INTERALS / EXERCISE 7.3 (NCERT)

QNo 1 $\int \sin^2(2x+5)\,dx = \int \frac{1-\cos 2(2x+5)}{2}\,dx$

$= \frac{1}{2}\int dx - \frac{1}{2}\int \cos(4x+10)\,dx.$

$= \frac{1}{2}x - \frac{1}{2}\cdot\frac{\sin(4x+10)}{4} + c = \frac{1}{2}x - \frac{1}{8}\sin(4x+10) + c$

QNo 2 $\int \sin 3x \cos 4x\,dx = \frac{1}{2}\int 2\sin 3x\cos 4x\,dx.$

$= \frac{1}{2}\int [\sin(3x+4x) + \sin(3x-4x)]\,dx$ $\left[\because 2\sin A\cos B = \sin(A+B) + \sin(A-B)\right.$

$= \frac{1}{2}\int \sin 7x\,dx - \frac{1}{2}\int \sin x\,dx.$

$= \frac{1}{2}\left(-\frac{\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + c = \frac{1}{2}\left(-\frac{\cos 7x}{7} + \cos x\right) + c.$

QNo 3. $\int \cos 2x\cos 4x\cos 6x\,dx = \frac{1}{2}\int(2\cos 6x\cos 2x)\cos 4x\,dx.$

$= \frac{1}{2}\int [\cos(6x+2x) + \cos(6x-2x)]\cos 4x\,dx.$ $\left[\because 2\cos A\cos B = \cos(A+B) + \cos(A-B)\right.$

$= \frac{1}{2}\int(\cos 8x + \cos 4x)\cos 4x\,dx = \frac{1}{2}\int(\cos 8x\cos 4x + \cos^2 4x)\,dx$

$= \frac{1}{4}\int(2\cos 8x\cos 4x + 2\cos^2 4x)\,dx$

$= \frac{1}{4}\int(\cos 12x + \cos 4x + 1 + \cos 8x)\,dx$

$= \frac{1}{4}\left(\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + \frac{\sin 4x}{4} + x\right) + c$

QNo 4. $\int \sin^3(2x+1)\,dx$

Sol. $I = \int \frac{3\sin(2x+1) - \sin(6x+3)}{4}\,dx$ $\left[\because \sin 3x = 3\sin x - 4\sin^3 x \Rightarrow \sin^3 x = \frac{3\sin x - \sin 3x}{4}\right.$

$= \frac{3}{4}\int \sin(2x+1)\,dx - \frac{1}{4}\int \sin(6x+3)\,dx$

$$= \frac{3}{4}\left[\frac{-\cos(2x+1)}{2}\right] - \frac{1}{4}\left[\frac{-\cos(6x+3)}{6}\right] + c$$

$$= -\frac{3}{8}\cos(2x+1) + \frac{1}{24}\cos(6x+3) + c.$$

QNo5. $\int \sin^3 x \cos^3 x \, dx.$

$$I = \int \sin^3 x \cos^2 x \cdot \cos x \, dx. \quad = \int \sin^3 x (1-\sin^2 x) \cos x \, dx$$

Put $\sin x = t \Rightarrow \cos x \, dx = dt$

$$I = \int t^3(1-t^2)\,dt = \int (t^3 - t^5)\,dt$$

$$= \frac{t^4}{4} - \frac{t^6}{6} + c$$

$$= \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + c$$

Alternatively: $I = \int \sin x \cdot \sin^2 x \cos^3 x \, dx = \int \cos^3 x (1-\cos^2 x) \sin x \, dx$

Put $\cos x = t \Rightarrow -\sin x \, dx = dt$.

$$\therefore I = \int t^3(1-t^2)(-dt) = \int (t^5 - t^3)\,dt$$

$$= \frac{t^6}{6} - \frac{t^4}{4} + c$$

$$= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + c$$

Alternatively: $\int \sin^3 x \cos^3 x \, dx = \frac{1}{8}\int 8 \sin^3 x \cos^3 x \, dx$

$$= \frac{1}{8}\int (2\sin x \cos x)^3 dx = \frac{1}{8}\int (\sin 2x)^3 dx.$$

$$= \frac{1}{8}\int \left[\frac{3\sin 2x - \sin 6x}{4}\right] dx = \frac{1}{32}\int (3\sin 2x - \sin 6x)\,dx$$

$$= \frac{1}{32}\left[\frac{3\cdot(-\cos 2x)}{2} - \frac{(-\cos 6x)}{6}\right] + c = -\frac{3}{64}\cos 2x + \frac{\cos 6x}{192} + c$$

Q.No 6. $\int \sin x \sin 2x \sin 3x \, dx$

Sol. Now $\sin x \sin 2x \sin 3x = \frac{1}{2}(2\sin 3x \sin x) \sin 2x$

$= \frac{1}{2}(\cos 2x - \cos 4x) \sin 2x$ $\left[\because -2\sin A \sin B = \cos(A+B) - \cos(A-B)\right]$

$= \frac{1}{4}(2\sin 2x \cos 2x - 2\cos 4x \cdot \sin 2x)$

$= \frac{1}{4}(\sin 4x - \sin 6x - \sin 2x)$

$\therefore \int \sin x \sin 2x \sin 3x \, dx = \frac{1}{4}\int (\sin 4x - \sin 6x - \sin 2x) \, dx.$

$= \frac{1}{4}\left(\frac{-\cos 4x}{4} - \frac{-\cos 6x}{6} - \frac{-\cos 2x}{2}\right) + C$

$= -\frac{\cos 4x}{16} + \frac{\cos 6x}{24} - \frac{\cos 2x}{8} + C.$

Q.No 7. $\int \sin 4x \sin 8x \, dx$

$= \frac{1}{2}\int 2\sin 8x \sin 4x \, dx = \frac{1}{2}\int(\cos 4x - \cos 12x) \, dx.$

$= \frac{1}{2}\left(\frac{\sin 4x}{4} - \frac{\sin 12x}{12}\right) + C = \frac{1}{8}\sin 4x - \frac{1}{24}\sin 12x + C$

Q.No 8. $\int \frac{1-\cos x}{1+\cos x} \, dx$

$= \int \frac{2\sin^2 x/2}{2\cos^2 x/2} \, dx = \int \tan^2 \frac{x}{2} \, dx = \int\left(\sec^2 \frac{x}{2} - 1\right) dx$

$= \frac{\tan x/2}{1/2} - x + C = 2\tan\frac{x}{2} - x + C.$

Q.No 9 $\int \frac{\cos x}{1+\cos x} \, dx = \int \frac{1+\cos x - 1}{1+\cos x} \, dx = \int \frac{1+\cos x}{1+\cos x} \, dx - \int \frac{1}{1+\cos x} \, dx$

$= \int 1 \cdot dx - \int \frac{1}{2\cos^2 x/2} \, dx = \int 1 \, dx - \frac{1}{2}\int \sec^2 \frac{x}{2} \, dx.$

$= x - \frac{1}{2} \cdot \frac{\tan x/2}{\frac{1}{2}} + C = x - \tan\frac{x}{2} + C.$

Q No 10 $\int \sin^4 x\, dx = \int (\sin^2 x)^2 dx$

$$= \int\left(\frac{1-\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int (1+\cos^2 2x - 2\cos 2x)\, dx$$

$$= \frac{1}{4}\int\left(1+\frac{1+\cos 4x}{2} - 2\cos 2x\right) dx$$

$$= \frac{1}{4}\int\left(\frac{2+1+\cos 4x - 4\cos 2x}{2}\right) dx$$

$$= \frac{1}{8}\int (3-4\cos 2x+\cos 4x)\, dx$$

$$= \frac{1}{8}\left(3x - 4\frac{\sin 2x}{2} + \frac{\sin 4x}{4}\right) + c$$

$$= \frac{3}{8}x - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c$$

Q No 11 $\int \cos^4 2x\, dx = \int (\cos^2 2x)^2 dx$

$$= \int\left(\frac{1+\cos 4x}{2}\right)^2 dx = \frac{1}{4}\int (1+2\cos 4x+\cos^2 4x)\, dx$$

$$= \frac{1}{4}\int\left(1+2\cos 4x + \frac{1+\cos 8x}{2}\right) dx$$

$$= \frac{1}{8}\int (3+4\cos 4x + \cos 8x)\, dx$$

$$= \frac{1}{8}\left[3x + 4\frac{\sin 4x}{4} + \frac{\sin 8x}{8}\right] + c$$

Q No 12 $\int \frac{\sin^2 x}{1+\cos x}\, dx = \int \frac{1-\cos^2 x}{1+\cos x}\, dx$

$$= \int \frac{(1-\cos x)(1+\cos x)}{(1+\cos x)}\, dx$$

$$= \int (1-\cos x)\, dx = x - \sin x + c.$$

Q No. 13 $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} dx = \int \frac{(2\cos^2 x - 1) - (2\cos^2\alpha - 1)}{\cos x - \cos\alpha} dx$

$= \int \frac{2\cos^2 x - 1 - 2\cos^2\alpha + 1}{\cos x - \cos\alpha} dx = 2\int \frac{(\cos x - \cos\alpha)(\cos x + \cos\alpha)}{(\cos x - \cos\alpha)} dx$

$= 2\int (\cos x + \cos\alpha) dx = 2\sin x - 2\cos\alpha \cdot x + C$

Q No. 14. $\int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{\cos^2 x + \sin^2 x + 2\sin x\cos x} dx$ $\left[\because \sin^2 x + \cos^2 x = 1\right.$

$= \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx = \int (\sin x + \cos x)^{-2} \frac{d}{dx}(\sin x + \cos x)\, dx$

$= \frac{(\sin x + \cos x)^{-2+1}}{-2+1} + C$ $\left[\int (f(x))^n f'(x) \text{ type}\right]$

$= -\frac{1}{\sin x + \cos x} + C.$

Q No. 15 $\int \tan^3 2x \cdot \sec 2x \cdot dx = \int (\tan^2 2x)(\tan 2x)(\sec 2x) dx.$

$= \int (\sec^2 2x - 1)(\tan 2x \cdot \sec 2x)\, dx.$

$= \frac{1}{2}\int \sec^2 2x \cdot \sec 2x \cdot \tan 2x \cdot 2\, dx - \int \sec 2x \cdot \tan 2x\, dx$

$= \frac{1}{2} \frac{(\sec 2x)^{2+1}}{2+1} - \frac{\sec 2x}{2} + C$

$= \frac{1}{2\times 3}(\sec 2x)^3 - \frac{\sec 2x}{2} + C$

$= \frac{1}{6}\sec^3 2x - \frac{\sec 2x}{2} + C.$

Q No 16 $\int \tan^4 x\, dx = \int \tan^2 x \cdot \tan^2 x\, dx$

$= \int \tan^2 x\,(\sec^2 x - 1)\, dx$

$= \int (\tan^2 x \sec^2 x - \tan^2 x)\, dx$

$= \int (\tan x)^2 \cdot \sec^2 x\, dx - \int (\sec^2 x - 1)\, dx$

$= \frac{(\tan x)^3}{3} - \tan x + x + c$

Q No. 17. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} \right) dx$

$= \int \frac{\sin x}{\cos x \cdot \cos x} dx + \int \frac{\cos x}{\sin x \cdot \sin x} dx$

$= \int \tan x \cdot \sec x\, dx + \int \cot x \cdot \operatorname{cosec} x\, dx$

$= \sec x - \operatorname{cosec} x + c$

Q No 18: $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x} dx$

$= \int \frac{1}{\cos^2 x} dx = \int \sec^2 x\, dx = \tan x + c$

Q No 19 $\int \frac{1}{\sin x \cdot \cos^3 x} dx = \int \frac{\cos x}{\sin x \cdot \cos^4 x} dx$

$= \int \frac{1}{\tan x} \cdot \sec^4 x\, dx = \int \frac{\sec^2 x \cdot \sec^2 x}{\tan x} dx$

$= \int \frac{(1 + \tan^2 x)}{\tan x} \cdot \sec^2 x \cdot dx$

Put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$.

$$\therefore I = \int \frac{1+t^2}{t} \cdot dt = \int \frac{1}{t} dt + \int t\, dt$$

$$= \log|t| + \frac{t^2}{2} + c$$

$$= \log|\tan x| + \frac{\tan^2 x}{2} + c$$

Q No 20 $\displaystyle\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2} dx$

$$= \int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)(\cos x + \sin x)} dx = \int \frac{-\sin x + \cos x}{\cos x + \sin x} dx$$

$$= \log|\cos x + \sin x| + c \quad \left[\frac{f'(x)}{f(x)} \text{ type} \right]$$

Q No. 21 $\displaystyle\int \sin^{-1}(\cos x)\, dx = \int \left(\frac{\pi}{2} - \cos^{-1}(\cos x)\right) dx$

$$\left[\because \sin^{-1} t + \cos^{-1} t = \pi/2 \text{ for } |t| \leq 1\right]$$

$$= \int \frac{\pi}{2} dx - \int x\, dx$$

$$= \frac{\pi}{2} x - \frac{x^2}{2} + c$$

Q No. 22 $\displaystyle\int \frac{1}{\cos(x-a)\cos(x-b)} dx = \frac{1}{\sin(b-a)} \int \frac{\sin(b-a)}{\cos(x-a)\cos(x-b)} dx$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin[(x-a)-(x-b)]}{\cos(x-a)\cos(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int \frac{\sin(x-a)\cos(x-b) - \cos(x-a)\sin(x-b)}{\cos(x-a)\cos(x-b)} dx$$

$$= \frac{1}{\sin(b-a)} \int \left(\tan(x-a) - \tan(x-b)\right) dx.$$

$$= \frac{1}{\sin(b-a)} \left(-\log. \cos|x-a| + \log|\cos(x-b)|\right) + c$$

$$= \frac{1}{\sin(b-a)} \log. \left| \frac{\cos(x-b)}{\cos(x-a)} \right| + c$$

QNo. 23 $\displaystyle\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}\right) dx$

$$= \int \left(\sec^2 x - \operatorname{cosec}^2 x\right) dx = \tan x + \cot x + c$$

$\therefore$ A is correct option.

QNo 24. $\displaystyle\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$

Sol. Put $x e^x = t \Rightarrow (x \cdot e^x + e^x) dx = dt$ i.e. $e^x(1+x)dx = dt$

$$\therefore I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t \, dt = \tan t + c$$

$$= \tan(x e^x) + c$$

$\therefore$ Option B is correct option.

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