Trigonometric Equations

Exercise 17

Q. 1. Find the principal solutions of each of the following equations:

(i) $\sin x = \frac{\sqrt{3}}{2}$

(ii) $\cos x = \frac{1}{2}$

(iii) $\tan x = \sqrt{3}$

(iv) $\cot x = \sqrt{3}$

(v) $\operatorname{cosec} x = 2$

(vi) $\sec x = \frac{2}{\sqrt{3}}$

Answer : To Find: Principal solution.

[NOTE: The solutions of a trigonometry equation for which $0 \leq x < 2\pi$ is called principal solution]

(i) Given: $\sin x = \frac{\sqrt{3}}{2}$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in I$

By using above formula, we have

$\sin x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} \Rightarrow x = n\pi + \frac{\pi}{3}(-1)^n$

Put n= 0 $\Rightarrow x = 0\times \pi + \frac{\pi}{3}(-1)^0 \Rightarrow x = \frac{\pi}{3}$

Put n= 1 $\Rightarrow x = 1\times \pi + \frac{\pi}{3}(-1)^1 \Rightarrow x = 1 \times \pi + \frac{\pi}{3}(-1)^1 \Rightarrow x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

So principal solution is x= $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

(ii) Given: $\cos x = \frac{1}{2}$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = \frac{1}{2} = \cos\frac{\pi}{3} \implies \theta = 2n\pi \pm \alpha$, $n \in I$

Put n= 0 $\implies x = 2n\pi \pm \frac{\pi}{3} \implies x = \frac{\pi}{3}$

Put n= 1 $\implies x = 2\pi \pm \frac{\pi}{3} \implies x = \frac{5\pi}{3}, \frac{7\pi}{3} \implies x = \frac{5\pi}{3}, \frac{7\pi}{3}$

[$\frac{7\pi}{3} > 2\pi$ So it is not include in principal solution]

So principal solution is x= $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

(iii) Given: $\tan x = \sqrt{3}$

Formula used: $\tan\theta = \tan\alpha \implies \theta = n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\tan x = \sqrt{3} = \tan\frac{\pi}{3} \implies x = n\pi + \alpha, n \in I$

Put n= 0 $\implies x = n\pi + \frac{\pi}{3} \implies x = \frac{\pi}{3}$

Put n= 1 $\implies x = \pi + \frac{\pi}{3} \implies x = \frac{4\pi}{3} \implies x = \frac{4\pi}{3}$

So principal solution is x= $\frac{\pi}{3}$ and $\frac{4\pi}{3}$

We know that tanθ × cotθ = 1

So cotx = $\sqrt{3}$ $\Rightarrow$ tanx = $\frac{1}{\sqrt{3}}$

The formula used: tanθ = tanα $\Rightarrow$ θ = n$\pi \pm \alpha$, n ∈ I

By using the above formula, we have

tanx = $\frac{1}{\sqrt{3}}$ = tan$\frac{\pi}{6}$ $\Rightarrow$ θ = nπ + α, n ∈ I

Put n= 0 $\Rightarrow$ x = nπ + $\frac{\pi}{6}$ $\Rightarrow$ x = $\frac{\pi}{6}$

Put n= 1 $\Rightarrow$ x = π + $\frac{\pi}{6}$ $\Rightarrow$ x = $\frac{7\pi}{6}$

So principal solution is x= $\frac{\pi}{6}$ and $\frac{7\pi}{6}$

(v) Given: cosec x = 2

We know that cosecθ $\times$ sinθ = 1

So sinx = $\frac{1}{2}$

Formula used: sinθ = sinα $\Rightarrow$ θ = nπ + $(-1)^n\alpha$, n $\in$

By using above formula, we have

sinx = $\frac{1}{2}$ = sin$\frac{\pi}{6}$ $\Rightarrow$ θ = nπ + $\frac{\pi}{6}(-1)^n$

Put n= 0 $\Rightarrow$ $\theta = 0\times\pi + \frac{\pi}{6}(-1)^0 \Rightarrow \theta = \frac{\pi}{6}$

Put n= 1 $\Rightarrow$ $\theta = 1\times \pi + \frac{\pi}{6}(-1)^1 \Rightarrow \theta = 1 \times \pi + \frac{\pi}{6}(-1)^1 \Rightarrow \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

So principal solution is x= $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

(vi) Given: sec x = $\frac{2}{\sqrt{3}}$

We know that sec$\theta \times$ cosθ = 1

So cosx = $\frac{\sqrt{3}}{2}$

Formula used: cosθ = cosα $\Rightarrow$ θ = 2n$\pi \pm \alpha$, n $\in$ I

By using the above formula, we have

$\cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \Rightarrow x = 2n\pi \pm \alpha, n \in I$

Put n= 0 $\Rightarrow x = 2n\pi \pm \frac{\pi}{6} \Rightarrow x = \frac{\pi}{6}$

Put n= 1 $\Rightarrow x = 2\pi \pm \frac{\pi}{6} \Rightarrow x = \frac{11\pi}{6}, \frac{13\pi}{6} \Rightarrow x = \frac{11\pi}{6}, \frac{13\pi}{6}$

[$\frac{13\pi}{6} > 2\pi$ So it is not include in principal solution]

So principal solution is x= $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

Q. 2. Find the principal solutions of each of the following equations :

(i) $\sin x = \frac{-1}{2}$

(ii) $\sqrt{2}\cos x + 1 = 0$

(iii) tan x = -1

(iv) $\sqrt{3}\operatorname{cosec} x + 2 = 0$

(v) $\tan x = -\sqrt{3}$

(vi) $\sqrt{3}\sec x + 2 = 0$

Answer : To Find: Principal solution.

(i) Given: $\sin x = \frac{-1}{2}$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in I$

By using above formula, we have

$\sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin(\pi + \frac{\pi}{6}) = \sin\frac{7\pi}{6} \implies x = n\pi + \frac{7\pi}{6}(-1)^n$

Put n= 0 $\implies x = 0\times \pi + \frac{7\pi}{6}(-1)^0 \implies x = \frac{7\pi}{6}$

Put n= 1 $\implies x = 1\times \pi + \frac{7\pi}{6}(-1)^1 \implies x = 1 \times \pi + \frac{7\pi}{6}(-1)^1 \implies x = \pi - \frac{7\pi}{6} = -\frac{\pi}{6}$

[NOTE: $-\frac{\pi}{6} = \frac{11\pi}{6}$]

So principal solution is x= $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

(ii) Given: $\sqrt{2}\cos x + 1 = 0 \Rightarrow \cos x = \frac{-1}{\sqrt{2}}$

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4} \Rightarrow x = 2n\pi \pm \alpha$, $n \in I$

Put n= 0 $\Rightarrow x = 2 \times 0 \times \pi \pm \frac{3\pi}{4} \Rightarrow x = \frac{3\pi}{4}$

Put n= 1 $\Rightarrow x = 2\pi \pm \frac{3\pi}{4} \Rightarrow x = \frac{5\pi}{4}, \frac{11\pi}{4} \Rightarrow x = \frac{5\pi}{4}, \frac{11\pi}{4}$

[$\frac{11\pi}{4} > 2\pi$ So it is not include in principal solution]

So principal solution is x= $\frac{3\pi}{4}$ and $\frac{5\pi}{4}$

(iii) Given: tan x = -1

Formula used: $\tan\theta = \tan\alpha \implies \theta = n\pi \pm \alpha$, n ∈ I

By using above formula, we have

tan x = -1 = $\tan\frac{3\pi}{4} \Rightarrow x = n\pi + \alpha$, n ∈ I

Put n= 0 $\Rightarrow x = n\pi + \frac{3\pi}{4} \Rightarrow x = \frac{3\pi}{4}$

Put n= 1 $\Rightarrow$ x = $\pi + \frac{3\pi}{4}$ $\Rightarrow$ x = $\frac{7\pi}{4}$ $\Rightarrow$ x = $\frac{7\pi}{4}$

So principal solution is x= $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$

(iv) Given: $\sqrt{3}\,\text{cosec}\,x + 2 = 0$ $\Rightarrow$ cosec x = $\frac{-2}{\sqrt{3}}$

We know that cosecθ × sinθ = 1

So sinx = $\frac{-\sqrt{3}}{2}$

Formula used: sinθ = sinα $\Rightarrow$ θ = nπ + $(-1)^n\alpha$, n ∈

By using above formula, we have

sinx = $\frac{-\sqrt{3}}{2}$ = sin$\frac{4\pi}{3}$ $\Rightarrow$ θ = nπ + $\frac{4\pi}{3}$(-1)^n

Put n= 0 $\Rightarrow$ x = 0$\times$ π + $\frac{4\pi}{3}$(-1)^0$\Rightarrow$ x = $\frac{4\pi}{3}$

Put n= 1 $\Rightarrow$ x = 1$\times$ π + $\frac{4\pi}{3}$(-1)^1$\Rightarrow$ x = $1 \times \pi + \frac{4\pi}{3}(-1)^1$ $\Rightarrow$ x = $\pi - \frac{4\pi}{3} = \frac{-\pi}{3}$

[NOTE: $\frac{-\pi}{3}$ =$\frac{5\pi}{3}$]

So principal solution is x= $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$

(v) Given: tan x = -$\sqrt{3}$

Formula used: tanθ = tanα $\Rightarrow$ θ = nπ $\pm$ α , n $\in$ I

By using above formula, we have

tan x = $-\sqrt{3}$ = tan$\frac{2\pi}{3}$ $\Rightarrow$ x = nπ + α, n ∈ I

Put n= 0 $\Rightarrow$ x = nπ + $\frac{2\pi}{3}$ $\Rightarrow$ x = $\frac{2\pi}{3}$

Put n= 1 $\Rightarrow$ x = π + $\frac{2\pi}{3}$ $\Rightarrow$ x = $\frac{5\pi}{3}$

So principal solution is x= $\frac{2\pi}{3}$ and $\frac{5\pi}{3}$

(vi) Given: $\sqrt{3}\sec x + 2 = 0$ $\Rightarrow$ sec x = $\frac{-2}{\sqrt{3}}$

We know that secθ × cosθ = 1

So cosx = $\frac{-\sqrt{3}}{2}$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

$\cos x = \frac{\sqrt{3}}{2} = \cos\frac{5\pi}{6} \implies x = 2n\pi \pm \alpha, n \in I$

Put n= 0 $\implies x = 2n\pi \pm \frac{5\pi}{6} \implies x = \frac{5\pi}{6}$

Put n= 1 $\implies x = 2\pi \pm \frac{5\pi}{6} \implies x = \frac{7\pi}{6}, \frac{17\pi}{6} \implies x = \frac{7\pi}{6}, \frac{17\pi}{6}$

[$\frac{17\pi}{6} > 2\pi$ So it is not include in principal solution]

So principal solution is x= $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

Q. 3. Find the general solution of each of the following equations:

(i) $\sin 3x = 0$

(ii) $\sin \frac{3x}{2} = 0$

(iii) $\sin\left(x + \frac{\pi}{5}\right) = 0$

(iv) $\cos 2x = 0$

(v) $\cos \frac{5x}{2} = 0$

(vi) $\cos\left(x + \frac{\pi}{10}\right) = 0$

(vii) $\tan 2x = 0$

(viii) $\tan\left(3x + \frac{\pi}{6}\right) = 0$

(ix) $\tan\left(2x - \frac{\pi}{4}\right) = 0$

Answer : To Find: General solution.

[NOTE: A solution of a trigonometry equation generalized by means of periodicity, is known as general solution]

(i) Given: sin 3x = 0

Formula used: $\sin \theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

By using above formula, we have

sin 3x = 0 $\Rightarrow$ 3x = nπ $\Rightarrow$ x = $\frac{n\pi}{3}$ where n $\in$ *I*

So general solution is x= $\frac{n\pi}{3}$ where n $\in$ *I*

(ii) Given: sin$\frac{3x}{2}$ = 0

Formula used: sinθ= 0 $\Rightarrow$ θ = nπ , n $\in$ *I*

By using above formula, we have

sin $\frac{3x}{2}$ = 0 $\Rightarrow$ $\frac{3x}{2}$ = nπ $\Rightarrow$ x = $\frac{2n\pi}{3}$ where n $\in$

So general solution is x= $\frac{2n\pi}{3}$ where n $\in$ *I*

(iii) Given: $\sin\left(x + \frac{\pi}{5}\right) = 0$

Formula used: $\sin\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

By using the above formula, we have

$\sin\left(x + \frac{\pi}{5}\right) = 0 \Rightarrow x + \frac{\pi}{5} = n\pi \Rightarrow x = n\pi - \frac{\pi}{5}$ where $n \in I$

So general solution is $x = n\pi - \frac{\pi}{5}$ where $n \in I$

(iv) Given: $\cos 2x = 0$

Formula used: $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$, $n \in I$

By using above formula, we have

$\cos 2x = 0 \Rightarrow 2x = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{4}$ where $n \in I$

So general solution is $x= (2n+1)\frac{\pi}{4}$ where $n \in I$

(v) Given: $\cos \frac{5x}{2} = 0$

Formula used: $\cos\theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$, $n \in I$

By using the above formula, we have

$\cos \frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = (2n+1)\frac{\pi}{2} \Rightarrow x = (2n+1)\frac{\pi}{5}$ where $n \in I$

So general solution is $x= (2n+1)\frac{\pi}{5}$ where $n \in I$

(vi) Given: cos $\left(x + \frac{\pi}{10}\right)$ = 0

Formula used: cosθ= 0 $\Rightarrow$ θ = (2n+1)$\frac{\pi}{2}$, n ∈ *I*

By using the above formula, we have

cos $\left(x + \frac{\pi}{10}\right)$ = 0 $\Rightarrow$ $x + \frac{\pi}{10}$ = (2n+1)$\frac{\pi}{2}$ $\Rightarrow$ x = (2n+1)$\frac{\pi}{2}$ - $\frac{\pi}{10}$ $\Rightarrow$ x = nπ + $\frac{2\pi}{5}$ where n ∈ *I*

So general solution is x= nπ + $\frac{2\pi}{5}$ where n ∈ *I*

(vii) Given: tan 2x = 0

Formula used: tanθ= 0 $\Rightarrow$ θ = nπ , n ∈ *I*

Formula used: tanθ= 0 $\Rightarrow$ θ = nπ , n $\in$ I

By using above formula, we have

tan 2x = 0 $\Rightarrow$ 2x = nπ $\Rightarrow$ x = $\frac{n\pi}{2}$ where n $\in$ I

So general solution is x= $\frac{n\pi}{2}$ where n $\in$ I

(viii) Given: tan $\left(3x + \frac{\pi}{6}\right)$ = 0

Formula used: tanθ= 0 $\Rightarrow$ θ = nπ , n $\in$ I

By using above formula, we have

$\tan\left(3x+\frac{\pi}{6}\right) = 0 \Rightarrow 3x+\frac{\pi}{6} = n\pi \Rightarrow 3x = n\pi - \frac{\pi}{6} \Rightarrow x = \frac{n\pi}{3} - \frac{\pi}{18}$ where $n \in I$

So general solution is $x = \frac{n\pi}{3} - \frac{\pi}{18}$ where $n \in I$

(ix) Given: $\tan\left(2x-\frac{\pi}{4}\right) = 0$

Formula used: $\tan\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

By using above formula, we have

$\tan\left(2x-\frac{\pi}{4}\right) = 0 \Rightarrow 2x-\frac{\pi}{4} = n\pi \Rightarrow 2x = n\pi - \frac{\pi}{4} \Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8}$ where $n \in I$

So general solution is $x = \frac{n\pi}{2} + \frac{\pi}{8}$ where $n \in I$

Q. 4. Find the general solution of each of the following equations:

(i) $\sin x = \frac{\sqrt{3}}{2}$

(ii) cos x = 1

(iii) $\sec x = \sqrt{2}$

Answer : To Find: General solution.

(i) Given: sin x = $\frac{\sqrt{3}}{2}$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in I$

By using above formula, we have

$\sin x = \frac{\sqrt{3}}{2} = \sin\frac{\pi}{3} \implies x = n\pi + (-1)^n . \frac{\pi}{3}$

So general solution is $x = n\pi + (-1)^n . \frac{\pi}{3}$ where $n \in I$

(ii) Given: cos x = 1

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = 1 = \cos(0°) \Rightarrow x = 2n\pi$, $n \in I$

So general solution is $x = 2n\pi$ where $n \in I$

(iii) Given: $\sec x = \sqrt{2}$

We know that $\sec\theta \times \cos\theta = 1$

So $\cos x = \frac{1}{\sqrt{2}}$

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} \Rightarrow x = 2n\pi \pm \frac{\pi}{4}$, $n \in I$

So general solution is $x = 2n\pi \pm \frac{\pi}{4}$ where $n \in I$

Q. 5. Find the general solution of each of the following equations:

(i) $\cos x = \frac{-1}{2}$

(ii) $\text{cosec}\, x = -\sqrt{2}$

(iii) tan x = -1

Answer : To Find: General solution.

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos x = \frac{-1}{2} = -\cos(\frac{\pi}{3}) = \cos(\pi - \frac{\pi}{3}) = \cos(\frac{2\pi}{3}) \implies x = 2n\pi \pm \frac{2\pi}{3}$, $n \in I$

So general solution is $x = 2n\pi \pm \frac{2\pi}{3}$ where $n \in I$

(ii) Given: cosec $x = -\sqrt{2}$

We know that $\text{cosec}\theta \times \sin\theta = 1$

So $\sin x = \frac{-1}{\sqrt{2}}$

Formula used: $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$, n ∈

By using above formula, we have

$\sin x = \frac{-1}{\sqrt{2}} = \sin\frac{5\pi}{4} \Rightarrow x = n\pi + (-1)^n . \frac{5\pi}{4}$

So general solution is $x = n\pi + (-1)^n . \frac{5\pi}{4}$ where $n \in I$

(iii) Given: tan x = -1

Formula used: $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$, $n \in I$

By using above formula, we have

$\tan x = -1 = \tan\frac{3\pi}{4} \Rightarrow x = n\pi + \frac{3\pi}{4}$, $n \in I$

So the general solution is $x = n\pi + \frac{3\pi}{4}$ where $n \in I$

Q. 6. Find the general solution of each of the following equations:

(i) $\sin 2x = \frac{1}{2}$

(ii) $\cos 3x = \frac{1}{\sqrt{2}}$

(iii) $\tan\frac{2x}{3} = \sqrt{3}$

Answer : To Find: General solution.

(i) Given: $\sin 2x = \frac{1}{2}$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in I$

By using above formula, we have

$\sin 2x = \frac{1}{2} = \sin\frac{\pi}{6} \implies 2x = n\pi + (-1)^n \cdot \frac{\pi}{6} \implies x = \frac{n\pi}{2} + (-1)^n \cdot \frac{\pi}{12}$, $n \in I$

So general solution is $x = \frac{n\pi}{2} + (-1)^n \cdot \frac{\pi}{12}$ where $n \in I$

(ii) Given: $\cos 3x = \frac{1}{\sqrt{2}}$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\cos 3x = \frac{1}{\sqrt{2}} = \cos(\frac{\pi}{4}) \Rightarrow 3x = 2n\pi \pm \frac{\pi}{4} \Rightarrow x = \frac{2n\pi}{3} \pm \frac{\pi}{12}, n \in I$

So the general solution is $x = \frac{2n\pi}{3} \pm \frac{\pi}{12}$ where $n \in I$

(iii) Given: $\tan \frac{2x}{3} = \sqrt{3}$

Formula used: $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$, $n \in I$

By using above formula, we have

$\tan \frac{2x}{3} = \sqrt{3} = \tan\frac{\pi}{3} \Rightarrow \frac{2x}{3} = n\pi + \frac{\pi}{3} \Rightarrow x = \frac{3n\pi}{2} + \frac{\pi}{2}, n \in I$

So general solution is $x = (3n+1)\frac{\pi}{2}$,where $n \in I$

Q. 7. Find the general solution of each of the following equations:

(i) sec 3x = -2
(ii) cot 4x = -1
(iii) $\operatorname{cosec} 3x = \frac{-2}{\sqrt{3}}$

Answer : To Find: General solution.

(i) Given: sec 3x = -2

We know that secθ × cosθ = 1

So cos 3x = $\frac{-1}{2}$

Formula used: cosθ = cosα $\Rightarrow$ θ = 2n$\pi \pm \alpha$, n ∈ I

By using above formula, we have

cos 3x = $\frac{-1}{2}$= -cos$\frac{\pi}{3}$= cos$\left(\pi - \frac{\pi}{3}\right)$= cos$\frac{2\pi}{3}$ $\Rightarrow$ 3x = 2n$\pi \pm \frac{2\pi}{3}$ $\Rightarrow$ x = $\frac{2n\pi}{3} \pm \frac{2\pi}{9}$, n ∈ I

So the general solution is x = $\frac{2n\pi}{3} \pm \frac{2\pi}{9}$, ,where n ∈ I

(ii) Given: cot 4x = -1

We know that $\tan\theta \times \cot\theta = 1$

So tan 4x = -1

Formula used: $\tan\theta = \tan\alpha \implies \theta = n\pi \pm \alpha$, $n \in I$

By using above formula, we have

$\tan 4x = -1 = \tan\frac{3\pi}{4} \implies 4x = n\pi + \frac{3\pi}{4} \implies x = \frac{n\pi}{4} + \frac{3\pi}{16}, n \in I$

So general solution is $x = (4n+3)\frac{\pi}{16}$,where $n \in I$

(iii) Given: cosec 3x = $\frac{-2}{\sqrt{3}}$

We know that cosecθ × sinθ = 1

So sin 3x = $\frac{-\sqrt{3}}{2}$

Formula used: sinθ = sinα $\Longrightarrow$ θ = nπ + $(-1)^n$.α , n ∈ I

By using above formula, we have

sin 3x = $\frac{-\sqrt{3}}{2}$ = sin$\frac{4\pi}{3}$ $\Longrightarrow$ 3x= nπ + $(-1)^n$. $\frac{4\pi}{3}$ $\Longrightarrow$ x= $\frac{n\pi}{3}$ + $(-1)^n$. $\frac{4\pi}{9}$, n ∈ I

So general solution is x = $\frac{n\pi}{3}$ + $(-1)^n$. $\frac{4\pi}{9}$, where n ∈ I

Q. 8. Find the general solution of each of the following equations:

(i) $4\cos^2 x = 1$
(ii) $4\sin^2 x - 3 = 0$
(iii) $\tan^2 x = 1$

Answer : To Find: General solution.

(i) Given: $4\cos^2 x = 1 \Rightarrow \cos^2 x = \left(\frac{1}{4}\right)$

$\therefore \cos^2 x = \cos^2\frac{\pi}{3}$

Formula used: $\cos^2\theta = \cos^2\alpha \;\Rightarrow\; \theta = n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

$x = n\pi \pm \frac{\pi}{3}$, $n \in I$

So the general solution is $x = n\pi \pm \frac{\pi}{3}$ where $n \in I$

(ii) Given: $4\sin^2 x - 3 = 0 \Rightarrow \sin^2 x = \frac{3}{4} = \sin^2\frac{\pi}{3}$

$\therefore \sin^2 x = \sin^2 \frac{\pi}{3}$

Formula used: $\sin^2\theta = \sin^2\alpha \implies \theta = n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

$x = n\pi \pm \frac{\pi}{3}$, $n \in I$

So the general solution is $x = n\pi \pm \frac{\pi}{3}$ where $n \in I$

(ii) Given: $\tan^2 x = 1 \Rightarrow \tan^2 x = \tan^2 \frac{\pi}{4}$

$\therefore \tan^2 x = \tan^2 \frac{\pi}{4}$

The formula used: $\tan^2\theta = \tan^2\alpha \implies \theta = n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

$x = n\pi \pm \frac{\pi}{4}$, $n \in I$

So the general solution is $x = n\pi \pm \frac{\pi}{4}$ where $n \in I$

So general solution is $x = \frac{n\pi}{3} + (-1)^n \cdot \frac{4\pi}{9}$, where $n \in I$

Q. 9. Find the general solution of each of the following equations:

(i) cos 3x = cos 2x
(ii) cos 5x = sin 3x
(iii) cos mx = sin nx

Answer : To Find: General solution.

(i) Given: $\cos 3x = \cos 2x \Rightarrow \cos 3x - \cos 2x = 0 \Rightarrow -2\sin\frac{(5x)}{2}\sin\frac{(x)}{2} = 0$

[NOTE: $\cos C - \cos D = -2\sin\frac{(C+D)}{2}\sin\frac{(C-D)}{2}$]

So, $\sin\frac{(5x)}{2} = 0$ or $\sin\frac{(x)}{2} = 0$

Formula used: $\sin\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$

$\frac{(5x)}{2} = n\pi$ or $\frac{(x)}{2} = m\pi$ where $n, m \in I$

$x = 2n\pi/5$ or $x = 2m\pi$ where $n, m \in I$

So general solution is $x = 2n\pi/5$ or $x = 2m\pi$ where $n, m \in I$

(ii) Given: cos 5x = sin 3x $\Rightarrow$ cos 5x = $\cos\left(\frac{\pi}{2} - 3x\right)$

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$, $n \in I$

By using the above formula, we have

$5x = 2n\pi + \left(\frac{\pi}{2} - 3x\right)$ or $5x = 2n\pi - \left(\frac{\pi}{2} - 3x\right)$

$8x = 2n\pi + \frac{\pi}{2}$ or $2x = 2n\pi - \frac{\pi}{2}$

$x = \frac{n\pi}{4} + \frac{\pi}{16}$ or $x = n\pi - \frac{\pi}{4}$ where $n \in I$

So general solution is $x = \frac{n\pi}{4} + \frac{\pi}{16}$ or $x = n\pi - \frac{\pi}{4}$ where $n \in I$

(iii) Given: cos mx = sin nx $\Rightarrow$ cos mx = $\cos\left(\frac{\pi}{2} - nx\right)$

Formula used: $\cos\theta = \cos a \Rightarrow \theta = 2k\pi \pm a$, $k \in I$

By using the above formula, we have

mx = $2k\pi + \left(\frac{\pi}{2} - \mathrm{nx}\right)$ or 5x = $2k\pi - \left(\frac{\pi}{2} - \mathrm{nx}\right)$

(m+n)x = $2k\pi + \frac{\pi}{2}$ or (m-n)x = $2k\pi - \frac{\pi}{2}$

x = $\frac{2k\pi}{(\mathrm{m+n})} + \frac{\pi}{2(\mathrm{m+n})}$ or x = $\frac{2k\pi}{(\mathrm{m-n})} + \frac{\pi}{2(\mathrm{m-n})}$ where k ∈ I

x = $\frac{(4k+1)\pi}{2(\mathrm{m+n})}$ or x = $\frac{(4k-1)\pi}{2(\mathrm{m-n})}$ where k ∈ I

So the general solution is x = $\frac{(4k+1)\pi}{2(\mathrm{m+n})}$ or x = $\frac{(4k-1)\pi}{2(\mathrm{m-n})}$ where k ∈ I

Q. 10. Find the general solution of each of the following equations:

sin x = tan x

Answer : To Find: General solution.

Given: sin x = tan x ⇒ sin x = sin x ÷ cos x

So sin x = 0 or cos x = 1 = cos(0)

Formula used: $\sin\theta = 0 \Rightarrow \theta = n\pi$, n ∈ I and $\cos\theta = \cos\alpha \Rightarrow \theta = 2k\pi \pm \alpha$, k ∈ I

x = $n\pi$ or x = $2k\pi$ where n, k ∈ I

So general solution is x =$n\pi$ or x = $2k\pi$ where n, k ∈ I

Q. 11. Find the general solution of each of the following equations:

4sin x cos x + 2sin x + 2cos x + 1 = 0

Answer : To Find: General solution.

Given: 4sin x cos x + 2sin x + 2cos x + 1 = 0 ⇒ 2sin x (2cos x + 1) + 2cos x + 1 = 0

So (2cos x + 1) (2sin x + 1) = 0

$\cos x = \frac{-1}{2} = \cos(\frac{2\pi}{3})$ or $\sin x = \frac{-1}{2} = \sin\frac{7\pi}{6}$

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2n\pi \pm \alpha$ or $\sin\theta = \sin\alpha \implies \theta = m\pi + (-1)^m\alpha$ where n,m ∈ I

$x = 2n\pi \pm \frac{2\pi}{3}$ or $x = m\pi + (-1)^m . \frac{7\pi}{6}$ where n, m ∈ I

So the general solution is $x = 2n\pi \pm \frac{2\pi}{3}$ or $x = m\pi + (-1)^m . \frac{7\pi}{6}$ where n, m ∈ I

Q. 12. Find the general solution of each of the following equations:

$\sec^2 2x = 1- \tan 2x$

Answer : To Find: General solution.

Given: $\sec^2 2x = 1- \tan 2x \Rightarrow 1 + \tan^2 2x + \tan 2x = 1 \Rightarrow \tan 2x\,(1+\tan 2x) = 0$

So, tan 2x = 0 or tan 2x = -1 = tan $(\frac{3\pi}{4})$

Formula used: : tanθ= 0 $\Rightarrow$ θ = nπ , n $\in$ I and tanθ = tanα $\Rightarrow$ θ = kπ $\pm$ α , k $\in$ I

By using above formula, we have

2x = nπ or 2x = kπ $\pm$ $\frac{3\pi}{4}$ $\Rightarrow$ x = $\frac{n\pi}{2}$ or x = $\frac{k\pi}{2} \pm \frac{3\pi}{8}$

So the general solution is x = $\frac{n\pi}{2}$ or x = $\frac{k\pi}{2} \pm \frac{3\pi}{8}$ where n, k $\in$ I

Q. 13. Find the general solution of each of the following equations:

$\tan^3 x - 3\tan x = 0$

Answer : To Find: General solution.

Given: $\tan^3 x - 3\tan x = 0 \Rightarrow \tan x(\tan^2 x - 3) = 0 \Rightarrow \tan x = 0$ or tanx = $\pm\sqrt{3}$

$\Rightarrow$ tan x = 0 or tanx = tan$(\frac{\pi}{3})$ or tan x = tan$(\frac{2\pi}{3})$

$\Rightarrow$ Formula used: tanθ= 0 $\Rightarrow$ θ = nπ , n $\in$ I, tanθ = tanα $\Rightarrow$ θ = kπ $\pm$ α , k $\in$ I

So x = nπ or x = kπ + $\frac{\pi}{3}$ or x = pπ + $\frac{2\pi}{3}$ where n, k, p $\in$ I

So general solution is x = nπ or x = kπ + $\frac{\pi}{3}$ or x = pπ + $\frac{2\pi}{3}$ where n, k, p $\in$ I

Q. 14. Find the general solution of each of the following equations:

sin x + sin 3x + sin 5x = 0

Answer : To Find: General solution.

Given: sin x + sin 3x + sin 5x = 0 ⇒ sin 3x + 2sin 3x cos 2x= 0 ⇒ sin 3x (1 + 2cos 2x) = 0

[NOTE: sin C + sin D = 2sin (C+D)/2 × cos (C-D)/2]

$\Rightarrow \sin 3x = 0 \text{ or } \cos 2x = \frac{-1}{2} = \cos(\frac{2\pi}{3})$

Formula used: $\sin\theta = 0 \Rightarrow \theta = n\pi$, $n \in I$, $\cos\theta = \cos\alpha \Rightarrow \theta = 2k\pi \pm \alpha$, $k \in I$

$\Rightarrow 3x = n\pi \text{ or } 2x = 2k\pi \pm \frac{2\pi}{3} \Rightarrow x = \frac{n\pi}{3} \text{ or } x = k\pi \pm \frac{\pi}{3}$ where $n, k \in I$

So general solution is $x = \frac{n\pi}{3}$ or $x = k\pi \pm \frac{\pi}{3}$ where n, k, $\in$ I

Q. 15. Find the general solution of each of the following equations:

sin x tan x – 1 = tan x – sin x

Answer : To Find: General solution.

Given: sin x tan x – 1 = tan x – sin x ⇒ sin x(tan x + 1) = tan x + 1

So $\sin x = 1 = \sin\left(\frac{\pi}{2}\right)$ or $\tan x = -1 = \tan\left(\frac{3\pi}{4}\right)$

Formula used: $\sin\theta = \sin\alpha \implies \theta = n\pi + (-1)^n\alpha$, $n \in I$ and $\tan\theta = \tan\alpha \implies \theta = k\pi \pm \alpha$, $k \in I$

$\implies x = n\pi + (-1)^n \frac{\pi}{2}$ or $x = k\pi \pm \frac{3\pi}{4}$ where $n, k \in I$

So general solution is $x = n\pi + (-1)^n \frac{\pi}{2}$ or $x = k\pi \pm \frac{3\pi}{4}$ where $n, k, \in I$

Q. 16. Find the general solution of each of the following equations:

cos x + sin x = 1

Answer : To Find: General solution.

Given: $\cos x + \sin x = 1 \implies \cos(x - \frac{\pi}{4}) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

[divide $\sqrt{2}$ on both sides and $\cos(x-y) = \cos x \cos y - \sin x \sin y$]

Formula used: $\cos\theta = \cos\alpha \implies \theta = 2k\pi \pm \alpha$, $k \in I$

$\implies x - \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{4} \implies x = 2k\pi \pm \frac{\pi}{4} + \frac{\pi}{4} \implies x = 2k\pi + \frac{\pi}{4} + \frac{\pi}{4}$ or $\implies x = 2k\pi - \frac{\pi}{4} + \frac{\pi}{4}$

$\implies x = 2k\pi + \frac{\pi}{2}$ or $x = 2k\pi$

So general solution is $x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi$ where $n \in I$

Q. 17. Find the general solution of each of the following equations:

cos x – sin x = -1

Answer : To Find: General solution.

Given: cos x - sin x = 1 $\Rightarrow$ $\cos(x + \frac{\pi}{4}) = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

[divide $\sqrt{2}$ on both sides and cos(x-y) = cos x cos y - sin x sin y]

So sin x = 0 or cos x = 0

Formula used: $\cos\theta = \cos\alpha \Rightarrow \theta = 2k\pi \pm \alpha$, $k \in I$

$\Rightarrow x + \frac{\pi}{4} = 2k\pi \pm \frac{3\pi}{4} \Rightarrow x = 2k\pi \pm \frac{3\pi}{4} - \frac{\pi}{4} \Rightarrow x = 2k\pi + \frac{3\pi}{4} - \frac{\pi}{4}$ or $\Rightarrow x = 2k\pi - \frac{3\pi}{4} - \frac{\pi}{4}$

$\Rightarrow x = 2k\pi - \pi$ or $x = 2k\pi + \frac{\pi}{2}$

So general solution is $x = 2n\pi + \frac{\pi}{2}$ or $x = (2n-1)\pi$ where $n \in I$

Q. 18. Find the general solution of each of the following equations:

$\sqrt{3}$ cos x + sin x = 1

Answer : To Find: General solution.

Given: $\sqrt{3}$ cos x + sin x = 1 $\Rightarrow$ cos (x - $\frac{\pi}{6}$) = $\frac{1}{2}$ = cos($\frac{\pi}{3}$) or cos($\frac{5\pi}{3}$)

[Divide $\sqrt{2}$ on both sides and cos(x-y) = cos x cos y - sin x sin y]

Formula used: cosθ = cosα $\Rightarrow$ θ = 2n$\pi \pm \alpha$

By using above formula, we have

$\Rightarrow$ x - $\frac{\pi}{6}$ = 2n$\pi \pm \frac{\pi}{3}$ $\Rightarrow$ x = 2n$\pi \pm \frac{\pi}{3} + \frac{\pi}{6}$

$\Rightarrow$ x = 2nπ + $\frac{\pi}{2}$ or x = 2nπ - $\frac{\pi}{6}$ where n $\in$ *I*

So general solution is x = 2nπ + $\frac{\pi}{2}$ or x = 2nπ - $\frac{\pi}{6}$ where n $\in$ *I*

Q. 19. Find the general solution of each of the following equations:

2 tan x – cot x + 1 = 0

Answer : To Find: General solution.

Given: 2 tan x – cot x + 1 = 0 ⇒ $2\tan^2 x$ – 1 + tan x = 0 ⇒ $2\tan^2 x$ – 1 + 2tan x – tanx = 0 ⇒ 2tanx(tanx +1) – (1+ tanx) = 0

$\Rightarrow$ (2tanx-1) (1+ tanx) = 0 $\Rightarrow$ tan x = $\frac{1}{2} = \tan^{-1}\frac{1}{2}$ or tan x = -1 = tan$\frac{3\pi}{4}$

Formula used: tanθ = tanα $\Rightarrow$ $\theta = n\pi + \alpha$, n ∈ I

x = $n\pi + \tan^{-1}\frac{1}{2}$ or x = $n\pi + \frac{3\pi}{4}$

So the general solution is x = $n\pi + \tan^{-1}\frac{1}{2}$ or x = $n\pi + \frac{3\pi}{4}$ where n ∈ I

Q. 20. Find the general solution of each of the following equations:

sin x tan x – 1 = tan x – sin x

Answer : To Find: General solution.

Given: sin x tan x – 1 = tan x – sin x ⇒ sin x(tan x + 1) = tan x + 1

So sin x = 1 = sin $(\frac{\pi}{2})$ or tan x = -1 = tan$(\frac{3\pi}{4})$

Formula used: sinθ = sinα $\Rightarrow$ $\theta = n\pi + (-1)^n\alpha$, n ∈ I and tanθ = tanα $\Rightarrow$ $\theta = k\pi + \alpha$, k ∈ I

$\Rightarrow$ x = $n\pi + (-1)^n\frac{\pi}{2}$ or x = $k\pi + \frac{3\pi}{4}$ where n, k ∈ I

So general solution is x = $n\pi + (-1)^n\frac{\pi}{2}$ or x = $k\pi + \frac{3\pi}{4}$ where n, k ∈ I

Q. 21. Find the general solution of each of the following equations:

cot x + tan x = 2 cosec x

Answer : To Find: General solution.

Given: cot x + tan x = 2 cosec x ⇒ $\cos^2 x + \sin^2 x$ = 2 sinx cosx cosec x ⇒ 1 = sin 2x cosec x

$\Longrightarrow$ cosec 2x = cosecx $\Longrightarrow$ sin x = sin 2x $\Longrightarrow$ sin x = 2 sin x cos x $\Longrightarrow$ sin x = 0 or cos x = $\frac{1}{2}$ = cos( )

Formula used: $\sin\theta = 0 \Longrightarrow \theta = n\pi$, $\cos\theta = \cos\alpha \Longrightarrow \theta = 2n$

By using above formula , we have

$x = n\pi$ or $x = 2m\pi \pm \frac{\pi}{3}$ where n, m I

So general solution is $x = n\pi$ or $x = 2m\pi \pm \frac{\pi}{3}$ where n, m$\in I$