

4. BIVARIATE FREQUENCY DISTRIBUTION AND CHI SQUARE STATISTIC

EXERCISE 4.1

1) Following tale gives income (X) and expenditure (Y) of 25 families:

	200 - 300	300 - 400	400 -500
Y	X		
200 - 300			
300 - 400	-		
400 - 500	-	-	

Find

(I) Marginal frequency distributions of income and expenditure.

(II) Conditional frequency distribution of X when Y is between 300-400.

(III) Conditional frequency distribution of Y when X is between 200-300.

(IV) How many families have their income Rs. 300 and more and expenses Rs. 400 and less?

Sol: The bivariate frequency distribution is

	200 - 300	300 - 400	400 -500	Total ()
Y	X			
200-300	6	6	1	13
300-400	-	4	6	10
400-500	-	-	2	2
Total ()	6	10	9	25

(I) The Marginal frequency distributions of income (X)

	200 - 300	300 - 400	400 -500	Total
X				
Frequency	6	10	9	25

The Marginal frequency distributions of expenditure (Y)

	200 - 300	300 - 400	400 -500	Total
Y				

Frequency	13	10	2	25
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(II) Conditional frequency distribution of X when Y is between 300-400.

	200 - 300	300 - 400	400 -500	Total
X				
Frequency	-	4	6	10

(III) Conditional frequency distribution of Y when X is between 200-300.

	200 - 300	300 - 400	400 -500	Total
Y				
Frequency	6	-	-	6

(IV) Number of families having their income Rs. 300 and expenses $\leq$ Rs. 400

$$= 6 + 1 + 4 + 6$$

$$= 17$$

2) Two dice are thrown simultaneously 25 times. The following pairs of observations are obtained.

(2, 3) (2, 5) (5, 5) (4, 5) (6, 4) (3, 2) (5, 2) (4, 1) (2, 5) (6, 1) (3, 1) (3, 3) (4, 3) (4, 5) (2,

5) (3, 4)

(2, 5) (3, 4) (2, 5) (4, 3) (5, 2) (4, 5) (4, 3) (2, 3) (4, 1)

Prepare a bivariate frequency distribution table for the above data. Also obtain the marginal distributions.

Sol: X: the number on the first die

Y: the number on the second die

X \ Y	2	3	4	5	6
1	-			-	
2	-		-		-
3				-	-
4	-		-	-	
5		-			-

Bivariate frequency distribution is

X \ Y	2	3	4	5	6	Total	O
1	-	1	2	-	1	4	
2	-	1	-	2	-	3	
3	2	1	3	-	-	6	
4	-	2	-	-	1	3	
5	5	-	3	1	-	9	
Total	7	5	8	3	2	25	

Marginal frequency distribution of X (number on first die)

X	2	3	4	5	6	Total
O	7	5	8	3	2	25

Margin frequency distribution of Y (number on second die)

Y	1	2	3	4	5	Total
O	4	3	6	3	9	25

3) Following data gives the age of husbands (X) and age of wives (Y) in years. Construct a bivariate frequency distribution table and find the marginal distributions.

X	27	25	28	26	29	27	28	26	25	25	27
Y	21	20	20	21	23	22	20	20	19	19	23
X	26	29	25	27	26	25	28	25	27	26	
Y	19	23	23	22	21	20	22	23	22	21	

Find conditional frequency distribution of age of husbands when the age of Wife is 23 years.

Sol: Let X: Age of husbands in years

Y: Age of Wives in years

X \ Y	25	26	27	28	29
1			-	-	-
2			-		-
3	-			-	-

4	-	-			-
5		-		-	

Bivariate frequency distribution is

X \ Y	2	3	4	5	6	Total	()
1	-	1	2	-	1	4	
2	-	1	-	2	-	3	
3	2	1	3	-	-	6	
4	-	2	-	-	1	3	
5	5	-	3	1	-	9	
Total ()	7	5	8	3	2	25	

Marginal frequency distribution of X (Age of husbands)

X	25	26	27	28	29	Total
()	6	5	5	3	2	21

Margin frequency distribution of Y (Age of wives)

Y	19	20	21	22	23	Total
()	3	5	4	4	5	21

Conditional distribution of X (age of husbands) when Y (age of wives) is 23 years.

X	25	26	27	28	29	Total
Frequency	2	-	1	-	2	5

4) Construct a bivariate frequency distribution table of the marks obtained by students in English (X) and Statistics (Y).

Marks in Statistics (X)	37	20	46	28	35	26	41	48
Marks in English (Y)	30	32	41	33	29	43	30	21

Marks in Statistics (X)	32	23	20	39	47	33	27	26
Marks in English (Y)	44	38	47	24	32	31	20	21

Construct a bivariate frequency distribution table for the above data by taking class intervals 20-30, 30-40, etc. for both X and Y. Also find the marginal distributions and conditional frequency distribution of Y when X lies between 30-40.

Sol: Let X: Marks in Statistics
Y: Marks in English

X \ Y	20-30	30-40	40-50
20-30			
30-40			
40-50			

Bivariate frequency distribution is

X \ Y	20-30	30-40	40-50	Total	O
20-30	2	2	1	5	
30-40	4	1	2	7	
40-50	2	1	1	4	
Total	8	4	4	16	

Marginal frequency distribution of X (marks in statistics)

X	20-30	30-40	40-50	Total
O	8	4	4	16

Margin frequency distribution of Y (marks in English)

Y	20-30	30-40	40-50	Total
O	5	7	4	16

Conditional distribution of Y when X lies between 30-40.

	20-30	30-40	40-50	Total
X				
Frequency	2	1	1	4

5) Following data gives height in cm (X) and weight in kgs (Y) of 20 boys.
Prepare a bivariate frequency table taking class intervals 150-154, 155-159...etc.
for X and 35-39, 40-44 etc. for Y. Also find

(I) Marginal frequency distributions.

(II) Conditional frequency distribution of Y when $155 \leq X \leq 159$

(152, 40) (160, 54) (163, 52) (150, 35) (154, 36) (160, 49) (166, 54) (157, 38)(159,

43) (153, 48) (152, 41) (158, 51)(155, 44) (156, 47) (156, 43) (166, 53) (160, 50)

(151, 39) (153, 50) (158, 46)

Sol: Let X: height in cms.

Y: weight in kgs.

X Y	150-154	155-159	160-164	165-169
35-39			-	-
40-44			-	-
45-49				-
50-54				

Bivariate frequency distribution is

X Y	150-154	155-159	160-164	165-169	Total ()
35-39	3	1	-	-	4
40-44	2	3	-	-	5
45-49	1	2	1	-	4
50-54	1	1	3	2	7
Total ()	7	7	4	2	20

(I) Marginal distribution for X (height in cms)

X	150-154	155-159	160-164	165-169	Total
O	7	7	4	2	20

Marginal distribution for Y (weight in kgs)

Y	150-154	155-159	160-164	165-169	Total
O	4	5	4	7	20

(II) Conditional distribution of Y when $155 \leq X \leq 159$

X	150-154	155-159	160-164	165-169	Total
Frequency	1	3	2	1	7

EXERCISE 4.2

1) Following table shows the classification of applications for secretarial and for sales positions according to gender. Calculate the value of Statistic.

	Offered	Denied
Male	75	150
Female	25	50

Sol:

	Offered	Denied	Total
Male	75	150	225
Female	25	50	75
Total	100	200	N = 300

$\therefore =$

$O_{11} = 75, E_{11} = 75$

$O_{12} = 150, E_{12} = 150$

$O_{21} = 25, E_{21} = 25$

$O_{22} = 50, E_{22} = 50$

$\chi^2 =$

$$= \frac{(75-75)^2}{75} + \frac{(150-150)^2}{150} + \frac{(25-25)^2}{25} + \frac{(50-50)^2}{50}$$

$$\chi^2 = 0$$

2) 200 teenagers were asked which take-out food do they prefer - French fries, burger or pizza. The results were -

	French fries	Denied	Pizza
Boys	75	150	24
Girls	25	50	92

Compute χ^2 Statistics.

Sol:

	French fries	Denied	Pizza	Total
Boys	75	150	24	50
Girls	25	50	92	150
Total	24	60	116	N = 200

$\therefore =$

$$O_{11} = 6, E_{11} = 6$$

$$O_{12} = 20, E_{12} = 15$$

$$O_{13} = 24, E_{13} = 29$$

$$O_{21} = 18, E_{21} = 18$$

$$O_{22} = 40, E_{22} = 45$$

$$O_{23} = 92, E_{23} = 87$$

$$\begin{aligned}
 \chi^2 &= \sum \frac{(O-E)^2}{E} \\
 &= \frac{(6-6)^2}{6} + \frac{(20-15)^2}{15} + \frac{(24-29)^2}{29} + \\
 &\quad \frac{(18-18)^2}{18} + \frac{(40-45)^2}{45} + \frac{(92-87)^2}{87} \\
 &= 0 + \frac{25}{15} + \frac{25}{29} + 0 + \frac{25}{45} + \frac{25}{87} \\
 &= 1.67 + 0.86 + 0.56 + 0.29
 \end{aligned}$$

$$\chi^2 = 3.38$$

3) A sample of men and women who had passed their driving test either in 1st attempt or in 2nd attempt surveyed. Compute statistics.

	1st Attempt	2nd attempt
Passed in		
Men	32	28
Women	8	12

Sol:

Passed in	1st Attempt	2nd attempt	Total
Men	32	28	60
Women	8	12	20
Total	40	40	N = 80

$\therefore =$

$$O_{11} = 32, E_{11} = 30$$

$$O_{12} = 28, E_{12} = 30$$

$$O_{21} = 8, E_{21} = 10$$

$$O_{22} = 12, E_{22} = 10$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(32-30)^2}{30} + \frac{(28-30)^2}{30} + \frac{(8-10)^2}{10} + \frac{(12-10)^2}{10}$$

$$= \frac{4}{30} + \frac{4}{10} + \frac{4}{10} + \frac{4}{10} + \frac{4}{15} + \frac{4}{5} = \frac{16}{15}$$

$$= 1.0667 = 1.07$$

4) 800 people were asked whether they wear glasses for reading with following results.

Age	Wear glasses	Do not wear glasses
≤ 30	310	90
> 30	290	110

Sol:

	Wear glasses	Do not wear glasses	Total
Age ≤ 30	310	90	400
30 < Age ≤ 50	290	110	400
Total	600	200	N = 800

$\therefore =$

$$O_{11} = 310, E_{11} = 300$$

$$O_{12} = 90, E_{12} = 100$$

$$O_{21} = 290, E_{21} = 300$$

$$O_{22} = 110, E_{22} = 100$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= \frac{(310-300)^2}{300} + \frac{(90-100)^2}{100} + \frac{(290-300)^2}{300} + \frac{(110-100)^2}{100}$$

$$= \frac{100}{300} + \frac{100}{100} + \frac{100}{300} + \frac{100}{100} + \frac{1}{3} + 1 + \frac{1}{3} + 1$$

$$= 2 + \frac{2}{3} = 2 + 0.666 = 2.666$$

$$= 2.67$$

5) Out of a sample of 120 persons in a village, 80 were administered a new drug for preventing influenza and out of them 18 were attacked by influenza. Out of those who were not administered the new drug, 10 persons were not attacked by influenza: Prepare: (I) a two-way table showing frequencies. (II) Compute the square statistic.

Sol: (I)

New Drug	Effect of influenza		
	Attacked	Not Attacked	Total
Administered	18	62	80
Not Administered	30	10	40
Total	48	72	120

(II) $\therefore$ =

$$O_{11} = 18, E_{11} = 32$$

$$O_{12} = 62, E_{12} = 48$$

$$O_{21} = 30, E_{21} = 16$$

$$O_{22} = 10, E_{22} = 24$$

$$\chi^2 =$$

$$\begin{aligned} \chi^2 &= \sum \frac{(O-E)^2}{E} \\ &= \frac{(18-32)^2}{32} + \frac{(62-48)^2}{48} + \frac{(30-16)^2}{16} + \frac{(10-24)^2}{24} \\ &= \frac{100}{32} + \frac{100}{48} + \frac{100}{16} + \frac{100}{24} \\ &= 3.125 + 2.083 + 6.25 + 4.167 \\ &= 15.625 \end{aligned}$$

$$= 15.625 + 4.083 + 12.25 + 8.167$$

$$= 30.125$$