. Each question carries 1 mark. For each correct response, the candidate will get 1 mark. For each incorrect response ¼ mark will be deducted. The maximum marks are 40.
2. This test is of 1 hr. duration.
3. Use Black Ball Point Pen only for writing particulars on OMR Answer Sheet and marking answer by darkening the circle '•'.
4. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
5. **On completion of the test, the candidate must handover the Answer Sheet to the Invigilator in the Room/Hall. The candidates are allowed to take away this Test Booklet with them.**
6. The Set No. for this Booklet is **05**. Make sure that the Set No. printed on the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately.
7. The candidate should ensure that the Answer Sheet is not folded. Do not make any stray marks on the Answer Sheet.
8. Do not write you Seat No. anywhere else, except in the specified space in the Test Booklet/Answer Sheet.
9. Use of White fluid for correction is not permissible on the Answer Sheet.
10. Each candidate must show on demand his/her Admission Card to the Invigilator.
11. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her sent.
12. Use of Manual Calculator is permissible.
13. The candidate should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and must sign the Attendance Sheet (Patrak - 01). Cases where a candidate has not signed the Attendance Sheet (Patrak - 01) will be deemed not to have handed over the Answer Sheet and will be dealt with as an unfair means case.
14. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
15. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
16. The candidates will write the Correct Test Booklet Set No. As given in the Test Booklet/Answer Sheet in the Attendance Sheet. (Patrak - 01)

Candidate's Name : ………………………………………………………………………………………………………

Exam. Seat No. (in figures)…………………………………….(in words)……………………………………………

Name of Exam. Centre : ……………………………………….Exam. Centre No. : …………………………………

Test Booklet Set No. : ……………………………………………Test Booklet No. : ………………………………..

Candidate's Sign……………………………………………Block Supervisor Sign……………………………………………

MATHEMATICS

1. Matrix $A_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$; r = 1, 2, 3, ….. If $\sum_{r=1}^{100} |A_r| = \left(\sqrt{10}\right)^k$, then K = ……….. ; ($|A_r| = \det(A_r)$)

(A) 2 (B) 6 (C) 4 (D) 8

Ans. **(D)**

Sol. $|A_r| = 2r - 1$

$\sum_{r=1}^{100}(2r-1) = 1 + 3 + \ldots + 199 = 10000 = 10^{k/2} \Rightarrow \frac{k}{2} = 4 \Rightarrow k = 8$

2. $\frac{d}{dx}\left(3\cos\left(\frac{\pi}{6}+x^\circ\right) - 4\cos^3\left(\frac{\pi}{6}+x^\circ\right)\right)$ = ……………

(A) $\cos(3x^\circ)$ (B) $\frac{\pi}{60}\sin(3x^\circ)$ (C) $\frac{\pi}{60}\cos(3x^\circ)$ (D) $-\frac{\pi}{60}\sin(3x^\circ)$

Ans. **(C)**

Sol. $\frac{d}{dx}\left(-\cos 3\left(\frac{\pi}{6}+x^\circ\right)\right) = \frac{d}{dx}\left(-\cos\left(\frac{\pi}{2}+3x^\circ\right)\right) = \frac{d}{dx}\left(\sin 3x\frac{\pi}{180}\right) = \frac{3\pi}{180}\cos 3x^\circ$

3. If $f(x) = 1 + x + x^2 + \ldots + x^{1000}$, then $f'(-1)$ = …….

(A) –50 (B) –500 (C) –100 (D) 500500

Ans. **(B)**

Sol. $f'(x) = 1 + 2x + 3x^2 + \ldots + 1000x^{999}$

$f'(-1) = 1 - 2 + 3 - 4 + \ldots - 1000 \Rightarrow -500$

4. Applying mean value theorem on f(x) = logx; $x \in [1, e]$ the value of c = ………

(A) log(e – 1) (B) e – 1 (C) 1 – e (D) 2

Ans. **(B)**

Sol. Using LMVT $f'(c) = \frac{f(e)-f(1)}{e-1}$

$\frac{1}{c} = \frac{1-0}{e-1} \Rightarrow c = e - 1$

5. If $\int \sin^{13}x\cos^3 x\,dx = A\sin^{14}x + B\sin^{16}x + C$, then A + B = ……..

(A) $\frac{1}{110}$ (B) $\frac{17}{112}$ (C) $\frac{15}{112}$ (D) $\frac{1}{112}$

Ans. **(D)**

Sol. Let sinx = t $\Rightarrow$ cosx dx = dt = $\int t^{13}(1-t^2)dt \Rightarrow \frac{t^{14}}{14} - \frac{t^{16}}{16} + c = \frac{1}{14}\sin^{14}x - \frac{1}{16}\sin^{16}x + c$

$A + B = \frac{1}{14} - \frac{1}{16} = \frac{1}{112}$

6. If $\int \frac{1+\cos x}{\cos x - \cos^2 x}dx$ = log|secx + tanx| – 2f'(x) + C, then f(x) = ……..

(A) $2\cot\left(\frac{x}{2}\right)$ (B) $2\log\left|\sin\frac{x}{2}\right|$ (C) $-2\cot\left(\frac{x}{2}\right)$ (D) $-2\log\left|\sin\frac{x}{2}\right|$

Ans. **(B)**

Sol. $\int \frac{1-\cos x+2\cos x}{\cos x(1-\cos x)}dx = \int \frac{1}{\cos x}dx+\int \frac{2}{1-\cos x}dx = \int \sec x dx + \int \text{cosec}^2 \frac{x}{2}dx$

$= \ell n|\sec x + \tan x| - 2\cot\frac{x}{2} + c$

Now $-2f'(x) = -2\cot\frac{x}{2}$ $\Rightarrow$ $f(x) = 2\ell n\left|\sin\frac{x}{2}\right|$

7. The probability that an event A occurs in a single trial of an experiment is 0.6. In the first three independent trials of the experiment, the probability that A occurs atleast once is ……….

(A) 0.930 (B) 0.936 (C) 0.925 (D) 0.927

Ans. **(B)**

Sol. $1 - {}^3C_0\,(.6)^\circ(.4)^3 = 0.936$

8. If $6P(A) = 8P(B) = 14P(A \cap B) = 1$, then the $P\left(\frac{A'}{B}\right)$ = ………..

(A) $\frac{3}{7}$ (B) $\frac{4}{7}$ (C) $\frac{3}{5}$ (D) $\frac{2}{5}$

Ans. **(A)**

Sol. $P\left(\frac{\bar{A}}{B}\right) = \frac{P(\bar{A}\cap B)}{P(B)} = \frac{P(B)-P(\bar{A}\cap B)}{P(B)} = 1-\frac{\frac{1}{14}}{\frac{1}{8}} = \frac{3}{7}$

9. The mean and variance of a random variable X having a binomial distribution are 6 and 3 respectively. The probability of variable X less than 2 is

(A) $\frac{13}{2048}$ (B) $\frac{13}{4096}$ (C) $\frac{15}{4096}$ (D) $\frac{25}{2048}$

Ans. **(B)**

Sol. $np = 6,\ npq = 3$ $\Rightarrow$ $q = \frac{1}{2},\ p = \frac{1}{2},\ n = 12$

so required probability $= {}^{12}C_0\, p^0\, q^{12} + {}^{12}C_1\, p^1\, q^{11} = \frac{13}{2^{12}} = \frac{13}{4096}$

10. The coordinates of the corner points of the bounded feasible region are (10, 0), (2, 4), (1, 5) and (0, 8). the maximum of objective function z = 60x + 10y is ………..

(A) 700 (B) 800 (C) 600 (D) 110

Ans. **(C)**

Sol.

Corner	$t = 60x + 10y$
(10, 0)	$t = 600$
(2, 4)	$t = 160$
(1, 5)	$t = 110$
(0, 8)	$t = 80$

so max = 600

11. If the rate of change of area of rhombus with respect to it's side is equal to the side of rhombus, then the angles of rhombus are …….

(A) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ (B) $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ (C) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ (D) $\frac{5\pi}{12}$ and $\frac{7\pi}{12}$

Ans. **(C)**

Sol. $A = x^2\sin\theta \Rightarrow \frac{dA}{dx} = 2x\sin\theta = x \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

12. The approximate value of $5^{2.01}$ is …….., where, ($\log_e 5 = 1.6095$).

(A) 25.4125 (B) 25.2525 (C) 25.5025 (D) 25.4024

Ans. **(D)**

Sol. Let $f(x) = 5^x$, $x = 2$, $\Delta x = 0.01$

$f(x + \Delta x) = f(x) + \frac{dy}{dx}.\Delta x$

$f(2 + 0.01) = f(2) + \frac{d}{dx}(5^x)\Delta x$

$= 5^2 + 5^x \ell n5\Delta x = 25 + (25)(1.6095)(.01) = 25.402375 = 25.4024$

13. $f(x) = \frac{x}{\log_x e}$ is increasing on the interval ……..; where $x \in R^+ - \{1\}$.

(A) $(-e, \infty)$ (B) $\left(\frac{1}{e}, 1\right) \cup (1, \infty)$ (C) $(0, \infty) - \{1\}$ (D) $\left(\frac{1}{e}, \infty\right)$

Ans. **(B)**

Sol. $f(x) = x\ell nx$

$f'(x) = \frac{x}{x} + \ell nx \Rightarrow \ell nx + 1$

increasing $\Rightarrow f'(x) \geq 0 \Rightarrow \ell nx + 1 \geq 0 \Rightarrow x \geq e^{-1} \Rightarrow \left(\frac{1}{e}, 1\right) \cup (1, \infty)$

14 $\int (2 + \log x)(ex)^x dx =$ ……. + C ; x > 1

(A) $(ex)^x$ (B) x^x (C) $(ex)^{-x}$ (D) e^{x^x}

Ans. **(A)**

Sol. Let $(ex)^x = t$

$x\ell nex = \ell nt$

$x(1 + \ell nx) = \ell nt$

$\left(x.\frac{1}{x} + (1 + \ell nx)\right)dx = \frac{1}{t}dt$

$(ex)^x (2 + \ell nx)dx = \frac{1}{t}dt$

$\int dt = t + c \Rightarrow (ex)^x + c$

15. $\int e^{\sqrt{x}} dx =$ ……….. + C ; x > 0

(A) $2\left(\sqrt{x} - 1\right)e^{\sqrt{x}}$ (B) $\left(1 - \sqrt{x}\right)e^{\sqrt{x}}$ (C) $2\left(1 - \sqrt{x}\right)e^{\sqrt{x}}$ (D) $\left(\sqrt{x} - 1\right)e^{\sqrt{x}}$

Ans. **(A)**

Sol. $\int e^{\sqrt{x}} dx$

$x = t^2$

$dx = 2tdt$

$\int e^t.2t\,dt = 2t.e^t - \int 2.e^t\,dt = 2t\,e^t - 2e^t = 2.(\sqrt{x} - 1)\,e^{\sqrt{x}} + c$

16. If $\int \frac{\sin x}{\sin(x-\alpha)}dx$ = px – qlog|sin(x – α)| + C, then pq = ……….

(A) $-\frac{1}{2}\sin 2\alpha$ (B) sin2α (C) $\frac{1}{2}\sin 2\alpha$ (D) $-\sin 2\alpha$

Ans. **(A)**

Sol. $\int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)}dx = \int \frac{\sin(x-\alpha)\cos\alpha}{\sin(x-\alpha)} + \frac{\cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)}dx$ = cosα.x + sinα.ln|sin(x – α)| + c

$pq = -\sin\alpha\cos\alpha = -\frac{1}{2}\sin 2\alpha$

17. $\int_1^3 \left(\frac{x^2+1}{4x}\right)^{-1} dx$ = ……

(A) log5 (B) $\frac{1}{2}$ log5 (C) log25 (D) log100

Ans. **(C)**

Sol. $\int_1^3 \frac{4x}{1+x^2}dx = 2\left[\ell n(1+x^2)\right]_1^3 = 2[\ell n10 - \ell n2] = \ell n25$

18. If $\int_1^k (2x-3)\,dx = 12$, then K = ……

(A) –2 and 5 (B) 5 (C) 2 (D) –5

Ans. **(A)**

Sol. $\int_1^k (2x-3)\,dx = \left[x^2-3x\right]_1^k = (k^2-3k)-(1-3) = k^2-3k+2$

$x^2 - 3k + 2 = 12 \Rightarrow x^2 - 3x - 10 = 0 \Rightarrow k = -2, 5$

19. $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 2x}{1+25^x}dx$ = …….

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{2}$ (C) $\frac{\pi}{2}$ (D) $-\frac{\pi}{4}$

Ans. **(A)**

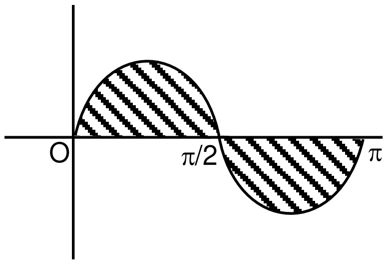
Sol. $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 2x}{1+25^x}dx \Rightarrow \int_0^{\pi/2} \frac{\cos^2 2x}{1+25^x} + \frac{\cos^2 2x}{1+25^{-x}}dx = \int_0^{\pi/2} \cos^2 2x\,dx$

$= \int_0^{\pi/2} \frac{1+\cos 4x}{2}dx = \frac{1}{2}\left[x+\frac{\sin 4x}{4}\right]_0^{\pi/2} = \frac{\pi}{4}$

20. The area bounded by curve y = sin2x (x = 0 to x = π) and X-axis is ………

(A) 4 (B) 2 (C) 1 (D) $\frac{3}{2}$

Ans. **(B)**

Sol. 

$A = 2\int_0^{\pi/2} \sin 2x dx = 2\left[\frac{-\cos 2x}{2}\right]_0^{\pi/2} = A = 2$

21. Area bounded by the ellipse $2x^2 + 3y^2 = 1$ is ……….

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{\sqrt{6}}$ (C) 6π (D) $\sqrt{6}\pi$

Ans. **(B)**

Sol. $A = \pi.a.b = \pi.\ \frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}} = \frac{\pi}{\sqrt{6}}$

22. The integrating factor (I.F.) of differential equation $\frac{dy}{dx}(1 + x) - xy = 1 - x$ is ………

(A) $(1 + x)e^x$ (B) $(x - 1)e^{-x}$ (C) $(1 + x)e^{-x}$ (D) $(1 - x)e^{-x}$

Ans. **(C)**

Sol. If = $e^{\int -\frac{x}{1+x}dx} = e^{-\int\left(1-\frac{1}{1+x}\right)dx} = e^{-(x-\ell n1+x)}$

$= e^{-x} . e^{\ell n(1+x)} = (1 + x)e^{-x}$

23. If the general solution of some differential equation is $y = a_1(a_2 + a_3).\cos(x + a_4) - a_5e^{x+a_6}$ then order of differential equation is ……… .

(A) 6 (B) 5 (C) 4 (D) 3

Ans. **(D)**

Sol. $y = \lambda_1\cos(x + a_4) - a^5.e^x.\ e^{a_6} = \lambda_1\cos(x + a_4) - \lambda_2e^x$

so order is 3.

24. If the length of the subnormal at any point of the curve is constant, then the eccentricity of this curve is ………. .

(A) $e = \sqrt{2}$ (B) $e > 1$ (C) $0 < e < 1$ (D) $e = 1$

Ans. **(D)**

Sol. $|y_1m| = a$

$y\frac{dy}{dx} = \pm a \quad \Rightarrow \quad \int ydy = \int \pm adx \quad \Rightarrow \quad \frac{y^2}{2} = \pm ax + c$ it is a parabola so e = 1.

25. If $|\vec{x}| = |\vec{y}| = |\vec{x} + \vec{y}| = 1$, then $|\vec{x} - \vec{y}| =$ ………… .

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 1 (D) 3

Ans. **(B)**

Sol. $|\vec{x}| = 1 \qquad |\vec{x} + \vec{y}| = 1 \quad \Rightarrow \quad |\vec{x}|^2 + |\vec{y}|^2 + 2\vec{x}.\vec{y} = 1$

$|\vec{y}| = 1 \qquad \Rightarrow \quad \vec{x}.\vec{y} = -\frac{1}{2}$

Now $|\vec{x} - \vec{y}|^2 = |\vec{x}|^2 + |\vec{y}|^2 - 2\vec{x}.\vec{y} = 1 + 1 - 2.\left(-\frac{1}{2}\right) = 3$

26. If x is a vector in the direction of (2, –2, 1) of magnitude 6 and y is a vector in the direction of (1, 1, –1) of magnitude $\sqrt{3}$, then $\left|\vec{x}+2\vec{y}\right|$ = ...

(A) 40 (B) $\sqrt{35}$ (C) $\sqrt{17}$ (D) $2\sqrt{10}$

Ans. **(D)**

Sol. $\vec{x} = \dfrac{6(2\hat{i}-2\hat{j}+\hat{k})}{3}$, $\vec{y} = \dfrac{\sqrt{3}(\hat{i}+\hat{j}-\hat{k})}{\sqrt{3}}$

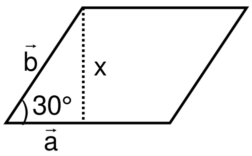
so $|\vec{x}+2\vec{y}| = |6\hat{i}-2\hat{j}| = \sqrt{40} = 2\sqrt{10}$

27. The angle between two adjacent sides $\vec{a}$ and $\vec{b}$ of parallelogram is $\dfrac{\pi}{6}$.If $\vec{a}$ = (2,-2,1) and $|\vec{b}| = 2|\vec{a}|$, then area of this parallelogram is ....

(A) 9 (B) 18 (C) $\dfrac{9}{2}$ (D) $\dfrac{3}{4}$

Ans. **(A)**

Sol.



$\vec{a} = 2\hat{i}-2\hat{j}+\hat{k}$, $|\vec{a}| = 3$

$\sin 30° = \dfrac{x}{|\vec{b}|} \quad \Rightarrow \quad x = 3$

area = 3 × 3 = 9

28. The perpendicular distance from the point of intersection of the line $\dfrac{x+1}{2} = \dfrac{y+2}{3} = \dfrac{z}{-1}$ and plane $2x - y + z = 0$ to the Z-axis is ....

(A) 1 (B) $\sqrt{5}$ (C) 2 (D) 5

Ans. **(B)**

Sol. $\dfrac{x+1}{2} = \dfrac{y+2}{3} = \dfrac{z}{-1} = \lambda$, $2x - y + z = 0$

$(2\lambda - 1, 3\lambda - 2, -\lambda)$ lies on plane

$2(2\lambda - 1) - (3\lambda - 2) + (-\lambda) = 0 \Rightarrow 0 = 0$

so point of intersection is (–1, –2, 0)

so dist from z axis = $\sqrt{(-1)^2+(-2)^2} = \sqrt{5}$

29. The measure of the angle between the line $\bar{r} = (2, -3, 1) + k(2, 2, 1);\ k \in R$ and the plane $2x - 2y + z + 7 = 0$ is ....

(A) $\cos^{-1}\dfrac{1}{9}$ (B) $\tan^{-1}\dfrac{1}{4\sqrt{5}}$ (C) $\sin^{-1}\dfrac{1}{3}$ (D) $\pi/2$

Ans. **(B)**

Sol. $\sin\theta = \left|\dfrac{4-4+1}{\sqrt{9}\sqrt{9}}\right| = \theta = \sin^{-1}\dfrac{1}{9} = \tan^{-1}\dfrac{1}{4\sqrt{5}}$

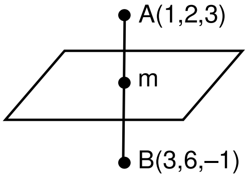
30. The image of the point A(1, 2, 3) relative to the plane is B(3, 6, –1), the equation of plane is -----------

(A) $x + 2y + 3z - 1 = 0$ (B) $x + 2y - 2z + 8 = 0$

(C) $x - 2y + 2z - 8 = 0$ (D) $x + 2y - 2z - 8 = 0$

Ans. **(D)**

Sol.



M(mid point) = (2, 4, 1)

DR(2, 4, –4)

equation of plane $2(x - 2) + 4(y - 4) - 4(z - 1) = 0$

$\Rightarrow$ $x - 2 + 2y - 8 - 2z + 2 = 0$

$x + 2y - 2z - 8 = 0$

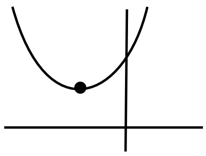
31. $f : R \to R$, $f(x) = x^2 + 3x + 4$ is ------------

(A) one-one and onto (B) one-one and not onto

(C) many-one and not onto (D) not one-one and onto

Ans. **(C)**

Sol.



many one and into (not onto)

32. If $a * b = \frac{ab}{10}$; $a, b \in Q^+$, then $(5 * 8)^{-1}$ = -----------

(A) 4 (B) $\frac{1}{25}$ (C) 10 (D) 25

Ans. **(D)**

Sol. $a * b = \frac{ab}{10}$, $5 * 8 = \frac{5.8}{10} = 4$

Now identity element of 4 is $4 * e = 4 = e * 4$ $\Rightarrow$ $\frac{4.e}{10} = 4 = \frac{e.4}{10} \Rightarrow e = 10$

Now, inverse element of 4 is $4 * b = e = b * 4 \Rightarrow \frac{4b}{10} = 10 = \frac{b4}{10} \Rightarrow b = 25$

33. If $f : N \to N$, $f(x) = x + 3$, then $f^{-1}(x)$ = ---------------

(A) $x + 3$ (B) does not exists (C) $x - 3$ (D) $3 - x$

Ans. **(B)**

Sol. Function is into so $f^{-1}(x)$ does not exist.

34. $\sin^2\left(\sin^{-1}\frac{1}{2}\right) + \tan^2(\sec^{-1} 2) + \cot^2(\operatorname{cosec}^{-1} 4)$ = --------------

(A) $\frac{73}{4}$ (B) $\frac{37}{2}$ (C) $\frac{89}{4}$ (D) 19

Ans. **(A)**

Sol. $\sin^2\left(\frac{\pi}{6}\right)$ + $\tan^2\left(\frac{\pi}{3}\right)$ + $\cot^2(\text{cosec}^{-1}4)$

$\frac{1}{4}+3+15=\frac{73}{4}$

(Let $\text{cosec}^{-1}4 = \theta$; $\text{cosec}\theta = 4$; so $\cot^2\theta = \text{cosec}^2\theta - 1 = 15$

35. $\tan\left(\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right)$ = -------------------

(A) $\frac{3}{17}$ (B) $\frac{17}{6}$ (C) $\frac{17}{4}$ (D) $\frac{6}{17}$

Ans. **(B)**

Sol. Let $\cos^{-1}\frac{4}{5}$ = A ⇒ $\tan^{-1}\frac{2}{3}$ = B ⇒ $\cos A = \frac{4}{5}$ ⇒ $\tan B = \frac{2}{3}$

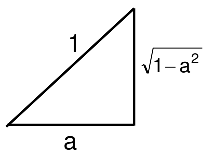
$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B} = \frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\frac{2}{3}} = \frac{17}{6}$

36. $\cos(\cot^{-1}(\text{cosec}(\cos^{-1}a)))$ = ------------- (where, $0 < a < 1$)

(A) $\frac{1}{\sqrt{2-a^2}}$ (B) $\sqrt{3-a^2}$ (C) $\sqrt{2-a^2}$ (D) $\frac{1}{\sqrt{2+a^2}}$

Ans. **(A)**

Sol.



Let $\cos^{-1}a = \theta$ ⇒ $\cos\theta = a$ ⇒ $\text{cosec}\theta = \frac{1}{\sqrt{1-a^2}}$

$\cos\left(\cot^{-1}\left(\frac{1}{\sqrt{1-a^2}}\right)\right)$

Let $\cot^{-1}\frac{1}{\sqrt{1-a^2}} = \phi$ ⇒ $\cot\phi = \frac{1}{\sqrt{1-a^2}}$ ⇒ $\cos\phi = \frac{1}{\sqrt{2-a^2}}$

37. $\begin{vmatrix}\sin^2\theta & \cos^2\theta \\ -\cos^2\theta & \sin^2\theta\end{vmatrix} =$

(A) $\cos 2\theta$ (B) $\frac{1}{2}(1+\cos^2 2\theta)$ (C) $\frac{1}{2}(1-\sin^2 2\theta)$ (D) $\frac{1}{2}\sin^2 2\theta$

Ans. **(B)**

Sol. $\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta = 1 - \frac{1}{2}\sin^2 2\theta = 1 - \frac{1}{2}(1-\cos^2 2\theta) = \frac{1}{2}[1+\cos^2 2\theta]$

38. $\begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix} = 2016K$, then K =

(A) 24 (B) 84 (C) $\frac{1}{24}$ (D) $\frac{1}{84}$

Ans. **(D)**

Sol. $3!2!\begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 12 \\ 1 & 4 & 20 \end{vmatrix} = 6.2.2\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 6 \\ 1 & 4 & 10 \end{vmatrix} \Rightarrow 24 = 2016K \Rightarrow K = \frac{1}{84}$

39. $\begin{vmatrix} 1+x & 1 & 1 \\ 1+y & 1+2y & 1 \\ 1+z & 1+z & 1+3z \end{vmatrix} = 10K\,xyz\left(3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$, then k = __________

$\left(\text{where } xyz \neq 0; 3+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \neq 0\right)$.

(A) $\frac{1}{5}$ (B) 2 (C) 5 (D) 1

Ans. **(A)**

Sol. Put x = –1, y = –1 & z = 1

$\begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1 \\ 2 & 2 & 4 \end{vmatrix} = 10K(-1)(-1)(1)(3-1-1+1) \Rightarrow K = \frac{1}{5}$

40. If the inverse of the matrix A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is $\frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & \alpha \\ 2 & 2 & -3 \end{bmatrix}$ then, α = ____________

(A) 3 (B) 4 (C) 2 (D) –2

Ans. **(C)**

Sol. $AA^{-1} = I \Rightarrow \frac{1}{5}\begin{bmatrix} 1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & \alpha \\ 2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Compare $a_{13} \Rightarrow \frac{1}{5}[2 + 2\alpha - 6] = 0 \Rightarrow \alpha = 2$