

## LESSON 6

### PRODUCTION FUNCTION

#### Introduction:

Generally, there is a direct relationship between the prices and supply of goods & services. There is increase in the supply of goods and services with increase in their prices and vice versa. The supply of goods and services in an economy depends upon their productions, which in turn depends on two things—

1. The prices of factors of productions or input.
2. The physical quantitative relationship between input and output.

Hence, it is essential to study the economic analysis of the relationship between input and output. Various economic variables are closely related as in demand–function, demand of good and price of goods and services are related and in supply-function, supply of goods and service are related to their prices, in the same way in production-function, there is relationship between production(output) and factors of production(input) for instance labour, capital, land, management, technology, entrepreneurship (L, K, N, T, E) are related to the production(output).

#### Meaning of Function -

‘Function’ is a technical term of maths. It is a quantitative relationship between two variables (independent and dependent variable), for example,  $Y = f(X)$  is expressed as Y is the function of X. It means Y, which a dependent variable, is quantitatively related to independent variable X. Here, f is a symbol for function.

In words of Alfa. C. Chiang, “Function in a specific order is a group of combination of variables (independent and dependent). The characteristic of this function is that any value of X determines the unique value of Y”.

#### Meaning of production function:-

Production function is a quantitative relationship. For instance one labour, machine worth ten thousand and 20 feet long and 20 feet broad land is required

to produce 1 meter cloth. Then the relationship between 1 meter cloth and one labour, machine worth ten thousands and 20 feet long and 20 feet broad land will be called as quantitative relation. This production function can be expressed as follow:

$$Q_{1 \text{ Meter Cloth}} = f(20 \times 20 \text{ Land, } 1 \text{ L, Rs.10,000 K})$$

where, Q= output, L= labour, K= machine

#### Definition of production function-

Production function has been defined by many economists. Some of the definitions are as follows:-

Henderson and Quandt- “Production function is an engineering concept, which explains relationship between inputs and outputs.

Dr. Balwant Kandoi- “If the quantity of production by a firm is Q, when factors of production, labour, capital, land, management technique, and enterprenur or enterprise (Ld, L, K, O) are put to production , then we will write this production function as  $Y = f(Ld., L, K, O)$ ”

N. Gregory Mankiw- “The relationship between the quantity of factors of production and quantity of production is known as production function”.

#### Assumptions of production function-

As we have seen previously assumptions are certain fundamental and essential conditions on which laws and theories are based. The fulfillment of these conditions are necessary to prove the validity of any law or theory, as such certain conditions are required to be fulfilled to prove the validity of the theory of production functions.

Production function is based on following assumptions:

1. Production function assumes technology as given.
2. The price of factor inputs are given.

3. It is related to a particular period of time.
4. The ratio between factor inputs can be changed to a certain extent.
5. The units of factors of productions are homogeneous.
6. The factors of production are variable.
7. The process of change in factors of production is done gradually.
8. The substitution between factors of production can be done upto certain limit.
9. In a short period the supply of fixed factor is inelastic.
10. The objective of firm is to maximise the profit and production.
11. The factors of production are used efficiently  
If there is any change in the above assumptions, then the production function ought to be changed.

### **Characteristics of production function-**

On the basis of various definitions, following are the features of production function-

1. Production function is an engineering concept.
2. Production function is related to flow of factors and production.
3. It depicts the relation of transformation of the inputs into output.
4. It depicts the physical quantity of production produced by the factor input .
5. It is related to specific period of time.
6. One unit of labour or one unit of capital can be substituted by their other units.
7. It is related to a given particular technology.
8. Production function includes only physical quantity of production and inputs, their prices are not included.
9. On the basis of time production function can be classified as long-run and short-run.

After knowing the above features it is necessary to know the changes in production, with the help of production function we can know the short run and

long run changes in production. On basis of time, changes in factors of production & their ratios are considered.

### **Difference between short run and long run production function-**

The main difference between short run and long run production function is regarding the input ratio. In short run, change in production occurs according to the conditions of short run production function. In short run, the ratio between fixed and variable factors changes with change in production. In the long run all the factors of production are changed in same proportion or ratio. The second difference between both types of production function is related to change in technology. In the short run, there is no change in the given technology, whereas in the long run, technology can be changed.

On the basis of time there are two types of production function-

1. Production function of fixed proportion.
2. Production function of variable proportion.

### **Production function of fixed proportion-**

In the long run all the factors of the production are variable. The long run production function is also called “fixed proportion production function.” and is related to “law of returns to scale”.

In the phrase “return to scale” it is necessary to know the meaning of scale. Here ‘Scale’ means a unit of measurement like metre, litre, kilogram, feet, etc. For instance a man buys 1 metre of cloth but if the unit of measurement is changed to centimetres then it is said that the man bought 100 centimetres of cloth. In this way with change in unit (scale) we show the measurement in the changed unit (scale).

Similarly for instance, 5 labourers with 2 acre of land produce 10 quintals of wheat. Hence here acre is the unit (scale) of measurement of land. Quintal is unit (scale) of measurements of output of

it is evident that there is twice increase in both the factors of production, land and labour and production measurement units ( scale). Hence when both factors of production are doubled, the production also doubles. Though there is increase in input and output in same proportion the ratio or coefficient of the output and necessary factors does not change. In every condition the ratio of land and labour remains 1:25 when in a land of 2 acres, 5 labourers produces 10 quintals wheat then, the proportion between land and output is 1:5 and proportion of labour and output is 1:2. On changing the amount of land and labour, the output also doubles, but ratio of land and labour remains 1:25 and of land and output ratio 1:5 and labour and output ratio 1:2 remains stable as earlier. The fixed proportion production function is explained in table 6.1 and figure 6.1 -

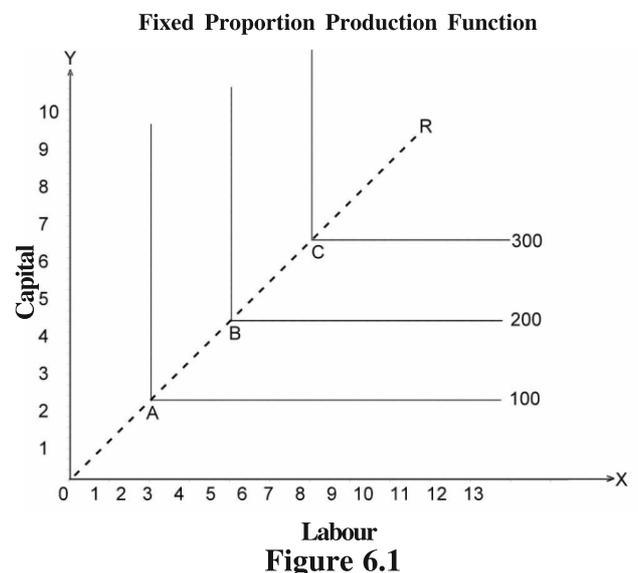
**Table 6.1**

**Change in factor inputs under Return to scale**

Land (in hectares)	Changes in of land Quantity	Labour hours (time)	Changes in labour
5	—	100	—
10	2 times	200	2 times
15	3 times	300	3 times
20	4 times	400	4 times
25	5 times	500	5 times
30	6 times	600	6 times
35	7 times	700	7 times
40	8 times	800	8 times

In fixed proportion production function the two factors must be used in fixed ratio such as 10% or 20 % or 200% In the above table 6.1 and figure 6.1 below we see that initially the quantity of land is 5 hectares, and quantity of labour is 100 man-hour. The quantity of land is increased to 10 hectares and labour to 200 man-hour. Thus when the quantity of land is changed 2 times, 3 times, and 8 times. The quantity of man hour is also changed, respectively 2, 3 and 8 times. Thus changing all factors of production in same ratio ensures that the ratio between them also remains

the same. Whatever the level of production, here labour cannot be substituted for capital. This is called the situation of fixed proportion production function.



**Figure 6.1**

There are various types of returns to scale in fixed proportion production function. For instance 1 unit labourer+1 acre of land produces 2 quintal of wheat. If the factors are doubled *i.e.* 2 labourer+2 land and production increase to 6 quintal, then this situation is called as increasing return to scale. The proportionate change in production is more than the proportionate change in the factor input. If on doubling the ratio of factors of production, the production increases to three quintals wheat, then it is called as “decreasing return to scale”. As the proportionate change in production is less than proportionate change in factors. When there is change in factor of production and production in same proportion, it is called “constant return to scale”. If 2 labourer + 2 acre land yield 4 quintal of wheat then it is the situation of constant return to scale.

Return to scale are related to long run, when all the factors of production are variable.

**Short run production function-**

The short run production function refers to the time period when the output can be changed only by changes in variable factors. The process of change in production in short run, is called short run production

function, which has various names, diminishing marginal production or law of variable proportion.

The short run production function is mainly known by two names “law of diminishing marginal production” and “law of variable proportion”. The word law of diminishing marginal production was first used by French economist Turgot. Later it was used by Marshall and David Ricardo in their theories. Marshall discussed the law of diminishing marginal production in relation to agriculture. Mrs. John Robinson also used this term and according to her the diminishing returns was due to imperfect substitution amongst the factors of production. Later on Stigler, Benham and present economists Pindyck and Rubinfeld coined the new name, “diminishing marginal production after certain limit”. Richard G Lipsey, and K.A. Crystal also used these words. K.E. Boulding described this law, as law of eventually diminishing marginal physical productivity. E.H. Chamberlin mentioned, that indivisibility of factors of production and change factor ratio are the chief determinants of changes of production in short run. Logical discussions continued amongst A.N. Mgleod, F.H. Hon, Thompson M Python, Morris H Paston, L. Harvey, Leibensleen and E.H. Chamberlin. Thus the behaviour of outputs in short run came to be known as law of variable proportion.

In the short run quantity of one factor changes, keeping the quantity of other factor constant shows the proportional changes in factors of productions. Thus the short period production function is called as law of variable proportion which is illustrated by table 6.2 given below.

**TABLE 6.2**

(Land In Hectare.)	Man Hours	Total Production
5	0	0
5	1	2
5	2	6
5	3	12
5	4	18
5	5	20
5	6	20
5	7	14

It is clear from table 6.2 that land is fixed and labour is variable. The proportion of both factors are

5:0, 5:1, 5:2, 5:3, 5:4, 5:5, 5:6 and 5:7 respectively. Similarly the proportion between land and output are 5:0, 5:2, 5:6, 5:12, 5:18, 5:20 and 5:14 respectively. In this way there is change in the proportion between factors and minimum factors required changes.

**The conclusions are –**

1. Initially the short period production was called as law of diminishing marginal productivity which in recent times is called as law of variable proportion.
2. This law is applicable only in short run.
3. In short run production theory only the variable factor (labour) changes.
4. It is possible to change the combinations of factor ratio in the production theory. These changes in factor ratio is not visualised in law of return to scale.

**Returns to Outlays -**

In returns to scale all factors change proportionately. There is no change in proportion or ratio of factors of production. For example factors cannot be changed as follows capital by 10% land by 20% or labourer by 200 or 300%, The expenses incurred on these factors of production is changed in equal or in different ratios then it is called as returns to outlays.

There is difference between returns to scale and returns to outlay. In returns to scale the proportion (ratio) combination of factors is constant, but in returns to outlays, the proportion combination of factors changes. The long run production function is shown as  $O = f(L, lab, K, T, E)$ . These factors do not have bars on top which shows that all factors of production are variable in the long run.

**Types of production function:**

In economics various production functions are well described. The economic problems are analysed with help of various production functions. Few production functions have been developed as improvement over the previous production functions.

In economics there are various forms of production

functions, some important ones are as follows:-

1. Linear Homogeneous production function.
2. Cobb-Douglas production function.
3. Input - output production function.
4. Activity - Analysis production function.
5. Constant elasticity of substitution production function (CES).
6. Variable elasticity of substitution production function.(VES)
7. Transcendental logarithmic production function.

The short run and long run production are differentiated. The output changes differently with changes in factors of production. The short run and long run changes in output are referred as -

1. Law of variable proportion.
2. Law of return to scale, respectively.

### Importance of production function:

Production function is related to engineering. In economics it is necessary to know the various alternative production function of any good or services to take decision regarding the efficient production. By comparison of various production function appropriate decision, the knowledge or perception of production function is very necessary.

### Important points

- Quantity of goods produced depends upon the following two factors:
  - 1) Prices of inputs or factors of production, and
  - 2) Physical or quantitative relationship between inputs and outputs.
- Normally, function means quantitative relationship between two variables (dependent and independent).
- According to Henderson and Quandt “Production function is an engineering concept which explains quantitative relationship between inputs and outputs with given technology”.
- In short run production function, only labour is variable and other factors of production are

constant. Therefore, it is denoted by drawing bar on the upper part of all fixed factors.

- For the first time in 1947, Prof Chamberlin gave the name of “law of variable proportion” to short run production function.
- Production function, in the long run, is known as returns to scale. In the long run, all factors are variable, however, improvement in technology is not considered. In return as to scale, the term scale means unit of measurement, like metre, litre, kilogram, acre etc.
- In short run and long run, change in quantity of production is known as:- Laws of variable proportions of factors of production. Laws of Returns to scale, respectively.
- Although production function has direct relationship with engineering, but in economics, it is important to have information about different alternatives of production functions while taking decision regarding production of goods and services at optimum output.

### Excercise Questions

#### Objective Type Questions :-

- 1) Production function is a function of which two variables?
  - (A) Inputs and outputs
  - (B) Demand and price
  - (C) Supply and price
  - (D) Consumption and Income
- 2) What type of relationship exists between inputs and outputs in production function?
  - (A) Quantitative
  - (B) Qualitative
  - (C) Economic
  - (D) None of these
- 3) On the basis of time period, production functions are:
  - (A) Short run
  - (B) Long run

- (C) Middle run  
 (D) Both (a) and (b)
- 4) Who did not use the term “law of diminishing marginal product”?  
 (A) Mrs. Joan Robinson  
 (B) Marshall  
 (C) Stigler  
 (D) E.H. Chamberlin
- 5) In the production function  $P = f(L, \bar{K}, \bar{N}, \bar{T}, \bar{E})$ , what does the line drawn above the factors of production mean?  
 (A) Factors below the straight line are variable.  
 (B) Factors below the straight line are constant.  
 (C) Factors below the straight line are homogenous.  
 (D) None of these

**Very Short Answer Type Questions :-**

- 1) What is function?
- 2) What is production function?
- 3) According to time period, what are the types of production functions?

- 4) What does input mean?
- 5) What is meant by scale?

**Short Answer Type Questions :-**

- 1) Explain in brief the concept of production function.
- 2) Explain in brief the characteristics of production function.
- 3) Mention the assumptions of production function.
- 4) From Law of Diminishing marginal production or Law of variable proportions of factors of production which name of the law, according to you, is appropriate and why? Explain in brief .
- 5) Explain in brief the difference between returns to scale and returns to outlay.

**Essay Type Questions :-**

- 1) Explain in detail the concept of production function
- 2) While differentiating the short run and long run production function, explain each in detail.

**Answer sheet**

<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>
<b>A</b>	<b>A</b>	<b>D</b>	<b>D</b>	<b>B</b>