* Gear Trains *

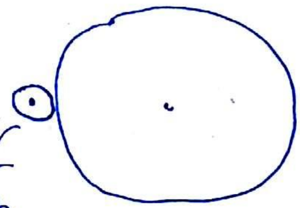
Gear train - Combination of Gears

Why Gear train ??

1. large centre distance
2. Very high/very low requirement of Velocity Ratio

~~Eg.~~ For eg.

$$\frac{\omega_1}{\omega_2} = \frac{10}{1} = \frac{R_2}{R_1}$$



tooth may break due to high inertia of large gear.

3. Multiple Velocity Ratio are require.

⇒ Any Gear train is a Combination of

1. Main Driver (DVR.)
2. Main Driven (DVN.)
3. Intermidiate Gear

} All Gear train

4. Arm (Not a Gear)

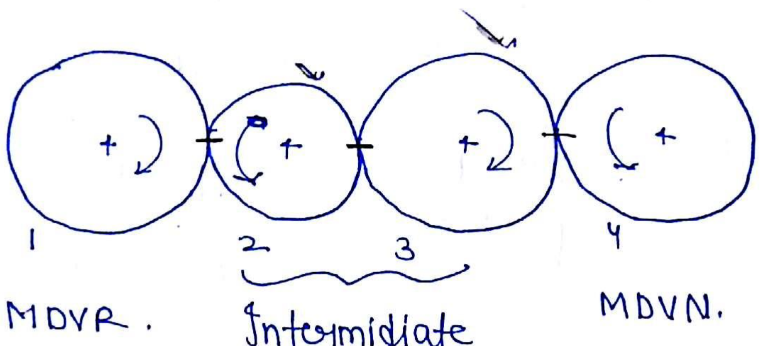
} epicyclic Gear train

* Apply law of Gearing only on touching bodies

$$\frac{\omega_{main\ DVR.}}{\omega_{main\ DVN.}} = \text{Speed Ratio (SR) of Gear train}$$

$$\frac{\omega_{main\ DVN.}}{\omega_{main\ DVR}} = \frac{1}{S.R.} = \text{Train Value.}$$

Simple Gear train :- Every shaft is having only one gear in use.



→ Idlers

(module - same)
All
touching one by one

Apply law of Gearing

(1,2) $\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$ — (1) (2,3) $\frac{\omega_2}{\omega_3} = \frac{T_3}{T_2}$ — (2) (3,4) $\frac{\omega_3}{\omega_4} = \frac{T_4}{T_3}$ — (3)

(1) × (2) × (3)

$$\frac{\omega_1}{\omega_4} = \frac{T_4}{T_1} = \text{Speed ratio}$$

$\boxed{\text{2 \& 3 are Idlers}}$ — Idlers are touching both side

If No. of bodies

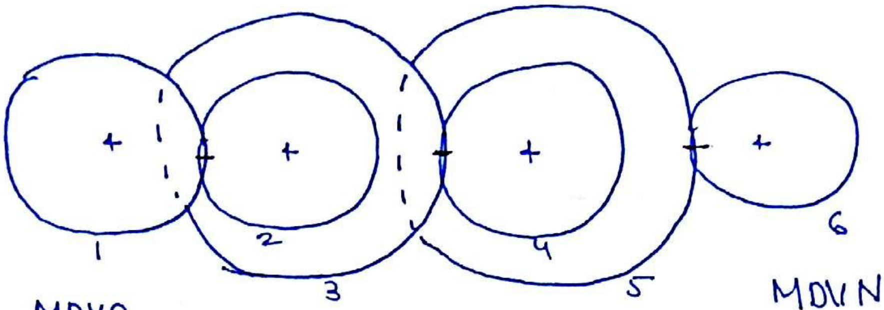
odd - dirn same

even - dirn opposite

Compound Gear train:- At least one ~~on~~ of the intermidiate shaft must have more than one gear in use.

modules

$$\left\{\begin{array}{l} m_1 = m_2 \\ m_3 = m_4 \\ m_5 = m_6 \end{array}\right\}$$



MDVR

(2-3) compound $\omega_2 = \omega_3$

{both are on same shaft}

(4-5) compound $\omega_4 = \omega_5$

MDVN

Group of Driver DVR (1, 3, 5)

Group of Driven DVN (2, 4, 6)

(1,2) $\dfrac{\omega_1}{\omega_2} = \dfrac{T_2}{T_1}$ — (1)

(3,4) $\dfrac{\omega_3}{\omega_4} = \dfrac{T_4}{T_1}$ (2)

(4,5,6) $\dfrac{\omega_5}{\omega_6} = \dfrac{T_6}{T_5}$ — (3)

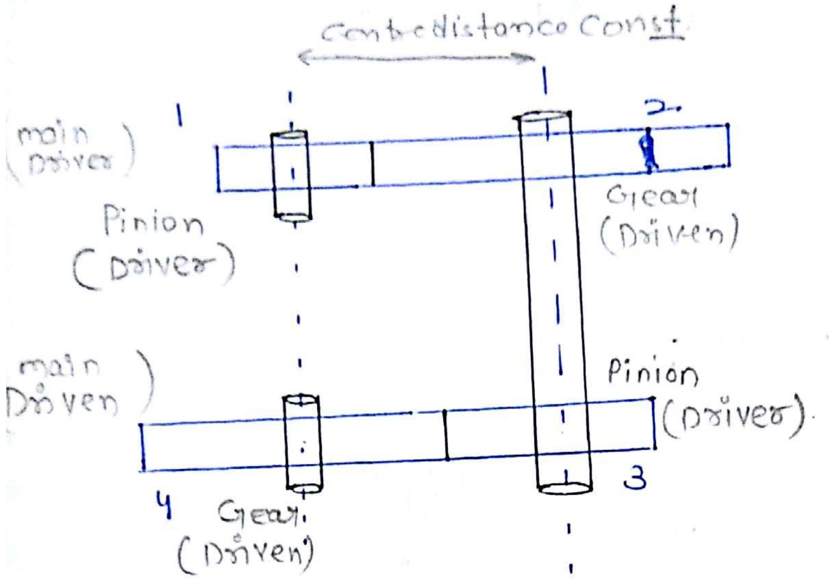
(1) x (2) x (3)

$$\boxed{\dfrac{\omega_1}{\omega_6} = \dfrac{T_2 \times T_4 \times T_6}{T_1 \times T_2 \times T_3} = \text{speed ratio}}$$

$$\boxed{S.R. = \dfrac{\text{Product of the No. of teeth on DVN}}{\text{Product of the No. of teeth on DVR}}}$$

* No idlears. (No Gear is touching both side)

Reverted Gear train:— That compound gear train which is used to connect co-axial shafts.



$m_1 = m_2 = m$

$m_3 = m_4 = m'$

DVR : (1, 3)

DVN : (2, 4)

$$\boxed{\text{Speed Ratio} = \frac{\omega_1}{\omega_2} = \left(\frac{T_2 \times T_4}{T_1 \times T_3}\right)}$$

Note:— If in the problem of Reverted Gear train, speed reduction is Given ~~to~~ same

then take $\left(\frac{T_2}{T_1}\right) = \left(\frac{T_4}{T_3}\right)$

A General Concept:—

$$r_1 + r_2 = r_3 + r_4$$

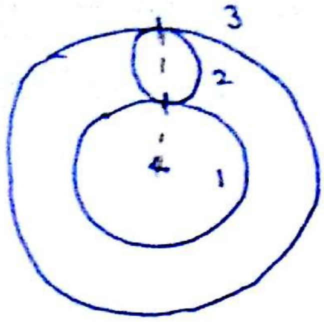
$$\frac{mT_1}{2} + \frac{mT_2}{2} = \frac{m'T_3}{2} + \frac{m'T_4}{2}$$

$$\boxed{m(T_1 + T_2) = m'(T_3 + T_4)}$$

if All gears have same module

$$\boxed{T_1 + T_2 = T_3 + T_4}$$

For eg.

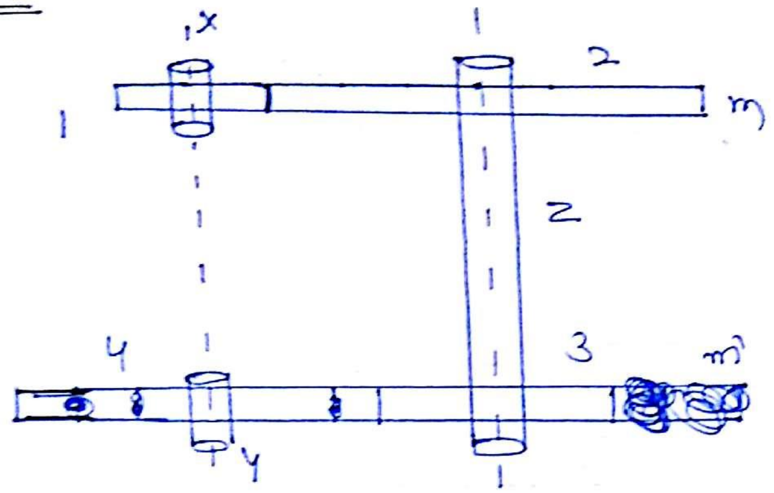


module - same

$$r_1 + 2r_2 = r_3$$

$$T_1 + 2T_2 = T_3$$

Q 48



~~small~~
Driver-pinion

$m = 2$

$m' = 3$

$$\omega_4 < \frac{\omega_1}{12}$$

$$\frac{\omega_1}{\omega_4} > 12$$

$$\frac{T_2 . T_4}{T_1 . T_3} > 12$$

$$T_1 = T_3 = 24 \text{ (Given)}$$

$$m(T_1 + T_2) = m'(T_3 + T_4)$$

$$2(24 + T_2) = 3(24 + T_4)$$

$$T_4 = \frac{2}{3}(T_2 - 12) \quad \text{— (1)}$$

$$T_2 . T_4 > 6912 \quad \text{— (2)}$$

from (1) & (2) $\quad T_2^2 - 12T_2 - 10368 > 0$

$$T_2 > 108$$

Assume $T_2 = 109$, $T_4 = \frac{2}{3}(109-12)$

$T_4 = 64.66$

Assume $T_4 = 65$

$[2(24+T_2) = 3(24+65)$ — we $\uparrow$ this $64.66 \to 65$

Assume $= T_3 = 24-1 = \underline{23}$

This is because we are assuming $64.66 \to 65$ to we have to reduce $\underline{\underline{T_3}}$

$$2(24+T_2) = 3(23+65)$$

$$T_2 = 108$$

$T_1 = 24$, $T_2 = 108$, $T_3 = 23$, $T_4 = 65$ Ans

$$\frac{\omega_1}{\omega_2} = \frac{108 \times 65}{24 \times 23} > 12.718$$

centre distance $= r_1 + r_2$

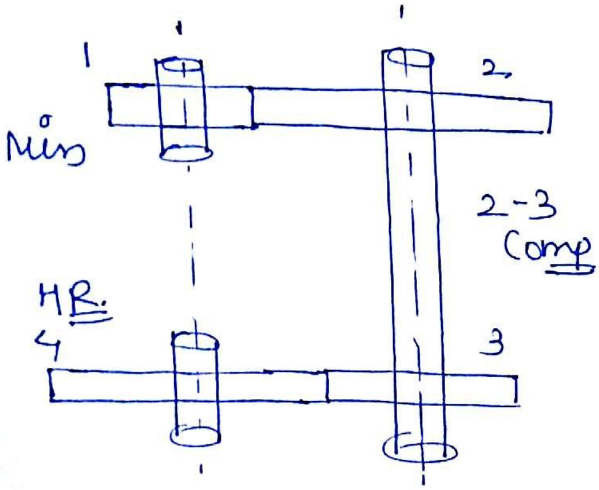
$$= \frac{m}{2}(T_1+T_2)$$

$$= \frac{2}{2}(24+108)$$

$$= \underline{\underline{132\ mm}}$$

Problem

A reverted gear train is designed to rotate hour hand of the clock with the help of minute hand. Considering the module of all the wheels same determine the appropiate No. of teeth one the wheels any wheel should not have 11 teeth ~~of~~ or less.



min head

$$\omega_1 = \frac{2\pi}{60 \times 60} \ rad/s$$

HR. head

$$\omega_4 = \frac{2\pi}{12 \times 60 \times 60} = rad/s$$

$$\frac{\omega_1}{\omega_4} = 12$$

$$\left(\frac{T_2}{T_1}\right) \times \left(\frac{T_4}{T_3}\right) = 12 \quad \text{—} (1)$$

$$T_1 + T_2 = T_3 + T_4 \quad \text{—} (2)$$

$\left.\begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{matrix}\right\} > 11$

Assume: $T_1 = 12$

Assume $\frac{T_2}{T_1} = 4$, $\frac{T_4}{T_3} = 3$

$T_2 = 48$, $12 + 48 = T_3 + 3T_3$

$T_3 = 15$

$T_4 = 45$

Ans

Assume $\frac{T_2}{T_1} = 3$, $\frac{T_4}{T_3} = 4$

$T_2 = 3T_1 = 36$

$12 + 36 = T_3 + 4T_3$

$T_3 = \frac{48}{5}$

Invalid soln

Assume

$\frac{T_2}{T_1} = 6, \quad \frac{T_4}{T_3} = 2$

$T_2 = 72$

$12 + 72 = T_3 + 2T_3$

$T_3 = 28$

$T_4 = 56$

Valid sol^n

Assume

$\frac{T_2}{T_1} = 2, \quad \frac{T_4}{T_3} = 6$

$T_2 = 2T_1 = 24$

$12 + 24 = T_3 + 6T_3$

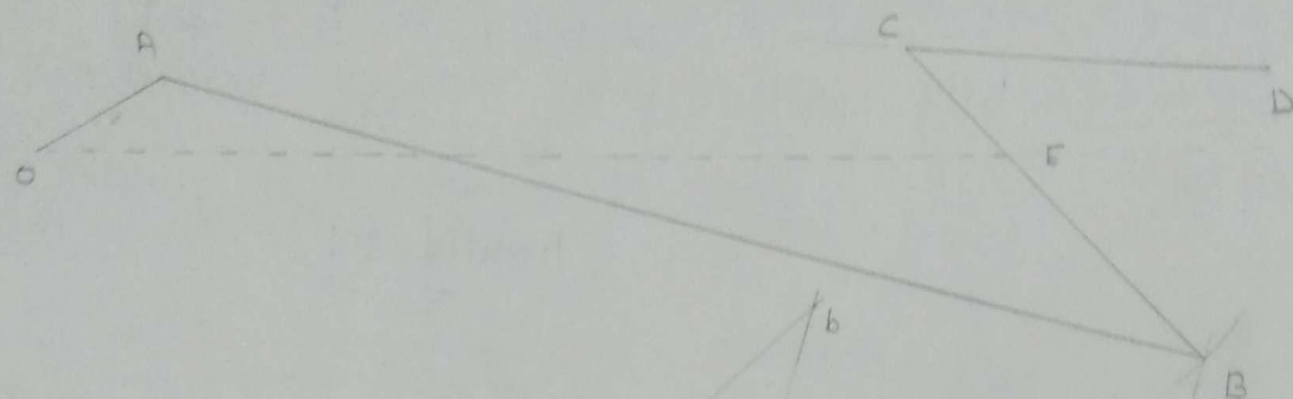
$T_3 = \frac{36}{7}$

Invalid sol^n

Q.27 $V_{AO} = \frac{0.200 \times 2\pi \times 120}{60}$

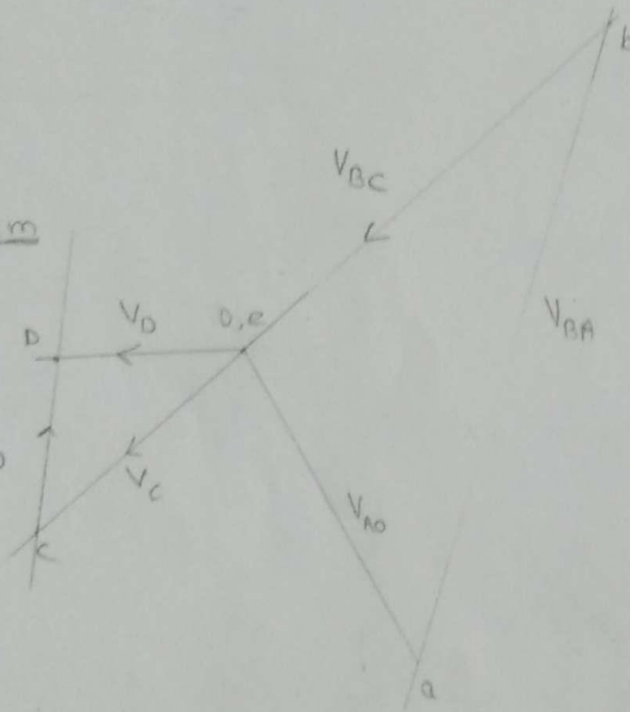
$V_{AO} = 2.5132$ m/s.

Scale = 100mm = 1 cm



Scale

0.5 m = 1 cm



$V_B = 4.4$ m/s $\quad V_{AB} = 5.11$ m/s $\quad \omega_{AB} = 3.40$ rad/s

$V_c = 2.4$ m/s $\quad V_{BC} = 5.6$ m/s $\quad \omega_{BC} = 9.33$ rad/s

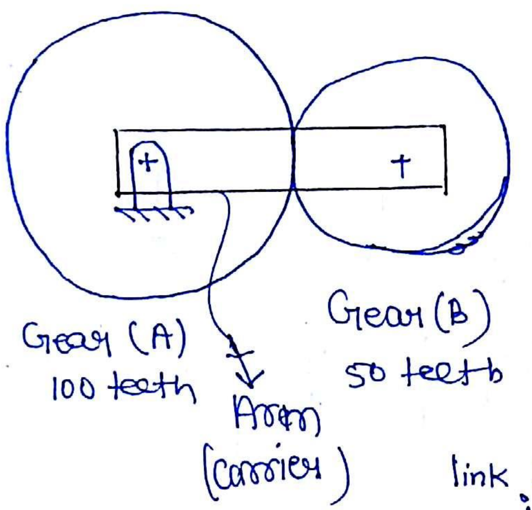
$V_D = 1.8$ m/s $\quad V_{CD} = 1.2$ m/s $\quad \omega_{CD} = 2.4$ rad/s.

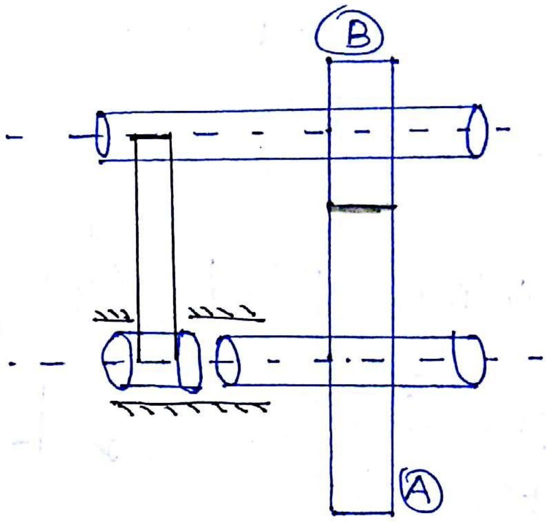
Epi-Cyclic Gear train:-

Epi - Axis
Cyclic - Rotation

Apart from the rotation of Gear if any Gear axis is rotating w.r.t. some other axis then the train will be known as Epi-cyclic Gear train. It may be simple epicyclic, compound, reverted, Bevel epicyclic and so on..

To rotate the axis of the Gear a link is used which is known as <u>arm</u> or <u>carrier</u>

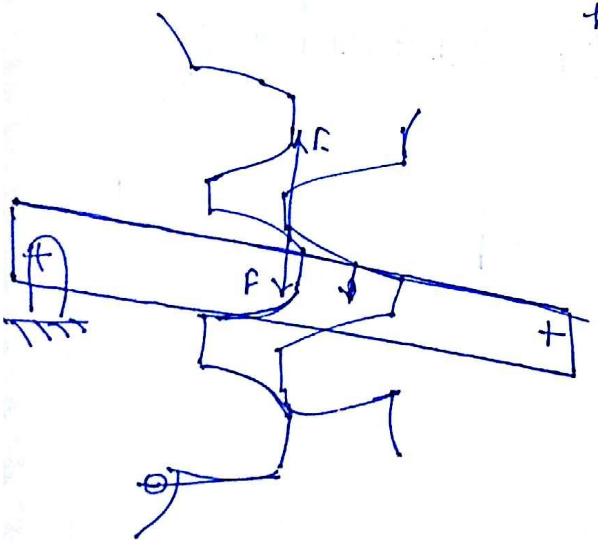




link $l = 4$
$j = 3$
$h = 1$

$F = 3(4-1) - 2\times3 - 1 \Rightarrow \boxed{F = 2}$

two input require for constraint output



<u>let</u> Gear A is rotating by 100 rpm (AC)

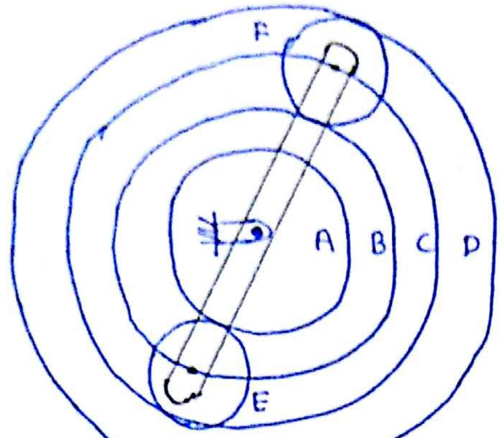
Gear B:
- By Gear A → 200 rpm (CW)
- Arm → 100 rpm (CW)

300 rpm (CW)

Arm → 200 rpm (ACW) — zero

Arm → 300 rpm (ACW) — 100 rpm (ACW)

Problem



A/B → Compound

All Gears have same module

$T_A = 20$ Gear D - fixed

$T_B = 30$

$T_E = T_F = 10$

Arm = 100 rpm (ACW)

Find N of ~~All rest~~ All 5 Gears.

Sol^n

$T_A + 2T_E = T_C$ $T_B + 2T_F = T_D$

$20 + 20 = T_C$ $30 + 20 = T_D$

$T_C = 40$ $T_D = 50$

External = opposite dir^n

Internal = same dir^n

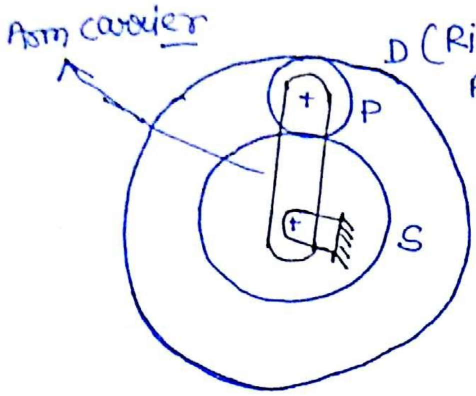
Motion	Arm	(Comp) A/B (20) (30)	E (10)	C (40)	F (10)	D (50)
1. Arm fixed let Gear (A) rotate +x rpm (CW)	0	$+x$	$-x\cdot\frac{\cancel{20}^{2}}{\cancel{10}}$	$-x\cdot\frac{\cancel{20}}{\cancel{10}}\cdot\frac{\cancel{10}}{\cancel{40}_{2}}$	$-x\cdot\frac{3\cancel{0}}{1\cancel{0}}$	$-x\cdot\frac{3\cancel{0}}{\cancel{10}}\cdot\frac{\cancel{10}}{5\cancel{0}}$
2. Arm	y	$y+x$	$(y-2x)$	$\left(y-\frac{x}{2}\right)$	$(y-3x)$	$\left(y-\frac{3x}{5}\right)$
Final sol^n →	-100					

D is fixed so $y - \frac{3x}{5} = 0$

$y = -100$

$x = \frac{-500}{3}$

Planetary Gear train (By Nature Epi-cyclic)



module will be same for All

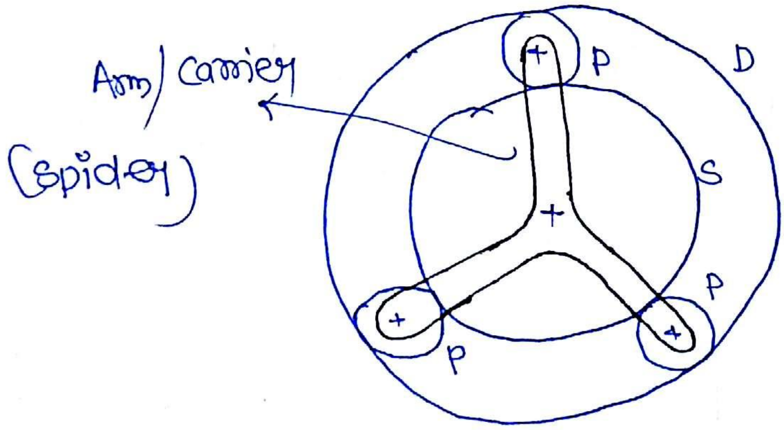
Ist Input :

Sun	Ring
→ fixed	input
→ input	fixed

IInd Input :. Arm.

- Generally in an epi-cyclic Gear train, No. of planets are more than one why ??

1. for the balancing of Gear train
2. For the load distribution among the no. of planets in high power transmission.



Q.49

IES 2000

	Arm	S (T_S)	P (T_P)	D (72)
Arm is fixed	0	$+x$	$-x \frac{T_S}{T_P}$	$-x \frac{T_S}{T_P} \times \frac{T_P}{72}$
Arm is rotating	y	$(y+x)$	$y - x \frac{T_S}{T_P}$	$\left(y - x \frac{T_S}{72}\right)$

Given $T_D = \frac{252}{3.5} = 72$ teeths. $\boxed{T_D = 72}$

$$T_S + 2T_P = 72$$

$$N_D = 0, \quad N_S = 5 N_A$$

$$\hookrightarrow (y+x) = 5y \Rightarrow x = 4y$$

$$y - x \frac{T_S}{72} = 0$$

$$y\left(1 - \frac{T_S}{18}\right) = 0, \quad y \neq 0$$

(Arm speed)

$$1 - \frac{T_S}{18} = 0$$

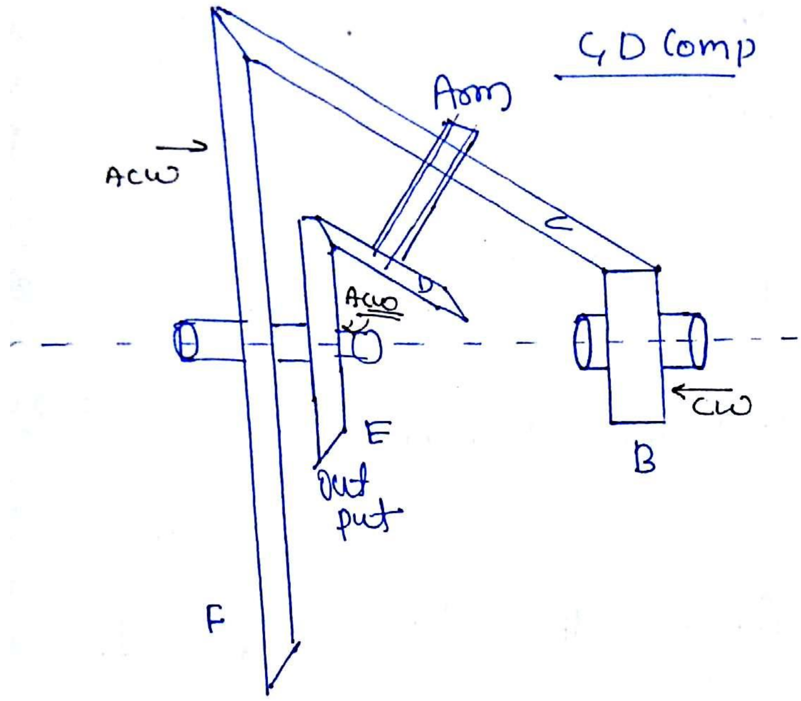
$$\boxed{T_S = 18}$$

$$18 + 2T_P = 72$$

$$\boxed{T_P = 27}$$

Direction Consideration in Bevel epi-cyclic Gear trains

Q.44 Humpuge Reduction Gear train in Lathe machine



$T_B = 19$

$T_C = 52$

$T_D = 20$

$T_E = 40$

$T_F = 76$

$N_B = 300$ rpm

$N_E = ?$

$N_F = -500$ rpm
↓
So it is not on output

Arm	B (19)	C & D (57) (20)	E (40)	F (76)
0	$+x$	$(\pm) x \cdot \frac{19}{57_3}$	$(-) x \cdot \frac{19}{57_3} \cdot \frac{20}{40_2}$	$(-) x \cdot \frac{19}{57} \cdot \frac{57}{76_4}$
y	$y+x$	$\left(y \pm \frac{x}{3}\right)$	$\left(y - \frac{x}{6}\right)$	$\left(y - \frac{x}{4}\right)$

$y + x = 300$ — (1)

$y - x/4 = -500$ — (2)

$x = 640$

$y = -340$

$N_E = y - x/6 = (-340) - \frac{640}{6}$

$N_E = -446.66$

$N_E = -447$ rpm

Fixing or holding Torque in an epicyclic Gear Train :-

$$\Sigma T = \left(T_{input} + T_{output} + T_{fixing} \right) = 0 \qquad -(1)$$

Power eqn $\quad \eta_{GT} = \dfrac{P_{output}}{P_{input}} \quad$ efficiency of Gear train

$$\eta_{GT} \, P_{input} + P_{output} = 0$$

$$\eta_{GT} \, T_{input} \cdot \omega_{input} + T_{output} \cdot \omega_{output} = 0 \qquad -(2)$$

Note:- If η_{GT} is not Given Then take $\eta_{GT} = 1$

Ques

$N_{output} = + \cancel{100}\ 250 \qquad T_{input} = +50$

$N_{input} = +100 \qquad T_{fixing} = ?$

$\eta_{GT} = 1$

$$(50)(100) + (T_{out})(250) = 0$$

$$T_{output} = -20$$

$$(50) + (-20) + T_{fixing} = 0$$

$$T_{fixing} = 30 \text{ kN-m}$$

ACW

Terminology of Helical or Spiral Gear :—

1 - Driver

2 - Driven

ϕ - pressure angle

μ - Coefficient of friction

$\boxed{\phi = \tan^{-1}\mu}$

ψ - Spiral angle

P - Pitch (circular pitch) P_c

P_n - Normal pitch

m - module

m_n - Normal module

λ - lead angle

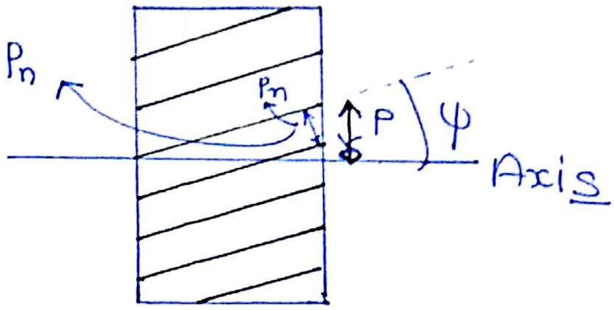
L - Lead — worm/worm wheel

= P (Single start)

= 2P (double start)

⋮

θ - Angle b/w the shafts Axis



Normal pitch

$$P_n = P\cos\psi$$

Normal Module

$$m_n = m\cos\psi$$

* For two mating Gears Normal pitch should be same. (module may diff.)

For two mating Gear

$$m_{n_1} = m_{n_2}$$

$$m_1 \cos\psi_1 = m_2 \cos\psi_2$$

$\theta = (\psi_1 + \psi_2) \Rightarrow$ If same hand gear are in contact

$\theta = (\psi_1 - \psi_2) \Rightarrow$ If opposite hand gear are in contact.

Note:- If in problem Nothing is mention take $\boxed{\theta = \psi_1 + \psi_2}$

Centre distance

$$= r_1 + r_2 \quad (\text{Always})$$

$$= \frac{m_1 T_1}{2} + \frac{m_2 T_2}{2}$$

$$= \frac{m_n T_1}{\cos\psi_1 \cdot 2} + \frac{m_n T_2}{\cos\psi_2 \cdot 2}$$

$$= \frac{m_n}{2}\left[\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right]$$

Velocity Ration (VR)

$$VR = \frac{\omega_1}{\omega_2} = \frac{V_1/r_1}{V_2/r_2} = \frac{V_1 \cdot r_2}{V_2 \cdot r_1}$$

law of gearing says $V_1 \cos\alpha = V_2 \cos\beta$

so $V_1 \cos\psi_1 = V_2 \cos\psi_2$

$$\frac{V_1}{V_2} = \frac{\cos\psi_2}{\cos\psi_1}$$

$$VR = \frac{\cos\psi_2}{\cos\psi_1} \times \frac{\frac{m_2 T_2}{2}}{\frac{m_1 T_1}{2}} = \frac{\cos\psi_2}{\cos\psi_1} \times \frac{\frac{m_n}{\cos\psi_2}}{\frac{m_n}{\cos\psi_1}} \times \frac{T_2}{T_1}$$

$$\boxed{VR = \frac{T_2}{T_1} = \frac{\omega_1}{\omega_2}}$$

$$VR = \frac{D_2 \cos\psi_2}{D_1 \cos\psi_1} = \frac{\omega_1}{\omega_2}$$

efficiency:–
(Remember)

$$\eta = \frac{P_2}{P_1} = \frac{F_2 \times r_2}{F_1 \times r_1}$$

$$\boxed{\eta = \frac{\cos(\psi_2 + \phi) \cdot \cos\psi_1}{\cos(\psi_1 - \phi) \cdot \cos\psi_2}}$$

max. eff (η_{max}) diff. wrt ψ_1 ~~or~~ ψ_2

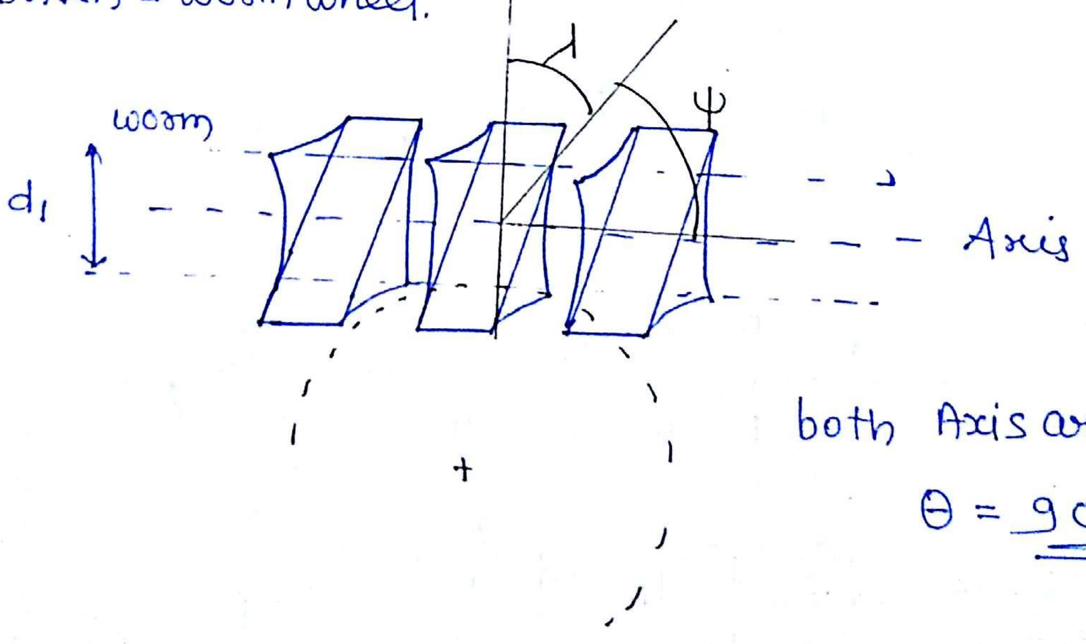
will ~~find~~ Get. $\psi_1 = \frac{\theta + \phi}{2}$, $\theta = \psi_1 + \psi_2$

$$\boxed{\eta_{max} = \frac{1 + \cos(\theta + \phi)}{1 + \cos(\theta - \phi)}}$$

Terminology of worm and worm wheel: – (W/WM)

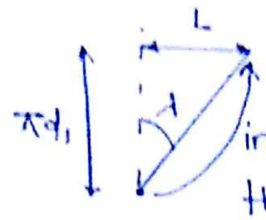
Driver – worm

Driven – Worm wheel.



both Axis are ⊥er

$\theta = 90°$



in one rotation this point how much travel is lead (L)

$$\boxed{\tan\lambda_1 = \frac{L}{\pi d_1}}$$

$L \rightarrow$ lead
$= p$ (single start)
$= 2p$ (double start)

$$\lambda_1 + \psi_1 = 90^\circ$$

$$\boxed{\psi_1 = 90^\circ - \lambda_1}$$

$$\theta = 90^\circ \Rightarrow \psi_1 + \psi_2 = 90^\circ$$

$$\boxed{\psi_2 = \lambda_1}$$

Centre distance $= (r_1 + r_2)$

$$= \frac{m_1 T_1}{2} + \frac{m_2 T_2}{2}$$

$$= \frac{m_2}{2}\left[\frac{m_1}{m_2} \cdot T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[\frac{m_{n_1}/\cos\psi_1}{m_{n_2}/\cos\psi_2} T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[\frac{\cos\lambda_1}{\cos(90-\lambda_1)} \cdot T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[T_1 . \cot\lambda_1 + T_2\right]$$

Angle

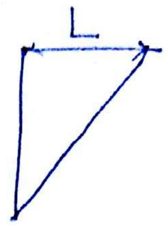
worm $= 2\pi$

wormwheel $= L/r_2$

Velocity Ratio: $VR = \dfrac{\omega_2}{\omega_1}$

$$V.R = \frac{\text{Angle turned by wheel in one revolution of worm}}{\text{Angle turn by worm in one revolution of worm.}}$$

$$VR = \frac{(L/r_2)}{2\pi}$$



$$\boxed{VR = \frac{L}{\pi d_2}}$$

efficiency :- $\eta = \dfrac{\cos(\psi_2 + \phi) \cdot \cos\psi_1}{\cos(\psi_1 - \phi)\ \cos\psi_2}$

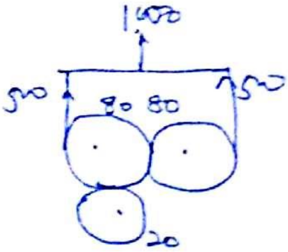
$$\eta = \frac{\cos(\lambda_1 + \phi) \cdot \cos(90 - \lambda_1)}{\cos(90^\circ - (\lambda_1 + \phi)) \cdot \cos\lambda_1}$$

$$\boxed{\eta = \frac{\tan\lambda_1}{\tan(\lambda_1 + \phi)}}$$

$$\eta_{max} = \frac{1 + \cos(\theta + \phi)}{1 + \cos(\theta - \phi)} \qquad \theta = 90^\circ$$

$$\boxed{\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi}} \qquad \boxed{\phi = \tan^{-1}(\mu)}$$

Q.30 (WB)



$$R = \frac{2 \times 80}{2} = 80\,mm$$

$$R = 0.080\ m$$

$$\eta = 0.80 = \frac{P_o}{P_i} = \frac{T_o \times \omega_o}{T_i \times \omega_i}$$

$$0.80 = \frac{500 \times 0.080 \times 2}{T_i} \times \frac{20}{80}$$

$$T_i = 25\ N\text{-}m$$

Q.31 $\phi = 20^\circ$

$F = ?$

$F \cos\phi = 500$

$$F = \frac{500}{\cos 20^\circ} = 532\,N$$

{ 1, 2, 3, 5, 6, 7, 8, 9
10, 11, 12, 13, 16, 19, 26
28, 30, 34, 35, 36, 37, 38
39, 40, 41, 42, 45, 46, 47
49, 50, 51-54, T1, T2