# MATHEMATICAL INDUCTION

- Principle of finite Mathematical Induction : $\{P(n) / n \varepsilon N\}$ is a set of statements.
  If (i) p (1) is true (ii) p (m) is true $\Rightarrow$ p (m+1) is true ; then p (n) is true for every n $\varepsilon$ N.
- Principle of complete induction: $\{P(n) / n \varepsilon N\}$ is a set of statements. If p (1) is true and p(1), p(2), p(3) .... p(m-1) are true => p(m) is true, then p (n) is true for every $n \in N$
- $\sum n = \frac{n(n+1)}{2}$;

  $\sum n^2 = \frac{n(n+1)(2n+1)}{6}; \sum n^3 = \frac{n^2(n+1)^2}{4}$
- $\sum n^2 \neq \left(\sum n\right)^2$ ... etc.,
- $a, (a+d), (a+2d), .............$ form an A.P.

  Here $t_n = a+(n-1)d$

  $S_n = \frac{n}{2}[2a+(n-1)d]$

  $= \frac{n}{2}[a+l]$, a = first term, $l$ = last term
- $a, ar, ar^2, ........$ form a G.P

  $t_n = a.r^{n-1}$ where a = first term

  r = common ratio

  $s_n = a\frac{(r^n-1)}{r-1}$; if $r > 1$

  $= a\left(\frac{1-r^n}{1-r}\right)$; if $r < 1$
- In Infinite G.P, Sum of Infinite terms is

  $S_\infty = \frac{a}{1-r}$
- In finding $n^{th}$ term of the series, In the suitable option, n = 1 gives the I term of the series
  n = 2 gives the II term of the series..etc
- In finding the Sum of 'n' terms of the series, In the suitable option,
  n = 1 gives the I term of the series
  n = 2 gives the sum of first two terms of the series.
  n = 3 gives the sum of the first three terms of the series.
- To test the divisibility of the given expression put n =1 in the given expression and observe the value and verify the options. If only one option divides that value then it is correct. If two or more options satisfy, then put n = 2 in the given expression and take the G.C.D of values obtained by substituting n=1, n=2.
- If the series is given upto 'n' terms, then begin the verification with n =1
  If the series is given upto (n-1) terms, then begin the verification with n =2
  Up to (n-2) terms $\Rightarrow$ take n = 3....etc.
- $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + ......'n'\, terms$

  $= -\frac{n(n+1)}{2}$; if 'n' is even.

  $= \frac{n(n+1)}{2}$; if 'n' is odd.
- Sum of the first 'n' odd +ve integers = $n^2$
  Sum of the first 'n' even +ve integers = n(n+1)
- The sum of cubes of three consecutive natural numbers is always divisible by 9
- $x^n - y^n$ is divisible by $x+y$ when 'n' is even.

## MULTIPLE CHOICE
## LEVEL - I

1. $\forall$ n $\varepsilon$ N , $\frac{n^4}{24}+\frac{n^3}{4}+\frac{11n^2}{24}+\frac{n}{4}$ is a
   1. Rational number 2. Integer
   3. Natural Number 4. Real Number
2. n>1, n even => digit in the units place of $2^{2n}+1$
   1. 5 2. 7 3. 6 4. 1
3. $1^4+2^4+3^4+....+n^4 =$

   1. $\frac{n(n+1)(2n+1)}{30}$

   2. $\frac{n(n+1)(2n+1)(3n^2+3n-1)}{6}$

   3. $\frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

   4. $\frac{n(n+1)(2n+1)^2}{30}$

4. $\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+....+\frac{1}{(4n-1)(4n+3)}=$

1. $\frac{n}{7(4n-1)}$ 2. $\frac{n}{3(4n+3)}$

3. $\frac{1}{3(4n-1)}$ 4. $\frac{1}{3(4n-3)}$

5. $\frac{1}{4.5}+\frac{1}{5.6}+\frac{1}{6.7}+...+\frac{1}{(n+3)(n+4)}=$

1. $\frac{n}{n+4}$ 2. $\frac{n}{4(n+4)}$

3. $\frac{n(n+1)}{4(n+4)}$ 4. $\frac{n}{4(n+3)}$

6. $\frac{1}{1.5}+\frac{1}{5.9}+....+\frac{1}{(4n-3)(4n+1)}=$

1. $\frac{n}{4n-3}$ 2. $\frac{n}{4n+3}$ 3. $\frac{n}{4n+1}$ 4. $\frac{n}{4n+5}$

7. $\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}+.....n\ \ terms=$

1. $\frac{n}{6n+4}$ 2. $\frac{n}{3n+2}$ 3. $\frac{n}{4n+6}$ 4. $\frac{1}{2(2n+3)}$

8. $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+$......(n-3) terms

1. $\frac{n}{n+2}$ 2. $\frac{n+1}{n(n+5)}$

3. $\frac{n-3}{2n-5}$ 4. $\frac{n-1}{n(2n-3)}$

9. In the series 3 +7 + 13+21+....... , nth term =
1. 4n-1 2. $n^2+2n$ 3. $n^2+n+1$ 4. $n^2+2$

10. nth term of the series 4 + 14+30+52+ ........=
1. 5n - 1 2. $2n^2+2n$ 3. $3n^2+n$ 4. $2n^2+2$

11. $4^3+8^3+12^3+$ ........n terms =
1. $\{4n(n+1)\}^2$ 2. $\{8n(n+1)\}^2$
3. $\{2n(n+1)\}^2$ 4. $\{16n(n+1)\}^2$

12. 1+3+6+10+ ........+ $\frac{(n-1)n}{2}+\frac{n(n+1)}{2}=$

1. $\frac{n(n+1)(n+2)}{3}$ 2. $\frac{(n+1)(n+2)}{6}$

3. $\frac{n(n+1)(n+2)}{6}$ 4. $\frac{(n+2)(n+1)^2}{3}$

13. 1+4+10+19+.........+ $\frac{3n^2-3n+2}{2}=$

1. $\frac{n^2(n^2+1)}{2}$ 2. $\frac{n(n^2+1)}{2}$

3. $\frac{n^2(n+1)}{2}$ 4. $\left\{\frac{n(n+1)}{2}\right\}^2$

14. 2+7+14+......+ $(n^2+2n-1)=$

1. $\frac{n(2n^2+9n+1)}{6}$ 2. $\frac{2n^2+9n+1}{6}$

3. $\frac{2n^2+9n+1}{12}$ 4. $\frac{2n^2+9n+1}{24}$

15. 1.2+2.3+.........+ n (n +1) =

1. $\frac{(n-1)n}{3}$ 2. $\frac{n(n+1)(n+2)}{3}$

3. $\frac{(n-1)n(n+1)}{3}$

4. $\frac{(n+1)(n+2)(n+3)}{3}$

16. 1.4+2.7+3.10+.......+ n(3n+1) =
1. $n(n+1)^2$ 2. $2n(n+1)^2$
3. $n^2(n+1)$ 4. (n+1) (n+2)

17. 3.6+6.9+9.12+.......+ 3n (3n+3) =

1. $\frac{n(n+1)(n+2)}{3}$ 2. 3n (n+1) (n+2)

3. $\frac{(n+1)(n+2)(n+3)}{3}$ 4. $\frac{(n+1)(n+2)(n+4)}{4}$

18. 1.3+3.5+5.7+........+ (2n-1) (2n +1) =

1. $\frac{n(4n^2+6n-1)}{3}$ 2. $\frac{n(3n^2+5n+1)}{3}$

3. $\frac{n(5n^2+7n-1)}{3}$ 4. $\frac{n(7n^2-5n+1)}{3}$

19. 1.3+2.4+3.5+.......+n(n+2) =

1. $\frac{n(n+1)(2n+7)}{6}$ 2. $\frac{2(n+2)(2n+3)}{6}$

3. $\frac{n(n+1)(2n+5)}{6}$ 4. $\frac{n(n+1)(2n+1)}{6}$

20. 1.4+2.5+........+ n(n+3) =

1. $\frac{n(n+3)(n+5)}{9}$ 2. $\frac{n(n+1)(n+5)}{3}$

3. $\frac{n(n+5)(n+7)}{6}$ 4. $\frac{n(n+3)(n+9)}{12}$

21. $1.6+2.9+3.12+.......+n(3n+3)=$
1. $n(n+1)(n+2)$ 2. $(n+1)(n+2)(n+3)$
3. $(n+2)(n+3)(n+4)$ 4. $(n-1)n(n+1)$

22. $2.4+4.7+6.10+.......(n-1)$ terms =
1. $2n^3-2n^2$ 2. $\frac{n^3+3n^2+1}{6}$
3. $2n^3+2n$ 4. $2n^3-n^2$

23. $2.3+3.4+4.5+............n$ terms=
1. $\frac{n(n^2+6n+11)}{6}$ 2. $\frac{n(n^2+6n+14)}{9}$
3. $\frac{n(n^2+6n+11)}{3}$ 4. $\frac{n(n^2+6n+17)}{12}$

24. $3.6+4.7+5.8+......+(n-2)$ terms =
1. $n^3+n^2+n+2$
2. $\frac{2n^3+12n^2+10n-84}{6}$
3. $2n^3+12n^2+10n+84$
4. $2n^3-12n^2+10n-84$

25. $1.2.3+2.3.4+.......+n(n+1)(n+2)=$
1. $\frac{n(n+1)(n+2)}{4}$
2. $\frac{n(n+1)(n+2)(n+3)}{4}$
3. $\frac{n(n+1)(n+2)(n+3)}{3}$
4. $\frac{n(n+1)(n+2)(n+3)}{6}$

26. $2.3.1+3.4.4+4.5.7+........n$ terms=
1. $\frac{n(9n^3+46n^2+51n-34)}{12}$
2. $\frac{n(8n^3+46n^2+52n-34)}{12}$
3. $\frac{n(9n^3+64n^2+15n-34)}{12}$
4. $\frac{n(8n^3+46n^2+52n-24)}{12}$

27. $2+3+5+6+8+9+........2n$ terms=
1. $3n^2+2n$ 2. $4n^2+2n$
3. $4n^2$ 4. $5n^2+2n$

28. $1^3+1^2+1+2^3+2^2+2+3^3+3^2+3+....3n$ terms=
1. $\frac{n(n+1)(n^2+12n+5)}{12}$
2. $\frac{n(n+1)(3n^2+7n+8)}{12}$
3. $\frac{n(n+1)(n+2)(n^2+5n+6)}{12}$
4. $\frac{(n+1)(n+2)(n+3)}{4}$

29. $1^2+1+2^2+2+3^2+3+.....+n^2+n=$
1. $\frac{(n+1)(n+2)}{3}$ 2. $\frac{n(n+1)(n+2)}{3}$
3. $\frac{n(n+2)(n+3)}{6}$ 4. $\frac{(n+1)(n+2)(n+3)}{4}$

30. $2.1^2+3.2^2+4.3^2+......n$ terms=
1. $\frac{n(n+1)(n+2)(3n+1)}{12}$
2. $\frac{n(n+2)(n+4)(n+11)}{6}$
3. $\frac{n(n+1)(n+3)(3n-1)}{3}$
4. $\frac{n(n+3)(n+5)(n+7)}{3}$

31. $1.2^2+2.3^2+3.4^2+.......n$ terms=
1. $\frac{n(n+1)(n+2)(3n+5)}{2}$
2. $\frac{n(n+1)(n+2)(3n+5)}{6}$
3. $\frac{n(n+1)(n+2)(3n+5)}{12}$
4. $\frac{n(n+1)(n+2)(3n+7)}{12}$

32. $\frac{1^2}{1}+\frac{1^2+2^2}{1+2}+\frac{1^2+2^2+3^2}{1+2+3}+.....+n$ terms $=$
1. $\frac{n(n+3)}{4}$ 2. $\frac{n(n+3)}{5}$
3. $\frac{n(n+2)}{3}$ 4. $\frac{n(n+5)}{6}$

33. $\frac{1^3}{1}+\frac{1^3+2^3}{2}+\frac{1^3+2^3+3^3}{3}+ \ldots\ldots\ldots +$
$\frac{1^3+2^3+....+n^3}{n}=$

1. $\frac{n(n+1)(n+2)(3n+5)}{48}$
2. $\frac{n(n+2)(n+3)(4n+5)}{48}$
3. $\frac{n(n+3)(n+5)(3n+7)}{48}$
4. $\frac{n(n+2)(n+3)(3n+5)}{48}$

34. $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+....n$ terms=

1. $\frac{n(2n^2+9n+13)}{24}$ 2. $\frac{n(2n^3+9n+13)}{8}$
3. $\frac{n(n^2+9n+13)}{24}$ 4. $\frac{n(n^2+9n+13)}{8}$

35. $\frac{1^2}{1.3}+\frac{2^2}{3.5}+...+\frac{n^2}{(2n-1)(2n+1)}=$

1. $\frac{n(n+1)}{2n+1}$ 2. $\frac{n(n+1)}{2(2n+1)}$
3. $\frac{n}{2(2n+1)}$ 4. $\frac{n+1}{2n+1}$

36. $\sum_{n=0}^{\infty}(-1)^n x^{n+1}=$

1. $\frac{x^n}{2(1+x)}$ 2. $\frac{x}{1+x}$ 3. $\frac{x}{x-1}$ 4. $\frac{x^n}{x-1}$

37. Sum of nth bracket of
(1) + (2+3+4)+(5+6+7+8+9) +........ is

1. $(n-1)^3+n^3$ 2. $(n-1)^3+8n^2$
3. $\frac{(n+1)(n+2)}{6}$ 4. $\frac{(n+3)(n+2)}{12}$

38. $1^2+(1^2+2^2)+(1^2+2^2+3^2)+....+$ n brackets=

1. $\frac{n(n+1)^2(n+2)^2}{12}$ 2. $\frac{n(n+1)^2(n+2)}{12}$
3. $\frac{n^2(n+1)(n+2)}{12}$ 4. $\frac{(n+1)}{2}$

39. Sum of nth bracket of
(1) + (1+3) + (1+3+5) + .... is =

1. $\frac{n(n+1)}{2}$ 2. $\frac{n(n+1)(2n+1)}{6}$
3. $\frac{n^2(n+1)^2}{4}$ 4. $n^2+n$

40. $(2^2)+(2^2+4^2)+(2^2+4^2+6^2)+$ ..... n brackets =

1. $\frac{n(n+2)^2(n+1)}{3}$ 2. $\frac{n(n+1)^2(n+2)}{3}$
3. $\frac{\{n(n+1)(n+2)\}^2}{3}$ 4. $\frac{n(n+1)(n+2)}{3}$

41. $\frac{1.2^2+2.3^2+3.4^2+....+n(n+1)^2}{1^2.2+2^2.3+3^2.4+....+n^2(n+1)}=$

1. $\frac{3n+1}{3n+5}$ 2. $\frac{3n+5}{3n+1}$
3. $(3n+1)(3n+5)$ 4. $\frac{3n+5}{3n+7}$

42. $\forall\ n\in N, x\in R,$
$\tan^{-1}\left[\frac{x}{1.2+x^2}\right]+\tan^{-1}\left[\frac{x}{2.3+x^2}\right]+..........+$
$\tan^{-1}\left[\frac{x}{n(n+1)+x^2}\right]=$

1. $\tan^{-1}\left[\frac{x}{n}\right]-\tan^{-1}\left[\frac{x}{n+1}\right]$
2. $\tan^{-1}[x]-\tan^{-1}\left[\frac{x}{n+1}\right]$
3. $\tan^{-1}[n+1]-\tan^{-1}[x]$ 4. $\tan^{-1}[x]$

43. $\forall n\varepsilon\ N,$
1. $|\sin(nx)|<|\sin x|$
2. $|\sin(nx)|<n|\sin x|$
3. $|\sin(nx)|\le n|\sin x|$
4. $\sin(nx)\le \sin n$

44. $A=\begin{bmatrix}3 & -4\\ 1 & -1\end{bmatrix}\Rightarrow A^n=$

1. $\begin{bmatrix}3n & -4n\\ n & -n\end{bmatrix}$ 2. $\begin{bmatrix}2+n & n-5\\ n & -n\end{bmatrix}$
3. $\begin{bmatrix}3^n & (-4)^n\\ 1^n & (-1)^n\end{bmatrix}$ 4. $\begin{bmatrix}2n+1 & -4n\\ n & 1-2n\end{bmatrix}$

45. The number of parts into which the plane is divided by n distinct st. lines drawn in the plane through a point is
1. n 2. 2n 3. 3n 4. 4

46. $4^3+5^3+6^3+.....+10^3=$
1. 1905 2. 2358 3. 2447 4. 2989

47. Every even power of every odd number greater than 1 when divided by 8 leaves the remainder
1. 7 2. 3 3. 5 4. 1

48. Sum of the cubes of three successive natural number is divisible by
1. 9 2. 27 3. 54 4. 99

49. The product of three consecutive natural numbers is divisible by
1. 2 2. 3 3. 4 4. 6

50. $n(n^2-1)$ is divisible by 24 $\Rightarrow$ n =
1. odd integer 2. even integer
3. integer 4. rational number

51. $\forall n \varepsilon N. n^2(n^4-1)$ is divisible by
1. 60 2. 120 3. 45 4. 90

52. $\forall n \varepsilon N, n^5 - n$ is divisible by
1. 30 2. 240 3. 60 4. 480

53. $\forall n \varepsilon N, 3^{2n}+7$ is divisible by
1. 8 2. 16 3. 24 4. 64

54. $\forall n\varepsilon\ N,\ 7^{2n}-48n-1$ is divisible by
1. 2304 2. 576 3. 24 4. 18

55. $\forall n \varepsilon N, 7^{2n}+3^{n-1}.2^{3n-3}$ is divisible by
1. 50 2. 25 3. 2425 4. 2550

56. $\forall n \varepsilon N, 10^n + 3.4^{n+2}+5$ is divisible by
1. 54 2. 207 3. 9 4. 208

57. $\forall n \varepsilon N, 2.4^{2n+1}+3^{3n+1}$ is divisible by
1. 19 2. 11 3. 209 4. 22

58. $\forall n \varepsilon N, 49^n + 16n - 1$ is divisible by
1. 64 2. 49 3. 132 4. 32

59. $\forall n\varepsilon N,\ 7.5^{2n}+12.6^n$ is divisible by
1. 13 2. 19 3. 247 4. 26

60. $\forall n\varepsilon N,\ 7^{2n}-4^{2n}$ is divisible by
1. 11 2. 3 3. 33 4. 7

61. $\forall n\varepsilon N, 2^{3n}-1$ is divisible by
1. 7 2. 9 3. 8 4. 3

62. $\forall n \varepsilon N, 5^{2n+2}-24n-25$ is divisible by
1. 576 2. 25 3. 24 4. 50

63. $\forall n \varepsilon N, 11^{n+2}+12^{2n+1}$ is divisible by
1. 121 2. 132 3. 133 4. 123

64. $\forall n \varepsilon N, x^{2n+1}+y^{2n+1}$ is divisible by
1. x - y 2. x+y 3. xy 4. $x^2+y^2$

65. $\frac{(n+2)!}{(n-1)!}$ is divisible by
1. 2 2. 3 3. 4 4. 6

# KEY

| | | | | |
|---|---|---|---|---|
| 1) 3 | 2) 2 | 3) 3 | 4) 2 | 5) 2 |
| 6) 3 | 7) 1 | 8) 3 | 9) 3 | 10) 3 |
| 11) 1 | 12) 3 | 13) 2 | 14) 1 | 15) 2 |
| 16) 1 | 17) 2 | 18) 1 | 19) 1 | 20) 2 |
| 21) 1 | 22) 1 | 23) 3 | 24) 2 | 25) 2 |
| 26) 1 | 27) 1 | 28) 2 | 29) 2 | 30) 1 |
| 31) 3 | 32) 3 | 33) 1 | 34) 1 | 35) 2 |
| 36) 2 | 37) 1 | 38) 2 | 39) 2 | 40) 2 |
| 41) 2 | 42) 2 | 43) 3 | 44) 4 | 45) 2 |
| 46) 4 | 47) 4 | 48) 1 | 49) 4 | 50) 1 |
| 51) 1 | 52) 1 | 53) 1 | 54) 1 | 55) 2 |
| 56) 3 | 57) 2 | 58) 1 | 59) 2 | 60) 3 |
| 61) 1 | 62) 1 | 63) 3 | 64) 2 | 65) 4 |

# HINTS

1. Put n = 1, n = 2 and verify the options.

2. Put n = 2, $2^4+1=17$.

5. Put n = 2, $S_2 = \frac{1}{4.5}+\frac{1}{5.6}=\frac{1}{12}$

option (2) $=\frac{2}{4(2+4)}=\frac{1}{12}$

8. Put n – 3 = 1, $S=\frac{1}{1.3}=\frac{1}{3}$
Substitute n = 4 and verify the options.

10. Given $T_2 = 14$
Put n = 2 and verify the options.

32. Put n = 2, $S_2 = \frac{1^2}{1}+\frac{1^2+2^2}{1+2}=8/3$
verify the options.

44. $A^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$ ; Put n = 2 and verify the options.

53. Put n = 1, $3^2 + 7 = 16$
Put n = 2, $3^4 + 7 = 88$
G.C.D. of 16 & 88 = 8

59. Put n = 1, $7.5^2 + 12.6 = 247$
Put n = 2, $7.5^4 + 12.6^2 = 4807$
G.C.D. of 247 & 4807 = 19

# LEVEL - II

66. If P is a natural number, then for every $n \in N$, $P^{n+1}+(P+1)^{2n-1}$ is divisible by
1. P 2. P + 1
3. $(P+1)^2$ 4. $P^2+P+1$

67. If $10^n + 3.4^n + x$ is divisible by 9 for all $n \in N$, then least positive value of x is

1. 1 2. 5 3. 14 4. 23

68. The value of the sum in the 50$^{th}$ bracket of $(1)+(2+3)+(4+5+6)+(7+8+9+10)+....$ is

1. 62525 2. 65225 3. 56255 4. 55625

69. $\left(\sum n^3\right)\left(\sum n\right)=\left(\sum n^2\right)^2$ iff

1. n = 3 2. n = 1 3. $n^2 = 3$ 4. n = -1

70. $n! < \left(\dfrac{n+1}{2}\right)^n$ is a true for

1. n=0 2. n < 1 3. n=1 4. n>1

71. If n is a positive integer, then $n.1+(n-1).2+(n-2).3+...+1.n=$

1. $\dfrac{n(n+1)}{2}$ 2. $\dfrac{n(n+1)(n+2)}{6}$

3. $\dfrac{(n+1)(n+2)}{2}$ 4. $\dfrac{n(n+1)(2n+1)}{6}$

72. If the sum to n terms of an A.P. is $\dfrac{4n^2-3n}{4}$, then the nth term of the A.P. is

1. $\dfrac{5n-1}{4}$ 2. $\dfrac{8n-7}{4}$

3. $\dfrac{3n^2-2}{4}$ 4. $\dfrac{7n-8}{4}$

73. $\forall n \in N$,

$Cos\theta\, Cos2\theta\, Cos4\theta\, Cos6\theta....Cos2^{n-1}\theta =$

1. $\dfrac{Sin2^{n-1}\theta}{2^{n-1}Sin\theta}$ 2. $\dfrac{Cos2^{n-1}\theta}{2^{n-1}Cos\theta}$

3. $\dfrac{Sin2^n\theta}{2^n Sin\theta}$ 4. $\dfrac{Cos2^n\theta}{2^n Cos\theta}$

74. If $a_k = \dfrac{1}{k(k+1)}$ for k = 1,2,3,...................n, then $\left(\sum_{k=1}^{n} a_k\right)^2 =$

1. $\dfrac{n}{n+1}$ 2. $\dfrac{n^2}{(n+1)^2}$

3. $\dfrac{n^4}{(n+1)^4}$ 4. $\dfrac{n^6}{(n+1)^6}$

75. $\dfrac{\frac{1}{2}.\frac{2}{2}}{1^3}+\dfrac{\frac{2}{2}.\frac{3}{2}}{1^3+2^3}+\dfrac{\frac{3}{2}.\frac{4}{2}}{1^3+2^3+3^3}+....n$ terms =

1. $\dfrac{n^2}{(n+1)^2}$ 2. $\dfrac{n^3}{(n+1)^3}$ 3. $\dfrac{n}{n+1}$ 4. $\dfrac{1}{n+1}$

76. If $2^3+4^3+6^3+.....+(2n)^3 = Kn^2(n+1)^2$ then k =

1. 1/2 2. 1 3. 3/2 4. 2

77. $1+\dfrac{x}{a_1}+\dfrac{x(x+a_1)}{a_1a_2}+........+\dfrac{x(x+a_1)(x+a_2)........(x+a_{n-1})}{a_1a_2......a_n}=$

1. $\dfrac{(x+a_1)(x+a_2)........(x+a_n)}{a_1a_2.......a_n}$

2. $\dfrac{(x-a_1)(x-a_2)........(x-a_n)}{a_1a_2.......a_n}$

3. $(x+a_1)(x+a_2)........(x+a_n)$

4. $(x-a_1)(x-a_2)........(x-a_n)$

78. Sum to n terms of the series $1+(1+x)+(1+x+x^2)+(1+x+x^2+x^3)+....$ is

1. $\dfrac{n}{1-x}-\dfrac{x(1-x^n)}{(1-x)^2}$ 2. $\dfrac{n}{1-x}+\dfrac{x(1-x^n)}{(1-x)^2}$

3. $\dfrac{n}{1-x}+\dfrac{x(1+x^n)}{(1-x)^2}$ 4. $\dfrac{n}{1-x}+\dfrac{x(1-x^n)}{(1-x)^2}$

79. For all $n \in N$, $1+\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}+.....+\dfrac{1}{\sqrt{n}}=$

1. $\sqrt{n}$ 2. $<\sqrt{n}$ 3. $\le\sqrt{n}$ 4. $\ge\sqrt{n}$

80. $\sum_{k=1}^{n} k\left(1+\dfrac{1}{n}\right)^{k-1} =$

1. $n(n-1)$ 2. $n(n+1)$ 3. $n^2$ 4. $(n+1)^2$

81. $7+77+777+......+(777......7$ n times$)=$

1. $\dfrac{7}{81}\left(10^{n+1}-9n-10\right)$

2. $\dfrac{7}{81}\left(10^{n}-9n-10\right)$

3. $\dfrac{7}{81}\left(10^{n+1}+9n+10\right)$

4. $\dfrac{7}{81}\left(10^{n+1}+9n-10\right)$

82. $3+33+333+....+(333......3$ n times$)=$

1. $\dfrac{1}{7}\left(10^{n+1}-9n-10\right)$

2. $\dfrac{1}{27}\left(10^{n+1}-9n-10\right)$

3. $\dfrac{1}{17}\left(10^{n+1}-9n-10\right)$

4. $\dfrac{1}{27}\left(10^{n+1}+9n-10\right)$

83. Mathematical Induction is the principle containing the set
1.R 2. N 3. Q 4. Z

84. $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then $A^n=...$

1. $\begin{bmatrix}\cosh n\theta & \sin hn\theta\\ -\sinh n\theta & \cosh n\theta\end{bmatrix}$
2. $\begin{bmatrix}\cos n\theta & \cos\theta\\ -\sin\theta & \sin\theta\end{bmatrix}$
3. $\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$
4. $\begin{bmatrix}\cos(n+1)\theta & \sin(n+1)\theta\\ -\sin(n+1)\theta & \sin(n+1)\theta\end{bmatrix}$

85. Let $P(n)$ be a statement and let
$P(n)=P(n+1)\ \forall n\in N$, then $P(n)$ is true.
1. for all 'n'
2. for all $n>1$
3. for all $n>m$, m being a fixed +ve integer
4. Nothing can be said

86. $\cos\theta+\cos 2\theta+\cos 3\theta+....+\cos\{(n-1)\theta\}$
$+\cos n\theta=$

1. $\dfrac{\cos\left\{\frac{1}{2}(n+1)\theta\right\}.\sin\left(\frac{n\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$
2. $\dfrac{\cos(n+1)\theta}{\sin\left(\frac{\theta}{2}\right)}$
3. $\dfrac{\cos\left(\frac{(n-1)\theta}{2}\right)\sin\left(\frac{n\theta}{2}\right)}{sin\frac{\theta}{2}}$
4. $\dfrac{\sin\left(\frac{n\theta}{2}\right).\cos\{(n+1)\theta\}}{\sin\left(\frac{\theta}{2}\right)}$

87. $Tan^{-1}\left(\frac{1}{1+1+1^2}\right)+Tan^{-1}\left(\frac{1}{1+2+2^2}\right)$
$+....+Tan^{-1}\left(\frac{1}{1+n+n^2}\right)=$

1. $Tan^{-1}(n+1)+\pi$ 2. $Tan^{-1}(n+1)+\frac{\pi}{4}$
3. $Tan^{-1}(n+1)$ 4. $Tan^{-1}(n+1)-\frac{\pi}{4}$

88. $\left(1-\frac{4}{1^2}\right)\left(1-\frac{4}{3^2}\right)\left(1-\frac{4}{5^2}\right)....\left[1-\frac{4}{(2n-1)^2}\right]=$

1. $\frac{1+n}{1-n}$ 2. $\frac{1+2n}{1-2n}$
3. $\frac{1-2n}{1+2n}$ 4. $\frac{1-2n}{1+3n}$

89. $\frac{1}{2}\tan\left(\frac{x}{2}\right)+\frac{1}{4}\left(\tan\frac{x}{4}\right)+...+\frac{1}{2^n}\tan\left(\frac{x}{2^n}\right)=$

1. $\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)$ 2. $\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)+\cot x$
3. $\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right)-\cot x$ 4. $\cot\left(\frac{x}{2^n}\right)-\cot x$

90. Suppose $x,y,u,v$ are four real numbers such that $x+y=u+v$; $x^2+y^2=u^2+v^2$ then $x^n+y^n=$

1. $u^{n-1}+v^{n-1}$ 2. $(u+v)^n$
3. $u^n+v^n$ 4. $u^n-v^n$

91. If $t_n=\frac{1}{4}(n+2)(n+3)$ for $n=1,2,3...$ then
$\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+.....+\frac{1}{t_{2005}}=$

1. $\frac{4004}{3005}$ 2. $\frac{3005}{4004}$
3. $\frac{8020}{6024}$ 4. $\frac{8008}{6015}$

92. If $a>0$; $x>0$ then
$\frac{1}{\sqrt{a}+\sqrt{a+x}}+\frac{1}{\sqrt{a+x}+\sqrt{a+2x}}+...$
$........+\frac{1}{\sqrt{a+(n-1)x}+\sqrt{a+nx}}=$

1. $\frac{\sqrt{a+nx}+\sqrt{a}}{x}$ 2. $\frac{\sqrt{a+nx}-\sqrt{a}}{x}$
3. $\frac{x}{\sqrt{a+nx}+\sqrt{a}}$ 4. $\frac{x}{\sqrt{a+nx}+\sqrt{a}}$

93. For all positive integers $n>1$,
$\left\{x\left(x^{n-1}-n.a^{n-1}\right)+a^n(n-1)\right\}$ is divisible by

1. $(x-a)^2$ 2. $x-a$
3. $2(x-a)$ 4. $x+a$

94. $2+3.2+4.2^2+..............n\,terms=$

1. $n.2^{n+1}$ 2. $(n+1)2^n$
3. $n.2^{n-1}$ 4. $n.2^n$

95. If $1^3+2^3+3^3+...+100^3=K^2$ then K =
1. 10100 2. 5000 3.5050 4. 1010

96. The greatest +ve integer which divides $(n+16)(n+17)(n+18)(n+19)$ $\forall n \in N$ is

1. $\lfloor 3$ 2. 4 3. $\lfloor 4$ 4. $\lfloor 5$

97 $S_n = \frac{1+2+3+.....+n}{n}$ then

$S_1^2 + S_2^2 + S_3^2 + ..... + S_n^2 =$

1. $\frac{n}{24}(2n^2+9n+13)$ 2. $\frac{1}{24}(2n^2+9n+13)$

3. $\frac{n^2}{24}(2n^2+9n+13)$ 4. $\frac{n}{24}(2n^2-9n+13)$

98. The greatest +ve integer which divides $(n+1)(n+2).......(n+r)$, for all $n \in N$ is

1. $(r+1)!$ 2. $r!$ 3. r 4. r-1

99. $\forall n \in N$, $1+2x+3x^2+....+n.x^{n-1} =$ $(x \in R, x \neq 1)$

1. $\frac{1-(n+1)x^n+n.x^{n+1}}{(1-x)^2}$

2. $\frac{1-(n+1)x^n+n.x^{n+1}}{(1+x)^2}$

3. $\frac{(n+1)x^n}{(1-x)^2}$ 4. $\frac{(n-1)x^n}{(1+x)^2}$

100. $2^n > n^2$ is true for $n$

1. $\geq 4$ 2. $> 5$ 3. $\geq 6$ 4. $\leq 5$

101. ${}^{2n}C_n > \frac{4^n}{n+1}$, is true for $n$

1. $\geq 1$ 2. $\geq 3$ 3. $\geq 2$ 4. $= 2$

102. $\log(x)^n = n.\log x$ is true for $n$.

1. $= 2$ 2. $\geq 2$ 3. $\geq 1$ 4. $= 1$

103. ${}^{2n}C_n < 4^n$ is true for n

1. $= 2$ 2 $\geq 2$ 3. $\geq 4$ 4. $\geq 1$

## LEVEL - III

104. 1. $49^n + 16n - 1$ is divisible by $A(n \in N)$

2. $3^{2n} + 7$ is divisible by $B(n \in N)$

3. $4^n - 3n - 1$ is divisible by $C(n \in N)$

4. $3^{3n} - 26n - 1$ is divisible by $D(n \in N)$

then the increasing order of A, B, C, D is

1.A, B, C, D 2. C, B, A, D

3. B, C, A, D 4. D, A, C, B

105. Statement I: For all $n \in N$ $x^n - y^n$ is divisible by $x-y$

Statement II: $x^n + y^n$ is divisible by $x+y$ if n is even natural number

Which of the above statement is true :

1. only I 2. only II

3. both I & II 4. neither I nor II

106. Statement I : For all $n \in N$, $x^{2n+1} + y^{2n+1}$ is divisible by $x+y$

Statement II: If $n \in N$ $n^3 + 2n$ is divisible by 6

Which of the above statement is true:

1. only I 2. only II

3. both I & II 4. neither I nor II

107. Assertion : For all +ve, integral values of 'n', $3^{2n} + 7$ is divisible by 8

Reason : G.C.F. of 16 and 88 is 8

1. A true, R true and R is the correct explanationof A

2. A true,R true and R is not a correct explanation of A

3. A true, R false 4. A false, R true.

108. Assertion : If $a_k = \frac{1}{k(k+1)}$ for k = 1,2,3,...n

then $\left(\sum_{k=1}^{n} a_k\right)^2 = \frac{n^2}{(n+1)^2}$

Reason : Sum of the n terms of a given series is $\frac{n}{n+1}$

1. A true, R true and R is the correct explanationof A

2. A true,R true and R is not a correct explanation of A

3. A true, R false 4. A false, R true.

109. Assertion : $n \in N$ product of $n(n+1)(n+2)$ is divisible by 6

Reason : Product of 3 consecutive +ve integers is divisible by 3!

1. A true, R true and R is the correct explanation of A

2. A true,R true and R is not a correct explanation of A

3. A true, R false 4. A false, R true.

## KEY

| | | | | |
|---|---|---|---|---|
| 66. 4 | 67. 2 | 68. 1 | 69. 2 | 70. 4 |
| 71. 2 | 72. 2 | 73.3 | 74.2 | 75.3 |
| 76. 4 | 77. 1 | 78. 1 | 79. 4 | 80. 3 |
| 81. 1 | 82. 2 | 83. 2 | 84. 3 | 85. 4 |
| 86. 1 | 87. 4 | 88. 2 | 89. 3 | 90. 3 |
| 91. 3 | 92. 2 | 93. 1 | 94. 4 | 95. 3 |
| 96. 3 | 97. 1 | 98. 2 | 99. 1 | 100.2 |
| 101. 3 | 102. 3 | 103. 4 | 104. 3 | 105. 1 |
| 106. 1 | 107. 1 | 108. 1 | 109. 1 | |

## HINTS

66. Put n = 1 then $P^2+P+1$
67. n = 1 $\Rightarrow 10+3.4+x=9m \Rightarrow x=5$
68. First term of 50th bracket = (1+2+3+ ....+49)+1
69. Put n = 1 and verify the options.
70. Put n = 2 and verify the options.
71. Put n = 2 verify the options.
72. $t_n = S_n - S_{n-1}$
73. Put n = 1 and verify the options.
74. Put n = 1 and verify the options.
75. Put n = 2 and verify the options.
76. Put n = 1 and verify the options.
77. $n=1 \Rightarrow LHS = \frac{x+a_1}{a_1}$ option (1) $= \frac{x+a_1}{a_1}$
78. Multiply Nr and Dr by (1 – x) and split the terms.
79. By verification.
80. By verification.
91. $\frac{1}{t_n} = \frac{4}{(n+2)(n+3)} = \frac{4}{n+2} - \frac{4}{n+3}$

Now $\left(\frac{4}{3}-\frac{4}{4}\right)+\left(\frac{4}{4}-\frac{4}{5}\right)+\left(\frac{4}{5}-\frac{4}{6}\right)+......$

$$+\left(\frac{4}{2007}-\frac{4}{2008}\right)=$$

$$\frac{4}{3}-\frac{4}{2008}=\frac{8020}{6024}$$

## PREVIOUS EAMCET QUESTIONS

1. $\sum_{k=1}^{5} \frac{1^3+2^3+.....+k^3}{1+3+5+......+(2k-1)} =$

**( EAMCET 2004)**

1. 22.5  2. 24.5  3. 28.5  4. 32.5

2. If $t_n = \frac{1}{4}(n+2)(n+3)$ for n = 1, 2, 3...

then $\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+........+\frac{1}{t_{2003}} =$

**( EAMCET 2003)**

1. $\frac{4006}{3006}$  2. $\frac{4003}{3007}$  3. $\frac{4006}{3008}$  4. $\frac{4006}{3009}$

3. In the sequence $\{1\},\{2,3\},\{4,5,6\},\{7,8,9,10\},....$ of sets the sum of elements in the 50th set is

**( EAMCET 2002)**

1. 62525  2. 65225  3.56255  4. 55625

4. $3^{2n+2}-2^{3n}-9$ is divisible by

**[ AMU - 02]**

1. 3  2. 9  3. 64  4. 81

5. The sum of integers from 1 to 100 that are divisible by 2 or 5 is

**[ AIEEE - 02]**

1. 3000  2. 3050  3. 3600  4. 3250

6. $1^3-2^3+3^3-4^3+.....+9^3=$

**[ AIEEE - 02]**

1. 425  2. -425  3. 475  4. -475

7. If $2^3+4^3+6^3+......+(2n)^3 = kn^2(n+1)^2$ then k=

**( EAMCET 2001)**

1. $\frac{1}{2}$  2. 1  3. $\frac{3}{2}$  4. 2

8. $\frac{\frac{1}{2}.\frac{2}{2}}{1^3}+\frac{\frac{2}{2}.\frac{3}{2}}{1^3+2^3}+\frac{\frac{3}{2}.\frac{4}{2}}{1^3+2^3+3^3}+......$ n terms

**( EAMCET 2000)**

1. $\frac{n^2}{(n+1)^2}$  2. $\frac{n^3}{(n+1)^3}$  3. $\frac{n}{n+1}$  4. $\frac{1}{n+1}$

9. If $a_k = \frac{1}{k(k+1)}$ for k = 1,2,3,....n then $\left(\sum_{k=1}^{n} a_k\right)^2 =$

**[EAM-2000]**

1. $\frac{n}{n+1}$  2. $\frac{n^2}{(n+1)^2}$

3. $\frac{n^4}{(n+1)^4}$  4. $\frac{n^6}{(n+1)^6}$

10. If n $\in$ N, then $n^3+2n$ is divisible by **[EAM-95]**

1. 3  2. 8  3. 9  4. 11

11. $x^n - a^n$ is divisible by x - a for n is any

1. positive integer 2. integer
3. odd positive integer 4. none

12. (1)+ (2+3+4)+ (5+6+7+8+9)+....n brackets= **[IIT-61, 63]**

1. n(n+1) (n+2)/6 2. $n^2(n^2+1)/2$
3. n (b+1) (n +2) 4. n(n +1) (2n +1)/6

13. $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+$...n terms = **[IIT-76]**

1. $n(2n^2 + 9n + 13)/24$ 2. $n(2n^3 + 9n + 13)/8$
3. $n(n^2 + 9n + 13)/24$ 4. $n(n^2 + 9n + 13)/8$

14. 1.3 + 2.4+3.5 + .....n terms = **[EAM-86]**

1. n(n+1) (n+5)/3 2. n (n+1) (n+2/3
3. $n(4n^2 + 6n - 1)/3$ 4. n (n +1) (n+2)

15. 1+3+7+15....n terms = **[CEE-83]**

1. $2^{n+1}-n-2$ 2. $n^2+n-2$
3. $2^n+n^2-2$ 4. none

16. 2 + 3 + 5 + 6 + 8 + 9 + .....2n terms = **[CEE-81]**

1. $3n^2 + 2n$ 2. $4n^2 + 2n$
3. $4n^2$ 4. none

17. $\left(\sum n^3\right)\left(\sum n\right)=\left(\sum n^2\right)^2$ iff **[CEE-80]**

1. n = 3 2. n = 1 3. $n^2 = 3$ 4. n = -1

18. $1^2+1+2^2+2+3^2+3+....+n^2+n$ = **[CEE- 78]**

1. $\frac{(n+1)(n-2)}{3}$ 2. $\frac{n(n+1)(n+2)}{3}$
3. $\frac{(n-1)(n+3)}{5}$ 4. none

19. The n th term of the series 3+7 + 13 + 21 +.. is **[CEE-77]**

1. 4n - 1 2. $n^2 + 2n$
3. $n^2 + n + 1$ 4. $n^2 + 2$

20. 1.3 + 2.4+3.5 + .....n terms = [JNTU-79]

1. $\frac{n(n+1)(2n+1)}{6}$ 2. $\frac{n^2(n+1)^2}{6}$
3. $\frac{n(n+1)(2n+7)}{6}$ 4. none

21. 2.4+4.7+6.10 + ...(n -1) terms = **[CEE-81]**

1. $2n^3 - 2n^2$ 2. $(n^3 + 3n^2 + 1)/6$
3. $2n^3 + 2n$ 4. none

## KEY

| | | | | |
|---|---|---|---|---|
| 1) 1 | 2) 4 | 3) 1 | 4) 3 | 5) 2 |
| 6) 1 | 7) 4 | 8) 3 | 9) 2 | 10) 1 |
| 11) 1 | 12) 2 | 13) 1 | 14) 3 | 15) 1 |
| 16) 1 | 17) 2 | 18) 2 | 19) 3 | 20) 3 |
| 21) 1 | | | | |