SIMPLE HARMONIC MOTION

1.Periodic motion:

The motion of a particle which repeats itself along the same path in equal intervals of time is called periodic motion

Ex:a. Rotation of earth around the sun.
b. Oscillations of simple pendulum
c. Vibrations of prongs of a tuning fork
d. Motion of hands in a watch.

2. Displacement of a particle in periodic motion can be mathematically expressed in terms of sine (or) cosine function so that periodic motion is also called haromonic motion.

3. Simple harmonic motion:

The to and fro motion of a particle along the straight line such that its acceleration is always directed towards a fixed point in its path and is directly proportional to the displacement from that fixed point is called simple harmonic motion.

$$a\alpha - y \Rightarrow a = -w^2 y \Rightarrow \frac{d^2 y}{dt^2} + w^2 y = 0 .$$

Simple harmonic motion is of two types:

i) Linear simple harmonic motion:

If the motion is along the straight line path it is called linear simple harmonic motion

Ex: a.Vibrations of string
b. Oscillations of liquid in U. tube
c. Oscillations of loaded spring
d. Vibrations of the prongs of a tuning fork.

ii) Angular simple harmonic motion

If vibrations are angular then it is called angular simple harmonic motion

Ex: a. Oscillations of a torsional pendulum
b. Oscillations of balance wheel in watchs

4. Characteristics of simple harmonic motions:

a. The motion is periodic
b. It execute to and fro motion
c. Acceleration is directed towards the mean point
d. The acceleration is always directly proportional to its displacement and which is opposite in direction to the dispalcement $a\alpha - y$

5. Simple harmonic motion is a periodic motion but every periodic motion is need not be a simple harmonic motion.

6. Displacement:

The maximum distance of the particle from the equilibrium position at any instant is called displacement.

7. Amplitude:

The maximum displacement of the vibrating particle from mean position is called amplitude represented by 'A' (or) a

8. Time period:

The time taken for one oscillation is called time period

$$T = \frac{2\pi}{\omega}$$

9. Frequency:

The number of oscillations completed by a particle in one second is called frequency.

Frequency $f = \frac{1}{T}$ cyeles/s.

10. Phase:

It represents the position and direction of vibrating particle at a particular instant

at the instant of time t = 0 the phase of the particle is called intial phase i.e. 'ϕ' it is also called 'epoch'

11. Displacement, velocity and acceleration of a particle axecuting simple harmonic motion.

a. Displacement of a particle executing simple harmonic motion is given by

$$y = A\sin(wt \pm \phi)$$

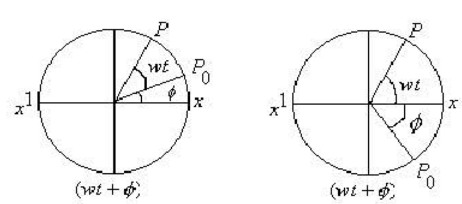
where $\phi \rightarrow$ is initial phase

$\omega \rightarrow$ angular frequency

$A \rightarrow$ ampletude

b. Velocity of the particle executing simple harmonic motion is given by

$$V = \omega\sqrt{A^2 - y^2}$$



Case (i) When particle at mean position :

$y = 0$

$\therefore V = \omega\sqrt{A^2 - 0}$

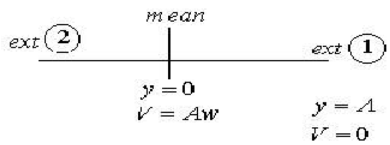
$V = A\omega =$ maximum

Case(ii) When particle at extreme end

$y = A$

$\therefore V = \omega\sqrt{A^2 - A^2}$

V = 0 = minimum



c. Acceleration of a particle executing simple harmonic motion is given by

$a = -A\omega^2 \sin(wt \pm \phi)$

$a = -\omega^2 y$

Case(i) Wehn particle at mean

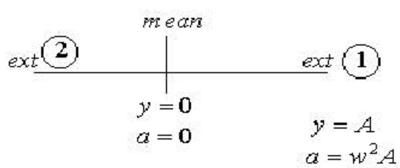
y = 0

a = 0

Case(ii) When particle at extreme

y = A

$a = \omega^2 A$ maximum



12. Time period of a particle executing simple harmonic motion is given by

$$T = 2\pi\sqrt{\frac{y}{a}}$$

Where y = displacement and

a = acceleration

13. Simple pendulum

1time period of simple pendulum executing simple harmonic motion is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where $l \rightarrow$ length of the simple pendulum

$g \rightarrow$ acceleration due to gravity

14. Laws of simple pendulum

a. Time period of a simple pendulum is directly proportional to square root of its lengths $T \propto \sqrt{l}$

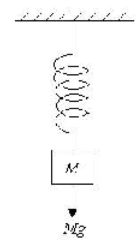
b. time period of simple pendulum is inversely proportion to square root of the acceleration due to gravity

$$T \propto \frac{1}{\sqrt{g}}$$

$$T^2 \alpha \frac{1}{g}$$

c. time period of simple pendulum is independent of mass and shape of the bob.

15. Loaded spring



i) If a mass M is suspended vertically from a spring and if the spring elongates ‘x’ then spring constant is

$$K = \frac{F}{x}$$

$$K = \frac{Mg}{x}$$

ii) time period of this loaded spring is

$$T = 2\pi\sqrt{\frac{M}{K}}$$

$$= 2\pi\sqrt{\frac{M}{\frac{Mg}{x}}} \Rightarrow T = 2\pi\sqrt{\frac{x}{g}}$$

iii) If a mass m is attached to the end of a spring and oscilates time period is

$$T = 2\pi\sqrt{\frac{m}{K}} \quad \text{....................(1)}$$

$$T = 2\pi\sqrt{\frac{m}{\frac{Mg}{x}}}$$

$$T = 2\pi\sqrt{\frac{mx}{Mg}} \text{ ....(2)}$$

If m_s is mass of the spring

then $T = 2\pi\sqrt{\frac{\left[m+\left(m_s/3\right)\right]x}{Mg}}$ ....(3)

16. Energy of a particle executing simple harmonic motion

Let us consider a particle executing simple harmonic motion it posses P. E. and K. E.

$$P.E. = \frac{1}{2}mw^2y^2 \text{ ....(1)}$$

$$K.E = \frac{1}{2}mw^2\left(A^2 - y^2\right) \text{........(2)}$$

total energy of the particle = PE + KE

$$TE = \frac{1}{2}mw^2A^2 \text{ ....(3)}$$