Statistics and Probability

Exercise 13.1 Multiple Choice Questions (MCQs)

Question 1:

In the formula $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$, for finding the mean of grouped data d_i's are deviation from a of

(a) lower limits of the classes (b) upper limits of the classes
(c) mid-points of the classes (d) frequencies of the class marks

Solution:
(c) We know that, $d_i = x_i - a$
i.e., d_i's are the deviation from a of mid-points of the classes.

Question 2:
While computing mean of grouped data, we assume that the frequencies are
(a) evenly distributed over all the classes
(b) centred at the class marks of the classes
(c) centred at the upper limits of the classes
(d) centred at the lower limits of the classes

Solution:
(b) In computing the mean of grouped data, the frequencies are centred at the class marks of the classes.

Question 3:

If x_i's are the mid-points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\bar{x}$ is the mean, then $\Sigma(f_i x_i - \bar{x})$ is equal to

(a) 0 (b) -1 (c) 1 (d) 2

Solution:

(a) $\because$ $\bar{x} = \frac{\Sigma f_i x_i}{n}$

$\therefore$ $\Sigma(f_i x_i - \bar{x}) = \Sigma f_i x_i - \Sigma \bar{x}$

$= n\bar{x} - n\bar{x}$ $[\because \Sigma \bar{x} = n\bar{x}]$

$= 0$

Question 4:

In the formula $\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$, for finding the mean of grouped frequency distribution u_i is equal to

(a) $\frac{x_i + a}{h}$ (b) $h(x_i - a)$ (c) $\frac{x_i - a}{h}$ (d) $\frac{a - x_i}{h}$

Solution:

(c) Given, $\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$

Above formula is a step deviation formula.

$$u_i = \frac{x_i - a}{h}$$

Question 5:

The abscissa of the point of intersection of the less than type and of the more than type cumulative frequency curves of a grouped data gives its

(a) mean (b) median (c) mode (d) All of these

Solution:

(b) Since, the intersection point of less than ogive and more than ogive gives the median on the abscissa.

Question 6:

For the following distribution,

Class	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
Frequency	10	15	12	20	9

the sum of lower limits of the median class and modal class is

(a) 15 (b) 25 (c) 30 (d) 35

Solution:

(b) Here,

Class	Frequency	Cumulative frequency
0-5	10	10
5-10	15	25
10-15	12	37
15-20	20	57
20-25	9	66

Now, $\frac{N}{2} = \frac{66}{2}$ = 33, which lies in the interval 10-15. Therefore, lower limit of the median class is 10.

The highest frequency is 20, which lies in the interval 15-20. Therefore, lower limit of modal class is 15. Hence, required sum is 10 + 15 = 25.

Question 7:

Consider the following frequency distribution

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

The upper limit of the median class is

(a) 7 (b) 17.5 (c) 18 (d) 18.5

Solution:

(b)

Class	Frequency	Cumulative frequency
–0.5-5.5	13	13
5.5-11.5	10	23
11.5-17.5	15	38
17.5-23.5	8	46
23.5-29.5	11	57

Question 8:

For the following distribution,

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

the modal class is
(a) 10-20 (b) 20-30 (c) 30-40 (d) 30-40

Solution:
(c)

Marks	Number of students	Cumulative frequency
Below 10	$3 = 3$	3
10-20	$(12 - 3) = 9$	12
20-30	$(27 - 12) = 15$	27
30-40	$(57 - 27) = 30$	57
40-50	$(75 - 57) = 18$	75
50-60	$(80 - 75) = 5$	80

Here,we see that the highest frequency is 30. which lies in the interval 30-40.

Question 9:
consider the data.

Class	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Frequency	4	5	13	20	14	7	4

The difference of the upper limit of the median class and the lower limit of the modal class is
(a) 0 (b) 19 (c) 20 (d) 38

Solution:

(c)

Class	Frequency	Cumulative frequency
65-85	4	4
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	7	63
185-205	4	67

Here, $\frac{N}{2} = \frac{67}{2}$ = 33.5 which lies in the interval 125 -145.
Hence, upper limit of median class is 145.
Here, we see that the highest frequency is 20 which lies in 125-145. Hence, the lower limit of modal class is 125.
Required difference = Upper limit of median class – Lower limit of modal class
= 145-125 = 20

Question 10:
The times (in seconds) taken by 150 atheletes to run a 110 m hurdle race are tabulated below

Class	13.8-14	14-14.2	14.2-14.4	14.4-14.6	14.6-14.8	14.8-15
Frequency	2	4	5	71	48	20

The number of atheletes who completed the race in less than 14.6 s is
(a) 11 (b) 71 (c) 82 (d) 130

Solution:
(c) The number of atheletes who completed the race in less than 14.6
= 2 + 4+ 5+71 =82

Question 11:
Consider the following distribution

Marks obtained	Number of students
More than or equal to 0	63
More than or equal to 10	58
More than or equal to 20	55
More than or equal to 30	51
More than or equal to 40	48
More than or equal to 50	42

the frequency of the class 30-40 is

(a) 3 (b) 4 (c) 3 (d) 4

Solution:

(a)

Marks obtained	Number of students
0-10	$(63-58)=5$
10-20	$(58-55)=3$
20-30	$(55-51)=4$
30-40	$(51-48)=3$
40-50	$(48-42)=6$
50...	$42=42$

Hence,frequency in the class interval 30-40 is 3

Question 12:

If an event cannot occur, then its probability is

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) 0

Solution:

(d) The event which cannot occur is said to be impossible event and probability of impossible event is zero.

Question 13:

Which of the following cannot be the probability of an event?

(a) $\frac{1}{2}$ (b) 0.1 (c) 3 (d) $\frac{17}{16}$

Solution:

(d) Since, probability of an event always lies between 0 and 1.

Question 14:
An event is very unlikely to happen. Its probability is closest to
(a) 0.0001 (b) 0.001 (c) 0.01 (d) 0.1

Solution:
(a) The probability of an event which is very unlikely to happen is closest to zero and from the given options 0.0001 is closest to zero.

Question 15:
If the probability of an event is P, then the probability of its completmentry event will be
(a) P -1 (b) P (c) 1 – P (d) $1 - \frac{1}{P}$

Solution:
(c) Since, probability of an event + probability of its complementry event = 1
So, probability of its complementry event = 1 – Probability of an event = 1 – P

Question 16:
The probability expressed as a percentage of a particular occurrence can never be
(a) less than 100 (b) less than 0
(c) greater than 1 (d) anything but a whole number

Solution:
(b) We know that, the probability expressed as a percentage always lie between 0 and 100. So, it cannot be less than 0.

Question 17:
If P (A) denotes the probability of an event A, then
(a) $P(A) < 0$ (b) $P(A) > 1$ (c) $0 \le P(A) \le 1$ (d) $-1 \le P(A) \le 1$

Solution:
(c) Since, probability of an event always lies between 0 and 1.

Question 18:
If a card is selected from a deck of 52 cards, then the probability of its being a red face card is
(a) $\frac{3}{26}$ (b) $\frac{3}{13}$ (c) $\frac{2}{13}$ (d) $\frac{1}{2}$

Solution:

(c) In a deck of 52 cards, there are 12 face cards i.e.,6 red and 6 black cards.

So, probability of getting a red face card = $\frac{6}{52} = \frac{3}{26}$

Question 19:

The probability that a non-leap your selected at random will contains 53 Sunday is '

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{5}{7}$

Solution:

(a) A non-leap year has 365 days and therefore 52 weeks and 1 day. This 1 day may be Sunday or Monday or Tuesday or Wednesday or Thursday or Friday or Saturday. Thus, out of 7 possibilities, 1 favourable event is the event that the one day is Sunday.

∴ Required probability = $\frac{1}{7}$

Question 20:

When a die is thrown, the probability of getting an odd number less than 3 is ,

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 0

Solution:

(a) When a die-is thrown,then total number of outcomes = 6 Odd number less than 3 is 1 only.

Number of possible outcomes = 1

Required probability = $\frac{1}{6}$

Question 21:

A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is

(a) 4 (b) 13 (c) 48 (d) 51

Solution:

(d) In a deck of 52 cards, there are 13 cards of heart and 1 is ace of heart.

Hence, the number of outcomes favourable to E = 51

Question 22:

The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is

(a) 7 (b) 14 (c) 21 (d) 28

Solution:

(b) Here, total number of eggs = 400

Probability of getting a bad egg = 0.035

$$\Rightarrow \quad \frac{\text{Number of bad eggs}}{\text{Total number of eggs}} = 0.035$$

$$\Rightarrow \quad \frac{\text{Number of bad eggs}}{400} = 0.035$$

$$\therefore \quad \text{Number of bad eggs} = 0.035 \times 400 = 14$$

Question 23:

A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, then how many tickets has she bought?

(a) 40 (b) 240 (c) 480 (d) 750

Solution:

(c) Given, total number of sold tickets = 6000

Let she bought x tickets.

Then, probability of her winning the first prize $= \frac{x}{6000} = 0.08$ [given]

$$\Rightarrow \quad x = 0.08 \times 6000$$

$$\therefore \quad x = 480$$

Hence, she bought 480 tickets.

Question 24:

One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 5 is

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{1}{3}$

Solution:

(a) Number of total outcomes = 40

Multiples of 5 between 1 to 40 = 5,10,15,20,25. 30 35, 40

$\therefore$ Total number of possible outcomes = 8

$$\therefore \quad \text{Required probability} = \frac{8}{40} = \frac{1}{5}$$

Question 25:

Someone is asked to take a number from 1 to 100. The probability that it is a prime,is

(a) $\frac{1}{5}$ (b) $\frac{6}{25}$ (c) $\frac{1}{4}$ (d) $\frac{13}{50}$

Solution:

(c) Total numbers of outcomes = 100

So, the prime numbers between 1 to 100 are 2, 3, 5, 7,11,13,17,19, 23, 29, 31,37, 41. 43, 47, 53, 56, 61, 67, 71, 73, 79, 83, 89 and 97.

∴ Total number of possible outcomes = 25

∴ Required probability $= \frac{25}{100} = \frac{1}{4}$

Question 26:

A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is

(a) $\frac{4}{23}$ (b) $\frac{6}{23}$ (c) $\frac{8}{23}$ (d) $\frac{17}{23}$

Solution:

(b) Total number of students = 23

Number of students in house A, B and C = 4+ 8 + 5 = 17

∴ Remains students = 23 – 17 = 6

So, probability that the selected student is not from A, B and $C = \frac{6}{23}$

Exercise 13.2 Very Short Answer Type Questions

Question 1:

The median of an ungrouped data and the median calculated when the same data is grouped are always the same. Do you think that this is a correct statement? Give reason.

Solution:

Not always, because for calculating median of a grouped data, the formula used is based on the assumption that the observations in the classes are uniformal distributed (or equally spaced).

Question 2:

In calculating the mean of grouped data, grouped in classes of equal width, we may use the formula,

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

Where, a is the assumed mean, a must be one of the mid-point of the classes. Is the last statement correct? Justify your answer.

Solution:

No, it is not necessary that assumed mean consider as the mid-point of the class interval. It is considered as any value which is easy to simplify it.

Question 3:
Is it true to say that the mean, mode and median of grouped data will always be different? Justify your answer

Solution:
the value of these three measures can be the same, it depends on the type of data.

Question 4:
Will the median class and modal class of grouped data always be different? Justify your answer.

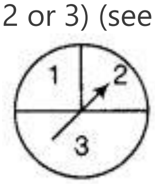
Solution:
Not always, It depends on the given data.

Question 5:
In a family having three children, there may be no girl, one girl, two girls or three girls. So, the probability of each is $\frac{1}{4}$. Is this correct? Justify your answer.

Solution:
No, the probability of each is not $\frac{1}{4}$ because the probability of no girl in three children is zero and probability of three girls in three children is one.

Justification
So, these events are not equally likely as outcome one girl, means gbb, bgb, bbg 'three girls' means 'ggg' and so on.

Question 6:
A game consists of spinning an arrow which comes to rest pointing at one of the regions (1, 2 or 3) (see figure). Are the outcomes 1, 2 and 3 equally likely to occur? Give reasons



Solution:
No, the outcomes are not equally likely, because 3 contains half part of the

total region, so it is more likely than 1 and 2, since 1 and 2, each contains half part of the remaining part of the region.

Question 7:
Apoorv throws two dice once and computes the product of the numbers appearing on the dice. Peehu throws one die and squares the number that appears on it. Who has the better chance of getting the number 36? Why?

Solution:
Apoorv throws two dice once.
So total number of outcomes = 36
Number of outcomes for getting product 36 = 1 (6 x 6)

$\therefore$ Probability for Apoorv $= \frac{1}{36}$

Also, Peehu throws one die,
So, total number of outcomes = 6
Number of outcomes for getting square 36 = 1 ($6^2 = 36$)

$\therefore$ Probability for Peehu $= \frac{1}{6} = \frac{6}{36}$

Hence, Peehu has better chance of getting the number 36.

Question 8:
When we toss a coin, there are two possible outcomes-head or tail. Therefore, the probability of each outcome is $\frac{1}{2}$. Justify your answer

Solution:
Yes, probability of each outcome is $\frac{1}{2}$ because head and tail both are equally likely events.

Question 9:
A student says that, if you throw a die, it will show up 1 or not 1. Therefore, the probability of getting 1 and the probability of getting not 1. each is equal to $\frac{1}{2}$. Is this correct? Give reasons.

Solution:
No, this is not correct.
Suppose we throw a die, then total number of outcomes = 6

Possible outcomes = 1 or 2 or 3 or 4 or 5 or 6

$\therefore$ Probability of getting 1 = $\frac{1}{6}$

Now, probability of getting not 1 = 1 − Probability of getting 1

$= 1 - \frac{1}{6} = \frac{5}{6}$

Question 10:
I toss three coins together. The possible outcomes are no heads, 1 head,2 head and 3 heads.
So, I say that probability of no heads is $\frac{1}{4}$. What is wrong with this conclusion?
Solution:
I toss three coins
together [given]
So, total number of outcomes = 2^3 = 8
and possible outcomes are (HHH), (HTT), (THT), (TTH),(HHT), (THH), (HIH)and (TTT)

Now, probability of getting no head = $\frac{1}{8}$

Hence, the given conclusion is wrong because the probability of no head is $\frac{1}{8}$ not $\frac{1}{4}$.

Question 11:
If you toss a coin 6 times and it comes down heads on each occasion. Can you say that the probability of getting a head is 1? Given reasons.

Solution:
No. if let we toss a coin, then we get head or tail, both are equally likely events
So,probability is $\frac{1}{2}$. If we toss a coin 6 times, then probability will be same in each case. So, the 2
probability of getting a head is not 1.

Question 12:
Sushma tosses a coin 3 times and gets tail each time. Do you think that the outcome of next toss will be a tail? Give reasons.

Solution:
The outcome of next toss may or may not be tail, because on tossing a coin, we get head or tail so both are equally likely events.

Question 13:
If I toss a coin 3 times and get head each time, should I expect a tail to have a higher chance

in the 4th toss? Give reason in support of your answer.

Solution:
No, let we toss a coin, then we get head or tail, both are equaly likely events i.e., probability of each event is $\frac{1}{2}$. So, no question of expecting a tail to have a higher chance in 4th toss.

Question 14:
A bag contains slips numbered from 1 to 100. If Fatima chooses a slip at random from the bag, it will either be an odd number or an even number. Since, this situation has only two possible outcomes, so the 1
probability of each is $\frac{1}{2}$. Justify.

Solution:
We know that, between 1 to 100 half numbers are even and half numbers are odd i.e., 50 numbers (2, 4, 6, 8.... 96, 98,100) are even and 50 numbers (1,3, 5, 7. . , 97, 99) are odd.
So, both events are equally likely.

So, probability of getting even number $= \frac{50}{100} = \frac{1}{2}$

and probability of getting odd number $= \frac{50}{100} = \frac{1}{2}$

Hence, the probability of each is $\frac{1}{2}$.

Exercise 13.3 Short Answer Type Questions

Question 1:
Find the mean of the distribution

Class	1-3	3-5	5-7	7-10
Frequency	9	22	27	17

Solution:
We first, find the class mark x_i, of each class and then proceed as follows.

Class	Class marks (x_i)	Frequency (f_i)	$f_i x_i$
1-3	2	9	18
3-5	4	22	88
5-7	6	27	162
7-10	8.5	17	144.5
		$\Sigma f_i = 75$	$\Sigma f_i x_i = 412.5$

Therefore, mean $(\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{412.5}{75} = 5.5$

Hence, mean of the given distribution is 5.5.

Question 2:

Calculate the mean of the scores of 20 students in a mathematics test

Marks	10-20	20-30	30-40	40-50	50-60
Number of students	2	4	7	6	1

Solution:

We first, find the class mark of each class and then proceed as follows

Marks	Class marks (x_i)	Frequency (f_i)	$f_i x_i$
10-20	15	2	30
20-30	25	4	100
30-40	35	7	245
40-50	45	6	270
50-60	55	1	55
		$\Sigma f_i = 20$	$\Sigma f_i x_i = 700$

Therefore, mean $(\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{700}{20} = 35$

Hence, the mean of scores of 20 students in mathematics test is 35.

Question 3:

Calculate the mean of the following data

Class	4-7	8-11	12-15	16-19
Frequency	5	4	9	10

Solution:

Since, given data is not continuous, so we subtract 0.5 from the lower limit and add 0.5 in the upper limit of each class.

Now, we first find the class mark x_i, of each class and then proceed as follows

Class	Class marks (x_i)	Frequency (f_i)	$f_i x_i$
3.5-7.5	5.5	5	27.5
7.5-11.5	9.5	4	38
11.5-15.5	13.5	9	121.5
15.5-19.5	17.5	10	175
		$\Sigma f_i = 28$	$\Sigma f_i x_i = 362$

Therefore, $\bar{x} \text{ (mean)} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{362}{28} = 12.93$

Hence, mean of the given data is 12.93.

Question 4:
The following table gives the number of pages written by Sarika for completing her own book for 30 days.

Number of pages written per day	16-18	19-21	22-24	25-27	28-30
Number of days	1	3	4	9	13

Find the mean number of pages written per day.

Solution:
Since,

Class mark	Mid-value (x_i)	Number of days (f_i)	$f_i x_i$
15.5-18.5	17	1	17
18.5-21.5	20	3	60
21.5-24.5	23	4	92
24.5-27.5	26	9	234
27.5-30.5	29	13	377
Total		30	780

Since, given data is not continuous, so we subtract 0.5 from the lower limit and add 0.5 in the upper limit of each class.

$$\therefore \quad \text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{780}{30} = 26$$

Hence, the mean of pages written per day is 26.

Question 5:
The daily income of a sample of 50 employees are tabulated as follows.

Income (in ₹)	1-200	201-400	401-600	601-800
Number of employees	14	15	14	7

Find the mean daily income of employees.

Solution:

Since, given data is not continuous, so we subtract 0.5 from the lower limit and add 0.5 in the upper limit of each class.

Now we first, find the class mark x_i, of each class and then proceed as follows

Income (in ₹)	Class marks (x_i)	Number of employees (f_i)	$u_i = \frac{x_i - a}{h} = \frac{x_i - 300.5}{200}$	$f_i u_i$
0.5-200.5	100.5	14	−1	−14
200.5-400.5	$a = 300.5$	15	0	0
400.5-600.5	500.5	14	1	14
600.5-800.5	700.5	7	2	14
		$N = \Sigma f_i = 50$		$\Sigma f_i u_i = 14$

∴ Assumed mean, $a = 300.5$

Class width, $h = 200$

and total observations, $N = 50$

By step deviation method,

$$\text{Mean} = a + h \times \frac{1}{N} \times \sum_{i=1}^{5} f_i u_i$$

$$= 300.5 + 200 \times \frac{1}{50} \times 14$$

$$= 300.5 + 56 = 356.5$$

Question 6:

An aircraft has 120 passenger seats. The number of seats occupied during 100 flights is given in the following table.

Number of seats	100-104	104-108	108-112	112-116	116-120
Frequency	15	20	32	18	15

Determine the mean number of seats occupied over the flights.

Solution:

We first, find the class mark x_i, of each class and then proceed as follows.

Number of seats	Class marks (x_i)	Frequency (f_i)	Deviation $d_i = x_i - a$	$f_i d_i$
100-104	102	15	−8	−120
104-108	106	20	−4	−80
108-112	$a = 110$	32	0	0
112-116	114	18	4	72
116-120	118	15	8	120
		$N = \Sigma f_i = 100$		$\Sigma f_i d_i = -8$

∴ Assumed mean, $a = 110$,

Class width, $h = 4$

and total observation, $N = 100$

By assumed mean method,

$$\text{Mean } (\bar{x}) = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 110 + \left(\frac{-8}{100}\right) = 110 - 0.08 = 109.92$$

Question 7:

The weights (in kg) of 50 wrestlers are recorded in the following table.
Find the mean weight of the wrestlers.

Weight (in kg)	100-110	110-120	120-130	130-140	140-150
Number of wrestlers	4	14	21	8	3

Solution:

We first find the class mark of each class and then proceed as follows

Weight (in kg)	Number of wrestlers (f_i)	Class marks (x_i)	Deviations $d_i = x_i - a$	$f_i d_i$
100-110	4	105	−20	−80
110-120	14	115	−10	−140
120-130	21	$a = 125$	0	0
130-140	8	135	10	80
140-150	3	145	20	60
	$N = \Sigma f_i = 50$			$\Sigma f_i d_i = -80$

∴ Assumed mean (a) = 125,
Class width (h) = 10 and total observation (N) = 50
By assumed mean method,

$$\text{Mean } (\bar{x}) = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$
$$= 125 + \frac{(-80)}{50}$$
$$= 125 - 1.6 = 123.4 \text{ kg}$$

Question 8:
The mileage (km per litre) of 50 cars of the same model was tested by a manufacturer and details are tabulated as given below

Mileage (kmL^{-1})	10-12	12-14	14-16	16-18
Number of cars	7	12	18	13

Find the mean mileage.
The manufacturer claimed that the mileage of the model was 16 kmL^{-1}.
Do you agree with this claim?

Solution:

Mileage (kmL^{-1})	**Class marks** (x_i)	**Number of cars** (f_i)	$f_i x_i$
10-12	11	7	77
12-14	13	12	156
14-16	15	18	270
16-18	17	13	221
Total		$\Sigma f_i = 50$	$\Sigma f_i x_i = 724$

Here, $\Sigma f_i = 50$

and $\Sigma f_i x_i = 724$

$\therefore$ $$\text{Mean } \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{724}{50} = 14.48$$

Hence, mean mileage is 14.48 kmL^{-1}.

No, the manufacturer is claiming mileage 1.52 kmh^{-1} more than average mileage.

Question 9:

The following is the distribution of weights (in kg) of 40 persons.

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80
Number of persons	4	4	13	5	6	5	2	1

Construct a cumulative frequency distribution (of the less than type) table for the data above.

Solution:

The cumulative distribution (less than type) table is shown below

Weight (in kg)	Cumulative frequency
Less than 45	4
Less than 50	4 + 4 = 8
Less than 55	8 + 13 = 21
Less than 60	21 + 5 = 26
Less than 65	26 + 6 = 32
Less than 70	32 + 5 = 37
Less than 75	37 + 2 = 39
Less than 80	39 + 1 = 40

Question 10:

The following table shows the cumulative frequency distribution of marks of 800 students in an examination.

Marks	Number of students
Below 10	10
Below 20	50
Below 30	130
Below 40	270
Below 50	440
Below 60	570
Below 70	670
Below 80	740
Below 90	780
Below 100	800

Construct a frequency distribution table for the data above.

Solution:

Here, we observe that 10 students have scored marks below 10 i.e., it lies between class interval 0-10. Similarly, 50 students have scored marks below 20. So, 50 -10 = 40 students lies in the interval 10-20 and so on. The table of a frequency distribution for the given data is

Class interval	Number of students
0-10	10
10-20	$50-10=40$
20-30	$130-50=80$
30-40	$270-130=140$
40-50	$440-270=170$
50-60	$570-440=130$
60-70	$670-570=100$
70-80	$740-670=70$
80-90	$780-740=40$
90-100	$800-780=20$

Question 11:

From the frequency distribution table from the following data.

Marks (Out of 90)	Number of candidates
More than or equal to 80	4
More than or equal to 70	6
More than or equal to 60	11
More than or equal to 50	17
More than or equal to 40	23
More than or equal to 30	27
More than or equal to 20	30
More than or equal to 10	32
More than or equal to 0	34

Solution:

Here, we observe that, all 34 students have scored marks more than or equal to 0. Since, 32 students have scored marks more than or equal to 10. So, 34- 32 =2 students lies in the interval 0-10 and so on.

Now, we construct the frequency distribution table.

Class interval	Number of candidates
0-10	$34-32=2$
10-20	$32-30=2$
20-30	$30-27=3$
30-40	$27-23=4$
40-50	$23-17=6$
50-60	$17-11=6$
60-70	$11-6=5$
70-80	$6-4=2$
80-90	4

Question 12:

Find the unknown entries o, b, c, d, e and f in the following distribution of heights of students in a class

Height (in cm)	Frequency	Cumulative frequency
150-155	12	a
155-160	b	25
160-165	10	c
165-170	d	43
170-175	e	48
175-180	2	f
Total	50	

Solution:

Height (in cm)	Frequency	Cumulative frequency (given)	Cumulative frequency
150-155	12	a	12
155-160	b	25	$12+b$
160-165	10	c	$22+b$
165-170	d	43	$22+b+d$
170-175	e	48	$22+b+d+e$
175-180	2	f	$24+b+d+e$
Total	50		

On comparing last two tables, we get

$$a = 12$$

$\therefore$ $$12 + b = 25$$

$\Rightarrow$ $$b = 25 - 12 = 13$$

$$22 + b = c$$

$\Rightarrow$ $$c = 22 + 13 = 35$$

$$22 + b + d = 43$$

$\Rightarrow$ $$22 + 13 + d = 43$$

$\Rightarrow$ $$d = 43 - 35 = 8$$

and $$22 + b + d + e = 48$$

$\Rightarrow$ $$22 + 13 + 8 + e = 48$$

$\Rightarrow$ $$e = 48 - 43 = 5$$

and $$24 + b + d + e = f$$

$\Rightarrow$ $$24 + 13 + 8 + 5 = f$$

$\therefore$ $$f = 50$$

Question 13:

The following are the ages of 300 patients getting medical treatment in a hospital on a particular day

Age (in year)	10-20	20-30	30-40	40-50	50-60	60-70
Number of patients	60	42	55	70	53	20

Form
(i) less than type cumulative frequency distribution.
(ii) More than type cumulative frequency distribution.

Solution:

(i) We observe that the number of patients which take medical treatment in a hospital on a particular day less than 10 is O'. Similarly, less than 20 include the number of patients which take medical treatment from 0-10 as well as the number of patients which take medical treatment from 10-20.
So, the total number of patients less than 20 is 0 + 60 = 60, we say that the cumulative frequency of the class 10-20 is 60. Similarly, for other class.

(ii) Also, we observe that all 300 patients which take medical treatment more than or equal to 10. Since, there are 60 patients which take medical treatment in the interval 10-20, this means that there are 300 – 60 = 240 patients which take medical treatment more than or equal to 20. Continuing in the same manner.

(i) Less than type		(ii) More than type	
Age (in year)	Number of students	Age (in year)	Number of students
Less than 10	0	More than or equal to 10	300
Less than 20	60	More than or equal to 20	240
Less than 30	102	More than or equal to 30	198
Less than 40	157	More than or equal to 40	143
Less than 50	227	More than or equal to 50	73
Less than 60	280	More than or equal to 60	60
Less than 70	300		

Question 14:
Given below is a cumulative frequency distribution showing the marks secured by 50 students of a class

Marks	Below 20	Below 40	Below 60	Below 80	Below 100
Number of students	17	22	29	37	50

Form the frequency distribution table for the data.

Solution:
Here, we observe that, 17 students have scored marks below 20 i.e., it lies between class interval 0-20 and 22 students have scored marks below 40, so 22 -17 = 5 students lies in the class interval 20-40 continuing in the same manner, we get the complete frequency distribution table for given data.

Marks	Number of students
0-20	17
20-40	$22-17=5$
40-60	$29-22=7$
60-80	$37-29=8$
80-100	$50-37=13$

Question 15:
Weekly income of 600 families is tabulated below

Weekly income (in ₹)	**Number of families**
0-1000	250
1000-2000	190
2000-3000	100
3000-4000	40
4000-5000	15
5000-6000	5
Total	**600**

Compute the median income.

Solution:
First we construct a cumulative frequency table

Weekly income (in ₹)	Number of families (f_i)	Cumulative frequency (cf)
0-1000	250	250
1000-2000 = mid class	190 = f	250 + 190 = 440
2000-3000	100	440 + 100 = 540
3000-4000	40	540 + 40 = 580
4000-5000	15	580 + 15 = 595
5000-6000	5	595 + 5 = 600

It is given that, $n = 600$

$$\therefore \quad \frac{n}{2} = \frac{600}{2} = 300$$

Since, cumulative frequency 440 lies in the interval 1000 - 2000.

Here, (lower median class) $l = 1000$,

$f = 190, cf = 250$, (class width) $h = 1000$

and (total observation) $n = 600$

$$\therefore \quad \text{Median} = l + \frac{\left\{\frac{n}{2} - cf\right\}}{f} \times h$$

$$= 1000 + \frac{(300 - 250)}{190} \times 1000$$

$$= 1000 + \frac{50}{190} \times 1000$$

$$= 1000 + \frac{5000}{19}$$

$$= 1000 + 263.15 = 1263.15$$

Hence, the median income is ₹ 1263.15.

Question 16:

The maximum bowling speeds, in km per hour, of 33 players at a cricket coaching centre are given as follows

Speed (in km/h)	85-100	100-115	115-130	130-145
Number of players	11	9	8	5

Caluculate the median bowling speed.

Solution:

First we construct the cumulative frequency table

Speed (in km/h)	Number of players	Cumulative frequency
85-100	11	11
100-115	9	11 + 9 = 20
115-130	8	20 + 8 = 28
130-145	5	28 + 5 = 33

It is given that, $n = 33$

$$\therefore \quad \frac{n}{2} = \frac{33}{2} = 16.5$$

So, the median class is 100-115.

where, lower limit $(l) = 100$,

frequency $(f) = 9$,

cumulative frequency $(cf) = 11$

and class width $(h) = 15$

$$\therefore \quad \text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

$$= 100 + \frac{(16.5 - 11)}{9} \times 15$$

$$= 100 + \frac{5.5 \times 15}{9} = 100 + \frac{82.5}{9} = 100 + 9.17$$

$$= 109.17$$

Hence, the median bowling speed is 109.17 km/h.

Question 17:

The monthly income of 100 families are given as below

Income (in ₹)	Number of families
0-5000	8
5000-10000	26
10000-15000	41
15000-20000	16
20000-25000	3
25000-30000	3
30000-35000	2
35000-40000	1

Caluculate the model income.

Solution:

In a given data, the highest frequency is 41, which lies in the interval 10000-15000.

Here, $l = 10000, f_m = 41, f_1 = 26, f_2 = 16$ and $h = 5000$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

$$= 10000 + \left(\frac{41 - 26}{2 \times 41 - 26 - 16}\right) \times 5000$$

$$= 10000 + \left(\frac{15}{82 - 42}\right) \times 5000$$

$$= 10000 + \left(\frac{15}{40}\right) \times 5000$$

$$= 10000 + 15 \times 125 = 10000 + 1875 = ₹\, 11875$$

Hence, the modal income is ₹ 11875.

Question 18:
The weight of coffee in 70 packets are shown in the following table

Weight (in g)	Number of packets
200-201	12
201-202	26
202-203	20
203-204	9
204-205	2
205-206	1

Determine the model weight .

Solution:
In the given data, the highest frequency is 26, which lies in the interval 201-202

Here, $l = 201, f_m = 26, f_1 = 12, f_2 = 20$ and (class width) $h = 1$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h = 201 + \left(\frac{26 - 12}{2 \times 26 - 12 - 20}\right) \times 1$$

$$= 201 + \left(\frac{14}{52 - 32}\right) = 201 + \frac{14}{20} = 201 + 0.7 = 201.7 \text{ g}$$

Hence, the modal weight is 201.7 g.

Question 19:
Two dice are thrown at the same time. Find the probability of getting
(i) same number on both dice.
(ii) different number on both dice.

Solution:
Two dice are thrown at the same

time. [given]
So, total number of possible outcomes = 36
(i) We have, same number on both dice.
So, possible outcomes are (1,1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6).
$\therefore$ Number of possible outcomes = 6

Now, required probability $= \frac{6}{36} = \frac{1}{6}$

(ii) We have, different number on both dice.
So, number of possible outcomes
= 36 – Number of possible outcomes for same number on both dice
= 36 – 6 = 30

$\therefore$ Required probability $= \frac{30}{36} = \frac{5}{6}$

Question 20:
Two dice are thrown simultaneously. What is the probability that the sum of the numbers appearing on the dice is
(i) 7 ? (ii) a prime number ? (iii) 1 ?

Solution:
Two dice are thrown simultaneously. [given]
So, total number of possible outcomes = 36

(i) Sum of the numbers appearing on the dice is 7.
So, the possible ways are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1).
Number of possible ways = 6

$\therefore$ Required probability $= \frac{6}{36} = \frac{1}{6}$

(ii) Sum of the numbers appearing on the dice is a prime number *i.e.*, 2, 3, 5, 7 and 11.
So, the possible ways are (1, 1), (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4,1), (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1), (5, 6) and (6, 5).
Number of possible ways = 15

$\therefore$ Required probability $= \frac{15}{36} = \frac{5}{12}$

(iii) Sum of the numbers appearing on the dice is 1.
It is not possible, so its probability is zero.

Question 21:
Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is
(i) 6 (ii) 12 (iii) 7

Solution:
Number of total outcomes = 36

(i) When product of the numbers on the top of the dice is 6.
So, the possible ways are (1, 6), (2, 3), (3, 2) and (6, 1).
Number of possible ways = 4

$\therefore$ Required probability = $\frac{4}{36} = \frac{1}{9}$

(ii) When product of the numbers on the top of the dice is 12.
So, the possible ways are (2, 6), (3, 4), (4, 3) and (6, 2).
Number of possible ways = 4

$\therefore$ Required probability = $\frac{4}{36} = \frac{1}{9}$

(iii) Product of the numbers on the top of the dice cannot be 7. So, its probability is zero.

Question 22:
Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.

Solution:
Number of total outcomes = 36
When product of numbers appearing on them is less than 9, then possible ways are (1,6), (1,5) (1,4), (1,3), (1,2), (1,1), (2, 2), (2, 3), (2, 4), (3, 2), (4, 2), (4,1), (3,1), (5,1), (6,1) and (2,1).
Number of possible ways = 16
Required probability = $\frac{16}{36} = \frac{4}{9}$

Question 23:
Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3, respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9, separately.

Solution:
Number of total outcomes = 36

(i) Let E_1 = Event of getting sum 2 = $\{(1, 1), (1, 1)\}$

$$n(E_1) = 2$$

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

(ii) Let E_2 = Event of getting sum 3 = $\{(1, 2), (1, 2), (2, 1), (2, 1)\}$

$$n(E_2) = 4$$

$$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

(iii) Let E_3 = Event of getting sum 4 = $\{(2, 2), (2, 2), (3, 1), (3, 1), (1, 3), (1, 3)\}$

$$\therefore \quad n(E_3) = 6$$

$$\therefore \quad P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(iv) Let E_4 = Event of getting sum 5 = $\{(2, 3), (2, 3), (4, 1), (4, 1), (3, 2), (3, 2)\}$

$$\therefore \quad n(E_4) = 6$$

$$\therefore \quad P(E_4) = \frac{n(E_4)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(v) Let E_5 = Event of getting sum 6 = $\{(3, 3), (3, 3), (4, 2), (4, 2), (5, 1), (5, 1)\}$

$$n(E_5) = 6$$

$$\therefore \quad P(E_5) = \frac{n(E_5)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(vi) Let E_6 = Event of getting sum 7 = $\{(4, 3), (4, 3), (5, 2), (5, 2), (6, 1), (6, 1)\}$

$$\therefore \quad n(E_6) = 6$$

$$\therefore \quad P(E_6) = \frac{n(E_6)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

(vii) Let E_7 = Event of getting sum 8 = $\{(5, 3), (5, 3), (6, 2), (6, 2)\}$

$$\therefore \quad n(E_7) = 4$$

$$\therefore \quad P(E_7) = \frac{n(E_7)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

(viii) Let E_8 = Event of getting sum 9 = $\{(6, 3), (6, 3)\}$

$$\therefore \quad n(E_8) = 2$$

$$\therefore \quad P(E_8) = \frac{n(E_8)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

Question 24:
A coin is tossed two times. Find the probability of getting atmost one head.

Solution:

The possible outcomes,if a coin is tossed 2 times is

$$S = \{(HH), (TT), (HT), (TH)\}$$

$\therefore$ $n(S) = 4$

Let E = Event of getting atmost one head

$$= \{(TT), (HT), (TH)\}$$

$\therefore$ $n(E) = 3$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{3}{4}$

Question 25:

A coin is tossed 3 times. List the possible outcomes. Find the probability of getting

(i) all heads (ii) atleast 2 heads

Solution:

The possible outcomes if a coin is tossed 3 times is

S = {(HHH), (TTT), (HTT), (THT), (TEH), (THH), (HTH), (HHT)}

(i) Let E_1 = Event of getting all heads = $\{(HHH)\}$

$\therefore$ $n(E_1) = 1$

$\therefore$ $P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{8}$

(ii) Let E_2 = Event of getting atleast 2 heads = $\{(HHT), (HTH), (THH), (HHH)\}$

$\therefore$ $n(E_2) = 4$

$\therefore$ $P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

Question 26:

Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.

Solution:

The total number of sample space in two dice, n (S) = 6 x 6 = 36

Let E = Event of getting the numbers whose difference is 2

$$= \{(1, 3), (2, 4), (3, 5), (4, 6), (3, 1), (4, 2), (5, 3), (6, 4)\}$$

$\therefore$ $n(E) = 8$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{8}{36} = \frac{2}{9}$

Question 27:

A bag contains 10 red, 5 blue and 7 green balls. A ball is drawn at random. Find the probability of this ball being a

(i) red ball (ii) green ball (iii) not a blue ball

solution:

if a ball is drawn out of 22 balls (5 blue + 7 green + 10 red), then the total number of outcomes are

$$n(S) = 22$$

(i) Let E_1 = Event of getting a red ball

$$n(E_1) = 10$$

$$\therefore \quad \text{Required probability} = \frac{n(E_1)}{n(S)} = \frac{10}{22} = \frac{5}{11}$$

(ii) Let E_2 = Event of getting a green ball

$$n(E_2) = 7$$

$$\therefore \quad \text{Required probability} = \frac{n(E_2)}{n(S)} = \frac{7}{22}$$

(iii) Let E_3 = Event getting a red ball or a green ball *i.e.*, not a blue ball.

$$n(E_3) = (10 + 7) = 17$$

$$\therefore \quad \text{Required probability} = \frac{n(E_3)}{n(S)} = \frac{17}{22}$$

Question 28:
The king, queen and jack of clubs are removed from a deck of 52 playing cards and then well shuffled. Now, one card is drawn at fandom from the remaining cards. Determine the probability that the card is
(i) a heart (ii) a king

Solution:
If we remove one king, one queen and one jack of clubs from 52 cards, then the remaining cards left, n(S) = 49

(i) Let E_1 = Event of getting a heart

$$n(E_1) = 13$$

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{13}{49}$$

(ii) Let E_2 = Event of getting a king

$n(E_2) = 3$ [since, out of 4 king, one club cards is already removed]

$$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{49}$$

Question 29:
Refer to Q.28. What is the probability that the card is
(i) a club (ii) 10 of hearts

Solution:

(i) Let E_3 = Event of getting a club

$n(E_3) = (13 - 3) = 10$

$\therefore$ Required probability $= \frac{n(E_3)}{n(S)} = \frac{10}{49}$

(ii) Let E_4 = Event of getting 10 of hearts

$n(E_4) = 1$

[because in 52 playing cards only 13 are the heart cards and only one10 in 13 heart cards]

$\therefore$ Required probability $= \frac{n(E_4)}{n(S)} = \frac{1}{49}$

Question 30:

All the jacks, queensapd kings are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 similar value for other cards, find the probability that the card has a value.

(i) 7 (ii) greater than 7(iii) Less than 7

Solution:

In out of 52 playing cards, 4 jacks, 4 queens and 4 kings are removed, then the remaining cards are left, n(S) = 52 – 3 x 4 = 40.

(i) Let E_1 = Event of getting a card whose value is 7

E = Card value 7 may be of a spade, a diamond, a club or a heart

$\therefore \quad n(E_1) = 4$

$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{40} = \frac{1}{10}$

(ii) Let E_2 = Event of getting a card whose value is greater than 7

= Event of getting a card whose value is 8, 9 or 10

$\therefore \quad n(E_2) = 3 \times 4 = 12$

$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{12}{40} = \frac{3}{10}$

(iii) Let E = Event of getting a card whose value is less than 7

= Event of getting a card whose value is 1, 2, 3, 4, 5 or 6

$\therefore \quad n(E_3) = 6 \times 4 = 24$

$\therefore \quad P(E_3) = \frac{n(E_3)}{n(S)} = \frac{24}{40} = \frac{3}{5}$

Question 31:

An integer is chosen between 0 and 100. What is the probability that it is

(i) divisible by 7? (ii) not divisible by 7?

Solution:
The number of integers between 0 and 100 is
n(S)= 99

(i) Let E_1 = Event of choosing an integer which is divisible by 7
= Event of choosing an integer which is multiple of 7
= {7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98}

$\therefore \quad n(E_1) = 14$

$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{14}{99}$

(ii) Let E_2 = Event of choosing an integer which is not divisible by 7

$\therefore \quad n(E_2) = n(S) - n(E_1)$
$= 99 - 14 = 85$

$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{85}{99}$

Question 32:
Cards with numbers 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has
(i) an even number (ii) a square number

Solution:
Total number of out comes with numbers 2 to 101, n(s) =100

(i) Let E_1 = Event of selecting a card which is an even number = {2, 4, 6,...100}
[in an AP, $l = a + (n-1)d$, here $l = 100$, $a = 2$ and $d = 2 \Rightarrow 100 = 2 + (n-1)2$
$\Rightarrow (n-1) = 49 \Rightarrow n = 50$]

$\therefore \quad n(E_1) = 50$

$\therefore \quad$ Required probability $= \frac{n(E_1)}{n(S)} = \frac{50}{100} = \frac{1}{2}$

(ii) Let E_2 = Event of selecting a card which is a square number
={4, 9, 16, 25, 36, 49, 64, 81, 100}
$= \{(2)^2, (3)^2, (4)^2, (5)^2, (6)^2, (7)^2, (8)^2, (9)^2, (10)^2\}$

$\therefore \quad n(E_2) = 9$

Hence, required probability $= \frac{n(E_2)}{n(S)} = \frac{9}{100}$

Question 33:
A letter of english alphabets is chosen at random. Determine the probability that the letter is a consonant
Solution:
We know that, in english alphabets, there are (5 vowels + 21 consonants)=26 letters. So, total number of outcomes in english alphabets

are, n(S) = 26

Let E = Event of choosing a english alphabet, which is a consonent

$= \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$

$\therefore \quad n(E) = 21$

Hence, required probability $= \frac{n(E)}{n(S)} = \frac{21}{26}$

Question 34:

There are 1000 sealed envelopes in a box, 10 of them contain a cash prize of ₹ 100 each, 100 of them contain a cash prize of ₹ 50 each and 200 of them contain a cash prize of ₹ 10 each and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out, what is the probability that it contains no cash prize?

Solution:

Total number of sealed envelopes in a box, n (S) = 1000

Number of envelopes containing cash prize = 10 + 100 + 200 = 310

Number of envelopes containing no cash prize,

$$n(E) = 1000 - 310 = 690$$

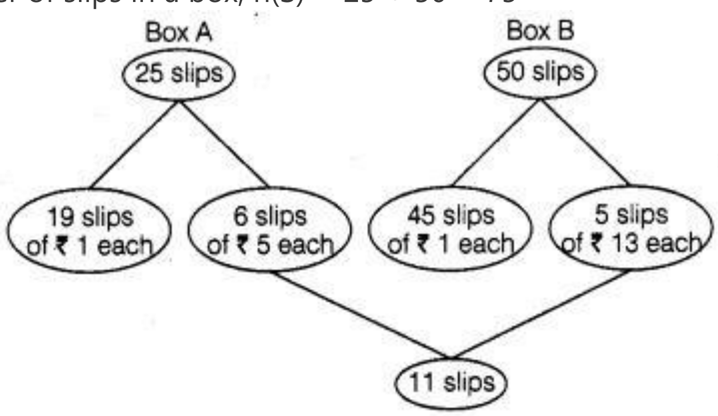
$$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{690}{1000} = \frac{69}{100} = 0.69$$

Question 35:

Box A contains 25 slips of which 19 are marked ₹ 1 and other are marked ₹ 5 each. Box B contains 50 slips of which 45 are marked ₹ 1 each and others are marked ₹ 13 each. Slips of both boxes are poured into a third box and resuffled. A slip is drawn at random. What is the probability that it is marked other than ₹ 1?

Solution:

Total number of slips in a box, n(S) = 25 + 50 = 75



From the chart it is clear that, there are 11 slips which are marked other than ₹ 1.

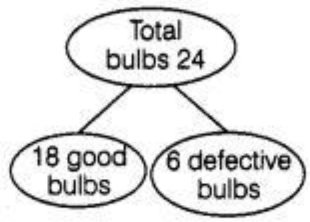
$$\therefore \quad \text{Required probability} = \frac{\text{Number of slips other than ₹ 1}}{\text{Total number of slips}} = \frac{11}{75}$$

Question 36:

A carton of 24 bulbs contain 6 defective bulbs. One bulb is drawn at random. What is the probability that the bulb is not defective? If the bulb selected is defective and it is not replaced and a second bulb is selected at random from the rest, what is the probability that the second bulb is defective?

Solution:

∴Total number of bulbs, n (S) = 24



Let E_1 = Event of selecting not defective bulb = Event of selecting good bulbs

$$n(E_1) = 18$$

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{18}{24} = \frac{3}{4}$$

Suppose, the selected bulb is defective and not replaced, then total number of bulbs remains in a carton, $n(S) = 23$.

In them, 18 are good bulbs and 5 are defective bulbs.

$\therefore P$ (selecting second defective bulb) $= \frac{5}{23}$

Question 37:

A child's game has 8 triangles of which 3 are blue and rest are red, and 10 squares of which 6 are blue and rest are red. One piece is lost at random. Find the probability that it is a

(i) triangle (ii) square(iii)square of blue colour (iv) triangle of red colour

Solution:

Total number of figures

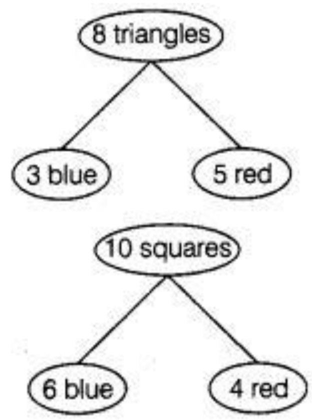
$$n(S) = 8 \text{ triangles} + 10 \text{ squares} = 18$$

(i) P (lost piece is a triangle) $= \frac{8}{18} = \frac{4}{9}$

(ii) P (lost piece is a square) $= \frac{10}{18} = \frac{5}{9}$

(iii) P (square of blue colour) $= \frac{6}{18} = \frac{1}{3}$

(iv) P (triangle of red colour) $= \frac{5}{18}$



Question 38:
In a game, the entry fee is of ₹ 5. The game consists of a tossing a coin 3 times. If one or two heads show, Sweta gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. For tossing a coin three times, find the probability that she
(i) loses the entry fee.
(ii) gets double entry fee.
(iii) just gets her entry fee.

Solution:
Total possible outcomes of tossing a coin 3 times,
S = {(HHH), (TTT), (HTT), (THT), (TTH), (THH), (HTH), (HHT)}
∴ n (S) = 8

(i) Let E_1 = Event that Sweta losses the entry fee
= She tosses tail on three times

$n(E_1) = \{(TTT)\}$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{8}$$

(ii) Let E_2 = Event that Sweta gets double entry fee
= She tosses heads on three times = $\{(HHH)\}$

$n(E_2) = 1$

$$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{8}$$

(iii) Let E_3 = Event that Sweta gets her entry fee back
= Sweta gets heads one or two times
= $\{(HTT), (THT), (TTH), (HHT), (HTH), (THH)\}$

$\therefore \quad n(E_3) = 6$

$$\therefore \quad P(E_3) = \frac{n(E_3)}{n(S)} = \frac{6}{8} = \frac{3}{4}$$

Question 39:
A die has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recorded.
(i) How many different scores are possible?
(ii) What is the probability of getting a total of 7?

Solution:
Given, a die has its six faces marked {0,1,1,1,6, 6}
Total sample space, $n(S) = 6^2 = 36$
(i) The different score which are possible are 6 scores e., 0,1,2,6,7 and12.

(ii) Let E = Event of getting a sum 7

$= \{(1,6),(1,6),(1,6),(1,6),(1,6),(1,6),(6,1),(6,1),(6,1),(6,1),(6,1),(6,1)\}$

$\therefore \quad n(E) = 12$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3}$

Question 40:

A lot consists of 48 mobile phones of which 42 are good, 3 have only minor defects and 3 have major defects. Varnika will buy a phone, if it is good but the trader will only buy a mobile, if it has no major defect. One phone is selected at random from the lot. What is the probability that it is

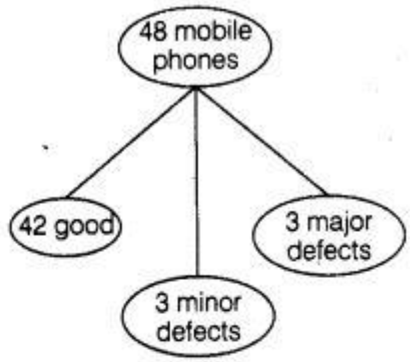
(i) acceptable to Varnika?

(ii) acceptable to the trader?

Solution:

Given, total number of mobile phones

$n(S) = 48$



(i) Let E_1 = Event that Varnika will buy a mobile phone

= Varnika buy only, if it is good mobile

$\therefore n(E_1) = 42$

$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{42}{48} = \frac{7}{8}$

(ii) Let E_2 = Event that trader will buy only when it has no major defects

= Trader will buy only 45 mobiles

$\therefore \quad n(E_2) = 45$

$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{45}{48} = \frac{15}{16}$

Question 41:

A bag contains 24 balls of which x are red, 2x are white and 3x am are. A ball is selected at random. What is the probability that it

(i) not red? (ii) white

Solution:

Given that, A bag contains total number of balls = 24 A bag contains number of red bails = 24

A bag contains number of white balls = 2x and a bag contains number of blue balls = x

By condition, $x + 2x + 3x = 24$

$\Rightarrow \quad 6x = 24$

$\therefore \quad x = 4$

∴Number of red balls = x = 4
Number of white balls = 2x = 2 x 4 = 8
and number of blue balls = 3x = 3 x 4 = 12
So, total number of outcomes for a ball is selected at random in a bag contains 24 balls.

$$\Rightarrow \quad n(S) = 24$$

(i) Let E_1 = Event of selecting a ball which is not red *i.e.*, can be white or blue.

∴ $n(E_1)$ = Number of white balls + Number of blue balls

$$\Rightarrow \quad n(E_1) = 8 + 12 = 20$$

$$\therefore \quad \text{Required probability} = \frac{n(E_1)}{n(S)} = \frac{20}{24} = \frac{5}{6}$$

(ii) Let E_2 = Event of selecting a ball which is white

∴ $n(E_2)$ = Number of white balls = 8

So, required probability $= \frac{n(E_2)}{n(S)} = \frac{8}{24} = \frac{1}{3}$

Question 42:
At a fete, cards bearing numbers 1 to 1000, one number on one card, are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square greater than 500, the player wins a prize. What is the probability that
(i) the first player wins a prize?
(ii) the second player wins a prize, if the first has won?

Solution:
Given that,, at a fete, cards bearing numbers 1 to 1000 one number on one card, are put in a box. Each player selects one card at random and that card is not replaced so, the total number of outcomes are n(S) = 1000

If the selected card has a perfect square greater than 500, then player wins a prize.

(i) Let E_1 = Event first player wins a prize = Player select a card which is a perfect square greater than 500

$= \{529, 576, 625, 676, 729, 784, 841, 900, 961\}$

$= \{(23)^2, (24)^2, (25)^2, (26)^2, (27)^2, (28)^2, (29)^2, (30)^2, (31)^2\}$

$\therefore \quad n(E_1) = 9$

So, required probability $= \dfrac{n(E_1)}{n(S)} = \dfrac{9}{1000} = 0.009$

(ii) First, has won *i.e.*, one card is already selected, greater than 500, has a perfect square. Since, repeatition is not allowed. So, one card is removed out of 1000 cards. So, number of remaining cards is 999.

$\therefore$ Total number of remaining outcomes, $n(S') = 999$

Let E_2 = Event the second player wins a rize, if the first has won.

= Remaining car s has a perfect square greater than 500 are 8.

$\therefore \quad n(E_2) = 9 - 1 = 8$

So, required probability $= \dfrac{n(E_2)}{n(S')} = \dfrac{8}{999}$

Exercise 13.4 Long Answer Type Questions

Question 1:

Find the mean marks of students for the following distribution

Marks	Number of students
0 and above	80
10 and above	77
20 and above	72
30 and above	65
40 and above	55
50 and above	43
60 and above	28
70 and above	16
80 and above	10
90 and above	8
100 and above	0

Solution:

Marks	Class marks (x_i)	Number of students (Cumulative frequency)	f_i	$f_i x_i$
0-10	5	80	3	15
10-20	15	77	5	75
20-30	25	72	7	175
30-40	35	65	10	350
40-50	45	55	12	540
50-60	55	43	15	825
60-70	65	28	12	780
70-80	75	16	6	450
80-90	85	10	2	170
90-100	95	8	8	760
100-110	105	0	0	0
				$\Sigma f_i x_i = 4140$

$$\text{Mean} = \frac{\Sigma f_i x_i}{N} = \frac{4140}{80} = 51.75$$

Question 2:

Marks	Number of students
Below 10	5
Below 20	9
Below 30	17
Below 40	29
Below 50	45
Below 60	60
Below 70	70
Below 80	78
Below 90	83
Below 100	85

Solution:

Here, we observe that, 5 students have scored marks below 10, i.e. it lies between class interval 0-10 and 9 students have scored marks below 20,

So, (9 – 5) = 4 students lies in the class interval 10-20. Continuing in the same manner, we get the complete frequency distribution table for given data.

Marks	Number of students (f_i)	Class marks (x_i)	$u_i = \frac{x_i - a}{h} = \frac{x_i - 45}{h}$	$f_i u_i$
0-10	5	5	−4	−20
10-20	$9-5=4$	15	−3	−12
20-30	$17-9=8$	25	−2	−16
30-40	$29-17=12$	35	−1	−12
40-50	$45-29=16$	$a=45$	0	0
50-60	$60-45=45$	55	1	15
60-70	$70-60=10$	65	2	20
70-80	$78-70=8$	75	3	24
80-90	$83-78=5$	85	4	20
90-100	$85-83=2$	95	5	10
	$N=\sum f_i=85$			$\sum f_i u_i=29$

Here, (assumed mean) $a = 45$
and (class width) $h = 10$
By step deviation method,

$$\text{Mean } (\bar{x}) = a + \frac{\sum f_i u_i}{\sum f_i} \times h = 45 + \frac{29}{85} \times 10 = 45 + \frac{58}{17}$$

$$= 45 + 3.41 = 48.41$$

Question 3:
Find the mean age of 100 residents of a town from the following data.

Age equal and above (in years)	0	10	20	30	40	50	60	70
Number of persons	100	90	75	50	25	15	5	0

Solution:
Here, we observe that, all 100 residents of a town have age equal and above 0. Since, 90 residents of a town have age equal and above 10.
So, 100 – 90 = 10 residents lies in the interval 0-10 and so on. Continue in this manner, we get frequency of all class intervals. Now, we construct the frequency distribution table.

Class interval	Number of persons (f_i)	Class marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0-10	$100 - 90 = 10$	5	-3	-30
10-20	$90 - 75 = 15$	15	-2	-30
20-30	$75 - 50 = 25$	25	-1	-25
30-40	$50 - 25 = 25$	$35 = a$	0	0
40-50	$25 - 15 = 10$	45	1	10
50-60	$15 - 5 = 10$	55	2	20
60-70	$5 - 0 = 5$	65	3	15
	$N = \sum f_i = 100$			$\sum f_i u_i = -40$

Here, (assumed mean) $a = 35$

and (class width) $h = 10$

By step deviation method,

$$\text{Mean } (\bar{x}) = a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$= 35 + \frac{(-40)}{100} \times 10$$

$$= 35 - 4 = 31.$$

< Hence, the required mean age is 31 yr.

Question 4:

The weights of tea in 70 packets are shown in the following table

Weight (in g)	**Number of packets**
200-201	13
201-202	27
202-203	18
203-204	10
204-205	1
205-206	1

Find the mean weight of packets.

Solution:

First,we find the class marks of the given data as follows,

Weight (in g)	Number of Packets (f_i)	Class marks (x_i)	Deviation ($d_i = x_i - a$)	$f_i d_i$
200-201	13	200.5	−3	−39
201-202	27	201.5	−2	−54
202-203	18	202.5	−1	−18
203-204	10	$a = 203.5$	0	0
204-205	1	204.5	1	1
205-206	1	205.5	2	2
	$N = \sum f_i = 70$			$\sum f_i d_i = -108$

Here, (assume mean) $a = 203.5$
and (class width) $h = 1$
By assumed mean method,

$$\text{Mean } (\bar{x}) = a + \frac{\sum f d_i}{\sum f_i}$$
$$= 203.5 - \frac{108}{70}$$
$$= 203.5 - 1.54 = 201.96$$

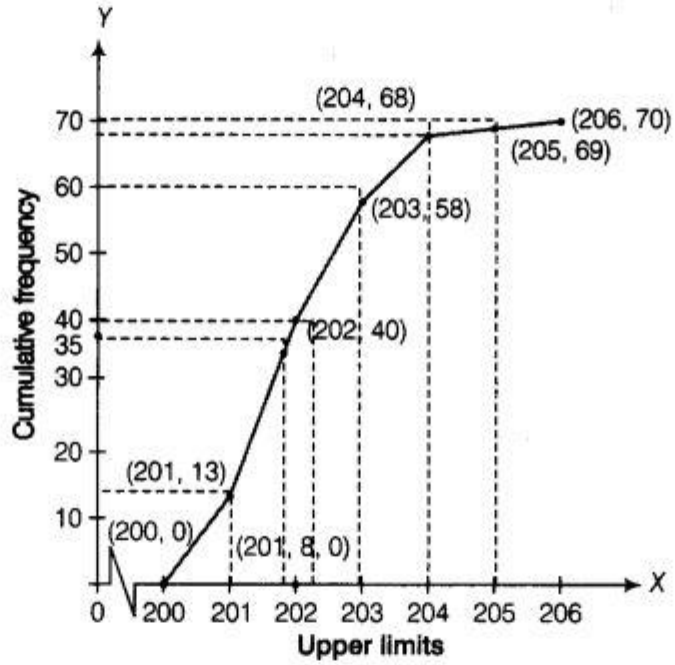
Hence, the required mean weight is 201.96 g.

Question 5:
Refer to Q.4 above. Draw the less than type ogive for this data and use it to find the median weight.

Solution:
We observe that, the number of packets less than 200 is 0, Similarly, less than 201 include the number of packets from 0-200 as well as the number of packets from 200-201.
So, the total number of packets less than 201 is 0 + 13 = 13. We say that, the cumulative frequency of the class 200-201 is 13. Similarly, for other class.

Less than type	
Weight (in g)	**Number of packets**
Less than 200	0
Less than 201	$0 + 13 = 13$
Less than 202	$27 + 13 = 40$
Less than 203	$18 + 40 = 58$
Less than 204	$10 + 58 = 68$
Less than 205	$1 + 68 = 69$
Less than 206	$1 + 69 = 70$

To draw the less than type ogive, we plot the points (200, 0), (201, 13), (202, 40) (203, 58), (204, 68), (205, 69) and (206, 70) on the paper and join by free hand, v Total number of packets (n) = 70



Now, $\frac{N}{2} = 35$

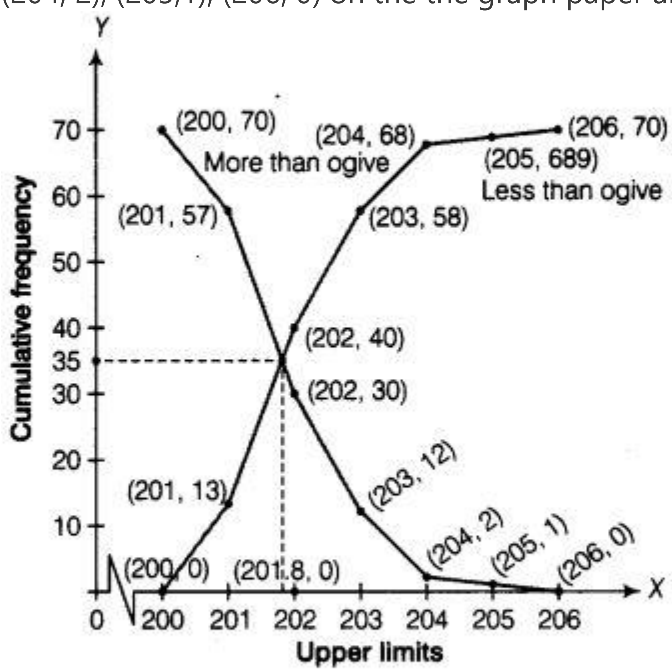
Firstly, we plot a point (0, 35) on Y-axis and draw a line y = 35 parallel to X-axis. The line cuts the less than ogive curve at a point. We draw a line on that point which is perpendicular to X-axis. The foot of the line perpendicular to X-axis is the required median.
Median weight = 201.8 g

Question 6:
Refer to Q.5 above. Draw the less than type and more than type ogives for the data and use them to find the meadian weight.

Solution:
For less than type table we follow the Q.5.
Here, we observe that, the weight of all 70 packets is more than or equal to 200. Since, 13 packets lie in the interval 200-201. So, the weight of 70 -13 = 57 packets is more than or equal to 201. Continuing in this manner we will get remaining more than or equal to 202, 203, 204, 205 and 206.

(i) Less than type		(ii) More than type	
Weight (in g)	**Number of packets**	**Number of packets**	**Number of students**
Less than 200	0	More than or equal to 200	70
Less than 201	13	More than or equal to 201	70 – 13 = 57
Less than 202	40	More than or equal to 202	57 – 27 = 30
Less than 203	58	More than or equal to 203	30 – 18 = 12
Less than 204	68	More than or equal to 204	12 – 10 = 2
Less than 205	69	More than or equal to 205	2 – 1 = 1
Less than 206	70	More than or equal to 206	1 – 1 = 0

To draw the less than type ogive, we plot the points (200, 0), (201,13), (202, 40), (203, 58), (204, 68), (205,69), (206, 70) on the paper and join them by free hand.
To draw the more than type ogive plot the points (200, 70), (201, 57), (202, 30), (203, 12), (204, 2), (205,1), (206, 0) on the the graph paper and join them by free hand.



Hence,required median weight = intersection point of x – axis = 201.8 g.

Question 7:

The table below shows the salaries of 280 persons.

Salary (in ₹ thousand)	Number of persons
5-10	49
10-15	133
15-20	63
20-25	15
25-30	6
30-35	7
35-40	4
40-45	2
45-50	1

caluculate the median and mode of the data.

Solution:

First, we construct a cumulative frequency table.

Salary (in ₹ thousand)	Number of persons (f_i)	Cumulative frequency (cf)
5-10	$49 = f_1$	$49 = cf$
10-15	$f_m = 133 = f$	$133 + 49 = 182$
15-20	$63 = f_2$	$182 + 63 = 245$
20-25	15	$245 + 15 = 260$
25-30	6	$260 + 6 = 266$
30-35	7	$266 + 7 = 273$
35-40	4	$273 + 4 = 277$
40-45	2	$277 + 2 = 279$
45-50	1	$279 + 1 = 280$
	$N = 280$	

$$\therefore \quad \frac{N}{2} = \frac{280}{2} = 140$$

(i) Here, median class is 10 – 15, because 140 lies in it.

Lower limit (l) = 10, Frequency (f) = 133,

Cumulative frequency (cf) = 49 and class width (h) = 5

$$\therefore \quad \text{Median} = l + \frac{\left(\frac{N}{2} - cf\right)}{f} \times h$$

$$= 10 + \frac{(140 - 49)}{133} \times 5$$

$$= 10 + \frac{91 \times 5}{133}$$

$$= 10 + \frac{455}{133} = 10 + 3.421$$

$$= ₹\ 13.421 \text{ (in thousand)}$$

$$= 13.421 \times 1000$$

$$= ₹\ 13421$$

(ii) Here, the highest frequency is 133, which lies in the interval 10-15, called modal class.

Lower limit (l) = 10, class width (h) = 5, f_m = 133, f_1 = 49, and f_2 = 63.

$$\therefore \quad \text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

$$= 10 + \left\{\frac{133 - 49}{2 \times 133 - 49 - 63}\right\} \times 5$$

$$= 10 + \frac{84 \times 5}{266 - 112} = 10 + \frac{84 \times 5}{154} = 10 + 2.727$$

$$= ₹\ 12.727 \text{ (in thousand)}$$

$$= 12.727 \times 1000 = ₹\ 12727$$

Hence,the median and modal salary are ₹13421 and ₹12727,respetively.

Question 8:

The mean of the following frequency distribution is 50 but the frequencies f_1 and f_2 in classes 20-40 and 60-80, respectively are not known. Find these frequencies, if the sum of all the frequencies is 120.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	f_1	32	f_2	19

Solution:

First we caluculate the class mark of given data

Class	Frequency (f_i)	Class marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0-20	17	10	-2	-34
20-40	f_1	30	-1	$-f_1$
40-60	32	$a = 50$	0	0
60-80	f_2	70	1	f_2
80-100	19	90	2	38
	$\Sigma f_i = 68 + f_1 + f_2$			$\Sigma f_i u_i = 4 + f_2 - f_1$

Given that, sum of all frequencies = 120

$\Rightarrow \quad \Sigma f_i = 68 + f_1 + f_2 = 120$

$\Rightarrow \quad f_1 + f_2 = 52 \quad$ ...(i)

Here, (assumed mean) $a = 50$

and (class width) $h = 20$

By step deviation method,

$$\text{Mean} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\Rightarrow \quad 50 = 50 + \frac{(4 + f_2 - f_1)}{120} \times 20$$

$\Rightarrow \quad 4 + f_2 - f_1 = 0$

$\Rightarrow \quad -f_2 + f_1 = 4 \quad$ ...(ii)

On adding Eqs. (i) and (ii), we get

$2f_1 = 56$

$\Rightarrow \quad f_1 = 28$

Put the value of f_1 in Eq. (i), we get

$f_2 = 52 - 28$

$\Rightarrow \quad f_2 = 24$

Hence, $f_1 = 28$ and $f_2 = 24$.

Question 9:

The median of the following data is 50. Find the values of p and q, if the sum of all the frequencies is 90.

Marks	Frequency
20-30	p
30-40	15
40-50	25
50-60	20
60-70	q
70-80	8
80-90	10

Solution:.

Marks	Frequency	Cumulative frequency
20-30	p	p
30-40	15	$15+p$
40-50	25	$40+p=cf$
50-60	$20=f$	$60+p$
60-70	q	$60+p+q$
70-80	8	$68+p+q$
80-90	10	$78+p+q$

Given, $N=90$

$\therefore$ $\frac{N}{2}=\frac{90}{2}=45$

which lies in the interval 50-60.

Lower limit, $l=50, f=20, cf=40+p, h=10$

$$\therefore \quad \text{Median} = l+\frac{\left(\frac{N}{2}-cf\right)}{f}\times h$$

$$=50+\frac{(45-40-p)}{20}\times 10$$

$$\Rightarrow \quad 50=50+\left(\frac{5-p}{2}\right)$$

$$\Rightarrow \quad 0=\frac{5-p}{2}$$

$$\therefore \quad p=5$$

Also, $78+p+q=90$ [given]

$\Rightarrow$ $78+5+q=90$

$\Rightarrow$ $q=90-83$

$\therefore$ $q=7$

Question 10:

The distribution of heights (in cm) of 96 children is given below

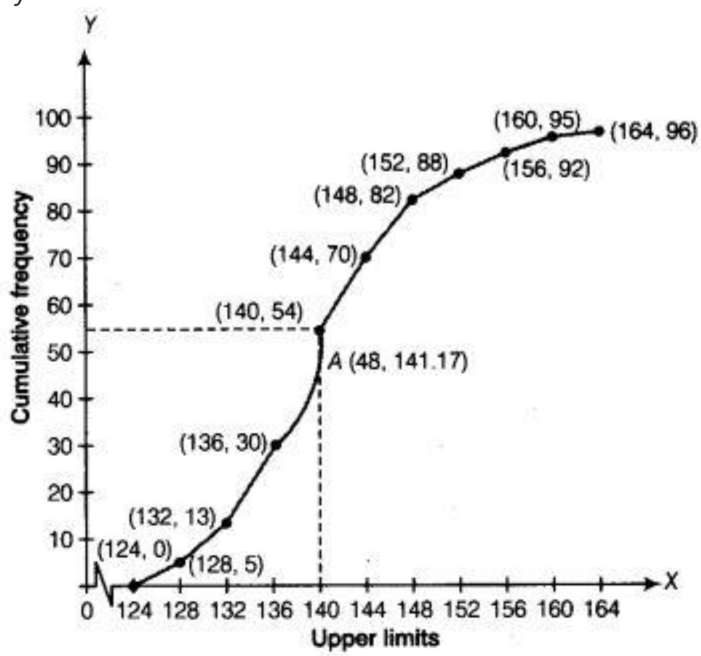
Height (in cm)	**Number of children**
124-128	5
128-132	8
132-136	17
136-140	24
140-144	16
144-148	12
148-152	6
152-156	4
156-160	3
160-164	1

Draw a less than type cumulative frequency curve for this data and use it to compute median height of the children.

Solution:

Height (in cm)	Number of children
Less than 124	0
Less than 128	5
Less than 132	13
Less than 136	30
Less than 140	54
Less than 144	70
Less than 148	82
Less than 152	88
Less than 156	92
Less than 160	95
Less than 164	96

To draw the less than type ogive, we plot the points (124, 0), (128, 5), (132, 13), (136, 30), (140, 54), (144, 70), (148, 82), (152, 88), (156, 92), (160, 95), (164, 96) and join all these point by free hand.



Here, $\frac{N}{2} = \frac{96}{2}$

We take, y = 48 in Y-coordinate and draw a line parallel to X-axis, meets the curve at A and draw a perpendicular line from point A to the X-axis and this line meets the X-axis at the point which is the median i.e., median = 141.17.

Question 11:
Size of agricultural holdings in a survey of 200 families is given in the following
Compute median and mode size of the holdings.

Size of agricultural holdings (in hec)	**Number of families**
0-5	10
5-10	15
10-15	30
15-20	80
20-25	40
25-30	20
30-35	5

Solution:

Size of agricultural holdings (in hec)	**Number of families** (f_i)	**Cumulative frequency**
0-5	10	10
5-10	15	25
10-15	30	55
15-20	80	135
20-25	40	175
25-30	20	195
30-35	5	200

(i) Here, $N = 200$

Now, $\frac{N}{2} = \frac{200}{2} = 100$, which lies in the interval 15-20.

Lower limit, $l = 15$, $h = 5$, $f = 80$ and $cf = 55$

$$\therefore \quad \text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h = 15 + \left(\frac{100 - 55}{80}\right) \times 5$$

$$= 15 + \left(\frac{45}{16}\right) = 15 + 2.81 = 17.81 \text{ hec}$$

(ii) In a given table 80 is the highest frequency.

So, the modal class is 15-20.

Here, $l = 15, f_m = 80, f_1 = 30, f_2 = 40$ and $h = 5$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times h$$

$$= 15 + \left(\frac{80 - 30}{2 \times 80 - 30 - 40}\right) \times 5$$

$$= 15 + \left(\frac{50}{160 - 70}\right) \times 5$$

$$= 15 + \left(\frac{50}{90}\right) \times 5 = 15 + \frac{25}{9}$$

$$= 15 + 2.77 = 17.77 \text{ hec}$$

Question 12:

The annual rainful record of a city for 66 days is given in the following table.

Rainfall (in cm)	0-10	10-20	20-30	30-40	40-50	50-60
Number of days	22	10	8	15	5	6

Calculate the median rainfall using ogives (or move than type and of less than type)

Solution:

We observe that, the annual rainfall record of a city less than 0 is 0. Similarly, less than 10 include the annual rainfall record of a city from 0 as well as the annual rainfall record of a city from 0-10.

So, the total annual rainfall record of a city for less than 10 cm is 0+ 22 =22 days.

Continuing in this manner, we will get remaining less than 20, 30, 40, 50, and 60.

Also, we observe that annual rainfall record of a city for 66 days is more than or equal to 0 cm. Since, 22

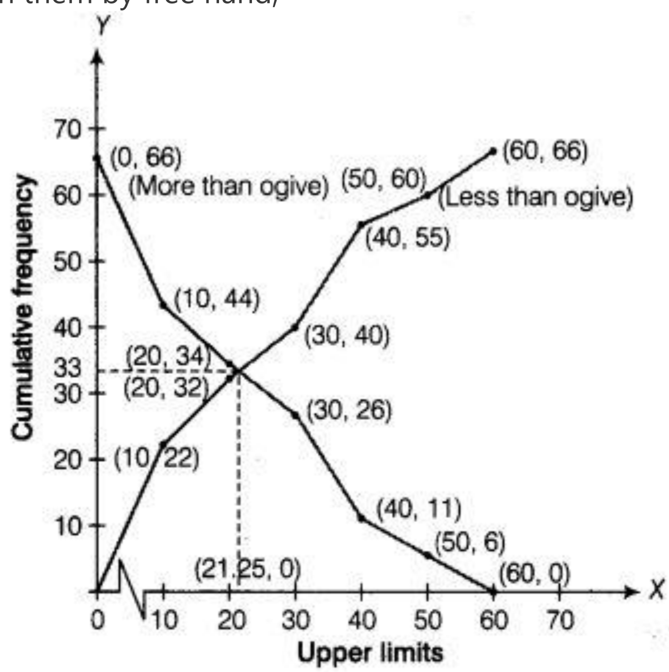
days lies in the interval 0-10. So, annual rainfall record for 66-22 = 44days is more than or equal to 10 cm.

Continuing in this manner we will get remaining more than or equal to 20, 30, 40, 50 and 60.

Now, we construct a table for less than and more than type.

(i) Less than type		(ii) More than type	
Rainfall (in cm)	Number of days	Rainfall (in cm)	Number of days
Less than 0	0	More than or equal to 0	66
Less than 10	$0+22=22$	More than or equal to 10	$66-22=44$
Less than 20	$22+10=32$	More than or equal to 20	$44-10=34$
Less than 30	$32+8=40$	More than or equal to 30	$34-8=26$
Less than 40	$40+15=55$	More than or equal to 40	$26-15=11$
Less than 50	$55+5=60$	More than or equal to 50	$11-5=6$
Less than 60	$60+6=66$	More than or equal to 60	$6-6=0$

To draw less than type ogive we plot the points (0, 0), (10, 22), (20, 32), (30, 40), (40, 55), (50, 60), (60, 66) on the paper and join them by free hand.To draw the more than type ogive we plot the points (0, 66), (10, 44), (20, 34), (30, 26), (40, 11), (50, 6) and (60, 0) on the graph paper and join them by free hand,



∵ Total number of days $(n) = 66$

Now, $\frac{n}{2} = 33$

Firstly, we plot a line parallel to X-axis at intersection point of both ogives, which further intersect at (0, 33) on Y-axis. Now, we draw a line perpendicular to X-axis at intersection point of both ogives, which further intersect at (21.25, 0) on X-axis. Which is the required

median using ogives.
Hence, median rainfall = 21.25 cm.

Question 13:
The following is the frequency distribution of duration for 100 calls made on a mobile phone.

Duration (in s)	Number of calls
95-125	14
125-155	22
155-185	28
185-215	21
215-245	15

Solution:
First, we calculate class marks as follows

Duration (in s)	Number of calls (f_i)	Class marks (x_i)	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
95-125	14	110	−2	−28
125-155	22	140	−1	−22
155-185	28	$a = 170$	0	0
185-215	21	200	1	21
215-245	15	230	2	30
	$\Sigma f_i = 100$			$\Sigma f_i u_i = 1$

Here, (assumed mean) a = 170,
and (class width) h = 30
By step deviation method,

$$\text{Average } (\bar{x}) = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h = 170 + \frac{1}{100} \times 30$$

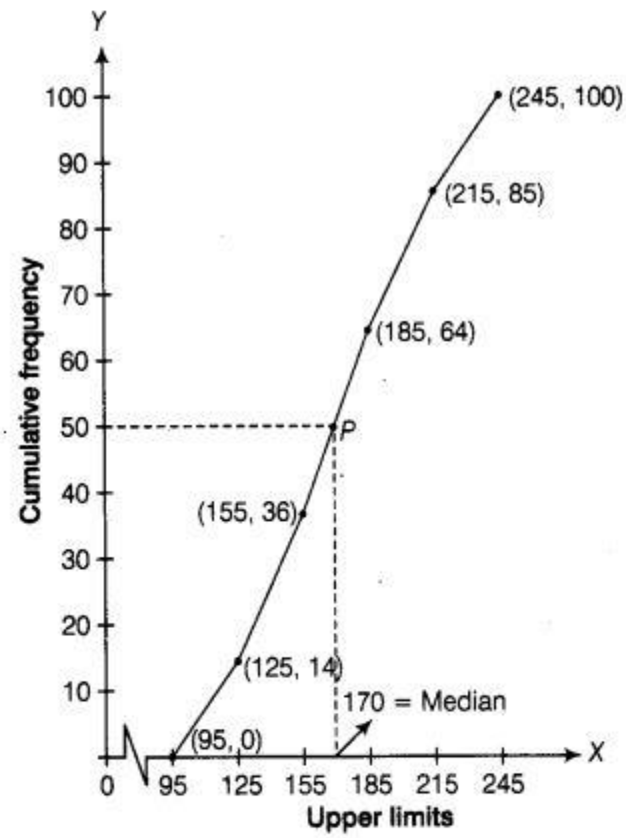
$$= 170 + 0.3 = 170.3$$

Hence, average duration is 170.3s.

For calculating median from a cumulative frequency curve
We prepare less than type or more than type ogive
We observe that, number of calls in less than 95 s is 0. Similarly, in less than 125 s include the number of calls in less than 95 s as well as the number of calls from 95-125.s So, the total number of calls less than 125 s is 0 + 14 = 14. Continuing in this manner, we will get remaining in less than 155,185, 215 and 245 s.
Now, we construct a table for less than ogive (cumulative frequency curve).

Less than type	
Duration (in s)	**Number of calls**
Less than 95	0
Less than 125	0 + 14 = 14
Less than 155	14 + 22 = 36
Less than 185	36 + 28 = 64
Less than 215	64 + 21 = 85
Less than 245	85 + 15 = 100

To draw less than type ogive we plot them the points (95, 0), (125, 14) (155, 36), (185, 64), (215, 85), (245, 100) on the paper and join them by free hand.



∵ Total number of calls $(n) = 100$

∴ $$\frac{n}{2} = \frac{100}{2} = 50.$$

Now, point 50 taking on Y-axis draw a line parallel to X-axis meet at a point P and draw a perpendicular line from P to the X-axis, the intersection point of X-axis is the median. Hence, required median is 170 .

Question 14:

50 students enter for a school javelin throw competition. The distance (in metre) thrown are

recorded below

Distance (in m)	0-20	20-40	40-60	60-80	80-100
Number of students	6	11	17	12	4

(i) Construct a cumulative frequency table.
(ii) Draw a cumulative frequency curve (less than type) and calculate the median distance drawn by using
this curve.
(iii) Calculate the median distance by using the formula for median.
(iv) Are the median distance calculated in (ii) and (iii) same?

Solution:
(i)

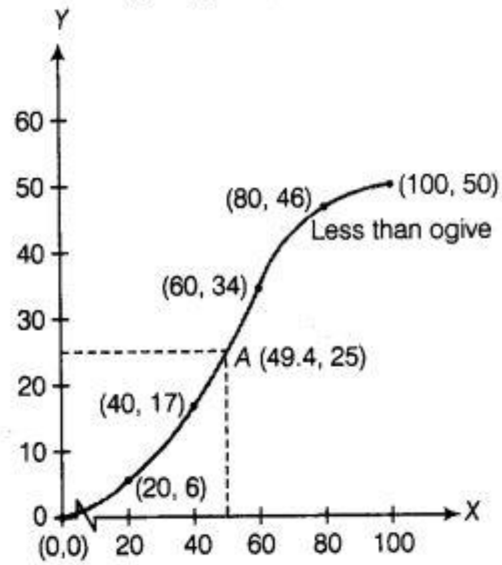
Distance (in m)	Number of students (f_i)	Cumulative frequency (cf)
0-20	6	6
20-40	11	17
40-60	17	34
60-80	12	46
80-100	4	50

(ii)

Distance (in m)	Cumulative frequency
0	0
Less than 20	6
Less than 40	17
Less than 60	34
Less than 80	46
Less than 100	50

To draw less than type ogive, we plot the points (0, 0), (20, 6), (40,17), (60, 34), (80, 46), (100, 50), join all these points by free hand.

Now, $$\frac{N}{2} = \frac{50}{2} = 25$$



Taking Y = 25 on y-axis and draw a line parallel to X-axis, which meets the curve at point A From point A we draw a line perpendicular to X-axis, where this meets that point is the required median i.e., 49.4.

(III) Now, $$\frac{N}{2} = \frac{50}{2} = 25$$

which lies is the interval 40-60.

$\therefore$ $$l = 40, h = 20, cf = 17 \text{ and } f = 17$$

$\therefore$ $$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f}\right) \times h$$

$$= 40 + \frac{(25 - 17)}{17} \times 20$$

$$= 40 + \frac{8 \times 20}{17}$$

$$= 40 + 9.41$$

$$= 49.41$$

(lv) Yes, median distance calculated by parts (ii) and (iii) are same.