PERMUTATIONS AND COMBINATIONS

- **Fundamental principle of counting** : If an event can occur in m different ways, following which another event can occur in n different ways, then the total number of occurence of the events in the given order is $m \times n$.

- **Permutations** : The number of permutations of n different things taken r at a time, where repetition is not allowed, is denoted by ^nP_r.

$$^nP_r = \frac{n!}{(n-r)!} \quad \text{where } 0 \le r \le n$$

- $n! = 1 \times 2 \times 3 \times \ldots \times n$

- $n! = n \times (n-1)!$ Factorial Notation (!) Ex: $3! = 1 \times 2 \times 3 = 6$

Theorem 1 : The number of permutations of n different things, taken r at a time, where $0 < r \le n$ and the objects do not repeat is $n(n-1)(n-2)\ldots(n-r+1)$ which is denoted by ^nP_r.

Theorem 2 : The number of permutations of n different things, taken r at a time, where repetition is allowed, is n^r.

Theorem 3 : The number of permutations of n objects, where p objects are of the same kind and rest are all different $= \frac{n!}{p!}$

Theorem 4 : The number of permutations of n objects taken all at a time, where p_1 objects are of first kind, p_2 objects are of the second kind, ..., p_k objects are of the k^{th} kind and rest, if any, are all different is $\frac{n!}{p_1!\,p_2!\ldots p_k!}$

- **Combinations** : The number of combinations of n different things taken r at a time, denoted by ^nC_r

$$^nC_r = \frac{n!}{r!(n-r)!} \;;\; 0 \le r \le n$$

Theorem 5 : $^nP_r = {^nC_r}\, r! \;;\; 0 < r \le n$

Theorem 6 : $^nC_r + {^nC_{r-1}} = {^{n+1}C_r}$

Note :

1. From above $\frac{n!}{(n-r)!} = {^nC_r} \times r!$, i.e. $^nC_r = \frac{n!}{r!(n-r)!}$

   In particular, if $r = n$, $^nC_n = \frac{n!}{n!\,0!} = 1$

2. We define $^nC_0 = 1$, i.e., the number of combinations of n different things taken nothing at all is considered to be 1. Counting combinations is merely counting the no. of ways in which some or all objects at a time are selected. Selecting nothing at all is the same as leaving behind all the objects and we know that there is only one way of doing so. This way we define $^nC_0 = 1$.

3. As $\frac{n!}{0!(n-0)!} = 1 = {^nC_0}$, the formula $^nC_r = \frac{n!}{r!(n-r)!}$ is applicable for $r = 0$ also. Hence $^nC_r = \frac{n!}{r!(n-r)!} \;;\; 0 \le r \le n$.

4. $^nC_{n-r} = \frac{n!}{(n-r)!(n-(n-r))!} = \frac{n!}{(n-r)!\,r!} = {^nC_r}$ i.e., selecting r objects out of n objects is same as rejecting $(n-r)$ objects.

5. $^nC_a \Rightarrow {^nC_a} \Rightarrow a = b$ or $a = n-b$, i.e., $n = a+b$