CLASS : XI CHAPTER-8 BINOMIAL THEOREM

EXERCISE : 8.2

Q.No.1 Find the coefficient of x^5 in $(x+3)^8$

The general Term of Expansion $(a+b)^n$ is given by

$$T_{r+1} = {}^nC_r\, a^{n-r}\, b^r.$$

Now Let x^5 occur in T_{r+1} of expansion $(x+3)^8$

The $T_{r+1} = {}^8C_r\, x^{8-r}\, 3^r.$

$\therefore\; 8-r = 5 \Rightarrow r = 3.$

$\therefore$ Coeff. of $x^5 = {}^8C_5 (3)^3 = \dfrac{8\times7\times6\times5\times4}{1\times2\times3\times4\times5}\times 27 = 1512.$

Q.No.2 Find Coeff of. a^5b^7 in $(a-2b)^{12}$.

Sol. Let a^5b^2 occur in T_{r+1} of expansion $(a-2b)^{12}$

$\therefore\; T_{r+1} = {}^{12}C_r\,(a)^{12-r}(-2b)^r = {}^{12}C_r\,(-2)^r\, a^{12-r}\, b^r$

$\therefore\; 12-r = 5 \Rightarrow r = 7.$

$\therefore$ Coeff of $a^5b^7 = {}^{12}C_7(-2)^7 = -\dfrac{12\times11\times10\times9\times8}{1\times2\times3\times4\times5}\times(128)\;\left[\because {}^{12}C_7 = {}^{12}C_5\right]$

$= -101376.$

Q.No.3. Write general term in Expansion of $(x^2-y)^6$.

Sol. Consider $(x^2-y)^6 = [x^2+(-y)]^6$

General Term $T_{r+1} = {}^6C_r (x^2)^{6-r}(-y)^r$

$= {}^6C_r\, x^{12-2r}\,(-1)^r\,(y)^r$

$= (-1)^r\, {}^6C_r\, x^{12-2r}\, y^r.$

Q No 4. Write general Term of expansion $(x^2-yx)^{12}$; $x \neq 0$

Sol.

$$(x^2-yx)^{12} = [x^2+(-yx)]^{12}$$

General Term $T_{r+1} = {}^{12}C_r (x^2)^{12-r} (-yx)^r$

$$= {}^{12}C_r\, x^{24-2r} (-1)^r y^r x^r$$

$$= (-1)^r\, {}^{12}C_r\, x^{24-2r+r} y^r = (-1)^r\, {}^{n}C_r\, x^{24-r} y^r.$$

Q No 5. Find the 4th term in Expansion of $(x-2y)^{12}$.

Sol. Here $(x-2y)^{12} = [x+(-2y)]^{12}$.

$$\therefore T_4 = T_{3+1} = {}^{12}C_3 (x)^{12-3} (-2y)^3 \quad \left[\because T_{r+1} = {}^{n}C_r\, a^{n-r} b^r\right]$$

$$= \frac{12\times11\times10}{1\times2\times3.} x^9 (-8y^3) = -1760\, x^9 y^3.$$

Q No. 6. Find 13th term in Expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$; $x \neq 0$

Sol.

Here $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18} = \left(9x + \frac{-1}{3\sqrt{x}}\right)^{18}$

$$\therefore T_{13} = T_{12+1} = {}^{18}C_{12} (9x)^6 \left(-\frac{1}{3\sqrt{x}}\right)^{12} = {}^{18}C_6 \times 9^6 x^6 \times \frac{1}{9^6 x^6} = {}^{18}C_6$$

$$= \frac{18\times17\times16\times15\times14\times13}{1\times2\times3\times4\times5\times6} = 18564.$$

Q No 7: Find the middle terms in expansions of $\left(3-\frac{x^3}{6}\right)^7$

Sol: Here $\left(3-\frac{x^3}{6}\right)^7 = \left[3+\left(-\frac{x^3}{6}\right)\right]^7$

Power of Binomial $= 7$.

$\therefore$ Number of terms in the expansion $= 7+1 = 8$.

$\therefore$ T_4, T_5 are two middle terms.

Now $T_4 = T_{3+1} = {}^{7}C_3 (3)^{7-3} \left(-\frac{x^3}{6}\right)^3 = {}^{7}C_3 (3^4)\left(-\frac{x^9}{216}\right)$

$$= -\frac{7\times6\times5}{1\times2\times3}\times 81\times\frac{x^9}{216} = -\frac{105}{8}x^9$$

and $$T_5 = {}^7C_4\,(3)^{7-4}\left(-\frac{x^3}{6}\right)^4 = {}^7C_3\times 3^3\times\frac{x^{12}}{1296}$$

$$= \frac{7\times6\times5}{1\times2\times3}\times 27\times\frac{x^{12}}{1296} = \frac{35}{48}x^{12}.$$

Q No. 8: Find the middle term in expansion of $\left(\frac{x}{3}+9y\right)^{10}$

Sol. Here power of Binomial = 10 (even no.)

∴ Number of terms = 10 + 1 = 11

∴ Middle term = $T_{\frac{10}{2}+1} = T_{5+1}$ ie T_6.

$$\therefore T_6 = {}^{10}C_5\left(\frac{x}{3}\right)^5(9y)^5 = {}^{10}C_5\times\frac{x^5}{243}\times 9^5y^5$$

$$= \frac{10\times9\times8\times7\times6}{1\times2\times3\times4\times5}\times\frac{x^5}{243}\times 9\times9\times9\times9\times9\times y^5 = 61236\,x^5y^5.$$

Q No. 9. In the expansion of $(1+a)^{m+n}$, prove that cofficients of a^m and a^n are equal.

Sol. T_{r+1} in expansion $(1+a)^{m+n}$

$$T_{r+1} = {}^{m+n}C_r\,a^r.$$

∴ Coeff. of $a^r = {}^{m+n}C_r$.

∴ Coeff of a^m in $(1+a)^{m+n} = {}^{m+n}C_m = \frac{\lfloor m+n}{\lfloor m+n-m\,\lfloor m} = \frac{\lfloor m+n}{\lfloor n\,\lfloor m}$.

Also Coeff of a^m in $(1+a)^{m+n} = {}^{m+n}C_n = \frac{\lfloor m+n}{\lfloor m+n-n\,\lfloor m} = \frac{\lfloor m+n}{\lfloor n\,\lfloor m}$

∴ Coeff of a^m and a^n in $(1+a)^{m+n}$ are equal.

Q No. 10. The coefficients of the $(r-1)^{th}$, r^{th} and $(r+1)^{th}$ terms in the expansion of $(x+1)^n$ are in ratio 1:3:5. Find n and r.

Sol: Coeff. of T_{r-1} in $(x+1)^n = {}^nC_{r-2}$

Coeff. of T_r in $(x+1)^n = {}^nC_{r-1}$

Coeff. of T_{r+1} in $(x+1)^n = {}^nC_r$

According to Question

$$^{n}C_{r-2} : {}^{n}C_{r-1} : {}^{n}C_{r} = 1:3:5$$

$$\therefore \frac{^{n}C_{r-2}}{^{n}C_{r-1}} = \frac{1}{3} \Rightarrow \frac{\dfrac{n!}{(n-r+2)!\,(r-2)!}}{\dfrac{n!}{(r-1)!\,(n-r+1)!}} = \frac{1}{3}.$$

$$\Rightarrow \frac{(r-1)!\,(n-r+1)!}{(r-2)!\,(n-r+2)!} = \frac{1}{3}.$$

$$\Rightarrow \frac{(r-1)(r-2)!\,(n-r+1)!}{(r-2)!\,(n-r+2)\,(n-r+1)!} = \frac{1}{3}.$$

$$\Rightarrow \frac{r-1}{n-r+2} = \frac{1}{3} \Longrightarrow 3r-3 = n-r+2 \Rightarrow n-4r+5=0 \quad \text{---(1)}$$

and
$$\frac{^{n}C_{r-1}}{^{n}C_{r}} = \frac{3}{5} \Longrightarrow \frac{\dfrac{n!}{(r-1)!\,(n-r+1)!}}{\dfrac{n!}{r!\,(n-r)!}} = \frac{3}{5}$$

$$\Rightarrow \frac{r!\,(n-r)!}{(r-1)!\,(n-r+1)!} = \frac{3}{5} \Longrightarrow \frac{(r)(r-1)!\,(n-r)!}{(r-1)!\,(n-r+1)\,(n-r)!} = \frac{3}{5}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{3}{5} \Longrightarrow 5r = 3n-3r+3 \Rightarrow 3n-8r+3=0 \quad \text{---(2)}$$

Solving (1) and (2) for n and r

$$\frac{n}{-12+40} = \frac{r}{15-3} = \frac{1}{-8+12}$$

$$\therefore \frac{n}{28} = \frac{r}{12} = \frac{1}{4}$$

$$\therefore n = \frac{28}{4} = 7 \quad \text{and} \quad r = \frac{12}{4} = 3.$$

Q No 11: Prove that the coeff. of x^n in the expansion of $(1+x)^{2n}$ is twice the coeff. of x^n in the expansion of $(1+x)^{2n-1}$

Sol:

$$T_{r+1} \text{ in } (1+x)^{2n} = {}^{2n}C_r \, x^r$$

$\therefore$ Coeff. of x^r in $(1+x)^{2n} = {}^{2n}C_r$

$\therefore$ Coeff. of x^n in $(1+x)^{2n} = {}^{2n}C_n$

Similarly Coeff. of x^n in $(1+x)^{2n-1} = {}^{2n-1}C_n$

Now $${}^{2n}C_n = \frac{\underline{\lfloor 2n}}{\underline{\lfloor n}\,\underline{\lfloor 2n-n}} = \frac{\underline{\lfloor 2n}}{\underline{\lfloor n}\,.\,\underline{\lfloor n}} = \frac{(2n)\,\underline{\lfloor 2n-1}}{(n)\,\underline{\lfloor n-1}\,\underline{\lfloor n}} = 2\times\frac{\underline{\lfloor 2n-1}}{\underline{\lfloor n-1}\,\underline{\lfloor n}} \quad -(1)$$

and $${}^{2n-1}C_n = \frac{\underline{\lfloor 2n-1}}{\underline{\lfloor 2n-1-n}\,\underline{\lfloor n}} = \frac{\underline{\lfloor 2n-1}}{\underline{\lfloor n-1}\,\underline{\lfloor n}} \quad -(2)$$

From (1) and (2)

$${}^{2n}C_n = 2\,.\,{}^{2n-1}C_n.$$

ie Coeff of x^n in $(1+x)^{2n}$ = $2\times$ Coeff of x^n in $(1+x)^{2n-1}$.

Q No. 12: Find a positive value of m for which the coeff. of x^2 in the expansion $(1+x)^m$ is 6.

Sol:

$$(1+x)^m = {}^mC_0 + {}^mC_1\,x + {}^mC_2\,x^2 + \cdots\; {}^mC_m\,x^m.$$

According to question. ${}^mC_2 = 6$

ie $$\frac{\underline{\lfloor m}}{\underline{\lfloor m-2}\,\underline{\lfloor 2}} = 6 \qquad \text{ie} \quad \frac{(m)(m-1)\,\cancel{\underline{\lfloor m-2}}}{\underline{\lfloor m-2} \times 2\times 1} = 6$$

ie $m(m-1) = 12$ ie $m^2 - m - 12 = 0$

ie $m^2 - 4m + 3m - 12 = 0$

ie $m[m-4] + 3[m-4] = 0$ ie $(m-4)(m+3) = 0$

$\Rightarrow m = 4$ or -3

$\Rightarrow m = 4$ [Rejecting -ve value]

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