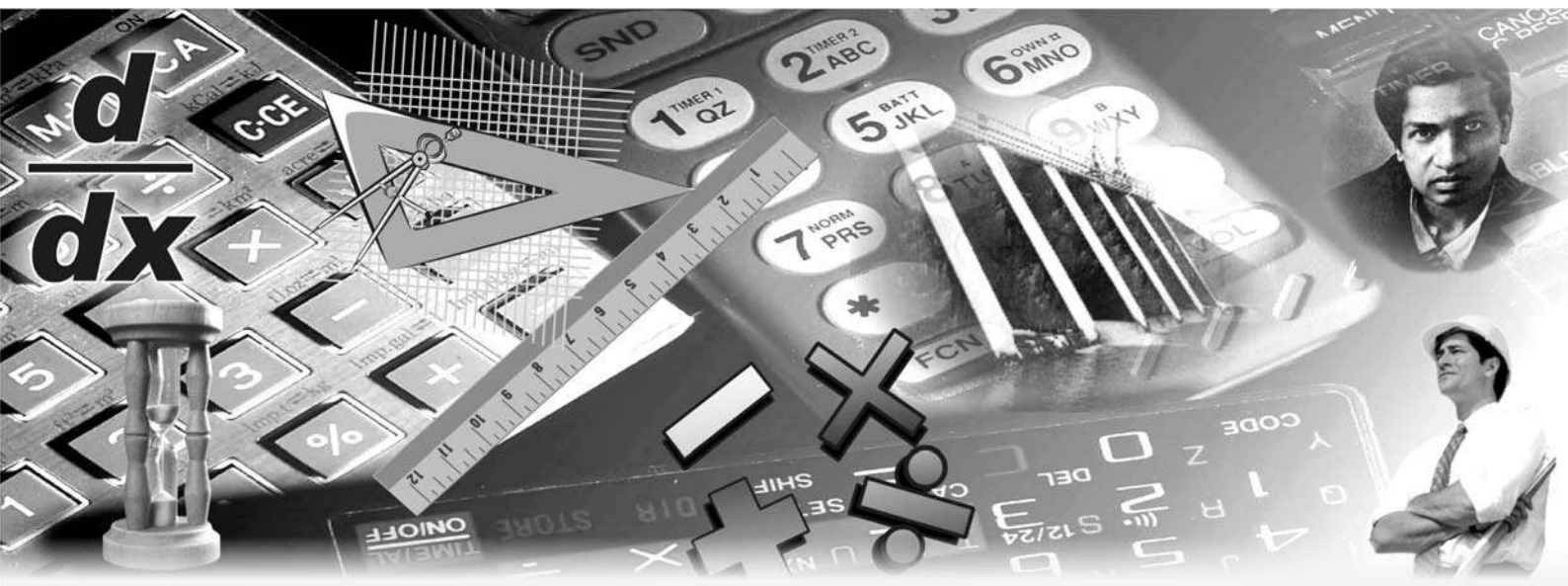




Chapter 23 अनिश्चित समाकलन

परिभाषा (Definition)

यदि $\phi'(x) = f(x)$ हो, तब मूल फलन $\phi(x)$ को $f(x)$ का पूर्वग या प्रतिअवकलज कहते हैं।

माना $\phi(x)$, फलन $f(x)$ का पूर्वग है तथा c कोई अचर है,

$$\text{तब } \frac{d}{dx}(\phi(x) + c) = f(x) \Rightarrow \int f(x) dx = \phi(x) + c$$

$\Rightarrow \phi(x) + c$ को भी $f(x)$ का प्रतिअवकलज कहते हैं, जहाँ c एक स्वेच्छ अचर है जिसके भिन्न-भिन्न मान हो सकते हैं।

समाकलन के कुछ मानक सूत्र

(Fundamental integration formulae)

- (1) (i) $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$
- (ii) $\int (ax + b)^n dx = \frac{1}{a} \cdot \frac{(ax + b)^{n+1}}{n+1} + c, n \neq -1$
- (2) (i) $\int \frac{1}{x} dx = \log |x| + c$
- (ii) $\int \frac{1}{ax + b} dx = \frac{1}{a} (\log |ax + b|) + c$
- (3) $\int e^x dx = e^x + c$
- (4) $\int a^x dx = \frac{a^x}{\log_e a} + c$
- (5) $\int \sin x dx = -\cos x + c$
- (6) $\int \cos x dx = \sin x + c$
- (7) $\int \sec^2 x dx = \tan x + c$
- (8) $\int \operatorname{cosec}^2 x dx = -\cot x + c$

- (9) $\int \sec x \tan x dx = \sec x + c$
- (10) $\int \operatorname{cosec} x \cot x dx = -\operatorname{cosec} x + c$
- (11) $\int \tan x dx = -\log |\cos x| + c = \log |\sec x| + c$
- (12) $\int \cot x dx = \log |\sin x| + c = -\log |\operatorname{cosec} x| + c$
- (13) $\int \sec x dx = \log |\sec x + \tan x| + c = \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) + c$
- (14) $\int \operatorname{cosec} x dx = \log |\operatorname{cosec} x - \cot x| + c = \log \tan \frac{x}{2} + c$
- (15) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c = -\cos^{-1} x + c$
- (16) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + c = -\cos^{-1} \frac{x}{a} + c$
- (17) $\int \frac{dx}{1+x^2} = \tan^{-1} x + c = -\cot^{-1} x + c$
- (18) $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c = \frac{-1}{a} \cot^{-1} \frac{x}{a} + c$
- (19) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c = -\operatorname{cosec}^{-1} x + c$
- (20) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + c = \frac{-1}{a} \operatorname{cosec}^{-1} \frac{x}{a} + c$

समाकलन के मानक सूत्र में यदि x की जगह $ax + b$ आता है तब वही सूत्र उपयोग होगा लेकिन उसमें x के गुणांक से या $ax + b$ के अवकलन अर्थात् a से भाग देते हैं।

सामान्यतया यदि $\int f(x) dx = \phi(x) + c$, तब

$$\int f(ax + b) dx = \frac{1}{a} \phi(ax + b) + c$$

$$\int \sin(ax + b) dx = \frac{-1}{a} \cos(ax + b) + c,$$

$$\int \sec(ax+b)dx = \frac{1}{a} \log |\sec(ax+b) + \tan(ax+b)| + c \text{ इत्यादि।}$$

कुछ महत्वपूर्ण परिणाम

$$(i) \int \frac{1}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c = \frac{-1}{a} \coth^{-1} \frac{x}{a} + c, \text{ जब } x > a$$

$$(ii) \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c = \frac{1}{a} \tanh^{-1} \frac{x}{a} + c, \text{ जब } x < a$$

$$(iii) \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \{ |x + \sqrt{x^2 - a^2}| \} + c = \cosh^{-1} \left(\frac{x}{a} \right) + c$$

$$(iv) \int \frac{dx}{\sqrt{x^2 + a^2}} = \log \{ |x + \sqrt{x^2 + a^2}| \} + c = \sinh^{-1} \left(\frac{x}{a} \right) + c$$

$$(v) \int \sqrt{a^2 - x^2} dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \sin^{-1} \left(\frac{x}{a} \right) + c$$

$$(vi) \int \sqrt{x^2 - a^2} dx = \frac{1}{2} x \sqrt{x^2 - a^2} - \frac{1}{2} a^2 \log \{ x + \sqrt{x^2 - a^2} \} + c$$

$$= \frac{1}{2} x \sqrt{x^2 - a^2} - \frac{1}{2} a^2 \cosh^{-1} \left(\frac{x}{a} \right) + c$$

$$(vii) \int \sqrt{x^2 + a^2} dx = \frac{1}{2} x \sqrt{x^2 + a^2} + \frac{1}{2} a^2 \log \{ x + \sqrt{x^2 + a^2} \} + c$$

$$= \frac{1}{2} x \sqrt{x^2 + a^2} + \frac{1}{2} a^2 \sinh^{-1} \left(\frac{x}{a} \right)$$

प्रतिस्थापन द्वारा समाकलन (Integration by substitution)

$$(i) \int f[\phi(x)] \phi'(x) dx \text{ रूप के फलन का समाकलन: माना } \phi(x) = t,$$

$\phi'(x)dx = dt$, तब समाकलन मानक रूप $\int f(t)dt$ में बदल जाता है।

(2) $I = \int f(x) \cdot f'(x) \cdot dx$ रूप के फलन का समाकलन: यहाँ दो घटक हैं जिनमें एक घटक, दूसरे का अवकलन है। इसमें $f(x) = t$ रखने पर समाकलन मानक रूप में बदल जाता है।

(3) $f(ax+b)$ रूप के फलन का समाकलन : इसमें $ax+b = t$ रखने पर समाकलन मानक रूप में बदल जाता है। यदि $\int f(x)dx = \phi(x)$ हो, तो

$$\int f(ax+b)dx = \frac{1}{a} \phi(ax+b) + c$$

$$(4) \frac{f'(x)}{f(x)} \text{ रूप के फलन का समाकलन : इसमें } f(x) = t \text{ रखने पर,}$$

$$f'(x)dx = dt \text{ हो जाता है। अतः } \int \frac{f'(x)}{f(x)} dx = \log[f(x)] + c$$

$$(5) [f(x)]^n f'(x) \text{ रूप के फलन का समाकलन : इसमें } f(x) = t \text{ रखने पर, } f'(x)dx = dt \int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c, [n \neq -1]$$

$$(6) \frac{f'(x)}{\sqrt{f(x)}} \text{ इस तरह के समाकलन में } f(x) = t \text{ रखने पर,}$$

$$\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$$

(7) मानक प्रतिस्थापन

सारणी : 23.1

	समाकलन रूप	प्रतिस्थापन
(i)	$\sqrt{a^2 - x^2}, \frac{1}{\sqrt{a^2 - x^2}}, a^2 - x^2$	$x = a \sin \theta$, या $x = a \cos \theta$
(ii)	$\sqrt{x^2 + a^2}, \frac{1}{\sqrt{x^2 + a^2}}, x^2 + a^2$	$x = a \tan \theta$ या $x = a \sinh \theta$
(iii)	$\sqrt{x^2 - a^2}, \frac{1}{\sqrt{x^2 - a^2}}, x^2 - a^2$	$x = a \sec \theta$ या $x = a \cosh \theta$
(iv)	$\sqrt{\frac{x}{a+x}}, \sqrt{\frac{a+x}{x}}, \sqrt{x(a+x)}, \frac{1}{\sqrt{x(a+x)}}$	$x = a \tan^2 \theta$
(v)	$\sqrt{\frac{x}{a-x}}, \sqrt{\frac{a-x}{x}}, \sqrt{x(a-x)}, \frac{1}{\sqrt{x(a-x)}}$	$x = a \sin^2 \theta$
(vi)	$\sqrt{\frac{x}{x-a}}, \sqrt{\frac{x-a}{x}}, \sqrt{x(x-a)}, \frac{1}{\sqrt{x(x-a)}}$	$x = a \sec^2 \theta$
(vii)	$\sqrt{\frac{a-x}{a+x}}, \sqrt{\frac{a+x}{a-x}}$	$x = a \cos 2\theta$
(viii)	$\sqrt{\frac{x-\alpha}{\beta-x}}, \sqrt{(x-\alpha)(\beta-x)}, (\beta > \alpha)$	$x = \alpha \cos^2 \theta + \beta \sin^2 \theta$

खण्डशः समाकलन (Integration by parts)

(i) जब समाकल्य में एक से अधिक फलन हो : इस प्रकार के समाकलनों को खण्डशः समाकलन द्वारा हल करते हैं।

$$\text{हम जानते हैं } \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\Rightarrow d(uv) = u dv + v du \Rightarrow \int d(uv) = \int u dv + \int v du$$

$$\text{यदि } u \text{ तथा } v, x \text{ के दो फलन हो, तो } \int u \frac{dv}{dx} dx = u \int \frac{v}{dx} dx -$$

$\int \left\{ \frac{du}{dx} \cdot \int v dx \right\} dx$ अर्थात् दो फलनों के गुणनफल का समाकलन = (प्रथम फलन) × (द्वितीय फलन का समाकलन) - {(प्रथम फलन का अवकलन) × (द्वितीय फलन का समाकलन)} का समाकलन

खण्डशः समाकलन में प्रथम फलन व द्वितीय फलन का उचित चयन करने से समाकलन शीघ्र व आसानी से हो जाता है अतः प्रथम फलन का चयन करने में निम्न बिन्दुओं का ध्यान रखते हैं :

(i) सामान्यतः प्रतिलोम व लघुगणक फलन को प्रथम फलन लेते हैं, जैसे $\log |x|, \sin^{-1} x, \cos^{-1} x, \tan^{-1} x, \dots$ इत्यादि।

(ii) यदि प्रतिलोम त्रिकोणमितीय एवं लघुगणक फलन के साथ अन्य कोई फलन विद्यमान नहीं हो, तो 1 को द्वितीय फलन लेकर खण्डशः समाकलन करते हैं। जैसे $\int \sin^{-1} x dx, \int \log x dx$, इत्यादि में।

(iii) यदि दोनों फलनों का समाकलन आसानी से हो सके, तब प्रथम फलन का चयन इस प्रकार करते हैं, कि इसके अवकलज से प्राप्त फलन का समाकलन आसानी से हो सके।

सामान्यतः प्रथम फलन का चयन "ILATE" नियम के अनुसार किया जाता है।

(2) $\int e^x \{f(x) + f'(x)\} dx$ रूप का समाकलन: यदि $\int e^x \{f(x) + f'(x)\} dx$, रूप का समाकलन हो तब इस समाकलन को दो समाकल्यों में विभक्त करते हैं। इनमें से एक समाकलन को खण्डशः समाकलन द्वारा हल करते हैं तथा दूसरे समाकलन को रोककर रखते हैं।

$$(i) \int e^x [f(x) + f'(x)] dx = e^x f(x) + c$$

$$(ii) \int e^{mx} [mf(x) + f'(x)] dx = e^{mx} f(x) + c$$

$$(iii) \int e^{mx} \left[f(x) + \frac{f'(x)}{m} \right] dx = \frac{e^{mx} f(x)}{m} + c$$

(3) $\int [x f'(x) + f(x)] dx$ रूप का समाकलन :

$\int [x f'(x) + f(x)] dx$ रूप के समाकलन को हल करने के लिए हम समाकल्य को दो भागों में विभक्त करते हैं। प्रथम भाग को खण्डशः समाकलन द्वारा हल करते हैं तथा दूसरे भाग को रोककर रखते हैं। अतः इस प्रकार, $\int [x f'(x) + f(x)] dx = x f(x) + c$

(4) $\int e^{ax} \sin bx dx$, $\int e^{ax} \cos bx dx$ रूप का समाकलन

$$\begin{aligned} \int e^{ax} \sin bx &= \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + c \\ &= \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin \left(bx - \tan^{-1} \frac{b}{a} \right) + c \end{aligned}$$

$$\begin{aligned} \int e^{ax} \cdot \cos bx dx &= \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + c \\ &= \frac{e^{ax}}{\sqrt{a^2 + b^2}} \cos \left(bx - \tan^{-1} \frac{b}{a} \right) + c \end{aligned}$$

$$\begin{aligned} \int e^{ax} \cdot \sin(bx + c) dx &= \frac{e^{ax}}{a^2 + b^2} [a \sin(bx + c) - b \cos(bx + c)] + k \\ &= \frac{e^{ax}}{\sqrt{a^2 + b^2}} \sin \left[(bx + c) - \tan^{-1} \left(\frac{b}{a} \right) \right] + k \end{aligned}$$

$$\begin{aligned} \int e^{ax} \cdot \cos(bx + c) dx &= \frac{e^{ax}}{a^2 + b^2} [a \cos(bx + c) + b \sin(bx + c)] + k \\ &= \frac{e^{ax}}{\sqrt{a^2 + b^2}} \cos \left[(bx + c) - \tan^{-1} \left(\frac{b}{a} \right) \right] + k \end{aligned}$$

मानक परिणामों के उपयोग से विभिन्न रूपों के समाकलन ज्ञात करना (Evaluation of the various forms of integrals by use of standard results)

(i) $\int \frac{dx}{ax^2 + bx + c}$ रूप का समाकलन

$$\text{कार्य विधि : } \int \frac{dx}{ax^2 + bx + c} = \frac{1}{a} \int \frac{dx}{x^2 + \frac{b}{a}x + \frac{c}{a}}$$

$$= \frac{1}{a} \int \frac{dx}{\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4c}}, \text{ जो कि } \int \frac{dx}{X^2 - A^2}, \int \frac{dx}{X^2 + A^2}$$

या $\int \frac{dx}{A^2 - X^2}$ रूप में होगा।

(2) $\int \frac{dx}{\sqrt{ax^2 + bx + c}}$ रूप का समाकलन: इसे निम्न रूप में

परिवर्तित कर सकते हैं $\int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{\sqrt{x^2 - a^2}}$ या $\int \frac{dx}{\sqrt{x^2 + a^2}}$.

(3) $\int \sqrt{ax^2 + bx + c} dx$ रूप का समाकलन: इसे निम्न रूप में

परिवर्तित कर सकते हैं $\int \sqrt{a^2 - x^2} dx, \int \sqrt{x^2 - a^2} dx$ या $\int \sqrt{a^2 + x^2} dx$

$$(4) \int \frac{(px + q) dx}{ax^2 + bx + c}, \int \frac{(px + q) dx}{\sqrt{ax^2 + bx + c}},$$

$$\int (px + q) \sqrt{ax^2 + bx + c} dx :$$

इस प्रकार के समाकलनों को हल करने के लिए

$$px + q = A \left\{ \frac{d}{dx} (ax^2 + bx + c) \right\} + B$$

दोनों पक्षों से x के गुणांक व अचर पद की तुलना करके A व B के मान ज्ञात करते हैं।

इस प्रकार समाकलन को दो भागों में विभक्त कर हल करते हैं।

$$(5) \int \frac{x^2 + 1}{x^4 + kx^2 + 1} dx, \int \frac{x^2 - 1}{x^4 + kx^2 + 1} dx, \int \frac{dx}{x^4 + kx^2 + 1}$$

जहाँ $k \in \mathbb{R}$ रूप का समाकलन

कार्य विधि : (i) इस प्रकार के समाकलन का मान ज्ञात करने के लिए अंश व हर में x^2 से भाग देते हैं।

$$(ii) x + \frac{1}{x} = t \text{ या } x - \frac{1}{x} = t \text{ रखते हैं।}$$

$$(6) \int \frac{x^2 + a^2}{x^4 + kx^2 + a^4} dx, \int \frac{x^2 - a^2}{x^4 + kx^2 + a^4} dx, \text{ जहाँ } k \text{ एक अचर, ऋणात्मक या शून्य है।}$$

इस प्रकार के समाकलनों को ज्ञात करने के लिए अंश व हर में x^2 से भाग देते हैं तथा $x - \frac{a^2}{x} = t$ तथा $x + \frac{a^2}{x} = t$ रखते हैं।

(7) अपरिमेय फलनों के लिए उचित प्रतिस्थापन

$$(i) \int \frac{dx}{\sqrt{(x - \alpha)(x - \beta)}}, \int \sqrt{\frac{x - \alpha}{\beta - x}} dx$$

$$\int \sqrt{(x - \alpha)(\beta - x)} dx, x = \alpha \cos^2 \theta + \beta \sin^2 \theta \text{ रखकर}$$

$$(ii) \int \frac{dx}{(px + q)\sqrt{ax + b}}, ax + b = t^2 \text{ रखकर}$$

$$(iii) \int \frac{dx}{(px + q)\sqrt{ax^2 + bx + c}}, px + q = \frac{1}{t} \text{ रखकर}$$

$$(iv) \int \frac{dx}{(px^2 + r)\sqrt{ax^2 + c}}, \text{ प्रथम में } x = \frac{1}{t} \text{ रखकर तथा } a + ct^2 = z^2.$$

$$(8) \int \frac{dx}{P\sqrt{Q}} \text{ रूप का समाकलन (जहाँ } P \text{ और } Q, x \text{ में रेखीय या वर्ग}$$

समीकरण हों) :

(i) जब Q रेखीय तथा P रेखीय या वर्ग समीकरण हों, तब $Q = t^2$ रखते हैं।

(ii) जब P रेखीय तथा Q वर्ग समीकरण हों, तब $P = \frac{1}{t}$ रखते हैं।

(iii) जब P और Q दोनों वर्ग समीकरण हों, तब $x = \frac{1}{t}$ रखते हैं।

$\int \frac{dx}{a + b \cos x}$ और $\int \frac{dx}{a + b \sin x}$ रूप का समाकलन

इस प्रकार के समाकलन को निम्न प्रकार से ज्ञात कर सकते हैं:

$$(i) \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \text{ तथा } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \text{ रखते हैं।}$$

$$(2) \text{ अंश में } 1 + \tan^2 \frac{x}{2} \text{ के स्थान पर } \sec^2 \frac{x}{2} \text{ रखते हैं।}$$

$$(3) \tan \frac{x}{2} = t \text{ तब } \frac{1}{2} \sec^2 \frac{x}{2} dx = dt \text{ रखते हैं।}$$

(4) अब प्राप्त समाकलन को, जो $\int \frac{dt}{at^2 + bt + c}$ के रूप में होगा, पहले दी गई विधि से हल करते हैं।

$$(i) \int \frac{dx}{a + b \cos x}$$

स्थिति I : जब $a > b$, तब

$$\int \frac{dx}{a + b \cos x} = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2} \right) + c$$

स्थिति II : जब $a < b$, तब

$$\int \frac{dx}{a + b \cos x} = \frac{1}{\sqrt{b^2 - a^2}} \log \left| \frac{\sqrt{b-a} \tan \frac{x}{2} + \sqrt{b+a}}{\sqrt{b-a} \tan \frac{x}{2} - \sqrt{b+a}} \right| + c$$

$$\text{स्थिति III : जब } a = b, \text{ तब } \int \frac{dx}{a + b \cos x} = \frac{1}{a} \tan \frac{x}{2} + c.$$

$$(ii) \int \frac{dx}{a + b \sin x}$$

स्थिति I : जब $a^2 > b^2$ या $a > 0$ तथा $a > b$, तब

$$\int \frac{dx}{a + b \sin x} = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left[\frac{a \tan \frac{x}{2} + b}{\sqrt{a^2 - b^2}} \right] + c$$

स्थिति II : जब $a^2 < b^2$, तब

$$\int \frac{dx}{a + b \sin x} = \frac{1}{\sqrt{b^2 - a^2}} \log \left| \frac{(a \tan \frac{x}{2} + b) - (\sqrt{b^2 - a^2})}{(a \tan \frac{x}{2} + b) + \sqrt{b^2 - a^2}} \right| + c$$

स्थिति III : जब $a^2 = b^2$

$$b = a \text{ या } b = -a$$

(a) जब $b = a$, तब

$$\int \frac{dx}{a + b \sin x} = \frac{-1}{a} \cot \left(\frac{\pi}{4} + \frac{x}{2} \right) + c = \frac{1}{a} [\tan x - \sec x] + c$$

$$(b) \text{ जब } b = -a, \text{ तब } \int \frac{dx}{a + b \sin x} = \frac{1}{a} \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) + c.$$

$\int \frac{dx}{a + b \cos x + c \sin x}$, $\int \frac{dx}{a \sin x + b \cos x}$ रूप का समाकलन

(1) $\int \frac{dx}{a + b \cos x + c \sin x}$ रूप का समाकलन : इस प्रकार के समाकलन को ज्ञात करने के लिए, सर्वप्रथम $b = r \cos \alpha$ और $c = r \sin \alpha$ रखने पर

$$\text{अतः } r^2 = b^2 + c^2 \text{ तथा } \alpha = \tan^{-1} \frac{c}{b}.$$

$$\therefore I = \int \frac{dx}{a + r(\cos \alpha \cos x + \sin \alpha \sin x)} = \int \frac{dx}{a + r \cos(x - \alpha)}$$

$$x - \alpha = t \Rightarrow dx = dt, \text{ रखने पर } I = \int \frac{dt}{a + r \cos t}$$

इस प्रकार के समाकलन को पहले दर्शाया जा चुका है।

(2) $\int \frac{dx}{a \sin x + b \cos x}$ रूप का समाकलन : इस प्रकार के समाकलन को ज्ञात करने के लिए, सर्वप्रथम $a = r \cos \theta$, $b = r \sin \theta$ रखने पर $r = \sqrt{a^2 + b^2}$, $\alpha = \tan^{-1} \frac{b}{a}$

$$\therefore \int \frac{dx}{a \sin x + b \cos x} = \frac{1}{r} \int \frac{dx}{\sin(x + \alpha)} = \frac{1}{r} \int \operatorname{cosec}(x + \alpha) dx$$

$$= \frac{1}{r} \log \left| \tan \left(\frac{x}{2} + \frac{\alpha}{2} \right) \right| = \frac{1}{\sqrt{a^2 + b^2}} \log \left| \tan \left(\frac{x}{2} + \frac{1}{2} \tan^{-1} \frac{b}{a} \right) \right| + c$$

$$\cos x = \frac{1 - \tan^2 x/2}{1 + \tan^2 x/2} \text{ और } \sin x = \frac{2 \tan(x/2)}{1 + \tan^2(x/2)} \text{ का उपयोग}$$

उपरोक्त समाकलन को ज्ञात करने के लिए करते हैं।

$$\int \frac{dx}{a + b \cos^2 x}, \int \frac{dx}{a + b \sin^2 x}, \int \frac{dx}{a \sin^2 x + b \cos^2 x},$$

$$\int \frac{dx}{(a \sin x + b \cos x)^2}, \int \frac{dx}{a + b \sin^2 x + c \cos^2 x}$$

रूप का समाकलन

इस प्रकार के समाकलन को ज्ञात करने के लिए निम्नांकित बिन्दुओं का अनुसरण करते हैं :

(1) अंश व हर दोनों को $\cos^2 x$ से भाग देते हैं।

(2) हर में $\sec^2 x$ की जगह $(1 + \tan^2 x)$ रखते हैं।

(3) अब $\tan x = t \Rightarrow \sec^2 x dx = dt$ रखते हैं।

(4) अब प्राप्त हुए समाकलन को पूर्व में दी गयी विधि द्वारा हल करते हैं।

$$\int \frac{a \sin x + b \cos x}{c \sin x + d \cos x} \text{ और } \int \frac{a \sin x + b \cos x + q}{c \sin x + d \cos x + r}$$

रूप का समाकलन

(1) $\int \frac{a \sin x + b \cos x}{c \sin x + d \cos x} dx$ रूप का समाकलन : इस प्रकार के

समाकलन को ज्ञात करने के लिए अंश को निम्न प्रकार से व्यक्त करते हैं।
अंश = M (हर का अवकलन) + N (हर)

अर्थात्

$$a \sin x + b \cos x = M \frac{d}{dx} (c \sin x + d \cos x) + N (c \sin x + d \cos x)$$

जहाँ M और N अचर हैं जिनका मान दोनों पक्षों में $\sin x$ तथा $\cos x$ के गुणांकों की तुलना कर ज्ञात करते हैं।

$$\begin{aligned} I &= \int \frac{a \sin x + b \cos x}{c \sin x + d \cos x} dx \\ &= \int \frac{M(c \cos x - d \sin x) + N(c \sin x + d \cos x)}{c \sin x + d \cos x} dx \\ &= M \int \frac{c \cos x - d \sin x}{c \sin x + d \cos x} dx + N \int \frac{1}{c \sin x + d \cos x} dx \\ &= M \log |c \sin x + d \cos x| + Nx + c. \end{aligned}$$

(2) $\int \frac{a \sin x + b \cos x + q}{c \sin x + d \cos x + r} dx$ रूप का समाकलन : इस प्रकार के समाकलन को ज्ञात करने के लिए अंश को निम्न प्रकार से व्यक्त करते हैं।

$$\text{अंश} = M(\text{हर}) + N(\text{हर का अवकलन}) + P$$

$$(c \sin x + b \cos x + q) = M(c \sin x + d \cos x + r) + N(c \cos x - d \sin x) + P.$$

जहाँ M, N, P अचर हैं, जिनका मान दोनों पक्षों में $\sin x, \cos x$ तथा अचर पद के गुणांकों की तुलना कर ज्ञात करते हैं।

$$\begin{aligned} \therefore \int \frac{a \sin x + b \cos x + q}{c \sin x + d \cos x + r} dx \\ &= \int M dx + N \int \frac{\text{हर का अवकलन}}{\text{हर}} dx + \int \frac{dx}{c \sin x - d \cos x + r} \\ &= Mx + N \log |\text{हर}| + P \int \frac{dx}{c \sin x + d \cos x + r}. \end{aligned}$$

आंशिक भिन्नों द्वारा परिमेय बीजीय फलनों का समाकलन (Integration of rational functions by using partial fractions)

यदि दिया गया फलन दो बहुपदों का भागफल है, तो इसके समाकलन के लिए इसे आंशिक भिन्नों में विभक्त करते हैं (यदि सम्भव है)। पूर्व अध्याय में आंशिक भिन्नों को हल करने की विधि हम पढ़ चुके हैं।

त्रिकोणमितीय फलनों का समाकलन

(Integration of trigonometric functions)

(i) $\int \sin^m x \cos^n x dx$ रूप का समाकलन : (i) इस प्रकार के समाकलन को ज्ञात करने के लिए $I = \int \sin^m x \cos^n x dx$, जहाँ m और n परिमेय संख्याएँ हैं

(a) यदि n विषम हो, तब $\sin x = t$, प्रतिस्थापित करते हैं।

(b) यदि m विषम हो, तब $\cos x = t$, प्रतिस्थापित करते हैं।

(c) यदि $m+n$ ऋणात्मक सम पूर्णांक हो, तब $\tan x = t$, प्रतिस्थापित करते हैं।

(d) यदि $\frac{1}{2}(n-1)$ एक पूर्णांक हो, तब $\cot x = t$ प्रतिस्थापित करते हैं।

(e) यदि m और n परिमेय संख्याएँ हो तथा $\left(\frac{m+n-2}{2}\right)$ एक ऋणात्मक पूर्णांक हो, तब $\cos x = t$ या $\tan x = t$ प्रतिस्थापित करते हैं।

(ii) $\int R(\sin x, \cos x) dx$ रूप का समाकलन :

$\int R(\sin x, \cos x) dx$, के रूप का समाकलन जहाँ $R, \sin x$ तथा $\cos x$ का परिमेय फलन है। इस प्रकार के समाकलन को $\tan \frac{x}{2} = t$ (जहाँ $-\pi < x < \pi$) रखकर, परिमेय फलन के समाकलन में परिवर्तित कर देते हैं।

कभी-कभी $\cot \frac{x}{2} = t$ (जहाँ $0 < x < 2\pi$) रखकर, परिमेय फलन के समाकलन के रूप में आसानी से परिवर्तित कर सकते हैं।

(a) यदि $\int R(\sin x) \cos x dx$ के रूप का समाकलन हो, तब $\sin x = t$ प्रतिस्थापित करते हैं।

(b) यदि $\int R(\cos x) \sin x dx$ के रूप का समाकलन हो, तब $\cos x = t$ प्रतिस्थापित करते हैं।

(c) यदि समाकलन केवल $\tan x$ पर आश्रित हों, तब $\tan x = t$, $dx = \frac{dt}{1+t^2}$ प्रतिस्थापित करते हैं।

(d) यदि $R(-\sin x, \cos x) = -R(\sin x, \cos x)$, तब $\cos x = t$ प्रतिस्थापित करते हैं।

(e) यदि $R(\sin x, -\cos x) = -R(\sin x, \cos x)$, तब $\sin x = t$ प्रतिस्थापित करते हैं।

(f) यदि $R(-\sin x, -\cos x) = -R(\sin x, \cos x)$, तब $\tan x = t$ प्रतिस्थापित करते हैं।

(2) विशेष प्रकार के समानयन सूत्र

$$(i) \int \sin^n x dx = \frac{-\cos x \cdot \sin^{n-1} x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$(ii) \int \cos^n x dx = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$(iii) \int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx$$

$$(iv) \int \cot^n x dx = \frac{-1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x dx$$

$$(v) \int \sec^n x dx = \frac{1}{(n-1)} \left[\sec^{n-2} x \cdot \tan x + (n-2) \int \sec^{n-2} x dx \right]$$

$$(vi) \int \operatorname{cosec}^n x dx = \frac{1}{(n-1)} \left[-\operatorname{cosec}^{n-2} x \cot x + (n-2) \int \operatorname{cosec}^{n-2} x dx \right]$$

$$\begin{aligned} (vii) \int \sin^p x \cos^q x dx &= -\frac{\sin^{p+1} x \cdot \cos^{q-1} x}{p+q} \\ &\quad + \frac{p-1}{p+q} \int \sin^{p-2} x \cdot \cos^q x dx \\ (viii) \int \sin^p x \cos^q x dx &= \frac{\sin^{p+1} x \cdot \cos^{q-1} x}{p+q} \\ &\quad + \frac{p-1}{p+q} \int \sin^p x \cdot \cos^{q-2} x dx \end{aligned}$$

$$(ix) \int \frac{dx}{(x^2+k)^n} = \frac{x}{k(2n-2)(x^2+k)^{n-1}} + \frac{(2n-3)}{k(2n-2)} \int \frac{dx}{(x^2+k)^{n-1}}$$

अतिपरवल्यिक फलनों का समाकलन

(Integration of hyperbolic functions)

$$(1) \int \sinh x dx = \cosh x + c \quad (2) \int \cosh x dx = \sinh x + c$$

$$(3) \int \sec^2 x dx = \tan x + c$$

$$(4) \int \operatorname{cosech}^2 x dx = -\coth x + c$$

$$(5) \int \sec hx \tan hx dx = \sec hx + c$$

$$(6) \int \operatorname{cosec} hx \cot h dx = -\operatorname{cosec} hx + c$$

Tips & Tricks

☞ यदि अन्तराल $[a, b]$ में फलन $f(x)$ के दो प्रतिअवकलज $F_1(x)$ तथा $F_2(x)$ हों, तब उनका अन्तर एक अचर होता है।

☞ सिग्नाम फलन का प्रति अवकलज उस अन्तराल में होता है, जिसमें बिन्दु $x=0$ शामिल नहीं है।

☞ प्रत्येक समफलन का प्रतिअवकलज विषम फलन होता है तथा प्रत्येक विषम फलन का प्रतिअवकलज सम फलन होता है।

☞ यदि $I_n = \int x^n \cdot e^{ax} dx$, तो $I_n = \frac{x^n e^{ax}}{a} - \frac{n}{a} I_{n-1}$

☞ यदि $I_n = \int (\log x) dx$, तो $I_n = x \log x - x$

☞ यदि $I_n = \int \frac{1}{\log x} dx$, तो

$$I_n = \log(\log x) + \log x + \frac{(\log x)^2}{2 \cdot (2!)} + \frac{(\log x)^3}{3(3!)} + \dots \dots \dots$$

☞ यदि $I_n = \int (\log x)^n dx$; तो $I_n = x(\log x)^n - n I_{n-1}$

☞ उत्तरोत्तर खण्डशः समाकलन का उपयोग तब कर सकते हैं, जबकि एक फलन x^n (n घनात्मक पूर्णांक है) है, जो उत्तरोत्तर अवकलित होगा एवं दूसरा फलन जो कि $\sin ax, \cos ax, e^{ax}, e^{-ax}, (x+a)^m$ में से कोई एक है, उत्तरोत्तर समाकलित होगा।

☞ श्रृंखला नियम : $\int u \cdot v dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \dots \dots \dots$
 $+ (-1)^{n-1} u^{n-1} v_n + (-1)^n \int u^n \cdot v_n dx$

जहाँ $u = \frac{d^n u}{dx^n}$ तथा v, v' के n वें समाकलन को व्यक्त करता है।

☞ $\int x e^{ax} \sin(bx+c) dx = \frac{x e^{ax}}{a^2+b^2} [a \sin(bx+c) - b \cos(bx+c)]$
 $- \frac{e^{ax}}{(a^2+b^2)^2} [(a^2-b^2) \sin(bx+c) - 2ab \cos(bx+c)] + k$

☞ $\int x \cdot e^{ax} \cos(bx+c) dx = \frac{x \cdot e^{ax}}{a^2+b^2} [a \cos(bx+c) + b \sin(bx+c)]$
 $- \frac{e^{ax}}{(a^2+b^2)^2} [(a^2-b^2) \cos(bx+c) + 2ab \sin(bx+c)] + k$

☞ $\int a^x \cdot \sin(bx+c) dx = \frac{a^x}{(\log a)^2 + b^2}$
 $[(\log a) \sin(bx+c) - b \cos(bx+c)] + k$

☞ $\int a^x \cdot \cos(bx+c) dx = \frac{a^x}{(\log a)^2 + b^2}$
 $[(\log a) \cos(bx+c) + b \sin(bx+c)] + k$

☞ $\int \frac{a \cos x + b \sin x}{c \cos x + d \sin x} dx$
 $= \frac{ac+bd}{c^2+d^2} x + \frac{ad-bc}{c^2+d^2} \log |c \cos x + d \sin x| + k$

☞ $\int \sin mx \cos nx dx, \int \sin mx \cdot \sin nx dx, \int \cos mx \cdot \cos nx dx$ तथा $\int \cos mx \cdot \sin nx dx$ रूप के समाकलन को ज्ञात करने के लिए निम्न त्रिकोणमितीय गुणधर्मों का उपयोग करते हैं।

☞ $\sin mx \cdot \cos nx = \frac{1}{2} [\sin(m-n)x + \sin(m+n)x]$

$\Rightarrow \cos mx \cdot \sin nx = \frac{1}{2} [\sin(m+n)x - \sin(m-n)x]$

☞ $\sin mx \cdot \sin nx = \frac{1}{2} [\cos(m-n)x - \cos(m+n)x]$

$\Rightarrow \cos mx \cdot \cos nx = \frac{1}{2} [\cos(m-n)x + \cos(m+n)x]$

☞ $I_{(n,m)} = \int \frac{\sin^n x}{\cos^m x} dx$ के लिए समानयन सूत्र

$$I_{(n,m)} = \frac{1}{m-1} \cdot \frac{\sin^{n-1} x}{\cos^{m-1} x} - \frac{(n-1)}{(m-1)} \cdot I_{(n-2,m-2)} \text{ है।}$$



Ordinary Thinking

Objective Questions

मूलभूत समाकलन

- $\int \sqrt{1 - \sin 2x} dx = \dots\dots\dots, x \in (0, \pi/4)$ [MP PET 1987]
 (a) $-\sin x + \cos x$ (b) $\sin x - \cos x$
 (c) $\tan x + \sec x$ (d) $\sin x + \cos x$
- $\int \frac{1 + \cos^2 x}{\sin^2 x} dx =$ [MP PET 1993; BIT Ranchi 1982]
 (a) $-\cot x - 2x + c$ (b) $-2 \cot x - 2x + c$
 (c) $-2 \cot x - x + c$ (d) $-2 \cot x + x + c$
- $\int \sin^{-1}(\cos x) dx =$
 (a) $\frac{\pi x}{2}$ (b) $\frac{\pi x^2}{2}$
 (c) $\frac{\pi x - x^2}{2}$ (d) $\frac{\pi x + x^2}{2}$
- $\int \frac{dx}{\tan x + \cot x} =$ [MP PET 1991]
 (a) $\frac{\cos 2x}{4} + c$ (b) $\frac{\sin 2x}{4} + c$
 (c) $-\frac{\sin 2x}{4} + c$ (d) $-\frac{\cos 2x}{4} + c$
- $\int (e^{a \log x} + e^{x \log a}) dx =$
 (a) $x^{a+1} + \frac{a^x}{\log a} + c$ (b) $\frac{x^{a+1}}{a+1} + a^x \log a + c$
 (c) $\frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$ (d) इनमें से कोई नहीं
- यदि $f'(x) = x^2 + 5$ और $f(0) = -1$, तब $f(x) =$
 (a) $x^3 + 5x - 1$ (b) $x^3 + 5x + 1$
 (c) $\frac{1}{3}x^3 + 5x - 1$ (d) $\frac{1}{3}x^3 + 5x + 1$
- $\int \tan^{-1} \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} dx =$
 (a) $2x^2 + c$ (b) $x^2 + c$

- (c) $\frac{x^2}{2} + c$ (d) $2x + c$
8. $\int \frac{dx}{\sqrt{x} + \sqrt{x-2}} =$ [MP PET 1990]
- (a) $\frac{1}{3}[x^{3/2} - (x-2)^{3/2}] + c$ (b) $\frac{2}{3}[x^{3/2} - (x-2)^{3/2}] + c$
- (c) $\frac{1}{3}[(x-2)^{3/2} - x^{3/2}] + c$ (d) $\frac{2}{3}[(x-2)^{3/2} - x^{3/2}] + c$
9. $\int \frac{\sin x}{\sin(x-\alpha)} dx =$ [RPET 1999; Kerala (Engg.) 2002; AIEEE 2004]
- (a) $x \cos \alpha - \sin \alpha \log \sin(x-\alpha) + c$
- (b) $x \cos \alpha + \sin \alpha \log \sin(x-\alpha) + c$
- (c) $x \sin \alpha - \sin \alpha \log \sin(x-\alpha) + c$
- (d) इनमें से कोई नहीं
10. $\int \frac{\cos x - 1}{\cos x + 1} dx =$ [MP PET 1989, 92]
- (a) $2 \tan \frac{x}{2} - x + c$ (b) $\frac{1}{2} \tan \frac{x}{2} - x + c$
- (c) $x - \frac{1}{2} \tan \frac{x}{2} + c$ (d) $x - 2 \tan \frac{x}{2} + c$
11. $\int \frac{dx}{1 - \sin x} =$ [MP PET 1991]
- (a) $x + \cos x + c$ (b) $1 + \sin x + c$
- (c) $\sec x - \tan x + c$ (d) $\sec x + \tan x + c$
12. यदि $\int (\sin 2x - \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x - a) + b$, तब [Roorkee 1978; MP PET 2001]
- (a) $a = \frac{\pi}{4}, b = 0$
- (b) $a = -\frac{\pi}{4}, b = 0$
- (c) $a = \frac{5\pi}{4}, b =$ कोई स्थिरांक
- (d) $a = -\frac{5\pi}{4}, b =$ कोई स्थिरांक
13. $\int \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right) dx =$
- (a) $-e^x + c$ (b) $e^x + c$
- (c) $e^{-x} + c$ (d) $-e^{-x} + c$
14. $\int \frac{\cot x \tan x}{\sec^2 x - 1} dx =$
- (a) $\cot x - x + c$ (b) $-\cot x + x + c$
- (c) $\cot x + x + c$ (d) $-\cot x - x + c$
15. $\int (\sec x + \tan x)^2 dx =$ [MP PET 1987, 92]
- (a) $2(\sec x + \tan x) - x + c$ (b) $1/3(\sec x + \tan x)^3 + c$
- (c) $\sec x(\sec x + \tan x) + c$ (d) $2(\sec x + \tan x) + c$
16. $\int x^{51}(\tan^{-1} x + \cot^{-1} x) dx =$ [MP PET 1991]
- (a) $\frac{x^{52}}{52}(\tan^{-1} x + \cot^{-1} x) + c$ (b) $\frac{x^{52}}{52} + \frac{\pi}{2} + c$
- (c) $\frac{\pi x^{52}}{104} + \frac{\pi}{2} + c$ (d) $\frac{x^{52}}{52} + \frac{\pi}{2} + c$
17. $\int 5 \sin x dx =$ [MP PET 1988]
- (a) $5 \cos x + c$ (b) $-5 \cos x + c$
- (c) $5 \sin x + c$ (d) $-5 \sin x + c$
18. $\int \frac{\tan x}{\sec x + \tan x} dx =$
- (a) $\sec x + \tan x - x + c$ (b) $\sec x - \tan x + x + c$
- (c) $\sec x + \tan x + x + c$ (d) $-\sec x - \tan x + x + c$
19. $\int \frac{dx}{\sin^2 x \cos^2 x} =$ [Roorkee 1976; RPET 1991]
- (a) $\tan x + \cot x + c$ (b) $\cot x - \tan x + c$
- (c) $\tan x - \cot x + c$ (d) इनमें से कोई नहीं
20. $\int \left(x + \frac{1}{x}\right)^3 dx =$
- (a) $\frac{1}{4} \left(x + \frac{1}{x}\right)^4 + c$
- (b) $\frac{x^4}{4} + \frac{3x^2}{2} + 3 \log x - \frac{1}{2x^2} + c$
- (c) $\frac{x^4}{4} + \frac{3x^2}{2} + 3 \log x + \frac{1}{x^2} + c$
- (d) इनमें से कोई नहीं
21. $\int \sqrt{1 + \sin \frac{x}{2}} dx =$ [IIT 1980; MP PET 1989; Pb. CET 2003]
- (a) $\frac{1}{4} \left(\cos \frac{x}{4} - \sin \frac{x}{4}\right) + c$ (b) $4 \left(\cos \frac{x}{4} - \sin \frac{x}{4}\right) + c$
- (c) $4 \left(\sin \frac{x}{4} - \cos \frac{x}{4}\right) + c$ (d) $4 \left(\sin \frac{x}{4} + \cos \frac{x}{4}\right) + c$
22. $\int (\sin^{-1} x + \cos^{-1} x) dx =$ [MP PET 1990]
- (a) $\frac{1}{2} \pi x + c$ (b) $x(\sin^{-1} x - \cos^{-1} x) + c$
- (c) $x(\cos^{-1} x + \sin^{-1} x) + c$ (d) $\frac{\pi}{2} + x + c$
23. $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx =$ [MP PET 1990]
- (a) $\sin x + c$ (b) $\cos x + c$
- (c) $x + c$ (d) $x^2 + c$

24. $\int \frac{x-1}{(x+1)^2} dx =$
 (a) $\log(x+1) + \frac{2}{x+1} + c$ (b) $\log(x+1) - \frac{2}{x+1} + c$
 (c) $\frac{2}{x+1} - \log(x+1) + c$ (d) इनमें से कोई नहीं
25. $\int \frac{\operatorname{cosec} \theta - \cot \theta}{\operatorname{cosec} \theta + \cot \theta} d\theta =$
 (a) $2\operatorname{cosec} \theta - 2\cot \theta - \theta + c$
 (b) $2\operatorname{cosec} \theta - 2\cot \theta + \theta + c$
 (c) $2\operatorname{cosec} \theta + 2\cot \theta - \theta + c$
 (d) इनमें से कोई नहीं
26. $\int (1 + 2x + 3x^2 + 4x^3 + \dots) dx =$
 (a) $(1+x)^{-1} + c$ (b) $(1-x)^{-1} + c$
 (c) $(1-x)^{-1} - 1 + c$ (d) इनमें से कोई नहीं
27. यदि $\int (\cos x - \sin x) dx = \sqrt{2} \sin(x + \alpha) + c$, तब $\alpha =$
 (a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$
 (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$
28. $\int \frac{3x^3 - 2\sqrt{x}}{x} dx =$ [Roorkee 1976]
 (a) $x^3 - \sqrt{x} + c$ (b) $x^3 + \sqrt{x} + c$
 (c) $x^3 - 2\sqrt{x} + c$ (d) $x^3 - 4\sqrt{x} + c$
29. यदि $\int \frac{dx}{1 + \sin x} = \tan\left(\frac{x}{2} + a\right) + b$, तब [Roorkee 1979]
 (a) $a = \frac{\pi}{4}, b = 3$
 (b) $a = -\frac{\pi}{4}, b = 3$
 (c) $a = \frac{\pi}{4}, b =$ कोई स्थिरांक
 (d) $a = -\frac{\pi}{4}, b =$ कोई स्थिरांक
30. $\int \frac{dx}{\sin x + \cos x} =$ [BIT Ranchi 1990; RPET 1997; Karnataka CET 1999; Orissa JEE 2004]
 (a) $\log \tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + c$ (b) $\log \tan\left(\frac{\pi}{8} - \frac{x}{2}\right) + c$
 (c) $\frac{1}{\sqrt{2}} \log \tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + c$ (d) इनमें से कोई नहीं
31. $\int \frac{1}{x\sqrt{x^2-1}} dx =$ [MP PET 1988]
 (a) $\cos^{-1} x + c$ (b) $\sec^{-1} x + c$
 (c) $\cot^{-1} x + c$ (d) $\tan^{-1} x + c$
32. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx =$
 (a) $2\sec x + c$ (b) $2\tan x + c$
 (c) $\tan x + c$ (d) इनमें से कोई नहीं
33. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx =$
 (a) $\tan x + \cot x + c$ (b) $\tan x - \cot x + c$
 (c) $\operatorname{cosec} x - \cot x + c$ (d) $\sec x - \operatorname{cosec} x + c$
34. $\int \left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2 dx =$
 (a) $x + \cos x + c$ (b) $2\cos^2 \frac{x}{2} + c$
 (c) $\frac{1}{3} \left(\cos \frac{x}{2} - \frac{x}{2}\right)^3 + c$ (d) $x - \cos x + c$
35. $\int \frac{1}{x^2} (2x+1)^3 dx =$
 (a) $4x^2 + 12x + 6\log x - \frac{1}{x} + c$
 (b) $4x^2 + 12x - 6\log x - \frac{2}{x} + c$
 (c) $2x^2 + 8x + 3\log x - \frac{2}{x} + c$
 (d) $8x^2 + 6x + 6\log x + \frac{2}{x} + c$
36. $\int \frac{5(x^6+1)}{x^2+1} dx =$
 (a) $5(x^7+x)\tan^{-1} x + c$
 (b) $x^5 - \frac{5}{3}x^3 + 5x + c$
 (c) $3x^4 - 5x^2 + 15x + c$
 (d) $5\tan^{-1}(x^2+1) + \log(x^2+1) + c$
37. $\int \frac{ax^{-2} + bx^{-1} + c}{x^{-3}} dx =$
 (a) $2ax^2 + 3bx^3 + 4cx^4 + k$
 (b) $6ax^2 + 4bx^3 + 3cx^4 + k$
 (c) $a + b + cx^2 + k$
 (d) $\frac{1}{2}ax^2 + \frac{1}{3}bx^3 + \frac{1}{4}cx^4 + k$
38. $\int \frac{dx}{\sin x + \sqrt{3} \cos x} =$
 (a) $\log \tan\left(\frac{x}{2} + \frac{\pi}{2}\right) + c$ (b) $\frac{1}{2} \log \tan\left(\frac{x}{2} + \frac{\pi}{6}\right) + c$
 (c) $\log \cot\left(\frac{x}{2} + \frac{\pi}{6}\right) + c$ (d) $\frac{1}{2} \log \cot\left(\frac{x}{2} + \frac{\pi}{6}\right) + c$
39. $\int \frac{x^2 + x - 6}{(x-2)(x-1)} dx =$
 (a) $x + 2\log(x-1) + c$ (b) $2x + 2\log(x-1) + c$

- (c) $x + 4 \log(1-x) + c$ (d) $x + 4 \log(x-1) + c$
40. $\int \frac{dx}{4 \cos^3 2x - 3 \cos 2x} =$
 (a) $\frac{1}{3} \log[\sec 6x + \tan 6x] + c$ (b) $\frac{1}{6} \log[\sec 6x + \tan 6x] + c$
 (c) $\log[\sec 6x + \tan 6x] + c$ (d) इनमें से कोई नहीं
41. $\int \frac{\sin 3x}{\sin x} dx =$
 (a) $x + \sin 2x + c$ (b) $3x + \sin 2x + c$
 (c) $3x + \sin^2 x + c$ (d) इनमें से कोई नहीं
42. यदि $\int \frac{f(x) dx}{\log \sin x} = \log \log \sin x$, तब $f(x) =$
 (a) $\sin x$ (b) $\cos x$
 (c) $\log \sin x$ (d) $\cot x$
43. $\int \frac{\sin x + \operatorname{cosec} x}{\tan x} dx =$
 (a) $\sin x - \operatorname{cosec} x + c$ (b) $\operatorname{cosec} x - \sin x + c$
 (c) $\log \tan x + c$ (d) $\log \cot x + c$
44. $\int \frac{1}{\sqrt{1+\sin x}} dx =$ [RPET 1996]
 (a) $2\sqrt{2} \log \tan\left(\frac{\pi}{8} + \frac{x}{4}\right) + c$ (b) $\frac{1}{\sqrt{2}} \log \tan\left(\frac{\pi}{8} + \frac{x}{4}\right) + c$
 (c) $\sqrt{2} \log \tan\left(\frac{\pi}{8} + \frac{x}{4}\right) + c$ (d) $\frac{1}{2\sqrt{2}} \log \tan\left(\frac{\pi}{8} + \frac{x}{4}\right) + c$
45. $\int (\tan x - \cot x)^2 dx =$
 (a) $\tan x + \cot x + c$ (b) $\sec x \tan x + c$
 (c) $\operatorname{cosec} x \cot x + c$ (d) इनमें से कोई नहीं
46. $\int \{1 + 2 \tan x (\tan x + \sec x)\}^{1/2} dx =$ [Roorkee 1987]
 (a) $\log(\sec x + \tan x) + c$
 (b) $\log(\sec x + \tan x)^{1/2} + c$
 (c) $\log \sec x (\sec x + \tan x) + c$
 (d) इनमें से कोई नहीं
47. $\int \frac{2x}{(2x+1)^2} dx =$ [DSSE 1985]
 (a) $\frac{1}{2} \log(2x+1) + \frac{1}{2(2x+1)} + c$
 (b) $\frac{1}{2} \log(2x+1) - \frac{1}{2(2x+1)} + c$
 (c) $2 \log(2x+1) + \frac{1}{2(2x+1)} + c$
 (d) $2 \log(2x+1) - \frac{1}{2(2x+1)} + c$
48. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx =$ [MP PET 1996]
 (a) $\tan x + \cot x + c$ (b) $\tan x + \operatorname{cosec} x + c$
 (c) $-\tan x + \cot x + c$ (d) $\tan x + \sec x + c$
49. $\int (3 \operatorname{cosec}^2 x + 2 \sin 3x) dx =$ [AI CBSE 1981]
 (a) $3 \cot x + \frac{2}{3} \cos 3x + c$
 (b) $-\left(3 \cot x + \frac{2}{3} \cos 3x\right) + c$
 (c) $3 \cot x - \frac{2}{3} \cos 3x + c$
 (d) इनमें से कोई नहीं
50. यदि $f'(x) = \frac{1}{x} + x$ और $f(1) = \frac{5}{2}$, तब $f(x) =$
 (a) $\log x + \frac{x^2}{2} + 2$ (b) $\log x + \frac{x^2}{2} + 1$
 (c) $\log x - \frac{x^2}{2} + 2$ (d) $\log x - \frac{x^2}{2} + 1$
51. $\int \frac{dx}{\sqrt{1+x} + \sqrt{x}} =$
 (a) $\frac{2}{3}(1+x)^{2/3} - \frac{2}{3}x^{2/3} + c$
 (b) $\frac{3}{2}(1+x)^{2/3} + \frac{3}{2}x^{2/3} + c$
 (c) $\frac{3}{2}(1+x)^{3/2} + \frac{3}{2}x^{3/2} + c$
 (d) $\frac{2}{3}(1+x)^{3/2} - \frac{2}{3}x^{3/2} + c$
52. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx =$ [MP PET 1994]
 (a) $2[\sin x + x \cos \alpha] + c$ (b) $2[\sin x + \sin \alpha] + c$
 (c) $2[-\sin x + x \cos \alpha] + c$ (d) $-2[\sin x + \sin \alpha] + c$
53. $\int \frac{1 - \tan x}{1 + \tan x} dx =$ [MP PET 1994]
 (a) $\log \sec\left(\frac{\pi}{4} - x\right) + c$ (b) $\log \cos\left(\frac{\pi}{4} + x\right) + c$
 (c) $\log \sin\left(\frac{\pi}{4} + x\right) + c$ (d) इनमें से कोई नहीं
54. $\int a^x da =$ [MP PET 1994, 96]
 (a) $\frac{a^x}{\log_e a} + c$ (b) $a^x \log_e a + c$
 (c) $\frac{a^{x+1}}{x+1} + c$ (d) $xa^{x-1} + c$
55. $\int \cot x dx$ का मान है [RPET 1995]
 (a) $\log \cos x + c$ (b) $\log \tan x + c$
 (c) $\log \sin x + c$ (d) $\log \sec x + c$
56. $\int \frac{1}{x^4} dx$ का मान है [RPET 1995]
 (a) $\frac{1}{-3x^3} + c$ (b) $\frac{1}{3x^3} + c$
 (c) $\frac{1}{-4x^3} + c$ (d) $-\frac{1}{3x^2} + c$

57. $\int \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} dx =$ [MNR 1985]
 (a) $e \cdot 3^{-3x} + c$ (b) $e^3 \log x + c$
 (c) $\frac{x^3}{3} + c$ (d) इनमें से कोई नहीं
58. $\int \frac{x^4 + x^2 + 1}{x^2 - x + 1} dx =$
 (a) $\frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c$ (b) $\frac{1}{3}x^3 - \frac{1}{2}x^2 + x + c$
 (c) $\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + c$ (d) इनमें से कोई नहीं
59. $\int \sec x dx =$ [MP PET 1988, 95; RPET 1996]
 (a) $\log \tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + c$ (b) $-\log(\sec x - \tan x) + c$
 (c) $\log(\sec x - \tan x) + c$ (d) इनमें से कोई नहीं
60. $\int \sqrt{1 + \sin x} dx =$ [MP PET 1995]
 (a) $\frac{1}{2}\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right) + c$ (b) $\frac{1}{2}\left(\sin \frac{x}{2} - \cos \frac{x}{2}\right) + c$
 (c) $2\sqrt{1 + \sin x} + c$ (d) $-2\sqrt{1 - \sin x} + c$
61. $\int \operatorname{cosec}^2 x dx$ का मान है [MP PET 1999]
 (a) $\cot x + c$ (b) $-\cot x + c$
 (c) $\tan^2 x + c$ (d) $-\cot^2 x + c$
62. $\int \left(2 \sin x + \frac{1}{x}\right) dx$ का मान है [MP PET 1999]
 (a) $-2 \cos x + \log x + c$ (b) $2 \cos x + \log x + c$
 (c) $-2 \sin x - \frac{1}{x^2} + c$ (d) $-2 \cos x + \frac{1}{x^2} + c$
63. $\int \sqrt{1 + \cos x} dx =$ [RPET 1996]
 (a) $2\sqrt{2} \sin \frac{x}{2} + c$ (b) $-2\sqrt{2} \sin \frac{x}{2} + c$
 (c) $-2\sqrt{2} \cos \frac{x}{2} + c$ (d) $2\sqrt{2} \cos \frac{x}{2} + c$
64. $\int 2 \sin x \cos x dx =$ [SCRA 1996]
 (a) $\cos 2x + c$ (b) $\sin 2x + c$
 (c) $\cos^2 x + c$ (d) $\sin^2 x + c$
65. $\int \tan^2 x dx =$ [SCRA 1996]
 (a) $\tan x + x + c$ (b) $\tan x - x + c$
 (c) $\sec x + x + c$ (d) $\sec x - x + c$
66. $\int e^{\log(\sin x)} dx =$ [MP PET 1995]
 (a) $\sin x + c$ (b) $-\cos x + c$
 (c) $e^{\log(\cos x)} + c$ (d) इनमें से कोई नहीं
67. $\int e^{x \log a} \cdot e^x dx =$ [Kerala (Engg.) 2005]
 (a) $(ae)^x + c$ (b) $\frac{(ae)^x}{\log(ae)} + c$
 (c) $\frac{e^x}{1 + \log a} + c$ (d) इनमें से कोई नहीं
68. $\int \frac{1}{\sqrt{1 + \cos x}} dx =$
 (a) $\sqrt{2} \log\left(\sec \frac{x}{2} + \tan \frac{x}{2}\right) + K$
 (b) $\frac{1}{\sqrt{2}} \log\left(\sec \frac{x}{2} + \tan \frac{x}{2}\right) + K$
 (c) $\log\left(\sec \frac{x}{2} + \tan \frac{x}{2}\right) + K$
 (d) इनमें से कोई नहीं
69. $\int \frac{\cos 2x - 1}{\cos 2x + 1} dx =$ [MP PET 2000]
 (a) $\tan x - x + c$ (b) $x + \tan x + c$
 (c) $x - \tan x + c$ (d) $-x - \cot x + c$
70. $\int \frac{ax^3 + bx^2 + c}{x^4} dx =$ [RPET 2001]
 (a) $a \log x + \frac{b}{x^2} + \frac{c}{3x^3} + c$ (b) $a \log x + \frac{b}{x} - \frac{c}{3x^3} + c$
 (c) $a \log x - \frac{b}{x} - \frac{c}{3x^3} + c$ (d) इनमें से कोई नहीं
71. $\int \frac{1}{(x-5)^2} dx$ का मान है [Karnataka CET 2001; Pb. CET 2002]
 (a) $\frac{1}{x-5} + c$ (b) $-\frac{1}{x-5} + c$
 (c) $\frac{2}{(x-5)^3} + c$ (d) $-2(x-5)^3 + c$
72. यदि $\int f(x) dx = f(x)$, तब $\int [f(x)]^2 dx =$ [DCE 2002]
 (a) $\frac{1}{2}[f(x)]^2$ (b) $[f(x)]^3$
 (c) $\frac{[f(x)]^3}{3}$ (d) $[f(x)]^2$
73. यदि $\int \sqrt{2} \sqrt{1 + \sin x} dx = -4 \cos(ax + b) + c$, तब (a, b) का मान है [UPSEAT 2002]
 (a) $\frac{1}{2}, \frac{\pi}{4}$ (b) $1, \frac{\pi}{2}$
 (c) $1, 1$ (d) इनमें से कोई नहीं
74. $\int 13^x dx =$ [Kerala (Engg.) 2002]
 (a) $\frac{13^x}{\log 13} + c$ (b) $13^{x+1} + c$
 (c) $14x + c$ (d) $14^{x+1} + c$
75. $\int a^x dx =$ [RPET 2003]

- (a) $\frac{a^x}{\log a} + c$ (b) $a^x \log a + c$ (c) $\log_e |x - \sqrt{x^2 - a^2}| + c$ (d) $\frac{x\sqrt{x^2 - a^2}}{2 + c}$
- (c) $\log a + c$ (d) $a^x + c$
76. $\int \sec x \tan x \, dx =$ [RPET 2003]
(a) $\sec x + \tan x + c$ (b) $\sec x + c$
(c) $\tan x + c$ (d) $-\sec x + c$
77. $\int (\sin^4 x - \cos^4 x) dx =$ [RPET 2003]
(a) $-\frac{\cos 2x}{2} + c$ (b) $-\frac{\sin 2x}{2} + c$
(c) $\frac{\sin 2x}{2} + c$ (d) $\frac{\cos 2x}{2} + c$
78. $\int \frac{(x+1)^2 dx}{x(x^2+1)} =$ [MP PET 2003]
(a) $\log_e x + c$ (b) $\log_e x + 2 \tan^{-1} x + c$
(c) $\log_e \frac{1}{x^2+1} + c$ (d) $\log_e \{x(x^2+1)\} + c$
79. $\int \frac{dx}{\sqrt{1-x}} =$ [Pb. CET 2001]
(a) $2\sqrt{1-x} + c$ (b) $-2\sqrt{1-x} + c$
(c) $-\sin^{-1} \sqrt{x} + c$ (d) $\sin^{-1} \sqrt{x} + c$
80. $\int \frac{dx}{1-x^2} =$ [MP PET 1987, 92, 2000]
(a) $\tan^{-1} x + c$ (b) $\sin^{-1} x + c$
(c) $\frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + c$ (d) $\frac{1}{2} \ln \left| \frac{1-x}{1+x} \right| + c$
81. $\int \frac{dx}{4x^2+9} =$ [MP PET 1991; Roorkee 1977; MNR 1974]
(a) $\frac{1}{2} \tan^{-1} \left(\frac{2x}{3} \right) + c$ (b) $\frac{3}{2} \tan^{-1} \left(\frac{2x}{3} \right) + c$
(c) $\frac{1}{6} \tan^{-1} \left(\frac{2x}{3} \right) + c$ (d) $\frac{1}{6} \tan^{-1} \left(\frac{3x}{2} \right) + c$
82. $\int \sqrt{1+x^2} \, dx =$ [MP PET 1987, 89]
(a) $\frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log(x + \sqrt{1+x^2}) + c$
(b) $\frac{2}{3} (1+x^2)^{3/2} + c$
(c) $\frac{2}{3} x(1+x^2)^{3/2} + c$
(d) इनमें से कोई नहीं
83. $\int \frac{dx}{\sqrt{x^2 - a^2}} =$ [SCRA 1996]
(a) $\sin^{-1} \left(\frac{x}{a} \right) + c$ (b) $\log_e |x + \sqrt{x^2 - a^2}| + c$
84. $\int \frac{x^2}{x^2+4} \, dx =$ [RPET 2001]
(a) $x - 2 \tan^{-1}(x/2) + c$ (b) $x + 2 \tan^{-1}(x/2) + c$
(c) $x - 4 \tan^{-1}(x/2) + c$ (d) $x + 4 \tan^{-1}(x/2) + c$
85. $\int \sqrt{x^2 + a^2} \, dx =$ [RPET 2001]
(a) $\frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \log \{x + \sqrt{x^2 + a^2}\} + c$
(b) $\frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \{x + \sqrt{x^2 + a^2}\} + c$
(c) $\frac{x}{2} \sqrt{x^2 + a^2} - \frac{a^2}{2} \log \{x - \sqrt{x^2 + a^2}\} + c$
(d) $\frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \{x - \sqrt{x^2 + a^2}\} + c$
86. $\int \frac{dx}{a^2 - x^2} =$ [EAMCET 2002]
(a) $\frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right)$ (b) $\frac{1}{2a} \sin^{-1} \left(\frac{a-x}{a+x} \right)$
(c) $\frac{1}{2a} \log \left(\frac{a+x}{a-x} \right)$ (d) $\frac{1}{2a} \log \left(\frac{a-x}{a+x} \right)$
87. $\int \frac{dx}{\sqrt{x^2 + a^2}} =$ [MP PET 2004]
(a) $\frac{1}{2} x \sqrt{x^2 + a^2} + \frac{1}{2} a^2 \log(x + \sqrt{x^2 + a^2}) + c$
(b) $\frac{1}{2} \log(x^2 + a^2) + c$
(c) $\log(x + \sqrt{x^2 + a^2}) + c$
(d) $\log(x - \sqrt{x^2 + a^2}) + c$

प्रतिस्थापन द्वारा समाकलन

1. $\int \frac{dx}{1+e^x} =$ [MP PET 1991; Roorkee 1977]
(a) $\log(1+e^x)$ (b) $-\log(1+e^{-x})$
(c) $-\log(1-e^{-x})$ (d) $\log(e^{-x} + e^{-2x})$
2. $\int \frac{dx}{e^x + e^{-x}} =$ [Bihar CEE 1976; MNR 1974]
(a) $\tan^{-1}(e^{-x})$ (b) $\tan^{-1}(e^x)$
(c) $\log(e^x - e^{-x})$ (d) $\log(e^x + e^{-x})$
3. $\int \frac{e^{\sqrt{x}} \cos e^{\sqrt{x}}}{\sqrt{x}} \, dx =$
(a) $2 \sin e^{\sqrt{x}}$ (b) $\sin e^{\sqrt{x}}$
(c) $2 \cos e^{\sqrt{x}}$ (d) $-2 \sin e^{\sqrt{x}}$

4. $\int \frac{dx}{x + x \log x} =$ [MP PET 1993; Roorkee 1977]
 (a) $\log(1 + \log x)$ (b) $\log \log(1 + \log x)$
 (c) $\log x + \log(\log x)$ (d) इनमें से कोई नहीं
5. $\int \frac{1 + \log x}{x} dx$ का मान ज्ञात करने के लिये उचित प्रतिस्थापन है [MP PET 1988]
 (a) $\log x = t$ (b) $1 + \log x = t$
 (c) $\frac{1}{x} = t$ (d) इनमें से कोई नहीं
6. $\int \frac{\sec x \, dx}{\sqrt{\cos 2x}} =$
 (a) $\sin^{-1}(\tan x)$ (b) $\tan x$
 (c) $\cos^{-1}(\tan x)$ (d) $\frac{\sin x}{\sqrt{\cos x}}$
7. $\int \frac{dx}{x\sqrt{2ax - x^2}}$ का मान ज्ञात करने के लिये उचित प्रतिस्थापन है
 (a) $x = a \cos t$ (b) $x = 2a \cos t$
 (c) $x = 2at$ (d) $x = 2a \sin^2 t$
8. $\int \frac{x \, dx}{1 - x \cot x} =$
 (a) $\log(\cos x - x \sin x) + c$ (b) $\log(x \sin x - \cos x) + c$
 (c) $\log(\sin x - x \cos x) + c$ (d) इनमें से कोई नहीं
9. $\int \frac{\sin 2x}{1 + \sin^2 x} dx =$ [Roorkee 1976]
 (a) $\log \sin 2x + c$ (b) $\log(1 + \sin^2 x) + c$
 (c) $\frac{1}{2} \log(1 + \sin^2 x) + c$ (d) $\tan^{-1}(\sin x) + c$
10. $\int \frac{x^3}{\sqrt{x^2 + 2}} dx =$
 (a) $\frac{1}{3}(x^2 + 2)^{3/2} + 2(x^2 + 2)^{1/2} + c$
 (b) $\frac{1}{3}(x^2 + 2)^{3/2} - 2(x^2 + 2)^{1/2} + c$
 (c) $\frac{1}{3}(x^2 + 2)^{3/2} + (x^2 + 2)^{1/2} + c$
 (d) $\frac{1}{3}(x^2 + 2)^{3/2} - (x^2 + 2)^{1/2} + c$
11. $\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx =$
 (a) $\log(x^e + e^x) + c$ (b) $e \log(x^e + e^x) + c$
 (c) $\frac{1}{e} \log(x^e + e^x) + c$ (d) इनमें से कोई नहीं
12. $\int \frac{\sin x \, dx}{a^2 + b^2 \cos^2 x} =$
 (a) $\log(a^2 + b^2 \cos^2 x) + c$ (b) $\frac{1}{ab} \tan^{-1}\left(\frac{a \cos x}{b}\right) + c$
- (c) $\frac{1}{ab} \cot^{-1}\left(\frac{b \cos x}{a}\right) + c$ (d) $\frac{1}{ab} \cot^{-1}\left(\frac{a \cos x}{b}\right) + c$
13. $\int \sec x \log(\sec x + \tan x) dx =$
 (a) $[\log(\sec x + \tan x)]^2 + c$
 (b) $\frac{1}{2}[\log(\sec x + \tan x)]^2 + c$
 (c) $\sec^2 x + \tan x \sec x + c$
 (d) इनमें से कोई नहीं
14. $\int \frac{x-2}{x^2 - 4x + 3} dx =$ [MP PET 1987]
 (a) $\log \sqrt{x^2 - 4x + 3} + c$ (b) $x \log(x-3) - 2 \log(x-2) + c$
 (c) $\log[(x-3)(x-1)]$ (d) इनमें से कोई नहीं
15. $\int \frac{3x^2}{x^6 + 1} dx =$ [MNR 198; MP PET 1988; RPET 1995]
 (a) $\log(x^6 + 1) + c$ (b) $\tan^{-1}(x^3) + c$
 (c) $3 \tan^{-1}(x^3) + c$ (d) $3 \tan^{-1}\left(\frac{x^3}{3}\right) + c$
16. $\int \frac{\cot x}{\log \sin x} dx =$ [MNR 1974]
 (a) $\log(\log \sin x) + c$ (b) $\log(\log \operatorname{cosec} x) + c$
 (c) $2 \log(\log \sin x) + c$ (d) इनमें से कोई नहीं
17. $\int \frac{(1 + \log x)^2}{x} dx =$ [Roorkee 1977]
 (a) $(1 + \log x)^3 + c$ (b) $3(1 + \log x)^3 + c$
 (c) $\frac{1}{3}(1 + \log x)^3 + c$ (d) इनमें से कोई नहीं
18. $\int \sec^p x \tan x \, dx =$
 (a) $\frac{\sec^{p+1} x}{p+1} + c$ (b) $\frac{\sec^p x}{p} + c$
 (c) $\frac{\tan^{p+1} x}{p+1} + c$ (d) $\frac{\tan^p x}{p} + c$
19. $\int \frac{dx}{e^x - 1} =$ [MP PET 1989]
 (a) $\ln(1 - e^{-x}) + c$ (b) $-\ln(1 - e^{-x}) + c$
 (c) $\ln(e^x - 1) + c$ (d) इनमें से कोई नहीं
20. $\int x^2 \sec x^3 \, dx =$ [MNR 1986; Roorkee 1975]
 (a) $\log(\sec x^3 + \tan x^3)$ (b) $3(\sec x^3 + \tan x^3)$
 (c) $\frac{1}{3} \log(\sec x^3 + \tan x^3)$ (d) इनमें से कोई नहीं
21. $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx =$ [RPET 1995]
 (a) $\cot^{-1}(\tan^2 x) + c$ (b) $\tan^{-1}(\tan^2 x) + c$

- (c) $\cot^{-1}(\cot^2 x) + c$ (d) $\tan^{-1}(\cot^2 x) + c$
22. $\int \frac{x-2}{x(2\log x - x)} dx =$
 (a) $\log(2\log x - x) + c$ (b) $\log\left(\frac{1}{2\log x - x}\right) + c$
 (c) $\log(x - 2\log x) + c$ (d) $\log\left(\frac{1}{x - 2\log x}\right) + c$
23. $\int x\sqrt{1+x^2} dx =$ [MP PET 1989]
 (a) $\frac{1+2x^2}{\sqrt{1+x^2}} + c$ (b) $\sqrt{1+x^2} + c$
 (c) $3(1+x^2)^{3/2} + c$ (d) $\frac{1}{3}(1+x^2)^{3/2} + c$
24. $\int \frac{e^x(x+1)}{\cos^2(xe^x)} dx =$ [Roorkee 1979; MP PET 1995; Pb. CET 2001]
 (a) $\tan(xe^x) + c$ (b) $\sec(xe^x)\tan(xe^x) + c$
 (c) $-\tan(xe^x) + c$ (d) इनमें से कोई नहीं
25. $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx =$ [MP PET 1987; IIT 1990; SCRA 1996; RPET 2001]
 (a) $2\cos \sqrt{x} + c$ (b) $2\sin \sqrt{x} + c$
 (c) $\sin \sqrt{x} + c$ (d) $\frac{1}{2}\cos \sqrt{x} + c$
26. $\int \frac{x+1}{\sqrt{1+x^2}} dx =$ [MP PET 1991]
 (a) $\sqrt{1+x^2} + \tan^{-1} x + c$
 (b) $\sqrt{1+x^2} - \log\{x + \sqrt{1+x^2}\} + c$
 (c) $\sqrt{1+x^2} + \log\{x + \sqrt{1+x^2}\} + c$
 (d) $\sqrt{1+x^2} + \log(\sec x + \tan x) + c$
27. $\int \frac{\sin x \cos x}{a \cos^2 x + b \sin^2 x} dx =$ [AI CBSE 1988, 89]
 (a) $\frac{1}{2(b-a)} \log(a \cos^2 x + b \sin^2 x) + c$
 (b) $\frac{1}{b-a} \log(a \cos^2 x + b \sin^2 x) + c$
 (c) $\frac{1}{2} \log(a \cos^2 x + b \sin^2 x) + c$
 (d) इनमें से कोई नहीं
28. $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx =$ [MP PET 1987]
 (a) $\log(1+x^2) + c$ (b) $\log e^{\tan^{-1} x} + c$
 (c) $e^{\tan^{-1} x} + c$ (d) $\tan^{-1} e^{\tan^{-1} x} + c$
29. $\int \frac{1}{x(\log x)^2} dx =$
- (a) $\frac{1}{\log x} + c$ (b) $-\frac{1}{\log x} + c$
 (c) $\log \log x + c$ (d) $-\log \log x + c$
30. $\int \frac{1}{\sqrt{x}} \tan^4 \sqrt{x} \sec^2 \sqrt{x} dx =$
 (a) $2 \tan^5 \sqrt{x} + c$ (b) $\frac{1}{5} \tan^5 \sqrt{x} + c$
 (c) $\frac{2}{5} \tan^5 \sqrt{x} + c$ (d) इनमें से कोई नहीं
31. $\int \frac{a^x}{\sqrt{1-a^{2x}}} dx =$ [MNR 1983, 87]
 (a) $\frac{1}{\log a} \sin^{-1} a^x + c$ (b) $\sin^{-1} a^x + c$
 (c) $\frac{1}{\log a} \cos^{-1} a^x + c$ (d) $\cos^{-1} a^x + c$
32. $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx =$ [Bihar CEE 1974; MP PET 2002; Kerala (Engg.) 2002]
 (a) $2\sqrt{\sec x} + c$ (b) $2\sqrt{\tan x} + c$
 (c) $\frac{2}{\sqrt{\tan x}} + c$ (d) $\frac{2}{\sqrt{\sec x}} + c$
33. $\int \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx =$ [Roorkee 1977]
 (a) $\frac{1}{b^2} \log(a^2 + b^2 \sin^2 x) + c$
 (b) $\frac{1}{b} \log(a^2 + b^2 \sin^2 x) + c$
 (c) $\log(a^2 + b^2 \sin^2 x) + c$
 (d) $b^2 \log(a^2 + b^2 \sin^2 x) + c$
34. $\int \frac{1}{x\sqrt{1+\log x}} dx =$ [Roorkee 1977]
 (a) $\frac{2}{3}(1+\log x)^{3/2} + c$ (b) $(1+\log x)^{3/2} + c$
 (c) $2\sqrt{1+\log x} + c$ (d) $\sqrt{1+\log x} + c$
35. $\int \frac{\sec^2 x}{1+\tan x} dx =$ [MP PET 1987]
 (a) $\log(\cos x + \sin x) + c$ (b) $\log(\sec^2 x) + c$
 (c) $\log(1+\tan x) + c$ (d) $-\frac{1}{(1+\tan x)^2} + c$
36. $\int \frac{e^{2x}-1}{e^{2x}+1} dx =$ [MP PET 1987]
 (a) $\frac{e^{2x}-1}{e^{2x}+1} + c$ (b) $\log(e^{2x}+1) - x + c$
 (c) $\log(e^{2x}+1) + c$ (d) इनमें से कोई नहीं
37. $\int \frac{\operatorname{cosec} x}{\log \tan \frac{x}{2}} dx =$

- (a) $\log\left(\log \tan \frac{x}{2}\right) + c$ (b) $2 \log\left(\log \tan \frac{x}{2}\right) + c$
 (c) $\frac{1}{2} \log\left(\log \tan \frac{x}{2}\right) + c$ (d) इनमें से कोई नहीं
38. $\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx =$
 (a) $\frac{1}{\tan x - 1} + c$ (b) $\frac{1}{1 - \tan x} + c$
 (c) $-\frac{1}{3} \frac{1}{(1 - \tan x)^3} + c$ (d) इनमें से कोई नहीं
39. $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx =$ [MNR 1979]
 (a) $-\frac{1}{2} \frac{1}{(10^x + x^{10})^2} + c$ (b) $\log(10^x + x^{10}) + c$
 (c) $\frac{1}{2} \frac{1}{(10^x + x^{10})^2} + c$ (d) इनमें से कोई नहीं
40. $\int \frac{1}{(e^x + e^{-x})^2} dx =$
 (a) $-\frac{1}{2(e^{2x} + 1)} + c$ (b) $\frac{1}{2(e^{2x} + 1)} + c$
 (c) $-\frac{1}{e^{2x} + 1}$ (d) इनमें से कोई नहीं
41. $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx =$
 (a) $\log \sqrt{\cos x + \sin x} + c$ (b) $\log(\cos x - \sin x) + c$
 (c) $\log(\cos x + \sin x) + c$ (d) $-\frac{1}{\cos x + \sin x} + c$
42. $\int \frac{\tan(\log x)}{x} dx =$
 (a) $\log \cos(\log x) + c$ (b) $\log \sin(\log x) + c$
 (c) $\log \sec(\log x) + c$ (d) $\log \operatorname{cosec}(\log x) + c$
43. $\int \cos^3 x e^{\log(\sin x)} dx =$
 (a) $-\frac{\sin^4 x}{4} + c$ (b) $-\frac{\cos^4 x}{4} + c$
 (c) $\frac{e^{\sin x}}{4} + c$ (d) इनमें से कोई नहीं
44. $\int \tan(3x - 5) \sec(3x - 5) dx =$ [MP PET 1988]
 (a) $\sec(3x - 5) + c$ (b) $\frac{1}{3} \sec(3x - 5) + c$
 (c) $\tan(3x - 5) + c$ (d) इनमें से कोई नहीं
45. $\int \frac{x}{1 + x^4} dx =$ [IIT 1978; UPSEAT 2002]
 (a) $\frac{1}{2} \cot^{-1} x^2 + c$ (b) $\frac{1}{2} \tan^{-1} x^2 + c$
 (c) $\cot^{-1} x^2 + c$ (d) $\tan^{-1} x^2 + c$
46. $\int \frac{e^{-x}}{1 + e^x} dx =$

- (a) $\log(1 + e^x) - x - e^{-x} + c$ (b) $\log(1 + e^x) + x - e^{-x} + c$
 (c) $\log(1 + e^x) - x + e^{-x} + c$ (d) $\log(1 + e^x) + x + e^{-x} + c$
47. $\int \frac{1}{\sqrt{1 - e^{2x}}} dx =$ [MP PET 1993, 2002; RPET 1999]
 (a) $x - \log[1 + \sqrt{1 - e^{2x}}] + c$ (b) $x + \log[1 + \sqrt{1 - e^{2x}}] + c$
 (c) $\log[1 + \sqrt{1 - e^{2x}}] - x + c$ (d) इनमें से कोई नहीं
48. $\int \frac{3x^2}{\sqrt{9 - 16x^6}} dx =$
 (a) $\frac{1}{4} \sin^{-1}\left(\frac{4x^3}{3}\right) + c$ (b) $\frac{1}{3} \sin^{-1}\left(\frac{4x^3}{3}\right) + c$
 (c) $\frac{1}{4} \sin^{-1} x^3 + c$ (d) $\frac{1}{3} \sin^{-1} x^3 + c$
49. $\int \cos x \sqrt{4 - \sin^2 x} dx =$
 (a) $\frac{1}{2} \sin x \sqrt{4 - \sin^2 x} - 2 \sin^{-1}\left(\frac{1}{2} \sin x\right) + c$
 (b) $\frac{1}{2} \sin x \sqrt{4 - \sin^2 x} + 2 \sin^{-1}\left(\frac{1}{2} \sin x\right) + c$
 (c) $\frac{1}{2} \sin x \sqrt{4 - \sin^2 x} + \sin^{-1}\left(\frac{1}{2} \sin x\right) + c$
 (d) इनमें से कोई नहीं
50. $\int x^2 (3)^{x^3+1} dx =$
 (a) $(3)^{x^3} + c$ (b) $\frac{(3)^{x^3}}{\log 3} + c$
 (c) $\log 3 (3)^{x^3} + c$ (d) इनमें से कोई नहीं
51. $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx =$
 (a) $-3(\tan x)^{1/3} + c$ (b) $-3(\tan x)^{-1/3} + c$
 (c) $3(\tan x)^{-1/3} + c$ (d) $(\tan x)^{-1/3} + c$
52. $\int \cos^5 x dx =$
 (a) $\sin x - \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c$
 (b) $\sin x + \frac{2}{3} \sin^3 x + \frac{1}{5} \sin^5 x + c$
 (c) $\sin x - \frac{2}{3} \sin^3 x - \frac{1}{5} \sin^5 x + c$
 (d) इनमें से कोई नहीं
53. $\int \sec x \tan^3 x dx =$
 (a) $\frac{1}{3} \sec^3 x - \sec x + c$ (b) $\sec^3 x - \sec x + c$
 (c) $\frac{1}{3} \sec^3 x + \sec x + c$ (d) इनमें से कोई नहीं
54. $\int \frac{d\theta}{\sin \theta \cos^3 \theta} =$

- (a) $\log \tan \theta + \tan^2 \theta + c$ (b) $\log \tan \theta - \frac{1}{2} \tan^2 \theta + c$
 (c) $\log \tan \theta + \frac{1}{2} \tan^2 \theta + c$ (d) इनमें से कोई नहीं
55. $\int \frac{1}{\cos^{-1} x \cdot \sqrt{1-x^2}} dx =$
 (a) $\log(\cos^{-1} x) + c$ (b) $-\log(\cos^{-1} x) + c$
 (c) $-\frac{1}{2(\cos^{-1} x)^2} + c$ (d) इनमें से कोई नहीं
56. $\int x^3 e^{3x^2+5} dx$ का मान ज्ञात करने के लिये सरलतम तरीका है
 (a) $x^2 = t$ के प्रतिस्थापन द्वारा
 (b) $(3x^2 + 5) = t$ के प्रतिस्थापन द्वारा
 (c) खण्डशः समाकलन द्वारा
 (d) इनमें से कोई नहीं
57. $\int \frac{\sec^2 x}{(1 + \tan x)(2 + \tan x)} dx$ का मान ज्ञात करने के लिये उचित प्रतिस्थापन है
 (a) $1 + \tan x = t$ (b) $2 + \tan x = t$
 (c) $\tan x = t$ (d) इनमें से कोई नहीं
58. $\int \frac{\operatorname{cosec}^2 x}{1 + \cot x} dx =$ [MNR 1973]
 (a) $\log(1 + \cot x) + c$ (b) $-\log(1 + \cot x) + c$
 (c) $\frac{1}{2(1 + \cot x)^2} + c$ (d) इनमें से कोई नहीं
59. $\int \frac{1}{\sqrt{x}} \sin \sqrt{x} dx =$ [MP PET 1989]
 (a) $-\frac{1}{2} \cos \sqrt{x} + c$ (b) $-2 \cos \sqrt{x} + c$
 (c) $\frac{1}{2} \cos \sqrt{x} + c$ (d) $2 \cos \sqrt{x} + c$
60. $\int e^x \tan^2(e^x) dx =$
 (a) $\tan(e^x) - x + c$ (b) $e^x (\tan e^x - 1) + c$
 (c) $\sec(e^x) + c$ (d) $\tan(e^x) - e^x + c$
61. $\int \frac{dx}{e^{-2x}(e^{2x} + 1)^2} =$
 (a) $\frac{-1}{2(e^{2x} + 1)} + c$ (b) $\frac{1}{2(e^{2x} + 1)} + c$
 (c) $\frac{1}{e^{2x} + 1} + c$ (d) $\frac{-1}{e^{2x} + 1} + c$
62. $\int \tan^4 x dx =$
 (a) $\tan^3 x - \tan x + x + c$ (b) $\frac{1}{3} \tan^3 x - \tan x + x + c$
 (c) $\frac{1}{3} \tan^3 x + \tan x + x + c$ (d) $\frac{1}{3} \tan^3 x + \tan x + 2x + c$
63. $\int \frac{dx}{x\sqrt{1-(\log x)^2}} =$
 (a) $\cos^{-1}(\log x) + c$ (b) $x \log(1 - x^2) + c$
- (c) $\sin^{-1}(\log x) + c$ (d) $\frac{1}{2} \cos^{-1}(\log x) + c$
64. $\int \frac{f'(x)}{[f(x)]^2} dx =$
 (a) $-[f(x)]^{-1} + c$ (b) $\log[f(x)] + c$
 (c) $e^{f(x)} + c$ (d) इनमें से कोई नहीं
65. निम्न में से किस फलन के लिये $x^2 = t$ प्रतिस्थापन करने योग्य है
 (a) $\int x^6 \tan^{-1} x^3 dx$ (b) $\int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$
 (c) $\int x^3 \cos x^2 dx$ (d) इनमें से कोई नहीं
66. $\int \tan x \sec^2 x \sqrt{1 - \tan^2 x} dx =$
 (a) $-\frac{1}{3}(1 - \tan^2 x)^{3/2} + c$ (b) $\frac{1}{3}(1 - \tan^2 x)^{3/2} + c$
 (c) $-\frac{2}{3}(1 - \tan^2 x)^{2/3} + c$ (d) इनमें से कोई नहीं
67. $\int \frac{\sin 2x}{\sin 5x \sin 3x} dx =$
 (a) $\log \sin 3x - \log \sin 5x + c$
 (b) $\frac{1}{3} \log \sin 3x + \frac{1}{5} \log \sin 5x + c$
 (c) $\frac{1}{3} \log \sin 3x - \frac{1}{5} \log \sin 5x + c$
 (d) $3 \log \sin 3x - 5 \log \sin 5x + c$
68. $\int \frac{e^x dx}{\sqrt{1 - e^{2x}}} =$
 (a) $\cos^{-1}(e^x) + c$ (b) $-\cos^{-1}(e^x) + c$
 (c) $\cos^{-1}(e^{2x}) + c$ (d) $\sqrt{1 - e^{2x}} + c$
69. $\int \frac{1}{\log a} (a^x \cos a^x) dx =$
 (a) $\sin a^x + c$ (b) $a^x \sin a^x + c$
 (c) $\frac{1}{(\log a)^2} \sin a^x + c$ (d) $\log \sin a^x + c$
70. $\int \frac{\sin x dx}{(a + b \cos x)^2} =$
 (a) $\frac{1}{b}(a + b \cos x) + c$ (b) $\frac{1}{b(a + b \cos x)} + c$
 (c) $\frac{1}{b} \log(a + b \cos x) + c$ (d) इनमें से कोई नहीं
71. $\int \frac{1}{x^3} [\log x^x]^2 dx =$
 (a) $\frac{x^3}{3} (\log x) + x + c$ (b) $\frac{1}{3} (\log x)^3 + c$
 (c) $3 \log(\log x) + c$ (d) इनमें से कोई नहीं
72. $\int \frac{1}{x} \sec^2(\log x) dx =$
 (a) $\tan(\log x) + c$ (b) $\log(\sec x) + c$
 (c) $\log(\tan x) + c$ (d) $\sec(\log x) \cdot \tan(\log x) + c$

73. $\int \frac{dx}{x \log x \log(\log x)} =$
 (a) $2 \log(\log x) + c$ (b) $\log[\log(\log x)] + c$
 (c) $\log(x \log x) + c$ (d) इनमें से कोई नहीं
74. $\int \frac{\sec^2 x \, dx}{\sqrt{\tan^2 x + 4}} =$
 (a) $\log\left[\tan x + \sqrt{\tan^2 x + 4}\right] + c$
 (b) $\frac{1}{2} \log\left[\tan x + \sqrt{\tan^2 x + 4}\right] + c$
 (c) $\log\left[\frac{1}{2} \tan x + \frac{1}{2} \sqrt{\tan^2 x + 4}\right] + c$
 (d) इनमें से कोई नहीं
75. $\int \frac{2x \tan^{-1} x^2}{1 + x^4} dx =$ [Roorkee 1982]
 (a) $[\tan^{-1} x^2]^2 + c$ (b) $\frac{1}{2} [\tan^{-1} x^2]^2 + c$
 (c) $2[\tan^{-1} x^2]^2 + c$ (d) इनमें से कोई नहीं
76. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx =$ [Roorkee 1990; MP PET 2001]
 (a) $2a^{\sqrt{x}} \log_e a + c$ (b) $2a^{\sqrt{x}} \log_a e + c$
 (c) $2a^{\sqrt{x}} \log_{10} a + c$ (d) $2a^{\sqrt{x}} \log_a 10 + c$
77. $\int \frac{x^3}{\sqrt{1-x^8}} dx =$
 (a) $\frac{1}{2} \sin^{-1}(x^4) + c$ (b) $\frac{1}{3} \sin^{-1}(x^4) + c$
 (c) $\frac{1}{4} \sin^{-1}(x^4) + c$ (d) इनमें से कोई नहीं
78. $\int 2x \cos^3 x^2 \sin x^2 dx =$
 (a) $-\frac{1}{4} \cos^4 x^2 + c$ (b) $\frac{1}{4} \cos^4 x^2 + c$
 (c) $\cos^4 x^2 + c$ (d) इनमें से कोई नहीं
79. $\int \sec^4 x \tan x \, dx =$ [AI CBSE 1980, 81; SCRA 1996]
 (a) $\frac{1}{4} \sec^4 x + c$ (b) $4 \sec^4 x + c$
 (c) $\frac{\sec^3 x}{3} + c$ (d) $3 \sec^3 x + c$
80. $\int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5) dx =$ [AISSE 1988]
 (a) $\frac{1}{2} \cot(2e^{-x} + 5) + c$ (b) $-\frac{1}{2} \cot(2e^{-x} + 5) + c$
 (c) $2 \cot(2e^{-x} + 5) + c$ (d) $-2 \cot(2e^{-x} + 5) + c$
81. $\int \sin^3 x \cdot \cos x \, dx =$ [SCRA 1996]
 (a) $\frac{\sin^4 x \cos^2 x}{8} + c$ (b) $\frac{\sin^4 x}{4} + c$
 (c) $\frac{\sin^2 x}{2} + c$ (d) $4 \sin^4 x + c$
82. $\int a^{3x+3} dx =$ [Roorkee 1977]
 (a) $\frac{a^{3x+3}}{\log a} + c$ (b) $\frac{a^{3x+3}}{3 \log a} + c$
 (c) $a^{3x+3} \log a + c$ (d) $3a^{3x+3} \log a + c$
83. $\int \frac{\cos 2x + x + 1}{x^2 + \sin 2x + 2x} dx =$ [AI CBSE 1980]
 (a) $\log(x^2 + \sin 2x + 2x) + c$
 (b) $-\log(x^2 + \sin 2x + 2x) + c$
 (c) $\frac{1}{2} \log(x^2 + \sin 2x + 2x) + c$
 (d) इनमें से कोई नहीं
84. $\int \frac{1 + \tan x}{x + \log \sec x} dx =$ [AI CBSE 1986]
 (a) $\log(x + \log \sec x) + c$ (b) $-\log(x + \log \sec x) + c$
 (c) $\log(x - \log \sec x) + c$ (d) इनमें से कोई नहीं
85. $\int \frac{(x+1)(x+\log x)^2}{x} dx =$ [AI CBSE 1986]
 (a) $\frac{1}{3}(x + \log x) + c$ (b) $\frac{1}{3}(x + \log x)^2 + c$
 (c) $\frac{1}{3}(x + \log x)^3 + c$ (d) इनमें से कोई नहीं
86. $\int \frac{1+x^2}{\sqrt{1-x^2}} dx =$ [IIT 1977]
 (a) $\frac{3}{2} \sin^{-1} x - \frac{1}{2} x \sqrt{1-x^2} + c$
 (b) $\frac{3}{2} \sin^{-1} x + \frac{1}{2} x \sqrt{1-x^2} + c$
 (c) $\frac{3}{2} \cos^{-1} x - \frac{1}{2} x \sqrt{1-x^2} + c$
 (d) $\frac{3}{2} \cos^{-1} x + \frac{1}{2} x \sqrt{1-x^2} + c$
87. $\int \frac{\cos x - \sin x}{1 + \sin 2x} dx =$ [AISSE 1985]
 (a) $-\frac{1}{\cos x + \sin x} + c$ (b) $\frac{1}{\cos x + \sin x} + c$
 (c) $\frac{1}{\cos x - \sin x} + c$ (d) इनमें से कोई नहीं
88. $\int x^3 \sqrt{3+5x^4} \, dx =$ [DSSE 1982]
 (a) $(3+5x^4)^{3/2} + c$ (b) $\frac{1}{5}(3+5x^4)^{3/2} + c$
 (c) $\frac{1}{30}(3+5x^4)^{3/2} + c$ (d) इनमें से कोई नहीं
89. $\int \sqrt{\frac{x}{a^3 - x^3}} dx =$
 (a) $\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c$ (b) $\frac{2}{3} \sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c$

- (c) $\frac{3}{2} \sin^{-1} \left(\frac{x}{a} \right)^{3/2} + c$ (d) $\frac{3}{2} \sin^{-1} \left(\frac{x}{a} \right)^{2/3} + c$
90. $\int \frac{1}{x \cos^2(1 + \log x)} dx =$
 (a) $\tan(1 + \log x) + c$ (b) $\cot(1 + \log x) + c$
 (c) $-\tan(1 + \log x) + c$ (d) $-\cot(1 + \log x) + c$
91. $\int \frac{1}{x^2 \sqrt{1+x^2}} dx =$
 (a) $-\frac{\sqrt{1+x^2}}{x} + c$ (b) $\frac{\sqrt{1+x^2}}{x} + c$
 (c) $-\frac{\sqrt{1-x^2}}{x} + c$ (d) $-\frac{\sqrt{x^2-1}}{x} + c$
92. $\int \frac{1}{(x^2-1)\sqrt{x^2+1}} dx =$
 (a) $\frac{1}{2\sqrt{2}} \log \left\{ \frac{\sqrt{1+x^2} + x\sqrt{2}}{\sqrt{1+x^2} - x\sqrt{2}} \right\} + c$
 (b) $\frac{1}{2\sqrt{2}} \log \left\{ \frac{\sqrt{1+x^2} - \sqrt{2}}{\sqrt{1+x^2} + \sqrt{2}} \right\} + c$
 (c) $\frac{1}{2\sqrt{2}} \log \left\{ \frac{\sqrt{1+x^2} - x\sqrt{2}}{\sqrt{1+x^2} + x\sqrt{2}} \right\} + c$
 (d) इनमें से कोई नहीं
93. $\int \frac{\log(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx =$
 (a) $\frac{1}{2} [\log(x + \sqrt{1+x^2})]^2 + c$ (b) $\log(x + \sqrt{1+x^2})^2 + c$
 (c) $\log(x + \sqrt{1+x^2}) + c$ (d) इनमें से कोई नहीं
94. $\int e^x \sin(e^x) dx =$ [MP PET 1995]
 (a) $-\cos e^x + c$ (b) $\cos e^x + c$
 (c) $-\operatorname{cosec} e^x + c$ (d) इनमें से कोई नहीं
95. $\int \frac{x^5 dx}{\sqrt{1+x^3}} =$ [IIT 1975]
 (a) $\frac{2}{3} \sqrt{1+x^3} (x^3+2)$ (b) $\frac{2}{9} \sqrt{1+x^3} (x^3-4)$
 (c) $\frac{2}{9} \sqrt{1+x^3} (x^3+4)$ (d) $\frac{2}{9} \sqrt{1+x^3} (x^3-2)$
96. $\int \frac{(x^4-x)^{1/4}}{x^5} dx =$
 (a) $\frac{4}{15} \left(1 - \frac{1}{x^3} \right)^{5/4} + c$ (b) $\frac{4}{5} \left(1 - \frac{1}{x^3} \right)^{5/4} + c$
 (c) $\frac{4}{15} \left(1 + \frac{1}{x^3} \right)^{5/4} + c$ (d) इनमें से कोई नहीं
97. $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}} dx =$
- (a) $\frac{4}{3} \left(\frac{x-1}{x+2} \right)^{1/4} + c$ (b) $\frac{4}{3} \left(\frac{x+2}{x-1} \right)^{1/4} + c$
 (c) $\frac{1}{3} \left(\frac{x-1}{x+2} \right)^{1/4} + c$ (d) $\frac{1}{3} \left(\frac{x+2}{x-1} \right)^{1/4} + c$
98. $\int \frac{1}{1+\sin^2 x} dx =$
 (a) $\frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2} \tan x) + k$ (b) $\sqrt{2} \tan^{-1}(\sqrt{2} \tan x) + k$
 (c) $-\frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2} \tan x) + k$ (d) $-\sqrt{2} \tan^{-1}(\sqrt{2} \tan x) + k$
99. $\int \frac{\sin x}{\cos^2 x} dx =$
 (a) $\sin x + k$ (b) $\tan x + k$
 (c) $\sec x + k$ (d) $\tan x + \sec x + k$
100. $\int e^x \sec^2(e^x) dx =$
 (a) $\tan(e^x) + k$ (b) $\tan(e^x) \cdot e + k$
 (c) $e^x \tan x + k$ (d) $\frac{\tan(e^x)}{e^x} + k$
101. $\int \frac{dx}{x\sqrt{x^4-1}} =$
 (a) $\frac{1}{2} \sec^{-1} x^2 + k$ (b) $\log x \sqrt{x^4-1} + k$
 (c) $x \log \sqrt{x^4-1} + k$ (d) $\log \sqrt{x^4-1} + k$
102. $\int \frac{t}{e^{3t^2}} dt =$ [MP PET 1997]
 (a) $\frac{1}{6} e^{3t^2} + c$ (b) $-\frac{1}{6} e^{3t^2} + c$
 (c) $\frac{1}{6} e^{-3t^2} + c$ (d) $-\frac{1}{6} e^{-3t^2} + c$
103. यदि $\int \frac{1}{(1+x)\sqrt{x}} dx = f(x) + A$, जहाँ A कोई स्वेच्छ नियतांक है, तब फलन $f(x)$ है [MP PET 1998]
 (a) $2 \tan^{-1} x$ (b) $2 \tan^{-1} \sqrt{x}$
 (c) $2 \cot^{-1} \sqrt{x}$ (d) $\log_e(1+x)$
104. $\int x \cos x^2 dx =$ [MP PET 1999; Pb. CET 2000]
 (a) $-\frac{1}{2} \sin^2 x + c$ (b) $\frac{1}{2} \sin^2 x + c$
 (c) $-\frac{1}{2} \sin x^2 + c$ (d) $\frac{1}{2} \sin x^2 + c$
105. $\int \frac{x^2 \tan^{-1} x^3}{1+x^6} dx =$ [MP PET 1999; UPSEAT 1999]
 (a) $\tan^{-1}(x^3) + c$ (b) $\frac{1}{6} (\tan^{-1} x^3)^2 + c$
 (c) $-\frac{1}{2} (\tan^{-1} x^3)^2 + c$ (d) $\frac{1}{2} (\tan^{-1} x^2)^3 + c$

106. $\int \frac{x^2 + 1}{x(x^2 - 1)} dx =$ [MP PET 1999]

- (a) $\log \frac{x^2 - 1}{x} + c$ (b) $-\log \frac{x^2 - 1}{x} + c$
(c) $\log \frac{x}{x^2 + 1} + c$ (d) $-\log \frac{x}{x^2 + 1} + c$

107. $\int \frac{e^{2x} + 1}{e^{2x} - 1} dx =$ [RPET 1996]

- (a) $\log(e^x - e^{-x}) + c$ (b) $\log(e^x + e^{-x}) + c$
(c) $\log(e^{-x} - e^x) + c$ (d) $\log(1 - e^{-x}) + c$

108. $\int \frac{\cos x - \sin x}{\sqrt{\sin 2x}} dx =$ [RPET 1996]

- (a) $\cosh^{-1}(\sin x + \cos x) + c$
(b) $\sinh^{-1}(\sin x + \cos x) + c$
(c) $-\cosh^{-1}(\sin x + \cos x) + c$
(d) $-\sinh^{-1}(\sin x + \cos x) + c$

109. $\int \left(1 + \frac{1}{x^2}\right) e^{\left(\frac{x-1}{x}\right)} dx =$ [Kurukshetra CEE 1998]

- (a) $e^{\frac{x-1}{x}} + c$ (b) $e^{\frac{x+1}{x}} + c$
(c) $e^{\frac{x^2-1}{x}} + c$ (d) $e^{\frac{x^2+1}{x^2}} + c$

110. $\int (x+3)(x^2+6x+10)^9 dx =$ [SCRA 1996]

- (a) $\frac{1}{20}(x^2+6x+10)^{10} + c$
(b) $\frac{1}{20}(x+3)^2(x^2+6x+10)^{10} + c$
(c) $\frac{1}{16}(x^2+6x+10)^8 + c$
(d) $\frac{1}{38}(x+3)^{19} + \frac{1}{2}(x+3) + c$

111. $\int \frac{x}{x^2+1} dx =$ [SCRA 1996]

- (a) $\log_e(x^2+1)$ (b) $x \tan^{-1} x$
(c) $\frac{\log_e(x^2+1)}{2}$ (d) $\frac{1}{2} x \tan^{-1} x$

112. $\int \sin^3 x dx =$ [SCRA 1996]

- (a) $\sin^2 x + 1$ (b) $\sin x^2 + x^2 + 1$
(c) $\frac{\cos^3 x}{3} - \cos x$ (d) $\frac{1}{4} \sin^4 x - \frac{3}{4} \sin^2 x$

113. $\int \frac{1}{x} \log x dx =$ [SCRA 1996]

- (a) $\frac{1}{2} \log x + c$ (b) $\frac{1}{2} (\log x)^2 + c$

(c) $\frac{1}{2} \log(x)^2 + c$ (d) $\log x + c$

114. $\int \sin^2 x \cos x dx =$ [SCRA 1996]

- (a) $\frac{\cos^2 x}{2} + c$ (b) $\frac{\sin^2 x}{3} + c$
(c) $\frac{\sin^3 x}{3} + c$ (d) $-\frac{\cos^2 x}{2} + c$

115. $\int e^{x^2} x dx =$ [SCRA 1996]

- (a) e^{x^2} (b) $\frac{1}{2} e^{x^2}$
(c) $2e^{x^2}$ (d) $\frac{e^{x^2} - x^2}{2}$

116. $\int \frac{x^3}{\sqrt{1+x^4}} dx =$ [SCRA 1996]

- (a) $(1+x^4)^{\frac{1}{2}} + c$ (b) $-(1+x^4)^{\frac{1}{2}} + c$
(c) $\frac{1}{2}(1+x^4)^{\frac{1}{2}} + c$ (d) $-\frac{1}{2}(1+x^4)^{\frac{1}{2}} + c$

117. $I = \int \frac{dx}{(1+e^x)(1+e^{-x})}$ का क्या मान है [DCE 1999]

- (a) $\frac{-1}{1+e^x}$ (b) $\frac{e^x}{1+e^x}$
(c) $\frac{1}{1+e^x}$ (d) इनमें से कोई नहीं

118. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx =$ [DCE 1999]

- (a) $e^{\sqrt{x}}$ (b) $\frac{e^{\sqrt{x}}}{2}$
(c) $2e^{\sqrt{x}}$ (d) $\sqrt{x} \cdot e^{\sqrt{x}}$

119. $\int \frac{\sin^3 2x}{\cos^5 2x} dx =$ [Karnataka CET 1999]

- (a) $\tan^4 x + C$ (b) $\tan 4x + C$
(c) $\tan^4 2x + x + C$ (d) $\frac{1}{8} \tan^4 2x + C$

120. $\int x^x (1 + \log x) dx =$ [RPET 2000]

- (a) x^x (b) x^{2x}
(c) $x^x \log x$ (d) $\frac{1}{2} (1 + \log x)^2$

121. $\int \frac{dx}{(a^2 + x^2)^{3/2}} =$ [RPET 2000]

- (a) $\frac{x}{(a^2 + x^2)^{1/2}}$ (b) $\frac{x}{a^2(a^2 + x^2)^{1/2}}$
(c) $\frac{1}{a^2(a^2 + x^2)^{1/2}}$ (d) इनमें से कोई नहीं

122. $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx =$ [RPET 2001]

- (a) $e^{\tan^{-1} x}$ (b) $\frac{1}{m} e^{\tan^{-1} x}$
(c) $\frac{1}{m} e^{m \tan^{-1} x}$ (d) इनमें से कोई नहीं

123. $\int \frac{1+\tan^2 x}{1-\tan^2 x} dx =$ [RPET 2001]

- (a) $\log\left(\frac{1-\tan x}{1+\tan x}\right) + c$ (b) $\log\left(\frac{1+\tan x}{1-\tan x}\right) + c$
(c) $\frac{1}{2} \log\left(\frac{1-\tan x}{1+\tan x}\right) + c$ (d) $\frac{1}{2} \log\left(\frac{1+\tan x}{1-\tan x}\right) + c$

124. $\int \frac{2 dx}{\sqrt{1-4x^2}}$ का मान है [Karnataka CET 2001; Pb. CET 2001]

- (a) $\tan^{-1}(2x) + c$ (b) $\cot^{-1}(2x) + c$
(c) $\cos^{-1}(2x) + c$ (d) $\sin^{-1}(2x) + c$

125. $\int e^{3 \log x} (x^4 + 1)^{-1} dx =$ [MP PET 2001]

- (a) $\log(x^4 + 1) + c$ (b) $\frac{1}{4} \log(x^4 + 1) + c$
(c) $-\log(x^4 + 1) + c$ (d) इनमें से कोई नहीं

126. $\int \frac{dx}{2\sqrt{x}(1+x)} =$ [RPET 2002]

- (a) $\frac{1}{2} \tan^{-1}(\sqrt{x}) + c$ (b) $\tan^{-1}(\sqrt{x}) + c$
(c) $2 \tan^{-1}(\sqrt{x}) + c$ (d) इनमें से कोई नहीं

127. $\int \operatorname{cosec}^4 x dx =$ [RPET 2002]

- (a) $\cot x + \frac{\cot^3 x}{3} + c$ (b) $\tan x + \frac{\tan^3 x}{3} + c$
(c) $-\cot x - \frac{\cot^3 x}{3} + c$ (d) $-\tan x - \frac{\tan^3 x}{3} + c$

128. $\int x e^{x^2} dx =$ [RPET 2003]

- (a) $-\frac{e^{x^2}}{2} + c$ (b) $\frac{e^{x^2}}{2} + c$
(c) $\frac{e^x}{2} + c$ (d) $-\frac{e^x}{2} + c$

129. यदि $\int f(x) dx = g(x)$, तब $\int f^{-1}(x) dx =$ [MP PET 2003]

- (a) $g^{-1}(x)$ (b) $xf^{-1}(x) - g(f^{-1}(x))$
(c) $xf^{-1}(x) - g^{-1}(x)$ (d) $f^{-1}(x)$

130. $\int \frac{e^x}{e^x + 1} dx$ का मान है [Pb. CET 2000]

- (a) $e^x + c$ (b) $(e^x + 1) + c$
(c) $\log(e^x + 1) + c$ (d) इनमें से कोई नहीं

131. $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ का मान है [Pb. CET 2000]

- (a) $\frac{1}{\sin x + \cos x} + c$ (b) $\frac{1}{\sin x - \cos x} + c$
(c) $\log(\sin x + \cos x) + c$ (d) $\log\left(\frac{1}{\sin x + \cos x}\right) + c$

132. $\int \frac{(\tan^{-1} x)^3}{1+x^2} dx =$ [UPSEAT 2004]

- (a) $(\tan^{-1} x)^4 + c$ (b) $\frac{(\tan^{-1} x)^4}{4} + c$
(c) $2 \tan^{-1} x + c$ (d) $2(\tan^{-1} x)^2 + c$

133. $\int \sqrt{\frac{1-x}{1+x}} dx =$ [IIT 1971]

- (a) $\sin^{-1} x - \frac{1}{2} \sqrt{1-x^2} + c$ (b) $\sin^{-1} x + \frac{1}{2} \sqrt{1-x^2} + c$
(c) $\sin^{-1} x - \sqrt{1-x^2} + c$ (d) $\sin^{-1} x + \sqrt{1-x^2} + c$

134. $\int \frac{\sqrt{x}}{1+x} dx =$

- (a) $\sqrt{x} - \tan^{-1} \sqrt{x} + c$ (b) $2(\sqrt{x} - \tan^{-1} \sqrt{x}) + c$
(c) $2(\sqrt{x} + \tan^{-1} x) + c$ (d) $\sqrt{1+x} + c$

135. $\int \frac{\sin x}{\sin x - \cos x} dx =$ [Roorkee 1988]

- (a) $\frac{1}{2} \log(\sin x - \cos x) + x + c$
(b) $\frac{1}{2} [\log(\sin x - \cos x) + x] + c$
(c) $\frac{1}{2} \log(\cos x - \sin x) + x + c$
(d) $\frac{1}{2} [\log(\cos x - \sin x) + x] + c$

136. $\int \sqrt{\frac{1+x}{1-x}} dx =$ [RPET 2002]

- (a) $-\sin^{-1} x - \sqrt{1-x^2} + c$ (b) $\sin^{-1} x + \sqrt{1-x^2} + c$
(c) $\sin^{-1} x - \sqrt{1-x^2} + c$ (d) $-\sin^{-1} x - \sqrt{x^2-1} + c$

137. $\int \frac{x}{\sqrt{4-x^4}} dx =$ [Roorkee 1976]

- (a) $\cos^{-1} \frac{x^2}{2}$ (b) $\frac{1}{2} \cos^{-1} \frac{x^2}{2}$
(c) $\sin^{-1} \frac{x^2}{2}$ (d) $\frac{1}{2} \sin^{-1} \frac{x^2}{2}$

138. $\int \frac{\sin x dx}{3+4 \cos^2 x} =$ [Karnataka CET 2000]

- (a) $\log(3+4 \cos^2 x) + c$ (b) $\frac{-1}{2\sqrt{3}} \tan^{-1}\left(\frac{\cos x}{\sqrt{3}}\right) + c$
(c) $\frac{-1}{2\sqrt{3}} \tan^{-1}\left(\frac{2 \cos x}{\sqrt{3}}\right) + c$ (d) $\frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{2 \cos x}{\sqrt{3}}\right) + c$

139. $\int \frac{dx}{\sqrt{x}(x+9)}$ का मान है [Pb. CET 2002]

(a) $\tan^{-1} \sqrt{x}$

(b) $\tan^{-1} \left(\frac{\sqrt{x}}{3} \right)$

(c) $\frac{2}{3} \tan^{-1} \sqrt{x}$

(d) $\frac{2}{3} \tan^{-1} \left(\frac{\sqrt{x}}{3} \right)$

140. $\int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx =$ [AIEEE 2005]

(a) $\frac{xe^x}{1+x^2} + c$

(b) $\frac{x}{(\log x)^2 + 1} + c$

(c) $\frac{\log x}{(\log x)^2 + 1} + c$

(d) $\frac{x}{x^2 + 1} + c$

141. $\int \frac{\sin 2x dx}{1 + \cos^2 x} =$ [Karnataka CET 2005]

(a) $\frac{1}{2} \log(1 + \cos^2 x) + c$

(b) $2 \log(1 + \cos^2 x) + c$

(c) $\frac{1}{2} \log(1 + \cos 2x) + c$

(d) $-\log(1 + \cos^2 x) + c$

142. यदि $\int \frac{\cos 4x + 1}{\cos x - \tan x} dx = k \cos 4x + c$ तब [DCE 2005]

(a) $k = -1/2$

(b) $k = -1/8$

(c) $k = -1/4$

(d) इनमें से कोई नहीं

143. यदि $\int \frac{1}{x+x^5} dx = f(x) + c$, तब $\int \frac{x^4}{x+x^5} dx$ का मान है [DCE 2005]

(a) $\log x - f(x) + c$

(b) $f(x) + \log x + c$

(c) $f(x) - \log x + c$

(d) इनमें से कोई नहीं

144. माना $f(x) = \int \frac{x^2 dx}{(1+x^2)(1+\sqrt{1+x^2})}$ और $f(0) = 0$, तब $f(1)$ का मान है [AMU 2005]

(a) $\log(1 + \sqrt{2})$

(b) $\log(1 + \sqrt{2}) - \frac{\pi}{4}$

(c) $\log(1 + \sqrt{2}) + \frac{\pi}{2}$

(d) इनमें से कोई नहीं

145. $\int \sqrt{e^x - 1} dx =$ [Kerala (Engg.) 2005]

(a) $2 \left[\sqrt{e^x - 1} - \tan^{-1} \sqrt{e^x - 1} \right] + c$

(b) $\sqrt{e^x - 1} - \tan^{-1} \sqrt{e^x - 1} + c$

(c) $\sqrt{e^x - 1} + \tan^{-1} \sqrt{e^x - 1} + c$

(d) $2 \left[\sqrt{e^x - 1} + \tan^{-1} \sqrt{e^x - 1} \right] + c$

(e) $2 \left[\sqrt{e^x - 1} - \tan^{-1} \sqrt{e^x + 1} \right] + c$

146. $\int \frac{dx}{\sin(x-a)\sin(x-b)} =$ [Kerala (Engg.) 2005]

(a) $\frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$

(b) $\frac{-1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$

(c) $\log \sin(x-a)\sin(x-b) + c$

(d) $\log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$

(e) $\frac{1}{\sin(x-a)} \log \sin(x-a)\sin(x-b) + c$

147. $\int \frac{(\sin \theta + \cos \theta)}{\sqrt{\sin 2\theta}} d\theta =$ [Kerala (Engg.) 2005]

(a) $\log \left| \cos \theta - \sin \theta + \sqrt{\sin 2\theta} \right|$

(b) $\log \left| \sin \theta - \cos \theta + \sqrt{\sin 2\theta} \right|$

(c) $\sin^{-1}(\sin \theta - \cos \theta) + c$

(d) $\sin^{-1}(\sin \theta + \cos \theta) + c$

(e) $\sin^{-1}(\cos \theta - \sin \theta) + c$

148. $\int \cos^{-3/7} x \sin^{-11/7} x dx =$ [Kerala (Engg.) 2005]

(a) $\log |\sin^{4/7} x| + c$

(b) $\frac{4}{7} \tan^{4/7} x + c$

(c) $\frac{-7}{4} \tan^{-4/7} x + c$

(d) $\log |\cos^{3/7} x| + c$

(e) $\frac{7}{4} \tan^{-4/7} x + c$

खण्डशः समाकलन, $\int e^x(F(x) + F'(x)) dx$ तथा $\int e^{kx}(kF(x) + F'(x)) dx$ के रूप का समाकलन

1. $\int x \sec^2 x dx =$

[RPET 1996, 2003; MP PET 1987, 97; Pb. CET 2002]

(a) $\tan x + \log \cos x + c$

(b) $\frac{x^2}{2} \sec^2 x + \log \cos x + c$

(c) $x \tan x + \log \sec x + c$

(d) $x \tan x + \log \cos x + c$

2. $\int \sin(\log x) dx =$

(a) $\frac{1}{2} x [\cos(\log x) - \sin(\log x)]$

(b) $\cos(\log x) - x$

(c) $\frac{1}{2} x [\sin(\log x) - \cos(\log x)]$

(d) $-\cos \log x$

3. यदि $\int x \sin x dx = -x \cos x + A$, तब $A =$

[MP PET 1992, 2000; RPET 1997]

(a) $\sin x + \text{अचर}$

(b) $\cos x + \text{अचर}$

- (c) अचर (d) इनमें से कोई नहीं
4. $\int x \log x dx =$ [MP PET 1987]
 (a) $\frac{x^2}{2} \log x - \frac{x^2}{2} + c$ (b) $\frac{x^2}{2} \log x - \frac{x^2}{4} + c$
 (c) $\frac{x^2}{2} \log x + \frac{x^2}{2} + c$ (d) इनमें से कोई नहीं
5. $\int x \cos x dx =$ [MP PET 1988]
 (a) $x \sin x + \cos x$ (b) $x \sin x - \cos x$
 (c) $x \cos x + \sin x$ (d) $x \cos x - \sin x$
6. $\int x \cos^2 x dx =$ [IIT 1972]
 (a) $\frac{x^4}{4} - \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x + c$
 (b) $\frac{x^2}{4} + \frac{1}{4} x \sin 2x + \frac{1}{8} \cos 2x + c$
 (c) $\frac{x^4}{4} - \frac{1}{4} x \sin 2x + \frac{1}{8} \cos 2x + c$
 (d) $\frac{x^4}{4} + \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x + c$
7. $\int \tan^{-1} x dx =$ [Roorkee 1977]
 (a) $x \tan^{-1} x + \frac{1}{2} \log(1+x^2)$ (b) $x \tan^{-1} x - \frac{1}{2} \log(1+x^2)$
 (c) $(x-1) \tan^{-1} x$ (d) $x \tan^{-1} x - \log(1+x^2)$
8. $\int x \tan^{-1} x dx =$ [Roorkee 1979]
 (a) $\frac{1}{2}(x^2+1) \tan^{-1} x - \frac{1}{2} x + c$
 (b) $\frac{1}{2}(x^2-1) \tan^{-1} x - \frac{1}{2} x + c$
 (c) $\frac{1}{2}(x^2+1) \tan^{-1} x + \frac{1}{2} x + c$
 (d) $\frac{1}{2}(x^2+1) \tan^{-1} x - x + c$
9. $\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx =$
 (a) $x \log(\log x) + \frac{x}{\log x} + c$ (b) $x \log(\log x) - \frac{x}{\log x} + c$
 (c) $x \log(\log x) + \frac{\log x}{x} + c$ (d) $x \log(\log x) - \frac{\log x}{x} + c$
10. $\int [\sin(\log x) + \cos(\log x)] dx =$ [MP PET 1991]
 (a) $x \cos(\log x) + c$ (b) $\sin(\log x) + c$
 (c) $\cos(\log x) + c$ (d) $x \sin(\log x) + c$
11. $\int \frac{\log x dx}{x^3} =$ [Roorkee 1986]
 (a) $\frac{1}{4x^2} (2 \log x - 1) + c$ (b) $-\frac{1}{4x^2} (2 \log x + 1) + c$
 (c) $\frac{1}{4x^2} (2 \log x + 1) + c$ (d) $\frac{1}{4x^2} (1 - 2 \log x) + c$
12. $\int e^{-2x} \sin 3x dx =$
 (a) $\frac{1}{13} e^{-2x} [\sin 3x + \cos 3x] + c$
 (b) $-\frac{1}{13} e^{-2x} [\sin 3x + \cos 3x] + c$
 (c) $\frac{1}{13} e^{-2x} [2 \sin 3x + 3 \cos 3x] + c$
 (d) $-\frac{1}{13} e^{-2x} [2 \sin 3x + 3 \cos 3x] + c$
13. $\int x \sin x \sec^3 x dx =$
 (a) $\frac{1}{2} [\sec^2 x - \tan x] + c$ (b) $\frac{1}{2} [x \sec^2 x - \tan x] + c$
 (c) $\frac{1}{2} [x \sec^2 x + \tan x] + c$ (d) $\frac{1}{2} [\sec^2 x + \tan x] + c$
14. $\int x \sin^2 x dx =$ [BIT Ranchi 1977; IIT 1972]
 (a) $\frac{x^2}{4} + \frac{x}{4} \sin 2x + \frac{1}{8} \cos 2x + c$
 (b) $\frac{x^2}{4} - \frac{x}{4} \sin 2x + \frac{1}{8} \cos 2x + c$
 (c) $\frac{x^2}{4} + \frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x + c$
 (d) $\frac{x^2}{4} - \frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x + c$
15. $\int e^{2x+\log x} dx =$
 (a) $\frac{1}{4} (2x-1) e^{2x} + c$ (b) $\frac{1}{4} (2x+1) e^{2x} + c$
 (c) $\frac{1}{2} (2x+1) e^{2x} + c$ (d) $\frac{1}{2} (2x-1) e^{2x} + c$
16. $\int \log(x+1) dx =$ [Roorkee 1974]
 (a) $(x+1) \log(x+1) - x + c$ (b) $(x+1) \log(x+1) + x + c$
 (c) $(x-1) \log(x+1) - x + c$ (d) $(x-1) \log(x+1) + x + c$
17. यदि $\int \ln(x^2+x) dx = x \ln(x^2+x) + A$, तब $A =$ [MP PET 1992]
 (a) $2x + \ln(x+1) + \text{अचर}$ (b) $2x - \ln(x+1) + \text{अचर}$
 (c) अचर (d) इनमें से कोई नहीं
18. $\int x^2 \sin 2x dx =$ [IIT 1974]
 (a) $\frac{1}{2} x^2 \cos 2x + \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$
 (b) $-\frac{1}{2} x^2 \cos 2x + \frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + c$

(c) $\frac{1}{2}x^2 \cos 2x - \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + c$

(d) इनमें से कोई नहीं

19. $\int \log x dx =$ [MNR 1979; BIT Ranchi 1992; SCRA 1996]

(a) $x + x \log x + c$

(b) $x \log x - x + c$

(c) $x^2 \log x + c$

(d) $\frac{1}{x} \log x + x + c$

20. $\int \log_{10} x dx =$ [Roorkee 1973]

(a) $x \log_{10} x + c$

(b) $x(\log_{10} x + \log_{10} e) + c$

(c) $\log_{10} x + c$

(d) $x(\log_{10} x - \log_{10} e) + c$

21. $\int \frac{1}{\log_x e} dx =$ [MP PET 1994]

(a) $\log \log_x e + c$

(b) $\frac{1}{(\log_x e)^2} + c$

(c) $x \log\left(\frac{x}{e}\right) + c$

(d) इनमें से कोई नहीं

22. यदि $\int e^x \sin x dx = \frac{1}{2}e^x \cdot a + c$, तब $a =$ [MP PET 1989]

(a) $\sin x - \cos x$

(b) $\cos x - \sin x$

(c) $-\cos x - \sin x$

(d) $\cos x + \sin x$

23. $\int x^n \log x dx =$

(a) $\frac{x^{n+1}}{n+1} \left\{ \log x + \frac{1}{n+1} \right\} + c$

(b) $\frac{x^{n+1}}{n+1} \left\{ \log x + \frac{2}{n+1} \right\} + c$

(c) $\frac{x^{n+1}}{n+1} \left\{ 2 \log x - \frac{1}{n+1} \right\} + c$

(d) $\frac{x^{n+1}}{n+1} \left\{ \log x - \frac{1}{n+1} \right\} + c$

24. $\int \log x (\log x + 2) dx =$

(a) $x(\log x)^2 + c$

(b) $x(1 + \log x)^2 + c$

(c) $x[1 + (\log x)^2] + c$

(d) इनमें से कोई नहीं

25. $\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx =$

(a) $\frac{1}{\log x} + c$

(b) $\frac{x}{\log x} + c$

(c) $\frac{x}{(\log x)^2}$

(d) इनमें से कोई नहीं

26. $\int \frac{\log x}{(1 + \log x)^2} dx =$

(a) $\frac{1}{1 + \log x} + c$

(b) $\frac{x}{(1 + \log x)^2} + c$

(c) $\frac{x}{1 + \log x} + c$

(d) $\frac{1}{(1 + \log x)^2} + c$

27. $\int \left(\frac{2 + \sin 2x}{1 + \cos 2x} \right) e^x dx =$ [AISSE 1982]

(a) $e^x \cot x + c$

(b) $-e^x \cot x + c$

(c) $-e^x \tan x + c$

(d) $e^x \tan x + c$

28. $\int e^x \sin x dx =$ [IIT 1978; AI CBSE 1980; MP PET 1999]

(a) $\frac{1}{2}e^x (\sin x + \cos x) + c$

(b) $\frac{1}{2}e^x (\sin x - \cos x) + c$

(c) $e^x (\sin x + \cos x) + c$

(d) $e^x (\sin x - \cos x) + c$

29. $\int (1 - x^2) \log x dx =$ [DSSE 1982]

(a) $\left(x - \frac{x^3}{3} \right) \log x - \left(x - \frac{x^3}{9} \right) + c$

(b) $\left(x - \frac{x^3}{3} \right) \log x + \left(x - \frac{x^3}{9} \right) + c$

(c) $\left(x + \frac{x^3}{3} \right) \log x + \left(x + \frac{x^3}{9} \right) + c$

(d) इनमें से कोई नहीं

30. $\int e^{2x} (-\sin x + 2 \cos x) dx =$ [DSSE 1987]

(a) $e^{2x} \sin x + c$

(b) $-e^{2x} \sin x + c$

(c) $-e^{2x} \cos x + c$

(d) $e^{2x} \cos x + c$

31. $\int [f(x)g''(x) - f''(x)g(x)] dx =$ [MP PET 2001]

(a) $\frac{f(x)}{g'(x)}$

(b) $f'(x)g(x) - f(x)g'(x)$

(c) $f(x)g'(x) - f'(x)g(x)$

(d) $f(x)g'(x) + f'(x)g(x)$

32. $\int x \sin kx dx =$

(a) $\frac{\sin kx}{k} + c$

(b) $\frac{\cos kx}{k} + c$

(c) $\frac{\sin x}{k} + c$

(d) $-\frac{x \cos kx}{k} + \frac{\sin kx}{k^2} + c$

33. $\int xe^x dx =$ [SCRA 1996]

(a) $(x+1)e^x + c$

(b) $(x-1)e^x + c$

(c) $xe^x + 1 + c$

(d) $xe^x - 1 + c$

34. $\int x^3 e^{x^2} dx =$ [MNR 1980]

(a) $\frac{1}{2}(x^2 + 1)e^{x^2} + c$

(b) $(x^2 + 1)e^{x^2} + c$

(c) $\frac{1}{2}(x^2 - 1)e^{x^2} + c$

(d) $(x^2 - 1)e^{x^2} + c$

35. $\int \frac{\log x}{(x+1)^2} dx$ का मान है [UPSEAT 1999]

(a) $-\frac{\log x}{x+1} + \log x - \log(x+1)$

(b) $\frac{\log x}{(x+1)} + \log x - \log(x+1)$

(c) $\frac{\log x}{x+1} - \log x - \log(x+1)$

- (d) $\frac{-\log x}{x+1} - \log x - \log(x+1)$
36. यदि $\int xe^{2x} dx$ का मान $e^{2x} f(x) + C$ है, जहाँ C समाकलन स्थिरांक है, तब $f(x) =$ [UPSEAT 2001]
 (a) $(3x-1)/4$ (b) $(2x+1)/2$
 (c) $(2x-1)/4$ (d) $(x-4)/6$
37. यदि $\frac{d}{dx} f(x) = x \cos x + \sin x$ और $f(0) = 2$, तब $f(x) =$ [MP PET 1989]
 (a) $x \sin x$ (b) $x \cos x + \sin x + 2$
 (c) $x \sin x + 2$ (d) $x \cos x + 2$
38. $\int \cos^{-1}\left(\frac{1}{x}\right) dx =$ [RPET 2002]
 (a) $x \sec^{-1} x + \cosh^{-1} x + C$ (b) $x \sec^{-1} x - \cosh^{-1} x + C$
 (c) $x \sec^{-1} x - \sin^{-1} x + C$ (d) इनमें से कोई नहीं
39. $\int x^3 \log x dx =$ [Karnataka CET 2002]
 (a) $\frac{x^4 \log x}{4} + C$ (b) $\frac{1}{16} [4x^4 \log x - x^4] + C$
 (c) $\frac{1}{8} [x^4 \log x - 4x^2] + C$ (d) $\frac{1}{16} [4x^4 \log x + x^4] + C$
40. $\int \cos(\log_e x) dx =$ [MP PET 2003]
 (a) $\frac{1}{2} x \{ \cos(\log_e x) + \sin(\log_e x) \}$
 (b) $x \{ \cos(\log_e x) + \sin(\log_e x) \}$
 (c) $\frac{1}{2} x \{ \cos(\log_e x) - \sin(\log_e x) \}$
 (d) $x \{ \cos(\log_e x) - \sin(\log_e x) \}$
41. $\int e^x (1 + \tan x) \sec x dx =$ [Karnataka CET 2005]
 (a) $e^x \cot x$ (b) $e^x \tan x$
 (c) $e^x \sec x$ (d) $e^x \cos x$
42. $\int \frac{xe^x}{(1+x)^2} dx =$ [MP PET 1997; UPSEAT 2001; RPET 2002]
 (a) $\frac{e^{-x}}{1+x} + C$ (b) $-\frac{e^{-x}}{1+x} + C$
 (c) $\frac{e^x}{1+x} + C$ (d) $-\frac{e^x}{1+x} + C$
43. $\int e^x [\tan x - \log(\cos x)] dx =$ [MP PET 1991]
 (a) $e^x \log(\sec x) + C$ (b) $e^x \log(\operatorname{cosec} x) + C$
 (c) $e^x \log(\cos x) + C$ (d) $e^x \log(\sin x) + C$
44. $\int e^x \sin x (\sin x + 2 \cos x) dx =$ [MP PET 1988]
 (a) $e^x \sin^2 x + C$ (b) $e^x \sin x + C$
- (c) $e^x \sin 2x + C$ (d) इनमें से कोई नहीं
45. $\int e^{2x} \frac{1 + \sin 2x}{1 + \cos 2x} dx =$
 (a) $e^{2x} \tan x + C$ (b) $e^{2x} \cot x + C$
 (c) $\frac{e^{2x} \tan x}{2} + C$ (d) $\frac{e^{2x} \cot x}{2} + C$
46. $\int \frac{e^x (x-1)}{x^2} dx =$
 (a) $\frac{1}{x} e^x + C$ (b) $xe^{-x} + C$
 (c) $\frac{1}{x^2} e^x + C$ (d) $\left(x - \frac{1}{x}\right) e^x + C$
47. $\int e^x \left[\frac{1+x \log x}{x} \right] dx =$
 (a) $e^x + \log x + C$ (b) $\frac{e^x}{\log x} + C$
 (c) $e^x - \log x + C$ (d) $e^x \log x + C$
48. $\int e^x \left[\sin^{-1} \frac{x}{a} + \frac{1}{\sqrt{a^2 - x^2}} \right] dx =$
 (a) $\frac{1}{a} e^x \sin^{-1} \frac{x}{a} + C$ (b) $ae^x \sin^{-1} \frac{x}{a} + C$
 (c) $e^x \sin^{-1} \frac{x}{a} + C$ (d) $\frac{e^x}{\sqrt{a^2 - x^2}} + C$
49. $\int e^x \frac{(x^2+1)}{(x+1)^2} dx =$
 (a) $\left(\frac{x-1}{x+1}\right) e^x + C$ (b) $e^x \left(\frac{x+1}{x-1}\right) + C$
 (c) $e^x (x+1)(x-1) + C$ (d) इनमें से कोई नहीं
50. $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx =$ [AISSE 1983; MP PET 1994, 96]
 (a) $-\frac{e^x}{x^2} + C$ (b) $\frac{e^x}{x^2} + C$
 (c) $\frac{e^x}{x} + C$ (d) $-\frac{e^x}{x} + C$
51. $\int e^x (1 + \tan x + \tan^2 x) dx =$ [Karnataka CET 1999]
 (a) $e^x \sin x + C$ (b) $e^x \cos x + C$
 (c) $e^x \tan x + C$ (d) $e^x \sec x + C$
52. $\int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx =$ [RPET 2000]
 (a) $-e^x \tan(x/2)$ (b) $-e^x \cot(x/2)$
 (c) $-\frac{1}{2} e^x \tan\left(\frac{x}{2}\right)$ (d) $\frac{1}{2} e^x \cot\left(\frac{x}{2}\right)$
53. $\int \frac{(x+3)e^x}{(x+4)^2} dx =$ [Karnataka CET 2000]

- (a) $\frac{1}{(x+4)^2} + c$ (b) $\frac{e^x}{(x+4)^2} + c$
- (c) $\frac{e^x}{x+4} + c$ (d) $\frac{e^x}{x+3} + c$
54. $\int \left(\frac{x+2}{x+4} \right)^2 e^x dx =$ [AMU 2000]
- (a) $e^x \left(\frac{x}{x+4} \right) + c$ (b) $e^x \left(\frac{x+2}{x+4} \right) + c$
- (c) $e^x \left(\frac{x-2}{x+4} \right) + c$ (d) $\left(\frac{2xe^x}{x+4} \right) + c$
55. $\int e^x [f(x) + f'(x)] dx =$ [DCE 2002]
- (a) $e^x f(x)$ (b) e^x
- (c) $e^x f'(x)$ (d) इनमें से कोई नहीं
56. $\int e^x (1 - \cot x + \cot^2 x) dx =$ [MP PET 2002]
- (a) $e^x \cot x + c$ (b) $e^x \operatorname{cosec} x + c$
- (c) $-e^x \cot x + c$ (d) $-e^x \operatorname{cosec} x + c$
57. $\int \sin^{-1} x dx =$ [MP PET 2004]
- (a) $\frac{1}{\sqrt{1-x^2}} + c$ (b) $x \sin^{-1} x - \sqrt{1-x^2} + c$
- (c) $\cos^{-1} x + c$ (d) $x \sin^{-1} x + \sqrt{1-x^2} + c$
58. $\int \frac{x - \sin x}{1 - \cos x} dx =$ [AISSE 1989]
- (a) $x \cot \frac{x}{2} + c$ (b) $-x \cot \frac{x}{2} + c$
- (c) $\cot \frac{x}{2} + c$ (d) इनमें से कोई नहीं
59. $\int \frac{x + \sin x}{1 + \cos x} dx =$ [Roorkee 1980; UPSEAT 1999]
- (a) $-x \tan \frac{x}{2} + c$ (b) $x \tan \frac{x}{2} + c$
- (c) $x \tan x + c$ (d) $\frac{1}{2} x \tan x + c$
60. यदि $\int \frac{e^x (1 + \sin x) dx}{1 + \cos x} = e^x f(x) + c$, तब $f(x) =$ [RPET 1997; Karnataka CET 2003, 05; Orissa JEE 2004]
- (a) $\sin \frac{x}{2}$ (b) $\cos \frac{x}{2}$
- (c) $\tan \frac{x}{2}$ (d) $\log \frac{x}{2}$
61. $\int \sqrt{x} e^{\sqrt{x}} dx =$ [Karnataka CET 2004]
- (a) $2\sqrt{x} - e^{\sqrt{x}} - 4\sqrt{x} e^{\sqrt{x}} + c$
- (b) $(2x - 4\sqrt{x} + 4)e^{\sqrt{x}} + c$
- (c) $(2x + 4\sqrt{x} + 4)e^{\sqrt{x}} + c$
- (d) $(1 - 4\sqrt{x})e^{\sqrt{x}} + c$
62. $\int 32x^3 (\log x)^2 dx =$ [MP PET 2004]
- (a) $x^4 \{8(\log x)^2 - 4(\log x) + 1\} + c$
- (b) $x^3 \{(\log x)^2 + 2 \log x\} + c$
- (c) $x^4 \{8(\log x)^2 - 4 \log x\} + c$
- (d) $8x^4 (\log x)^2 + c$
63. $\int \sin^{-1}(3x - 4x^3) dx =$ [AISSE 1986; DSSE 1984]
- (a) $x \sin^{-1} x + \sqrt{1-x^2} + c$ (b) $x \sin^{-1} x - \sqrt{1-x^2} + c$
- (c) $2[x \sin^{-1} x + \sqrt{1-x^2}] + c$ (d) $3[x \sin^{-1} x + \sqrt{1-x^2}] + c$
64. $\int \cos \sqrt{x} dx =$ [BIT Ranchi 1990; IIT 1977; RPET 1999]
- (a) $2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$
- (b) $2[\sqrt{x} \sin \sqrt{x} - \cos \sqrt{x}] + c$
- (c) $2[\cos \sqrt{x} - \sqrt{x} \sin \sqrt{x}] + c$
- (d) $-2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$
65. $\int \tan^{-1} \frac{2x}{1-x^2} dx =$ [MP PET 1991]
- (a) $x \tan^{-1} x + c$
- (b) $x \tan^{-1} x - \log(1+x^2) + c$
- (c) $2x \tan^{-1} x + \log(1+x^2) + c$
- (d) $2x \tan^{-1} x - \log(1+x^2) + c$
66. $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx =$ [MNR 1978; EAMCET 1982; IIT 1984]
- (a) $x - \sqrt{1-x^2} \sin^{-1} x + c$ (b) $x + \sqrt{1-x^2} \sin^{-1} x + c$
- (c) $\sqrt{1-x^2} \sin^{-1} x - x + c$ (d) इनमें से कोई नहीं
67. $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$ [AISSE 1983, 87]
- (a) $\frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \frac{1}{2} \log(1-x^2) + c$
- (b) $\frac{x}{\sqrt{1-x^2}} \sin^{-1} x - \frac{1}{2} \log(1-x^2) + c$
- (c) $\frac{1}{\sqrt{1-x^2}} \sin^{-1} x - \frac{1}{2} \log(1-x^2) + c$
- (d) $\frac{1}{\sqrt{1-x^2}} \sin^{-1} x + \frac{1}{2} \log(1-x^2) + c$
68. $\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx =$
- (a) $\frac{x + \tan^{-1} x}{\sqrt{1+x^2}} + c$ (b) $\frac{x - \tan^{-1} x}{\sqrt{1+x^2}} + c$
- (c) $\frac{\tan^{-1} x - x}{\sqrt{1+x^2}} + c$ (d) इनमें से कोई नहीं

69. $\int x^5 \cdot e^{x^2} dx =$
- (a) $\frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} + c$
 (b) $\frac{1}{2} x^4 e^{x^2} + x^2 e^{x^2} + e^{x^2} + c$
 (c) $\frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} - e^{x^2} + c$
 (d) इनमें से कोई नहीं
70. $\int e^{\tan^{-1} x} \left(\frac{1+x+x^2}{1+x^2} \right) dx$ का मान है
- (a) $x e^{\tan^{-1} x} + c$ (b) $x^2 e^{\tan^{-1} x} + c$
 (c) $\frac{1}{x} e^{\tan^{-1} x} + c$ (d) इनमें से कोई नहीं
71. $\int e^{\sqrt{x}} dx$ का मान है [MP PET 1998]
- (a) $e^{\sqrt{x}} + A$ (b) $\frac{1}{2} e^{\sqrt{x}} + A$
 (c) $2(\sqrt{x}-1)e^{\sqrt{x}} + A$ (d) $2(\sqrt{x}+1)e^{\sqrt{x}} + A$
 (जहाँ A कोई स्वेच्छ अचर है)
72. $I_1 = \int \sin^{-1} x dx$ और $I_2 = \int \sin^{-1} \sqrt{1-x^2} dx$, तब [Kerala (Engg.) 2005]
- (a) $I_1 = I_2$ (b) $I_2 = \pi / 2 I_1$
 (c) $I_1 + I_2 = \pi / 2x$ (d) $I_1 + I_2 = \pi / 2$
 (e) $I_1 - I_2 = \pi / 2x$
73. यदि $f(x)$ का प्रतिअवकलज e^x और $g(x)$ का प्रतिअवकलज $\cos x$ है तब $\int f(x) \cos x dx + \int g(x) e^x dx =$ [Kerala (Engg.) 2005]
- (a) $f(x)g(x)+c$ (b) $f(x)+g(x)+c$
 (c) $e^x \cos x + c$ (d) $f(x) - g(x)+c$
 (e) $e^x \cos x + f(x)g(x)+c$
- आंशिक भिन्नों द्वारा परिमेय फलनों के समाकलन, समाकलन के विभिन्न रूपों का मान ज्ञात करना**
1. $\int \frac{dx}{(x-x^2)} =$ [Roorkee 1982]
- (a) $\log x - \log(1-x) + c$ (b) $\log(1-x^2) + c$
 (c) $-\log x + \log(1-x) + c$ (d) $\log(x-x^2) + c$
2. $\int \frac{dx}{1+x+x^2+x^3} =$ [MP PET 1991]
- (a) $\log \sqrt{1+x} - \frac{1}{2} \log \sqrt{1+x^2} + \frac{1}{2} \tan^{-1} x + c$
 (b) $\log \sqrt{1+x} - \log \sqrt{1+x^2} + \tan^{-1} x + c$
 (c) $\log \sqrt{1+x^2} - \log \sqrt{1+x} + \frac{1}{2} \tan^{-1} x + c$
 (d) $\log \sqrt{1+x} + \tan^{-1} x + \log \sqrt{1+x^2} + c$
3. $\int \frac{x-1}{(x-3)(x-2)} dx =$ [Roorkee 1978]
- (a) $\log(x-3) - \log(x-2) + c$
 (b) $\log(x-3)^2 - \log(x-2) + c$
 (c) $\log(x-3) + \log(x-2) + c$
 (d) $\log(x-3)^2 + \log(x-2) + c$
4. $\int \frac{1}{\cos x(1+\cos x)} dx =$
- (a) $\log(\sec x + \tan x) + 2 \tan \frac{x}{2} + c$
 (b) $\log(\sec x + \tan x) - 2 \tan \frac{x}{2} + c$
 (c) $\log(\sec x + \tan x) + \tan \frac{x}{2} + c$
 (d) $\log(\sec x + \tan x) - \tan \frac{x}{2} + c$
5. $\int \frac{dx}{(x+1)(x+2)} =$ [MP PET 1987]
- (a) $\log \frac{x+2}{x+1} + c$ (b) $\log(x+1) + \log(x+2) + c$
 (c) $\log \frac{x+1}{x+2} + c$ (d) इनमें से कोई नहीं
6. $\int \frac{x}{(x-2)(x-1)} dx$ का सही मान है [MP PET 1993]
- (a) $\log_e \frac{(x-2)^2}{(x-1)} + p$ (b) $\log_e \frac{(x-1)}{(x-2)} + p$
 (c) $\frac{x-1}{x-2} + p$ (d) $2 \log_e \left(\frac{x-2}{x-1} \right) + p$
 (जहाँ p एक स्वेच्छ अचर है)
7. $\int \frac{1}{(x-1)(x^2+1)} dx =$ [Roorkee 1984]
- (a) $\frac{1}{2} \log(x-1) - \frac{1}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + c$
 (b) $\frac{1}{2} \log(x-1) + \frac{1}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + c$
 (c) $\frac{1}{2} \log(x-1) - \frac{1}{2} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + c$
 (d) इनमें से कोई नहीं
8. $\int \frac{x^2+x-1}{x^2+x-6} dx =$ [AISSSE 1988]
- (a) $x + \log(x+3) + \log(x-2) + c$
 (b) $x - \log(x+3) + \log(x-2) + c$
 (c) $x - \log(x+3) - \log(x-2) + c$
 (d) इनमें से कोई नहीं
9. $\int \frac{x^2}{(x^2+2)(x^2+3)} dx =$ [AISSSE 1990]

- (a) $-\sqrt{2} \tan^{-1} x + \sqrt{3} \tan^{-1} x + c$
 (b) $-\sqrt{2} \tan^{-1} \frac{x}{\sqrt{2}} + \sqrt{3} \tan^{-1} \frac{x}{\sqrt{3}} + c$
 (c) $\sqrt{2} \tan^{-1} \frac{x}{\sqrt{2}} + \sqrt{3} \tan^{-1} \frac{x}{\sqrt{3}} + c$
 (d) इनमें से कोई नहीं

10. $\int \frac{dx}{(x^2+1)(x^2+4)} =$ [MP PET 1995]

- (a) $\frac{1}{3} \tan^{-1} x - \frac{1}{3} \tan^{-1} \frac{x}{2} + c$
 (b) $\frac{1}{3} \tan^{-1} x + \frac{1}{3} \tan^{-1} \frac{x}{2} + c$
 (c) $\frac{1}{3} \tan^{-1} x - \frac{1}{6} \tan^{-1} \frac{x}{2} + c$
 (d) $\tan^{-1} x - 2 \tan^{-1} \frac{x}{2} + c$

11. $\int \frac{1}{x-x^3} dx =$ [MP PET 1996]

- (a) $\frac{1}{2} \log \frac{(1-x^2)}{x^2} + c$ (b) $\log \frac{(1-x)}{x(1+x)} + c$
 (c) $\log x(1-x^2) + c$ (d) $\frac{1}{2} \log \frac{x^2}{(1-x^2)} + c$

12. यदि $\int \sin 5x \cos 3x dx = -\frac{\cos 8x}{16} + A$, तब $A =$ [MP PET 1992]

- (a) $\frac{\sin 2x}{16} + \text{अचर}$ (b) $-\frac{\cos 2x}{4} + \text{अचर}$
 (c) अचर (d) इनमें से कोई नहीं

13. $\int \sin^3 x \cos^2 x dx =$

- (a) $\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + c$ (b) $\frac{\cos^5 x}{5} + \frac{\cos^3 x}{3} + c$
 (c) $\frac{\sin^5 x}{5} - \frac{\sin^3 x}{3} + c$ (d) $\frac{\sin^5 x}{5} + \frac{\sin^3 x}{3} + c$

14. $\int \sin 2x \cos 3x dx =$ [Roorkee 1976]

- (a) $\frac{1}{2} \left(\cos x + \frac{1}{5} \cos 5x \right) + c$ (b) $\frac{1}{2} \left(\cos x - \frac{1}{5} \cos 5x \right) + c$
 (c) $\cos x + \frac{1}{5} \cos 5x + c$ (d) $\cos x - \frac{1}{5} \cos 5x + c$

15. $\int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx =$ [Roorkee 1979]

- (a) $\log[(1+\sin x)(2+\sin x)] + c$
 (b) $\log \frac{2+\sin x}{1+\sin x} + c$
 (c) $\log \frac{1+\sin x}{2+\sin x} + c$
 (d) इनमें से कोई नहीं

16. $\int \frac{e^x}{(1+e^x)(2+e^x)} dx =$

(a) $\log[(1+e^x)(2+e^x)] + c$ (b) $\log \left[\frac{1+e^x}{2+e^x} \right] + c$

(c) $\log[(1+e^x)\sqrt{2+e^x}] + c$ (d) इनमें से कोई नहीं

17. $\int \frac{dx}{e^x + 1 - 2e^{-x}} =$

- (a) $\log(e^x - 1) - \log(e^x + 2) + c$
 (b) $\frac{1}{2} \log(e^x - 1) - \frac{1}{3} \log(e^x + 2) + c$
 (c) $\frac{1}{3} \log(e^x - 1) - \frac{1}{3} \log(e^x + 2) + c$
 (d) $\frac{1}{3} \log(e^x - 1) + \frac{1}{3} \log(e^x + 2) + c$

18. $\int \frac{x}{x^4 - 1} dx =$

- (a) $\frac{1}{4} \log \left[\frac{x^2 - 1}{x^2 + 1} \right] + c$ (b) $\frac{1}{4} \log \left[\frac{x^2 + 1}{x^2 - 1} \right] + c$
 (c) $\frac{1}{2} \log \left[\frac{x^2 - 1}{x^2 + 1} \right] + c$ (d) $\frac{1}{2} \log \left[\frac{x^2 + 1}{x^2 - 1} \right] + c$

19. $\int \sin^5 x \cos^4 x dx =$

- (a) $-\frac{1}{5} \cos^5 x + \frac{2}{7} \cos^7 x - \frac{1}{9} \cos^9 x + c$
 (b) $\frac{1}{5} \cos^5 x + \frac{2}{7} \cos^7 x - \frac{1}{9} \cos^9 x + c$
 (c) $\frac{1}{5} \cos^5 x + \frac{2}{7} \cos^7 x + \frac{1}{9} \cos^9 x + c$
 (d) इनमें से कोई नहीं

20. $\int \sqrt{x^2 - 8x + 7} dx =$

- (a) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} + 9 \log[x-4+\sqrt{x^2-8x+7}] + c$
 (b) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - 3\sqrt{2} \log[x-4+\sqrt{x^2-8x+7}] + c$
 (c) $\frac{1}{2}(x-4)\sqrt{x^2-8x+7} - \frac{9}{2} \log[x-4+\sqrt{x^2-8x+7}] + c$
 (d) इनमें से कोई नहीं

21. $\int \frac{dx}{\sqrt{2x-x^2}} =$ [MP PET 1991; Karnataka CET 2002]

- (a) $\cos^{-1}(x-1) + c$ (b) $\sin^{-1}(x-1) + c$
 (c) $\cos^{-1}(1+x) + c$ (d) $\sin^{-1}(1-x) + c$

22. $\int \frac{x dx}{(x^2-a^2)(x^2-b^2)} =$ [Roorkee 1976]

- (a) $\frac{1}{a^2-b^2} \log \left(\frac{x^2-a^2}{x^2-b^2} \right) + c$
 (b) $\frac{1}{a^2-b^2} \log \left(\frac{x^2-b^2}{x^2-a^2} \right) + c$
 (c) $\frac{1}{2(a^2-b^2)} \log \left(\frac{x^2-a^2}{x^2-b^2} \right) + c$

$$(d) \frac{1}{2(a^2 - b^2)} \log \left(\frac{x^2 - b^2}{x^2 - a^2} \right) + c$$

$$23. \int \frac{dx}{5 + 4 \cos x} = \quad \text{[Roorkee 1983; RPET 1997]}$$

$$(a) \frac{2}{3} \tan^{-1} \left(\frac{1}{3} \tan x \right) + c \quad (b) \frac{1}{3} \tan^{-1} \left(\frac{1}{3} \tan x \right) + c$$

$$(c) \frac{2}{3} \tan^{-1} \left(\frac{1}{3} \tan \frac{x}{2} \right) + c \quad (d) \frac{1}{3} \tan^{-1} \left(\frac{1}{3} \tan \frac{x}{2} \right) + c$$

$$24. \int \frac{1}{(x^2 + a^2)(x^2 + b^2)} dx =$$

$$(a) \frac{1}{(a^2 - b^2)} \left[\frac{1}{b} \tan^{-1} \left(\frac{x}{b} \right) - \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) \right] + c$$

$$(b) \frac{1}{(b^2 - a^2)} \left[\frac{1}{b} \tan^{-1} \left(\frac{x}{b} \right) - \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) \right] + c$$

$$(c) \frac{1}{b} \tan^{-1} \left(\frac{x}{b} \right) - \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$$

$$(d) \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) - \frac{1}{b} \tan^{-1} \left(\frac{x}{b} \right) + c$$

$$25. \int \frac{1}{1 + \cos^2 x} dx =$$

$$(a) \frac{1}{\sqrt{2}} \tan^{-1}(\tan x) + c \quad (b) \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{1}{2} \tan x \right) + c$$

$$(c) \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{1}{\sqrt{2}} \tan x \right) + c \quad (d) \text{ इनमें से कोई नहीं }$$

$$26. \int \frac{dx}{1 + 3 \sin^2 x} = \quad \text{[Roorkee 1989; DCE 2001]}$$

$$(a) \frac{1}{3} \tan^{-1}(3 \tan^2 x) + c \quad (b) \frac{1}{2} \tan^{-1}(2 \tan x) + c$$

$$(c) \tan^{-1}(\tan x) + c \quad (d) \text{ इनमें से कोई नहीं }$$

$$27. \int \frac{dx}{2x^2 + x + 1} = \quad \text{[RPET 1997]}$$

$$(a) \frac{1}{\sqrt{7}} \tan^{-1} \left(\frac{4x+1}{\sqrt{7}} \right) + c \quad (b) \frac{1}{2\sqrt{7}} \tan^{-1} \left(\frac{4x+1}{\sqrt{7}} \right) + c$$

$$(c) \frac{1}{2} \tan^{-1} \left(\frac{4x+1}{\sqrt{7}} \right) + c \quad (d) \text{ इनमें से कोई नहीं }$$

$$28. \int \frac{dx}{7 + 5 \cos x} = \quad \text{[EAMCET 2002]}$$

$$(a) \frac{1}{\sqrt{6}} \tan^{-1} \left(\frac{1}{\sqrt{6}} \tan \frac{x}{2} \right) + c \quad (b) \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$$

$$(c) \frac{1}{4} \tan^{-1} \left(\tan \frac{x}{2} \right) + c \quad (d) \frac{1}{7} \tan^{-1} \left(\tan \frac{x}{2} \right) + c$$

$$29. \int \frac{dx}{x^2 + 4x + 13} = \quad \text{[Kerala (Engg.) 2002]}$$

$$(a) \log(x^2 + 4x + 13) + c \quad (b) \frac{1}{3} \tan^{-1} \left(\frac{x+2}{3} \right) + c$$

$$(c) \log(2x + 4) + c \quad (d) \frac{2x + 4}{(x^2 + 4x + 13)^2} + c$$

$$30. \int \frac{dx}{\cos x - \sin x} = \quad \text{[AIEEE 2004]}$$

$$(a) \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + c$$

$$(b) \frac{1}{\sqrt{2}} \log \left| \cot \left(\frac{x}{2} \right) \right| + c$$

$$(c) \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{3\pi}{8} \right) \right| + c$$

$$(d) \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} - \frac{\pi}{8} \right) \right| + c$$

$$31. \int \frac{dx}{x^2 + 2x + 2} = \quad \text{[Karnataka CET 2004]}$$

$$(a) \sin^{-1}(x + 1) + c \quad (b) \sinh^{-1}(x + 1) + c$$

$$(c) \tanh^{-1}(x + 1) + c \quad (d) \tan^{-1}(x + 1) + c$$

$$32. \int \frac{3 \sin x + 2 \cos x}{3 \cos x + 2 \sin x} dx =$$

$$(a) \frac{12}{13} x - \frac{5}{13} \log(3 \cos x + 2 \sin x)$$

$$(b) \frac{12}{13} x + \frac{5}{13} \log(3 \cos x + 2 \sin x)$$

$$(c) \frac{13}{12} x + \frac{5}{13} \log(3 \cos x + 2 \sin x)$$

$$(d) \text{ इनमें से कोई नहीं }$$

$$33. \int \frac{dx}{x[(\log x)^2 + 4 \log x - 1]} =$$

$$(a) \frac{1}{2\sqrt{5}} \log \left[\frac{\log x + 2 - \sqrt{5}}{\log x + 2 + \sqrt{5}} \right] + c$$

$$(b) \frac{1}{\sqrt{5}} \log \left[\frac{\log x + 2 - \sqrt{5}}{\log x + 2 + \sqrt{5}} \right] + c$$

$$(c) \frac{1}{2\sqrt{5}} \log \left[\frac{\log x + 2 + \sqrt{5}}{\log x + 2 - \sqrt{5}} \right] + c$$

$$(d) \frac{1}{\sqrt{5}} \log \left[\frac{\log x + 2 + \sqrt{5}}{\log x + 2 - \sqrt{5}} \right] + c$$

$$34. \int \frac{dx}{x(x^n + 1)} = \quad \text{[Roorkee 1979]}$$

$$(a) n \log \frac{x^n}{x^n + 1} + c \quad (b) n \log \frac{x^n + 1}{x^n} + c$$

$$(c) \frac{1}{n} \log \frac{x^n}{x^n + 1} + c \quad (d) \frac{1}{n} \log \frac{x^n + 1}{x^n} + c$$

$$35. \int \frac{dx}{x(x^7 + 1)} = \quad \text{[Karnataka CET 2004]}$$

$$(a) \log \left(\frac{x^7}{x^7 + 1} \right) + c \quad (b) \frac{1}{7} \log \left(\frac{x^7}{x^7 + 1} \right) + c$$

$$(c) \log \left(\frac{x^7 + 1}{x^7} \right) + c \quad (d) \frac{1}{7} \log \left(\frac{x^7 + 1}{x^7} \right) + c$$

36. $\int \frac{dx}{x(x^5 + 1)} =$ [UPSEAT 2004]

(a) $\frac{1}{5} \log x^5(x^5 + 1) + c$ (b) $\frac{1}{5} \log x^5 \left(\frac{1 + x^5}{x^5} \right) + c$

(c) $\frac{1}{5} \log x^5 \left(\frac{x^5}{x^5 + 1} \right) + c$ (d) इनमें से कोई नहीं

37. $\int \frac{x^2 + 1}{x^4 + 1} dx =$ [AISCSE 1990]

(a) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{2x} \right) + c$ (b) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + c$

(c) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{2\sqrt{x}} \right) + c$ (d) $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + c$

38. $\int \frac{x^2 - 1}{x^4 + x^2 + 1} dx =$ [AISCSE 1990]

(a) $\frac{1}{2} \log \left(\frac{x^2 + x + 1}{x^2 - x + 1} \right) + c$ (b) $\frac{1}{2} \log \left(\frac{x^2 - x - 1}{x^2 + x + 1} \right) + c$

(c) $\log \left(\frac{x^2 - x + 1}{x^2 + x + 1} \right) + c$ (d) $\frac{1}{2} \log \left(\frac{x^2 - x + 1}{x^2 + x + 1} \right) + c$

39. $\int \frac{x dx}{x^2 + 4x + 5} =$ [RPET 2002]

(a) $\frac{1}{2} \log(x^2 + 4x + 5) + 2 \tan^{-1}(x) + c$

(b) $\frac{1}{2} \log(x^2 + 4x + 5) - \tan^{-1}(x + 2) + c$

(c) $\frac{1}{2} \log(x^2 + 4x + 5) + \tan^{-1}(x + 2) + c$

(d) $\frac{1}{2} \log(x^2 + 4x + 5) - 2 \tan^{-1}(x + 2) + c$

40. यदि $\int \frac{2x^2 + 3dx}{(x^2 - 1)(x^2 - 4)} = \log \left(\frac{x - 2}{x + 2} \right)^a \left(\frac{x + 1}{x - 1} \right)^b + c$, तब a व b

के मान क्रमशः हैं [AMU 2005]

(a) $\frac{11}{12}, \frac{5}{6}$ (b) $\frac{11}{12}, -\frac{5}{6}$

(c) $-\frac{11}{12}, \frac{5}{6}$ (d) इनमें से कोई नहीं

(d) $\operatorname{cosec}(a - b) \log \frac{\cos(x - b)}{\cos(x - a)} + c$

2. $\int \frac{dx}{\sqrt{x + a} + \sqrt{x + b}} =$ [AISCSE 1989]

(a) $\frac{2}{3(b - a)} [(x + a)^{3/2} - (x + b)^{3/2}] + c$

(b) $\frac{2}{3(a - b)} [(x + a)^{3/2} - (x + b)^{3/2}] + c$

(c) $\frac{2}{3(a - b)} [(x + a)^{3/2} + (x + b)^{3/2}] + c$

(d) इनमें से कोई नहीं

3. $\int \frac{3 \cos x + 3 \sin x}{4 \sin x + 5 \cos x} dx =$ [EAMCET 1991]

(a) $\frac{27}{41} x - \frac{3}{41} \log(4 \sin x + 5 \cos x)$

(b) $\frac{27}{41} x + \frac{3}{41} \log(4 \sin x + 5 \cos x)$

(c) $\frac{27}{41} x - \frac{3}{41} \log(4 \sin x - 5 \cos x)$

(d) इनमें से कोई नहीं

4. यदि $\int (\sin 2x + \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x - c) + a$, तब a व c के

मान हैं [Roorkee 1978]

(a) $c = \pi/4$ और $a = k$ (एक स्वेच्छ अचर)

(b) $c = -\pi/4$ और $a = \pi/2$

(c) $c = \pi/2$ और a एक स्वेच्छ अचर है

(d) इनमें से कोई नहीं

5. $\int \frac{x^3 - x - 2}{(1 - x^2)} dx =$ [AI CBSE 1985]

(a) $\log \left(\frac{x + 1}{x - 1} \right) - \frac{x^2}{2} + c$ (b) $\log \left(\frac{x - 1}{x + 1} \right) + \frac{x^2}{2} + c$

(c) $\log \left(\frac{x + 1}{x - 1} \right) + \frac{x^2}{2} + c$ (d) $\log \left(\frac{x - 1}{x + 1} \right) - \frac{x^2}{2} + c$

6. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2 \sin^2 x \cos^2 x} dx =$ [IIT 1986]

(a) $\sin 2x + c$ (b) $-\frac{1}{2} \sin 2x + c$

(c) $\frac{1}{2} \sin 2x + c$ (d) $-\sin 2x + c$

7. $\int \frac{x^2 dx}{(a + bx)^2} =$ [IIT 1979]

(a) $\frac{1}{b^2} \left[x + \frac{2a}{b} \log(a + bx) - \frac{a^2}{b} \frac{1}{a + bx} \right]$

(b) $\frac{1}{b^2} \left[x - \frac{2a}{b} \log(a + bx) + \frac{a^2}{b} \frac{1}{a + bx} \right]$

(c) $\frac{1}{b^2} \left[x + \frac{2a}{b} \log(a + bx) + \frac{a^2}{b} \frac{1}{a + bx} \right]$

(d) $\frac{1}{b^2} \left[x + \frac{a}{b} - \frac{2a}{b} \log(a + bx) - \frac{a^2}{b} \frac{1}{a + bx} \right]$

Critical Thinking

Objective Questions

1. $\int \frac{dx}{\cos(x - a) \cos(x - b)} =$

(a) $\operatorname{cosec}(a - b) \log \frac{\sin(x - a)}{\sin(x - b)} + c$

(b) $\operatorname{cosec}(a - b) \log \frac{\cos(x - a)}{\cos(x - b)} + c$

(c) $\operatorname{cosec}(a - b) \log \frac{\sin(x - b)}{\sin(x - a)} + c$

8. $\int \frac{dx}{(1+x^2)\sqrt{p^2+q^2(\tan^{-1}x)^2}} =$
- (a) $\frac{1}{q} \log[q \tan^{-1}x + \sqrt{p^2+q^2(\tan^{-1}x)^2}] + c$
 (b) $\log[q \tan^{-1}x + \sqrt{p^2+q^2(\tan^{-1}x)^2}] + c$
 (c) $\frac{2}{3q}(p^2+q^2 \tan^{-1}x)^{3/2} + c$
 (d) इनमें से कोई नहीं
9. $\int \frac{x^5}{\sqrt{1+x^3}} dx =$ [IIT 1985]
- (a) $\frac{2}{9}(1+x^3)^{3/2} + c$
 (b) $\frac{2}{9}(1+x^3)^{3/2} + \frac{2}{3}(1+x^3)^{1/2} + c$
 (c) $\frac{2}{9}(1+x^3)^{3/2} - \frac{2}{3}(1+x^3)^{1/2} + c$
 (d) इनमें से कोई नहीं
10. $\int \frac{dx}{\sin x - \cos x + \sqrt{2}} =$ [MP PET 2002]
- (a) $-\frac{1}{\sqrt{2}} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ (b) $\frac{1}{\sqrt{2}} \tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$
 (c) $\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ (d) $-\frac{1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$
11. $\int \frac{a dx}{b + ce^x} =$ [MP PET 1988; BIT Ranchi 1979]
- (a) $\frac{a}{b} \log\left(\frac{e^x}{b + ce^x}\right) + c$ (b) $\frac{a}{b} \log\left(\frac{b + ce^x}{e^x}\right) + c$
 (c) $\frac{b}{a} \log\left(\frac{e^x}{b + ce^x}\right) + c$ (d) $\frac{b}{a} \log\left(\frac{b + ce^x}{e^x}\right) + c$
12. $\int \sin \sqrt{x} dx =$ [Roorkee 1977]
- (a) $2[\sin \sqrt{x} - \cos \sqrt{x}] + c$ (b) $2[\sin \sqrt{x} - \sqrt{x} \cos \sqrt{x}] + c$
 (c) $2[\sin \sqrt{x} + \cos \sqrt{x}] + c$ (d) $2[\sin \sqrt{x} + \sqrt{x} \cos \sqrt{x}] + c$
13. $\int \frac{x^2}{(9-x^2)^{3/2}} dx =$
- (a) $\frac{x}{\sqrt{9-x^2}} - \sin^{-1} \frac{x}{3} + c$ (b) $\frac{x}{\sqrt{9-x^2}} + \sin^{-1} \frac{x}{3} + c$
 (c) $\sin^{-1} \frac{x}{3} - \frac{x}{\sqrt{9-x^2}} + c$ (d) इनमें से कोई नहीं
14. $\int x \sqrt{\frac{1-x^2}{1+x^2}} dx =$
- (a) $\frac{1}{2}[\sin^{-1} x^2 + \sqrt{1-x^4}] + c$ (b) $\frac{1}{2}[\sin^{-1} x^2 + \sqrt{1-x^2}] + c$
 (c) $\sin^{-1} x^2 + \sqrt{1-x^4} + c$ (d) इनमें से कोई नहीं
15. यदि $\int f(x) \sin x \cos x dx = \frac{1}{2(b^2-a^2)} \log(f(x)) + c$, तब $f(x) =$
- (a) $\frac{1}{a^2 \sin^2 x + b^2 \cos^2 x}$ (b) $\frac{1}{a^2 \sin^2 x - b^2 \cos^2 x}$
 (c) $\frac{1}{a^2 \cos^2 x + b^2 \sin^2 x}$ (d) $\frac{1}{a^2 \cos^2 x - b^2 \sin^2 x}$
16. $\int \frac{dx}{4 \sin^2 x + 5 \cos^2 x} =$ [AISSE 1986]
- (a) $\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{2 \tan x}{\sqrt{5}}\right) + c$ (b) $\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right) + c$
 (c) $\frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{2 \tan x}{\sqrt{5}}\right) + c$ (d) इनमें से कोई नहीं
17. $\int \frac{x^2+1}{x^4-x^2+1} dx =$ [MP PET 1991]
- (a) $\tan^{-1}\left(\frac{1+x^2}{x}\right) + c$ (b) $\cot^{-1}\left(\frac{1+x^2}{x}\right) + c$
 (c) $\tan^{-1}\left(\frac{x^2-1}{x}\right) + c$ (d) $\cot^{-1}\left(\frac{x^2-1}{x}\right) + c$
18. $\int (\log x)^2 dx =$ [IIT 1971, 77]
- (a) $x(\log x)^2 - 2x \log x - 2x + c$
 (b) $x(\log x)^2 - 2x \log x - x + c$
 (c) $x(\log x)^2 - 2x \log x + 2x + c$
 (d) $x(\log x)^2 - 2x \log x + x + c$
19. $\int \frac{\sqrt{(x^2-a^2)}}{x} dx$ का मान होगा [UPSEAT 1999]
- (a) $\sqrt{(x^2-a^2)} - a \tan^{-1}\left[\frac{\sqrt{(x^2-a^2)}}{a}\right]$
 (b) $\sqrt{(x^2-a^2)} + a \tan^{-1}\left[\frac{\sqrt{(x^2-a^2)}}{a}\right]$
 (c) $\sqrt{(x^2-a^2)} + a^2 \tan^{-1}[\sqrt{x^2-a^2}]$
 (d) $\tan^{-1} x / a + c$
20. $\int \tan^3 2x \sec 2x dx =$ [IIT 1977]
- (a) $\frac{1}{6} \sec^3 2x - \frac{1}{2} \sec 2x + c$ (b) $\frac{1}{6} \sec^3 2x + \frac{1}{2} \sec 2x + c$
 (c) $\frac{1}{9} \sec^2 2x - \frac{1}{3} \sec 2x + c$ (d) इनमें से कोई नहीं
21. $\int x \sin^{-1} x dx =$ [MP PET 1991]
- (a) $\left(\frac{x^2}{2} - \frac{1}{4}\right) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + c$

- (b) $\left(\frac{x^2}{2} + \frac{1}{4}\right) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + c$
- (c) $\left(\frac{x^2}{2} - \frac{1}{4}\right) \sin^{-1} x - \frac{x}{4} \sqrt{1-x^2} + c$
- (d) $\left(\frac{x^2}{2} + \frac{1}{4}\right) \sin^{-1} x - \frac{x}{4} \sqrt{1-x^2} + c$
22. $\int \sqrt{\frac{a-x}{x}} dx =$
- (a) $a \left[\sin^{-1} \sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a}} \sqrt{\frac{a-x}{a}} \right] + c$
- (b) $\sin^{-1} \frac{x}{a} + \frac{x}{a} \sqrt{a^2 - x^2} + c$
- (c) $a \left[\sin^{-1} \frac{x}{a} - \frac{x}{a} \sqrt{a^2 - x^2} \right] + c$
- (d) $\sin^{-1} \frac{x}{a} - \frac{x}{a} \sqrt{a^2 - x^2} + c$
23. यदि $x \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$, तब $\int \frac{\sin x - \cos x}{\sqrt{1 - \sin 2x}} e^{\sin x} \cos x dx =$
- (a) $e^{\sin x} + c$ (b) $e^{\sin x - \cos x} + c$
- (c) $e^{\sin x + \cos x} + c$ (d) $e^{\cos x - \sin x} + c$
24. यदि $\int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}} dx = Ax + B \log(9e^{2x} - 4) + C$, तब A, B और C हैं [IIT 1990]
- (a) $A = \frac{3}{2}, B = \frac{36}{35}, C = \frac{3}{2} \log 3 + \text{अचर}$
- (b) $A = \frac{3}{2}, B = \frac{35}{36}, C = \frac{3}{2} \log 3 + \text{अचर}$
- (c) $A = -\frac{3}{2}, B = -\frac{35}{36}, C = -\frac{3}{2} \log 3 + \text{अचर}$
- (d) इनमें से कोई नहीं
25. $\int \sec^3 x dx$ का मान होगा [UPSEAT 1999]
- (a) $\frac{1}{2} [\sec x \tan x + \log(\sec x + \tan x)]$
- (b) $\frac{1}{3} [\sec x \tan x + \log(\sec x + \tan x)]$
- (c) $\frac{1}{4} [\sec x \tan x + \log(\sec x + \tan x)]$
- (d) $\frac{1}{8} [\sec x \tan x + \log(\sec x + \tan x)]$
26. $\int \frac{x-1}{(x+1)^3} e^x dx =$ [IIT 1983; MP PET 1990]
- (a) $\frac{-e^x}{(x+1)^2} + c$ (b) $\frac{e^x}{(x+1)^2} + c$
- (c) $\frac{e^x}{(x+1)^3} + c$ (d) $\frac{-e^x}{(x+1)^3} + c$
27. यदि $I = \int e^x \sin 2x dx$, तब K के किस मान के लिये $KI = e^x (\sin 2x - 2 \cos 2x) + \text{अचर होगा}$ [MP PET 1992]
- (a) 1 (b) 3
- (c) 5 (d) 7
28. $\int \frac{dx}{3-2x-x^2}$ का मान होगा [UPSEAT 1999]
- (a) $\frac{1}{4} \log \left(\frac{3+x}{1-x} \right)$ (b) $\frac{1}{3} \log \left(\frac{3+x}{1-x} \right)$
- (c) $\frac{1}{2} \log \left(\frac{3+x}{1-x} \right)$ (d) $\log \left(\frac{1-x}{3+x} \right)$
29. $\int x \sqrt{2x+3} dx =$ [AISSSE 1985]
- (a) $\frac{x}{3} (2x+3)^{3/2} - \frac{1}{15} (2x+3)^{5/2} + c$
- (b) $\frac{x}{3} (2x+3)^{3/2} + \frac{1}{15} (2x+3)^{5/2} + c$
- (c) $\frac{x}{2} (2x+3)^{3/2} + \frac{1}{6} (2x+3)^{5/2} + c$
- (d) इनमें से कोई नहीं
30. $\int \cos 2\theta \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta =$ [IIT 1994]
- (a) $(\cos \theta - \sin \theta)^2 \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right)$
- (b) $(\cos \theta + \sin \theta)^2 \log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right)$
- (c) $\frac{(\cos \theta - \sin \theta)^2}{2} \log \left(\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right)$
- (d) $\frac{1}{2} \sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta \right) - \frac{1}{2} \log \sec 2\theta$
31. $\int \frac{x^2}{(x \sin x + \cos x)^2} dx =$ [MNR 1989; RPET 2000]
- (a) $\frac{\sin x + \cos x}{x \sin x + \cos x}$ (b) $\frac{x \sin x - \cos x}{x \sin x + \cos x}$
- (c) $\frac{\sin x - x \cos x}{x \sin x + \cos x}$ (d) इनमें से कोई नहीं
32. यदि $u = \int e^{ax} \cos bx dx$ और $v = \int e^{ax} \sin bx dx$, तब $(a^2 + b^2)(u^2 + v^2) =$
- (a) $2e^{ax}$ (b) $(a^2 + b^2)e^{2ax}$
- (c) e^{2ax} (d) $(a^2 - b^2)e^{2ax}$
33. यदि $I_n = \int (\log x)^n dx$, तब $I_n + nI_{n-1} =$ [Karnataka CET 2003]
- (a) $x(\log x)^n$ (b) $(x \log x)^n$
- (c) $(\log x)^{n-1}$ (d) $n(\log x)^n$
34. $\int e^{x/2} \sin \left(\frac{x}{2} + \frac{\pi}{4} \right) dx =$ [Roorkee 1982]
- (a) $e^{x/2} \cos \frac{x}{2} + c$ (b) $\sqrt{2} e^{x/2} \cos \frac{x}{2} + c$

(c) $e^{x/2} \sin \frac{x}{2} + c$ (d) $\sqrt{2}e^{x/2} \sin \frac{x}{2} + c$

35. यदि $\int \frac{2x+3}{x^2-5x+6} dx = 9 \ln(x-3) - 7 \ln(x-2) + A$, तब

$A =$ [MP PET 1992]

(a) $5 \ln(x-2) + \text{अचर}$ (b) $-4 \ln(x-3) + \text{अचर}$

(c) अचर (d) इनमें से कोई नहीं

36. $\int \frac{dx}{2 + \cos x} =$

(a) $2 \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$ (b) $\frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$

(c) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{1}{\sqrt{3}} \tan \frac{x}{2} \right) + c$ (d) इनमें से कोई नहीं

37. $\int \frac{x}{x^4 + x^2 + 1} dx =$ [MP PET 2004]

(a) $\frac{1}{3} \tan^{-1} \left(\frac{2x^2+1}{3} \right)$ (b) $\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x^2+1}{\sqrt{3}} \right)$

(c) $\frac{1}{\sqrt{3}} \tan^{-1} (2x^2 + 1)$ (d) इनमें से कोई नहीं

38. $\int \frac{dx}{(\sin x + \sin 2x)} =$ [IIT 1984]

(a) $\frac{1}{6} \log(1 - \cos x) + \frac{1}{2} \log(1 + \cos x) - \frac{2}{3} \log(1 + 2 \cos x)$

(b) $6 \log(1 - \cos x) + 2 \log(1 + \cos x) - \frac{2}{3} \log(1 + 2 \cos x)$

(c) $6 \log(1 - \cos x) + \frac{1}{2} \log(1 + \cos x) + \frac{2}{3} \log(1 + 2 \cos x)$

(d) इनमें से कोई नहीं

39. यदि $\int \frac{2x+3}{(x-1)(x^2+1)} dx = \log_e \left\{ (x-1)^{\frac{5}{2}} (x^2+1)^a \right\} - \frac{1}{2} \tan^{-1} x + A$,

जहाँ A कोई स्वेच्छ अचर है, तब ' a ' का मान है [MP PET 1998]

(a) $5/4$

(b) $-5/3$

(c) $-5/6$

(d) $-5/4$

40. यदि $\int \frac{(2x^2+1) dx}{(x^2-4)(x^2-1)} = \log \left[\left(\frac{x+1}{x-1} \right)^a \left(\frac{x-2}{x+2} \right)^b \right] + C$, तब

a व b के मान क्रमशः हैं

[Roorkee 2000]

(a) $\frac{1}{2}, \frac{3}{4}$

(b) $-1, \frac{3}{2}$

(c) $1, \frac{3}{2}$

(d) $-\frac{1}{2}, \frac{3}{4}$

Answers

मूलभूत समाकलन

1	d	2	c	3	c	4	d	5	c
6	c	7	c	8	a	9	b	10	d
11	d	12	d	13	b	14	d	15	a
16	a	17	b	18	b	19	c	20	b
21	c	22	a,c	23	c	24	a	25	a
26	b	27	c	28	d	29	d	30	c
31	b	32	c	33	d	34	a	35	a
36	b	37	d	38	b	39	d	40	b
41	a	42	d	43	a	44	c	45	d
46	c	47	a	48	a	49	b	50	a
51	d	52	a	53	c	54	c	55	c
56	a	57	c	58	a	59	b	60	d
61	b	62	a	63	a	64	d	65	b
66	b	67	b	68	a	69	c	70	c
71	b	72	a	73	a	74	a	75	a
76	b	77	b	78	b	79	b	80	c
81	c	82	a	83	b	84	a	85	b
86	c	87	c						

प्रतिस्थापन द्वारा समाकलन

1	b	2	b	3	a	4	a	5	b
6	a	7	d	8	c	9	b	10	b
11	c	12	c	13	b	14	a	15	b
16	a	17	c	18	b	19	a	20	c
21	b	22	b	23	d	24	a	25	b
26	c	27	a	28	c	29	b	30	c
31	a	32	b	33	a	34	c	35	c
36	b	37	a	38	b	39	b	40	a
41	c	42	c	43	b	44	b	45	b
46	a	47	a	48	a	49	b	50	b
51	b	52	a	53	a	54	c	55	b
56	b	57	c	58	b	59	b	60	d
61	a	62	b	63	c	64	a	65	c
66	a	67	c	68	b	69	c	70	b
71	b	72	a	73	b	74	a	75	b
76	b	77	c	78	a	79	a	80	a
81	b	82	b	83	c	84	a	85	c
86	a	87	a	88	c	89	b	90	a
91	a	92	c	93	a	94	a	95	d
96	a	97	a	98	a	99	c	100	a
101	a	102	d	103	b	104	d	105	b
106	a	107	a	108	a	109	a	110	a

111	c	112	c	113	b	114	c	115	b
116	c	117	a	118	c	119	d	120	a
121	b	122	c	123	d	124	d	125	b
126	b	127	c	128	b	129	b	130	c
131	d	132	b	133	d	134	b	135	b
136	c	137	d	138	c	139	d	140	b
141	d	142	b	143	a	144	b	145	a
146	a	147	c	148	c				

खण्डशः समाकलन, $\int e^x(F(x) + F'(x)) dx$ तथा
 $\int e^{kx}(kF(x) + F'(x)) dx$ के रूप का समाकलन

1	d	2	c	3	a	4	b	5	a
6	b	7	b	8	a	9	b	10	d
11	b	12	d	13	b	14	d	15	a
16	a	17	d	18	b	19	b	20	d
21	c	22	a	23	d	24	a	25	b
26	c	27	d	28	b	29	a	30	d
31	c	32	d	33	b	34	c	35	a
36	c	37	c	38	b	39	b	40	a
41	c	42	c	43	a	44	a	45	c
46	a	47	d	48	c	49	a	50	c
51	c	52	b	53	c	54	a	55	a
56	c	57	d	58	b	59	b	60	c
61	b	62	a	63	d	64	a	65	d
66	a	67	a	68	b	69	a	70	a
71	c	72	c	73	c				

आंशिक भिन्नों द्वारा परिमेय फलनों के समाकलन, समाकलन के
विभिन्न रूपों का मान ज्ञात करना

1	a	2	a	3	b	4	d	5	c
6	a	7	a	8	b	9	b	10	c
11	d	12	b	13	a	14	b	15	c
16	b	17	c	18	a	19	a	20	c
21	b	22	c	23	c	24	a	25	c
26	b	27	d	28	a	29	b	30	a
31	d	32	a	33	a	34	c	35	b
36	d	37	d	38	d	39	d	40	a

Critical Thinking Questions

1	b	2	b	3	a	4	a	5	d
6	b	7	d	8	a	9	c	10	d
11	a	12	b	13	a	14	a	15	a
16	c	17	c	18	c	19	a	20	a
21	a	22	a	23	a	24	d	25	a
26	b	27	c	28	a	29	a	30	d
31	c	32	c	33	a	34	d	35	c
36	b	37	b	38	a	39	d	40	a

Answers and Solutions

मूलभूत समाकलन

1. (d) $\int \sqrt{1 - \sin 2x} \, dx$
 $= \int (\cos x - \sin x) dx = \sin x + \cos x + c.$
2. (c) $\int \frac{1 + \cos^2 x}{\sin^2 x} dx = \int (\operatorname{cosec}^2 x + \cot^2 x) dx$
 $= \int (2\operatorname{cosec}^2 x - 1) dx = -2 \cot x - x + c.$
3. (c) $\int \sin^{-1}(\cos x) dx = \int \left\{ \frac{\pi}{2} - \cos^{-1}(\cos x) \right\} dx$
 $= \frac{\pi}{2} x - \frac{x^2}{2} = \frac{\pi x - x^2}{2}.$
4. (d) $\int \frac{dx}{\tan x + \cot x} = \int \frac{dx}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$
 $= \frac{1}{2} \int 2 \sin x \cos x \, dx = \frac{-\cos 2x}{4} + c.$
5. (c) $\int (e^{a \log x} + e^{x \log a}) dx = \int (e^{\log_e x^a} + e^{\log_e a^x}) dx$
 $= \int (x^a + a^x) dx = \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c.$
6. (c) दिया गया है, $f'(x) = x^2 + 5$ और $f(0) = -1$
 $\Rightarrow f(x) = \frac{x^3}{3} + 5x + c.$ यदि $x = 0$, तब $f(0) = c \Rightarrow c = -1.$
अतः $f(x) = \frac{x^3}{3} + 5x - 1.$
7. (c) $\int \tan^{-1} \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}} dx = \int \tan^{-1} \tan x \, dx$
 $= \int x \, dx = \frac{x^2}{2} + c.$
8. (a) $\int \frac{dx}{\sqrt{x} + \sqrt{x-2}} = \frac{1}{2} \int \frac{x - (x-2)}{\sqrt{x} + \sqrt{x-2}} dx$
 $= \frac{1}{2} \int (\sqrt{x} - \sqrt{x-2}) dx = \frac{1}{2} \left[\frac{x^{3/2}}{3/2} - \frac{(x-2)^{3/2}}{3/2} \right] + c$
 $= \frac{1}{3} \left\{ x^{3/2} - (x-2)^{3/2} \right\} + c.$
9. (b) $\int \frac{\sin x}{\sin(x-\alpha)} dx = \int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)} dx$
 $= \int \frac{\{\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha\}}{\sin(x-\alpha)} dx$

$$= \int \cos \alpha \, dx + \int \sin \alpha \cdot \cot(x-\alpha) dx$$

$$= x \cos \alpha + \sin \alpha \cdot \log \sin(x-\alpha) + c.$$

10. (d) $\int \frac{\cos x - 1}{\cos x + 1} dx = - \int \tan^2 \frac{x}{2} dx$
 $= - \int \left(\sec^2 \frac{x}{2} - 1 \right) dx = \int \left(1 - \sec^2 \frac{x}{2} \right) dx = x - 2 \tan \frac{x}{2} + c.$
11. (d) $\int \frac{dx}{1 - \sin x} = \int \frac{(1 + \sin x)}{1 - \sin^2 x} dx$
 $= \int \sec^2 x \, dx + \int \tan x \cdot \sec x \, dx = \tan x + \sec x + c.$
12. (d) $\int (\sin 2x - \cos 2x) dx = \frac{1}{\sqrt{2}} \sin(2x - a) + b$
 $\Rightarrow -\frac{1}{2} (\sin 2x + \cos 2x) = \frac{1}{\sqrt{2}} \sin(2x - a) + b$
 $\Rightarrow \left[\frac{1}{\sqrt{2}} \sin 2x + \frac{1}{\sqrt{2}} \cos 2x \right] = \sin(2x - a) + b\sqrt{2}$
 $\Rightarrow \sin \left(2x + \frac{5\pi}{4} \right) = \sin(2x - a) + b\sqrt{2}$
 $\Rightarrow b$ कोई अचर है तथा $a = \frac{-5\pi}{4}.$
13. (b) $\int \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right) dx = \int e^x dx = e^x + c.$
14. (d) $\int \frac{\cot x \tan x}{\sec^2 x - 1} dx = \int \frac{1}{\tan^2 x} dx = \int \cot^2 x \, dx$
 $= \int (\operatorname{cosec}^2 x - 1) dx = -\cot x - x + c.$
15. (a) $\int (\sec x + \tan x)^2 dx$
 $= \int (\sec^2 x + \tan^2 x + 2 \sec x \tan x) dx$
 $= \int (2 \sec^2 x - 1 + 2 \sec x \tan x) dx$
 $= 2 \tan x + 2 \sec x - x + c = 2(\sec x + \tan x) - x + c.$
16. (a) $\int x^{51} (\tan^{-1} x + \cot^{-1} x) dx = \int x^{51} \cdot \frac{\pi}{2} dx,$
 $\left\{ \because \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \right\}$
 $= \frac{\pi x^{52}}{104} + c = \frac{x^{52}}{52} (\tan^{-1} x + \cot^{-1} x) + c.$
17. (b) $\int 5 \sin x \, dx = -5 \cos x + c.$
18. (b) $\int \frac{\tan x}{(\sec x + \tan x)} dx = \int \frac{\tan x (\sec x - \tan x)}{(\sec x + \tan x)(\sec x - \tan x)} dx$
अंश व हर को $(\sec x - \tan x)$ से गुणा करने पर,
 $= \int \frac{\tan x (\sec x - \tan x)}{(\sec^2 x - \tan^2 x)} dx = \int (\sec x \tan x - \tan^2 x) dx$
 $= \int \sec x \tan x \, dx - \int (\sec^2 x - 1) dx$

$$= \int \sec x \tan x \, dx - \int \sec^2 x \, dx + \int 1 \, dx$$

$$= \sec x - \tan x + x + c.$$

ट्रिक : जाँच द्वारा,

$$\frac{d}{dx} \{\sec x + \tan x\} = \sec x \tan x + \sec^2 x$$

$$= \sec x(\sec x + \tan x) = \frac{\sec x}{\sec x - \tan x}$$

$$\Rightarrow \frac{d}{dx} \{\sec x - \tan x + x + c\} = \sec x \tan x - \sec^2 x + 1$$

$$= -\sec x(\sec x - \tan x) + 1 = \frac{-\sec x}{\sec x + \tan x} + 1$$

$$= \frac{\tan x}{\sec x + \tan x}.$$

$$19. \quad (c) \quad \int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{(\cos^2 x + \sin^2 x) dx}{\cos^2 x \sin^2 x}$$

$$= \int \operatorname{cosec}^2 x \, dx + \int \sec^2 x \, dx = -\cot x + \tan x + c.$$

$$20. \quad (b) \quad \int \left(x + \frac{1}{x}\right)^3 dx = \int \left(x^3 + \frac{1}{x^3} + 3x + \frac{3}{x}\right) dx$$

$$= \frac{x^4}{4} - \frac{1}{2x^2} + \frac{3x^2}{2} + 3 \log x + c$$

$$= \frac{x^4}{4} + \frac{3x^2}{2} + 3 \log x - \frac{1}{2x^2} + c.$$

$$21. \quad (c) \quad \int \sqrt{1 + \sin \frac{x}{2}} \, dx = \int \sqrt{\left(\sin^2 \frac{x}{4} + \cos^2 \frac{x}{4} + 2 \sin \frac{x}{4} \cos \frac{x}{4}\right)} dx$$

$$= \int \left(\sin \frac{x}{4} + \cos \frac{x}{4}\right) dx = 4 \left(\sin \frac{x}{4} - \cos \frac{x}{4}\right) + c.$$

$$22. \quad (a, c) \quad \int (\sin^{-1} x + \cos^{-1} x) dx = \int \left(\frac{\pi}{2}\right) dx = \frac{\pi x}{2} + c$$

$$\left(\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right)$$

साथ ही, $\int (\sin^{-1} x + \cos^{-1} x) dx = x(\cos^{-1} x + \sin^{-1} x) + c.$

$$23. \quad (c) \quad \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx = \int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}} dx = \int dx = x + c.$$

$$24. \quad (a) \quad \int \frac{x-1}{(x+1)^2} dx = \int \frac{x+1-2}{(x+1)^2} dx$$

$$= \int \frac{1}{x+1} dx - \int \frac{2}{(x+1)^2} dx = \log(x+1) + \frac{2}{(x+1)} + c.$$

$$25. \quad (a) \quad \int \frac{\operatorname{cosec} \theta - \cot \theta}{\operatorname{cosec} \theta + \cot \theta} d\theta = \int (\operatorname{cosec} \theta - \cot \theta)^2 d\theta$$

$$= \int \operatorname{cosec}^2 \theta d\theta + \int \cot^2 \theta d\theta - 2 \int \operatorname{cosec} \theta \cot \theta d\theta$$

$$= \int (2 \operatorname{cosec}^2 \theta - 1) d\theta - 2 \int \operatorname{cosec} \theta \cot \theta d\theta$$

$$= 2 \operatorname{cosec} \theta - 2 \cot \theta - \theta + c.$$

$$26. \quad (b) \quad \int (1 + 2x + 3x^2 + 4x^3 + \dots) dx$$

$$= \int (1-x)^{-2} dx = (1-x)^{-1} + c.$$

$$27. \quad (c) \quad \text{दिया गया है, } \int (\cos x - \sin x) dx = \sqrt{2} \sin(x + \alpha) + c$$

$$\Rightarrow \sin x + \cos x + c = \sqrt{2} \sin(x + \alpha) + c$$

$$\Rightarrow \sqrt{2} \left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}} \right) + c = \sqrt{2} \sin(x + \alpha) + c$$

$$\Rightarrow \sqrt{2} \sin \left(x + \frac{\pi}{4} \right) + c = \sqrt{2} \sin(x + \alpha) + c$$

$$\Rightarrow \sin \left(x + \frac{\pi}{4} \right) = \sin(x + \alpha) \Rightarrow \alpha = \frac{\pi}{4}.$$

$$28. \quad (d) \quad \int \frac{3x^3 - 2\sqrt{x}}{x} dx = \int 3x^2 dx - 2 \int x^{-1/2} dx = x^3 - 4\sqrt{x} + c.$$

$$29. \quad (d) \quad \int \frac{dx}{1 + \sin x} = \tan x - \sec x + c = -\frac{1 - \sin x}{\cos x}$$

$$= -\frac{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}} + c = -\frac{1 - \tan \frac{x}{2}}{1 + \tan \frac{x}{2}} + c$$

$$= \frac{\tan \frac{x}{2} - 1}{1 + \tan \frac{x}{2}} + c = \tan \left(\frac{x}{2} - \frac{\pi}{4} \right) + c$$

$$\Rightarrow a = -\frac{\pi}{4}, b = \text{स्वेच्छ अचर}.$$

$$30. \quad (c) \quad \int \frac{dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}}$$

$$= \frac{1}{\sqrt{2}} \int \operatorname{cosec} \left(x + \frac{\pi}{4} \right) dx = \frac{1}{\sqrt{2}} \log \tan \left(\frac{\pi}{8} + \frac{x}{2} \right) + c.$$

$$31. \quad (b) \quad \int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1} x + c \quad (\text{सूत्र}).$$

$$32. \quad (c) \quad \int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} dx = \int \frac{2(\cos^2 x + \sin^2 x) - 1}{\cos^2 x} dx$$

$$= \int \sec^2 x \, dx = \tan x + c.$$

$$33. \quad (d) \quad \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} \right) dx$$

$$= \int (\sec x \tan x + \operatorname{cosec} x \cot x) dx = \sec x - \operatorname{cosec} x + c.$$

$$34. \quad (a) \quad \int \left(\cos \frac{x}{2} - \sin \frac{x}{2} \right)^2 dx$$

$$= \int \left\{ \cos^2 \frac{x}{2} + \sin^2 \frac{x}{2} - 2 \sin \frac{x}{2} \cos \frac{x}{2} \right\} dx$$

$$= \int (1 - \sin x) dx = x + \cos x + c.$$

$$35. (a) \int \frac{1}{x^2} (2x+1)^3 dx = \int \frac{(8x^3 + 1 + 12x^2 + 6x)}{x^2} dx$$

$$= \int \left(8x + 12 + \frac{6}{x} + \frac{1}{x^2} \right) dx = 4x^2 + 12x + 6 \log x - \frac{1}{x} + c.$$

$$36. (b) \int \frac{5(x^6 + 1)}{x^2 + 1} dx = \int \frac{5(x^2 + 1)(x^4 - x^2 + 1)}{(x^2 + 1)} dx$$

$$= \int 5(x^4 - x^2 + 1) dx = x^5 - \frac{5}{3}x^3 + 5x + c.$$

$$37. (d) \int \frac{ax^{-2} + bx^{-1} + c}{x^{-3}} dx = \int (ax + bx^2 + cx^3) dx$$

$$= \frac{1}{4}cx^4 + \frac{1}{3}bx^3 + \frac{1}{2}ax^2 + k.$$

$$38. (b) \int \frac{dx}{\sin x + \sqrt{3} \cos x} = \frac{1}{2} \int \frac{dx}{\frac{\sin x}{2} + \frac{\sqrt{3}}{2} \cos x}$$

$$= \frac{1}{2} \int \frac{dx}{\sin\left(x + \frac{\pi}{3}\right)} = \frac{1}{2} \int \operatorname{cosec}\left(x + \frac{\pi}{3}\right)$$

$$= \frac{1}{2} \log \tan\left(\frac{x}{2} + \frac{\pi}{6}\right) + c.$$

$$39. (d) \int \frac{x^2 + x - 6}{(x-2)(x-1)} dx = \int \frac{(x+3)(x-2)}{(x-2)(x-1)} dx = \int \frac{x+3}{x-1} dx$$

$$= \int \frac{x-1}{x-1} dx + \int \frac{4}{x-1} dx = x + 4 \log(x-1) + c.$$

$$40. (b) \int \frac{dx}{4 \cos^3 2x - 3 \cos 2x} = \int \frac{dx}{\cos 6x} = \int \sec 6x dx$$

$$= \frac{1}{6} \log(\sec 6x + \tan 6x) + c.$$

$$41. (a) \int \frac{\sin 3x}{\sin x} dx = \int \frac{3 \sin x - 4 \sin^3 x}{\sin x} dx$$

$$= \int 3 dx - 4 \int \sin^2 x dx = 3x - 2 \int (1 - \cos 2x) dx + c$$

$$= 3x - 2x + \sin 2x + c = x + \sin 2x + c.$$

$$42. (d) \int \frac{f(x) dx}{\log \sin x} = \log \log \sin x$$

दोनों पक्षों का अवकलन करने पर,

$$\frac{f(x)}{\log \sin x} = \frac{\cot x}{\log \sin x} \Rightarrow f(x) = \cot x.$$

$$43. (a) \int \frac{\sin x + \operatorname{cosec} x}{\tan x} dx = \int (\cos x + \operatorname{cosec} x \cot x) dx$$

$$= \sin x - \operatorname{cosec} x + c.$$

$$44. (c) \int \frac{1}{\sqrt{1 + \sin x}} dx = \int \frac{1}{\sqrt{2} \sin\left(\frac{\pi}{4} + \frac{x}{2}\right)} dx$$

$$= \frac{1}{\sqrt{2}} \int \operatorname{cosec}\left(\frac{x}{2} + \frac{\pi}{4}\right) dx = \sqrt{2} \log \tan\left(\frac{\pi}{8} + \frac{x}{4}\right) + c.$$

$$45. (d) \int (\tan x - \cot x)^2 dx = \int (\tan^2 x + \cot^2 x - 2) dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx - \int 4 dx$$

$$= \tan x - \cot x - 4x + c.$$

$$46. (c) \int (1 + 2 \tan^2 x + 2 \tan x \sec x)^{1/2} dx$$

$$= \int (\sec^2 x + \tan^2 x + 2 \tan x \sec x)^{1/2} dx$$

$$= \int (\sec x + \tan x) dx = \log(\sec x + \tan x) + \log \sec x + c$$

$$= \log \sec x (\sec x + \tan x) + c.$$

$$47. (a) \int \frac{2x}{(2x+1)^2} dx = \int \frac{2x+1-1}{(2x+1)^2} dx$$

$$= \int \frac{1}{(2x+1)} dx - \int (2x+1)^{-2} dx$$

$$= \frac{1}{2} \log(2x+1) + \frac{1}{2(2x+1)} + c.$$

$$48. (a) \int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \sec^2 x dx - \int \operatorname{cosec}^2 x dx$$

$$= \tan x + \cot x + c.$$

$$49. (b) \int (3 \operatorname{cosec}^2 x + 2 \sin 3x) dx$$

$$= -3 \cot x - \frac{2 \cos 3x}{3} + c = -[3 \cot x + \frac{2}{3} \cos 3x] + c.$$

$$50. (a) f(x) = \int f'(x) dx = \int \left(\frac{1}{x} + x \right) dx = \log x + \frac{x^2}{2} + c$$

$x = 1$ रखने पर, $\frac{5}{2} = 0 + \frac{1}{2} + c \Rightarrow c = 2$

अतः $f(x) = \log x + \frac{x^2}{2} + 2.$

$$51. (d) \int \frac{dx}{\sqrt{1+x} + \sqrt{x}} = \int \left[\frac{(x+1)-x}{\sqrt{1+x} + \sqrt{x}} \right] dx$$

$$= \int (\sqrt{x+1} - \sqrt{x}) dx = \frac{(x+1)^{3/2}}{3/2} - \frac{x^{3/2}}{3/2} + c$$

$$= \frac{2}{3} [(x+1)^{3/2} - x^{3/2}] + c = \frac{2}{3} (x+1)^{3/2} - \frac{2}{3} x^{3/2} + c.$$

$$52. (a) \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx = \int \frac{2(\cos^2 x - \cos^2 \alpha)}{\cos x - \cos \alpha} dx$$

$$= 2 \int (\cos x + \cos \alpha) dx = 2(\sin x + x \cos \alpha).$$

$$53. (c) \int \frac{1 - \tan x}{1 + \tan x} dx = \int \tan\left(\frac{\pi}{4} - x\right) dx$$

$$= \int \frac{\sin\left(\frac{\pi}{4} - x\right)}{\cos\left(\frac{\pi}{4} - x\right)} dx = \log \cos\left(\frac{\pi}{4} - x\right) + c$$

$$= \log \sin\left(\frac{\pi}{4} + x\right) + c.$$

$$54. (c) \int a^x da = \frac{a^{x+1}}{x+1} + c.$$

$$55. (c) \text{ यह स्पष्ट है।}$$

$$56. (a) \int x^{-4} dx = \frac{x^{-3}}{-3} + c = -\frac{1}{3x^3} + c.$$

57. (c) $\int \frac{e^{5 \log x} - e^{4 \log x}}{e^{3 \log x} - e^{2 \log x}} dx = \int \frac{x^5 - x^4}{x^3 - x^2} dx$
 $= \int \frac{x^4(x-1)}{x^2(x-1)} dx = \int x^2 dx = \frac{x^3}{3} + c.$
58. (a) $\int \frac{x^4 + x^2 + 1}{x^2 - x + 1} dx = \int (x^2 + x + 1) dx = \frac{x^3}{3} + \frac{x^2}{2} + x + c.$
59. (b) $\int \sec x dx = \log(\sec x + \tan x) + c$
 $= \log\left(\frac{1}{\sec x - \tan x}\right) + c = -\log(\sec x - \tan x) + c.$
60. (d) $\int \sqrt{1 + \sin x} dx = \int \sqrt{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2} dx$
 $= \int \sin \frac{x}{2} dx + \int \cos \frac{x}{2} dx = -2 \cos \frac{x}{2} + 2 \sin \frac{x}{2} + c$
 $= -2\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right) + c = -2\sqrt{1 - \sin x} + c.$
61. (b) $\int \operatorname{cosec}^2 x dx = -\cot x + c.$
62. (a) $\int \left(2 \sin x + \frac{1}{x}\right) dx = -2 \cos x + \log x + c.$
63. (a) $I = \int \sqrt{1 + \cos x} dx = \int \sqrt{2 \cos^2(x/2)} dx$
 $I = \sqrt{2} \int \cos(x/2) dx = 2\sqrt{2} \sin(x/2) + c.$
64. (d) $I = \int 2 \sin x \cdot \cos x dx = \int \sin 2x dx$
 $= -\frac{\cos 2x}{2} + c = -\frac{(1 - 2 \sin^2 x)}{2} + c$
 $= -\frac{1}{2} + \sin^2 x + c = \sin^2 x + c.$
65. (b) $\int \tan^2 x dx = \int (\sec^2 x - 1) dx = \tan x - x + c.$
66. (b) $\int e^{\log(\sin x)} dx = \int \sin x dx = -\cos x + c.$
67. (b) $\int e^{x \log a} e^x dx = \int e^{\log a^x} \cdot e^x dx = \int a^x e^x dx$
 $= \int (ae)^x dx = \frac{(ae)^x}{\log(ae)} + C.$
68. (a) $\int \frac{1}{\sqrt{1 + \cos x}} dx = \int \frac{dx}{\sqrt{2 \cos^2(x/2)}} = \frac{1}{\sqrt{2}} \int \sec \frac{x}{2} dx$
 $= \frac{1}{\sqrt{2}} \left\{ \log \left(\sec \frac{x}{2} + \tan \frac{x}{2} \right) \right\} \cdot \frac{1}{1/2} = \sqrt{2} \log \left(\sec \frac{x}{2} + \tan \frac{x}{2} \right) + K.$
69. (c) $\int \frac{\cos 2x - 1}{\cos 2x + 1} dx$
 $\Rightarrow I = -\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = -\int \frac{2 \sin^2 x}{2 \cos^2 x} dx$
 $\Rightarrow I = -\int \tan^2 x dx = -\int (\sec^2 x - 1) dx$
 $\Rightarrow I = -\int \sec^2 x dx + \int 1 dx = -\tan x + x + c$
 $\Rightarrow I = x - \tan x + c$
70. (c) $I = \int \frac{ax^3 + bx^2 + c}{x^4} dx = \int \left[\frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^4} \right] dx$
 $= a \log x - \frac{b}{x} - \frac{c}{3x^3} + c.$
71. (b) $I = \int \frac{1}{(x-5)^2} dx = \frac{(x-5)^{-2+1}}{-2+1} + c = \frac{(x-5)^{-1}}{-1} + c$
 $= -\frac{1}{(x-5)} + c.$
72. (a) ट्रिक : माना $f(x) = e^x$, तब $\int f(x) dx = f(x)$
 $\therefore \int (e^x)^2 dx = \frac{e^{2x}}{2} = \frac{1}{2} [f(x)]^2.$
73. (a) $I = \int \sqrt{2} \sqrt{1 + \sin x} dx = \sqrt{2} \int \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right) dx$
 $= 2 \int \sin \left(\frac{\pi}{4} + \frac{x}{2} \right) dx = -4 \cos \left(\frac{x}{2} + \frac{\pi}{4} \right) + c$
 तुलना करने पर, $a = \frac{1}{2}, b = \frac{\pi}{4}.$
74. (a) यह स्पष्ट है।
75. (a) यह स्पष्ट है।
76. (b) यह स्पष्ट है।
77. (b) $\int (\sin^4 x - \cos^4 x) dx = \int (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) dx$
 $= \int (\sin^2 x - \cos^2 x) dx = -\int (\cos^2 x - \sin^2 x) dx$
 $= -\int \cos 2x dx = \frac{-\sin 2x}{2} + c.$
78. (b) $\int \frac{(x+1)^2}{x(x^2+1)} dx = \int \frac{x^2+1+2x}{x(x^2+1)} dx$
 $= \int \frac{x^2+1}{x(x^2+1)} dx + 2 \int \frac{x}{x(x^2+1)} dx$
 $= \int \frac{dx}{x} + 2 \int \frac{dx}{x^2+1} = \log_e x + 2 \tan^{-1} x + c.$
79. (b) यहाँ $\int \frac{dx}{\sqrt{1-x}}$ या $I = \int (1-x)^{-1/2} dx$
 $I = \frac{(1-x)^{-1/2+1}}{(-1/2+1)} + c \Rightarrow I = -2\sqrt{1-x} + c.$
80. (c) $\int \frac{dx}{1-x^2} = \frac{1}{2} \log \left(\frac{1+x}{1-x} \right) + c = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + c.$
81. (c) $\int \frac{dx}{4x^2+9} = \frac{1}{4} \int \frac{dx}{x^2+(3/2)^2}$
 $= \frac{1}{4} \cdot \frac{2}{3} \cdot \tan^{-1} \left(\frac{2x}{3} \right) + c = \frac{1}{6} \tan^{-1} \left(\frac{2x}{3} \right) + c.$
82. (a) $\int \sqrt{1+x^2} dx = \frac{x}{2} \sqrt{x^2+1} + \frac{1}{2} \log(x + \sqrt{x^2+1}) + c.$
83. (b) $I = \int \frac{dx}{\sqrt{x^2-a^2}}$
 $x = a \sec \theta$ रखने पर $\Rightarrow dx = a \sec \theta \cdot \tan \theta d\theta$

$$\begin{aligned}\therefore I &= \int \frac{a \sec \theta \cdot \tan \theta d\theta}{a \tan \theta} = \int \sec \theta d\theta \\ &= \log(\sec \theta + \tan \theta) + \log \left(\frac{x}{a} + \frac{\sqrt{x^2 - a^2}}{a} \right) + c \\ &= \log(x + \sqrt{x^2 - a^2}) + c.\end{aligned}$$

$$\begin{aligned}84. (a) \quad I &= \int \frac{x^2}{x^2 + 4} dx = \int \frac{x^2 + 4 - 4}{(x^2 + 4)} dx \\ \Rightarrow I &= \int \left[1 - \frac{4}{x^2 + 4} \right] dx = \int dx - \int \frac{4}{x^2 + 4} dx \\ \Rightarrow I &= x - 4 \int \frac{dx}{x^2 + (2)^2} = x - \frac{4}{2} \tan^{-1}(x/2) + c \\ \Rightarrow I &= x - 2 \tan^{-1} \frac{x}{2} + c.\end{aligned}$$

$$\begin{aligned}85. (b) \quad I &= \int \sqrt{x^2 + a^2} dx = \int \sqrt{x^2 + a^2} \cdot 1 dx \\ &= \sqrt{x^2 + a^2} \int 1 dx - \int \left[\frac{d}{dx} (\sqrt{x^2 + a^2}) \int 1 dx \right] dx \\ &= x\sqrt{x^2 + a^2} - \int \left[\frac{2x}{2\sqrt{x^2 + a^2}} x \right] dx \\ &= x\sqrt{x^2 + a^2} - \int \left[\frac{x^2 + a^2 - a^2}{\sqrt{x^2 + a^2}} \right] dx \\ &= x\sqrt{x^2 + a^2} - \int \left[\sqrt{x^2 + a^2} - \frac{a^2}{\sqrt{x^2 + a^2}} \right] dx \\ &= x\sqrt{x^2 + a^2} - \int \sqrt{x^2 + a^2} dx + a^2 \int \frac{dx}{\sqrt{x^2 + a^2}} \\ &= \sqrt{x^2 + a^2} - I + a^2 \log \left[x + \sqrt{x^2 + a^2} \right] + C \\ \Rightarrow 2I &= x\sqrt{x^2 + a^2} + a^2 \log \left[x + \sqrt{x^2 + a^2} \right] + C \\ \Rightarrow I &= \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left[x + \sqrt{x^2 + a^2} \right] + C\end{aligned}$$

86. (c) यह सूत्र है।

87. (c) यह आधारभूत तथ्य है।

प्रतिस्थापन द्वारा समाकलन

$$\begin{aligned}1. (b) \quad \int \frac{dx}{1 + e^x} &= \int \frac{e^{-x}}{1 + e^{-x}} dx \\ 1 + e^{-x} &= t \text{ रखने पर, } e^{-x} dx = -dt \\ \therefore -\int \frac{dt}{t} &= -\log t = -\log(1 + e^{-x}) + c. \\ 2. (b) \quad \int \frac{dx}{e^x + e^{-x}} &= \int \frac{e^x}{e^{2x} + 1} dx = \int \frac{dt}{t^2 + 1} = \tan^{-1}(t) \\ &= \tan^{-1}(e^x) + c, \{ e^x = t \text{ रखने पर, } e^x dx = dt \}. \\ 3. (a) \quad e^{\sqrt{x}} &= t \text{ रखने पर, } \frac{e^{\sqrt{x}}}{\sqrt{x}} = 2dt, \text{ (स्वयं हल करें).}\end{aligned}$$

$$\begin{aligned}4. (a) \quad \int \frac{dx}{x + x \log x} &= \int \frac{dx}{x(1 + \log x)} \\ 1 + \log x &= t \text{ रखने पर, } \frac{1}{x} dx = dt \\ \therefore \int \frac{dt}{t} &= \log(t) = \log(1 + \log x) + c. \\ 5. (b) \quad \text{माना } 1 + \log x &= t \Rightarrow \frac{1}{x} dx = dt \\ \text{अतः, } \int \frac{1 + \log x}{x} dx &= \int t dt = \frac{t^2}{2} = \frac{(1 + \log x)^2}{2} + c. \\ 6. (a) \quad \int \frac{\sec x dx}{\sqrt{\cos 2x}} &= \int \frac{\sec x}{\sqrt{\cos^2 x - \sin^2 x}} dx \\ &= \int \frac{\sec^2 x dx}{\sqrt{1 - \tan^2 x}} \{ \text{अंश व हर को } \sec x \text{ से गुणा करने पर} \} \\ \text{अब } \tan x &= t \text{ रखने पर, } \sec^2 x dx = dt, \\ \text{अतः समाकलन का मान} &= \sin^{-1} t = \sin^{-1}(\tan x) + c \\ \text{ट्रिक : चूँकि } \frac{d}{dx} \{ \sin^{-1}(\tan x) \} &= \frac{\sec^2 x}{\sqrt{1 - \tan^2 x}} \\ &= \frac{\sec^2 x \cdot \cos x}{\sqrt{\cos^2 x - \sin^2 x}} = \frac{\sec x}{\sqrt{\cos 2x}}. \\ 7. (d) \quad \int \frac{dx}{x\sqrt{2ax - x^2}} &\text{ का मान ज्ञात करने के लिये उचित प्रतिस्थापन } 2a \sin^2 t = x \text{ है।} \\ 8. (c) \quad \int \frac{x dx}{1 - x \cot x} &= \int \frac{x dx}{1 - x \frac{\cos x}{\sin x}} = \int \frac{x \sin x}{\sin x - x \cos x} dx \\ \{ \sin x - x \cos x &= t \text{ रखने पर,} \\ [\cos x - (-x \sin x + \cos x)] dx &= dt \Rightarrow x \sin x dx = dt \} \\ &= \int \frac{dt}{t} = \log t = \log(\sin x - x \cos x) + c. \\ 9. (b) \quad (1 + \sin^2 x) &= t \text{ रखने पर, } \sin 2x dx = dt \\ \text{अतः } \int \frac{\sin 2x}{1 + \sin^2 x} dx &= \int \frac{1}{t} dt = \log(1 + \sin^2 x) + c. \\ 10. (b) \quad \int \frac{x^3}{\sqrt{x^2 + 2}} dx &= \int \frac{x^2 \cdot x}{\sqrt{x^2 + 2}} dx \\ x^2 + 2 &= t^2 \text{ रखने पर, } x dx = t dt \text{ और } x^2 = t^2 - 2, \\ \therefore \int \frac{(t^2 - 2)t dt}{t} &= \int (t^2 - 2) dt \\ &= \frac{t^3}{3} - 2t + c = \frac{(x^2 + 2)^{3/2}}{3} - 2(x^2 + 2)^{1/2} + c. \\ 11. (c) \quad x^e + e^x &= t \text{ रखने पर, } e(x^{e-1} + e^{x-1}) dx = dt \\ \text{अतः } \int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx &= \frac{1}{e} \int \frac{dt}{t} = \frac{1}{e} \log t = \frac{1}{e} \log(x^e + e^x) + c \\ 12. (c) \quad b \cos x &= t \text{ रखकर हल करें।} \\ 13. (b) \quad \text{माना } \log(\sec x + \tan x) &= t \Rightarrow \sec x dx = dt \\ \therefore \int \sec x \log(\sec x + \tan x) dx &= \int t dt\end{aligned}$$

$$= \frac{t^2}{2} + c = \frac{[\log(\sec x + \tan x)]^2}{2} + c.$$

14. (a) $x^2 - 4x + 3 = t$ रखने पर, $(2x - 4)dx = dt$

$$\Rightarrow (x - 2)dx = \frac{1}{2}dt,$$

$$\therefore \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \log t + c = \frac{1}{2} \log(x^2 - 4x + 3) + c.$$

15. (b) माना $x^3 = t \Rightarrow 3x^2 dx = dt$,

$$\therefore \int \frac{3x^2}{x^6 + 1} dx = \int \frac{1}{t^2 + 1} dt = \tan^{-1}(t) + c = \tan^{-1}(x^3) + c.$$

16. (a) $\log \sin x = t$ रखकर हल करें।

17. (c) $(1 + \log x) = t$ रखने पर, $\frac{1}{x} dx = dt$

$$\therefore \int \frac{(1 + \log x)^2}{x} dx = \int t^2 dt = \frac{t^3}{3} + c = \frac{(1 + \log x)^3}{3} + c.$$

18. (b) $\sec x = t$ रखने पर, $\sec x \tan x dx = dt$,

$$\therefore \int \sec^p x \tan x dx = \int t^{p-1} dt = \frac{t^p}{p} + c = \frac{\sec^p x}{p} + c.$$

19. (a) $\int \frac{dx}{e^x - 1} = \int \frac{e^{-x}}{1 - e^{-x}} dx$

$$1 - e^{-x} = t \text{ रखने पर, } e^{-x} dx = dt,$$

$$\therefore \int \frac{dt}{t} = \log t + c = \log(1 - e^{-x}) + c.$$

20. (c) $x^3 = t$ रखने पर, $3x^2 dx = dt$

अब आगे स्वयं हल करें।

21. (b) $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$

$$= \int \frac{2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx = \int \frac{2 \tan x \sec^2 x}{1 + \tan^4 x} dx$$

$$\tan^2 x = t \text{ रखने पर, } 2 \tan x \sec^2 x dx = dt,$$

$$\therefore \int \frac{dt}{1+t^2} = \tan^{-1} t + c = \tan^{-1}(\tan^2 x) + c.$$

ट्रिक : जॉच द्वारा,

$$\frac{d}{dx} \{ \cot^{-1}(\tan^2 x) \} = - \frac{1(2 \tan x \cdot \sec^2 x)}{1 + \tan^4 x} = - \frac{\sin 2x}{\cos^4 x + \sin^4 x}$$

$$\Rightarrow \frac{d}{dx} \{ \tan^{-1}(\tan^2 x) \} = \frac{\sin 2x}{\sin^4 x + \cos^4 x}.$$

22. (b) $\int \frac{x-2}{x(2 \log x - x)} dx = - \int \frac{\left(\frac{2}{x} - 1\right)}{(2 \log x - x)} dx$

$$(2 \log x - x) = t \text{ रखने पर, } \left(\frac{2}{x} - 1\right) dx = dt,$$

$$\therefore - \int \frac{1}{t} dt = - \log t = - \log(2 \log x - x)$$

$$= \log \left(\frac{1}{2 \log x - x} \right) + c.$$

23. (d) $1 + x^2 = t$ रखने पर, $x dx = \frac{dt}{2}$

$$\therefore \frac{1}{2} \int t^{1/2} dt = \frac{1}{2} \times \frac{t^{3/2}}{3/2} = \frac{1}{3} (1 + x^2)^{3/2} + c.$$

24. (a) $\int \frac{e^x(x+1)}{\cos^2(xe^x)} = \int e^x(x+1) \sec^2(xe^x) dx$

$$xe^x = t \text{ रखने पर, } (x+1)e^x dx = dt$$

$$\therefore \int \sec^2 t dt = \tan t + c = \tan(xe^x) + c.$$

25. (b) $\sqrt{x} = t$ रखने पर, $\frac{1}{2\sqrt{x}} dx = dt$.

अब आगे स्वयं हल करें।

26. (c) $\int \frac{x+1}{\sqrt{x^2+1}} dx = \int \frac{x}{\sqrt{x^2+1}} dx + \int \frac{1}{\sqrt{x^2+1}} dx$

$$x^2 + 1 = t \text{ रखने पर, } 2x dx = dt$$

$$\therefore \frac{1}{2} \int \frac{dt}{t^{1/2}} + \int \frac{1}{\sqrt{x^2+1}} dx = \frac{1}{2} \cdot 2 \cdot t^{1/2} + \log(x + \sqrt{x^2+1}) + c$$

$$= (x^2 + 1)^{1/2} + \log(x + \sqrt{x^2 + 1}) + c.$$

27. (a) $a \cos^2 x + b \sin^2 x = t$ रखने पर $\Rightarrow 2(b-a) \sin x \cos x = dt$,

$$\text{तब } \int \frac{\sin x \cos x dx}{a \cos^2 x + b \sin^2 x} = \frac{1}{2(b-a)} \int \frac{1}{t} dt$$

$$= \frac{1}{2(b-a)} \log(a \cos^2 x + b \sin^2 x) + c.$$

28. (c) $t = \tan^{-1} x$ रखने पर, $dt = \frac{1}{1+x^2} dx$

$$\text{अतः } \int \frac{e^{\tan^{-1} x}}{1+x^2} dx = \int e^t dt = e^t + c = e^{\tan^{-1} x} + c.$$

29. (b) $\log x = t$ रखने पर, $\frac{1}{x} dx = dt$, तब

$$\int \frac{1}{x(\log x)^2} dx = \int \frac{1}{t^2} dt = -\frac{1}{t} + c = -\frac{1}{\log x} + c.$$

30. (c) $\int \frac{1}{\sqrt{x}} \tan^4 \sqrt{x} \cdot \sec^2 \sqrt{x} dx$

$$\tan \sqrt{x} = t \text{ रखने पर, } \frac{\sec^2 \sqrt{x}}{2\sqrt{x}} dx = dt$$

$$\therefore 2 \int t^4 dt = \frac{2}{5} (\tan \sqrt{x})^5 + c = \frac{2}{5} \tan^5 \sqrt{x} + c.$$

31. (a) $a^x = t$ रखने पर, $a^x \log_e a dx = dt$ तब

$$\int \frac{a^x}{\sqrt{1-a^{2x}}} dx = \frac{1}{\log_e a} \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \frac{1}{\log_e a} \sin^{-1}(t) + c = \frac{\sin^{-1}(a^x)}{\log_e a} + c.$$

32. (b) $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx = \int \frac{\tan x}{\sqrt{\tan x} \sin x \cos x} dx$

$$= \int \frac{\sin x \sec x}{\sqrt{\tan x} \sin x \cos x} dx = \int \frac{\sec^2 x}{\sqrt{\tan x}} dx$$

$$t = \tan x \text{ रखने पर, } dt = \sec^2 x dx$$

$$\therefore \int \frac{1}{\sqrt{t}} dt = 2t^{1/2} + c = 2\sqrt{\tan x} + c.$$

33. (a) $a^2 + b^2 \sin^2 x = t$ रखने पर, $b^2 \sin 2x dx = dt$ तब

$$\begin{aligned} \int \frac{\sin 2x}{a^2 + b^2 \sin^2 x} dx &= \frac{1}{b^2} \int \frac{dt}{t} = \frac{1}{b^2} \log t + c \\ &= \frac{1}{b^2} \log(a^2 + b^2 \sin^2 x) + c. \end{aligned}$$

34. (c) $t = 1 + \log x$ रखने पर, $dt = \frac{1}{x} dx$

$$\text{अतः } \int \frac{dx}{x\sqrt{1+\log x}} = \int \frac{dt}{t^{1/2}} = 2t^{1/2} + c = 2(1+\log x)^{1/2} + c.$$

35. (c) $t = 1 + \tan x$ रखने पर, $dt = \sec^2 x dx$

$$\therefore \int \frac{\sec^2 x}{1 + \tan x} dx = \int \frac{1}{t} dt = \log t + c = \log(1 + \tan x) + c.$$

36. (b) $\int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$
 $e^x + e^{-x} = t$ रखने पर, $(e^x - e^{-x}) dx = dt$

$$\therefore \int \frac{dt}{t} = \log t = \log(e^x + e^{-x}) = \log(e^{2x} + 1) - x + c.$$

37. (a) $\log \tan \frac{x}{2} = t$ रखने पर, $\frac{1}{\tan \frac{x}{2}} \cdot \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$

$$\Rightarrow \operatorname{cosec} x dx = dt,$$

$$\therefore \int \frac{\operatorname{cosec} x}{\log \tan \frac{x}{2}} dx = \int \frac{1}{t} dt = \log t + c = \log \left(\log \tan \frac{x}{2} \right) + c$$

38. (b) $\int \frac{1}{\cos^2 x (1 - \tan x)^2} dx = \int \frac{\sec^2 x dx}{(\tan x - 1)^2}$

$$\tan x - 1 = t \text{ रखने पर, } \sec^2 x dx = dt$$

$$\therefore \int \frac{1}{t^2} dt = \frac{-1}{\tan x - 1} + c = \frac{1}{1 - \tan x} + c.$$

39. (b) $x^{10} + 10^x = t$ रखने पर, $(10x^9 + 10^x \log_e 10) dx = dt$

$$\begin{aligned} \text{तब } \int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx &= \int \frac{1}{t} dt = \log t + c \\ &= \log(x^{10} + 10^x) + c. \end{aligned}$$

40. (a) $\int \frac{1}{(e^x + e^{-x})^2} dx = \int \frac{e^{2x}}{(e^{2x} + 1)^2} dx$

$$e^{2x} + 1 = t \text{ रखने पर, } 2e^{2x} dx = dt$$

$$\therefore \frac{1}{2} \int \frac{1}{t^2} dt = -\frac{1}{2} \cdot \frac{1}{t} + c = -\frac{1}{2(e^{2x} + 1)} + c.$$

41. (c) $\int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2} dx$

$$= \int \frac{\cos x - \sin x}{\cos x + \sin x} dx$$

$$t = \sin x + \cos x \text{ रखने पर, } dt = (\cos x - \sin x) dx$$

$$\therefore \int \frac{1}{t} dt = \log t + c = \log(\sin x + \cos x) + c.$$

42. (c) $\log x = t$ रखने पर, $\frac{1}{x} dx = dt$

$$\begin{aligned} \therefore \int \frac{\tan(\log x)}{x} dx &= \int \tan t dt \\ &= \log \sec t + c = \log \sec(\log x) + c. \end{aligned}$$

43. (b) $\int \cos^3 x e^{\log \sin x} dx = \int \cos^3 x \sin x dx$

$$= -\int t^3 dt = -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c, \{t = \cos x \text{ रखने पर}\}.$$

44. (b) $t = 3x - 5$ रखने पर, $dt = 3dx$

$$\begin{aligned} \therefore \int \tan(3x - 5) \sec(3x - 5) dx &= \frac{1}{3} \int \tan t \cdot \sec t dt \\ &= \frac{\sec t}{3} + c = \frac{\sec(3x - 5)}{3} + c. \end{aligned}$$

45. (b) $t = x^2$ रखने पर, $dt = 2x dx$

$$\therefore \int \frac{x}{1+x^4} dx = \frac{1}{2} \int \frac{1}{1+t^2} dt = \frac{1}{2} \tan^{-1} t + c = \frac{1}{2} \tan^{-1} x^2 + c.$$

46. (a) $\int \frac{e^{-x}}{1+e^x} dx = \int \frac{e^{-x} e^{-x}}{e^{-x} + 1} dx$

$$e^{-x} + 1 = t \text{ रखने पर, } -e^{-x} dx = dt$$

$$\begin{aligned} \therefore -\int \frac{(t-1)}{t} dt &= \int \left(\frac{1}{t} - 1 \right) dt \\ &= \log t - t + c = \log(e^{-x} + 1) - (e^{-x} + 1) + c \\ &= \log(e^x + 1) - x - e^{-x} - 1 + c \\ &= \log(e^x + 1) - x - e^{-x} + c, \quad (\because 1 = \text{अचर}). \end{aligned}$$

47. (a) $\int \frac{1}{\sqrt{1-e^{2x}}} dx = \int \frac{e^{-x}}{\sqrt{e^{-2x}-1}} dx$

$$e^{-x} = t \text{ रखने पर, } -e^{-x} dx = dt$$

$$\therefore -\int \frac{1}{\sqrt{t^2-1}} dt = -\log \left[t + \sqrt{t^2-1} \right] + c$$

$$= -\log \left[e^{-x} + \sqrt{e^{-2x}-1} \right] = -\log \left[\frac{1}{e^x} + \frac{\sqrt{1-e^{2x}}}{e^x} \right]$$

$$= -\log \left[1 + \sqrt{1-e^{2x}} \right] + \log e^x + c$$

$$= x - \log \left[1 + \sqrt{1-e^{2x}} \right] + c.$$

48. (a) $\int \frac{3x^2}{\sqrt{9-16x^6}} dx = \int \frac{3x^2}{\sqrt{(3)^2-(4x^3)^2}} dx$

$$4x^3 = t \text{ रखने पर, } 12x^2 dx = dt$$

$$\therefore \frac{1}{4} \int \frac{dt}{\sqrt{(3)^2-t^2}} = \frac{1}{4} \cdot \frac{1}{1} \sin^{-1} \left(\frac{t}{3} \right) + c = \frac{1}{4} \sin^{-1} \left(\frac{4x^3}{3} \right) + c.$$

49. (b) $\sin x = t$ रखने पर, $\cos x dx = dt$

$$\therefore \int \cos x \sqrt{4-\sin^2 x} dx = \int \sqrt{4-t^2} dt = \int \sqrt{(2)^2-t^2} dt$$

$$= \frac{t}{2} \sqrt{4-t^2} + \frac{4}{2} \sin^{-1} \frac{t}{2} + c$$

$$= \frac{1}{2} \sin x \sqrt{4 - \sin^2 x} + 2 \sin^{-1} \left(\frac{1}{2} \sin x \right) + c.$$

50. (b) $\int x^2 (3)^{x^3+1} dx = \int 3x^2 \cdot (3)^{x^3} dx$

$$x^3 = t \text{ रखने पर, } 3x^2 dx = dt$$

$$= \int 3^t dt = \frac{3^t}{\log 3} + c = \frac{(3)^{x^3}}{\log 3} + c.$$

51. (b) $\int \sec^{2/3} x \operatorname{cosec}^{4/3} x dx = \int \frac{dx}{\sin^{4/3} x \cos^{2/3} x}$

$$\text{अंश व हर को } \cos^2 x, \text{ से गुणा करने पर,}$$

$$\{ \tan x = t \text{ रखने पर, } \sec^2 x dx = dt \}$$

$$\therefore \int \frac{\sec^2 x dx}{\tan^{4/3} x} = \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{(-1/3)} + c = -3(\tan x)^{-1/3} + c$$

52. (a) $\int \cos^5 x dx = \int \cos^4 x \cos x dx = \int (1 - \sin^2 x)^2 \cos x dx$

$$\sin x = t \text{ रखने पर, } \cos x dx = dt$$

$$\therefore \int (1 - t^2)^2 dt = \int (1 + t^4 - 2t^2) dt = \frac{t^5}{5} - \frac{2t^3}{3} + t + c$$

$$= \frac{\sin^5 x}{5} - \frac{2 \sin^3 x}{3} + \sin x + c.$$

53. (a) $\int \sec x \tan^3 x dx = \int \sec x (\sec^2 x - 1) \tan x dx$

$$= \int \sec x \tan x \sec^2 x dx - \int \sec x \tan x dx$$

$$= \frac{\sec^3 x}{3} - \sec x + c, \text{ (प्रथम भाग में } \sec x = t \text{ रखने पर).}$$

54. (c) $\int \frac{d\theta}{\sin \theta \cos^3 \theta} = \int \frac{\sec^2 \theta d\theta}{\sin \theta \cos \theta} = \int \frac{\sec^2 \theta (1 + \tan^2 \theta)}{\tan \theta} d\theta$

$$t = \tan \theta \text{ रखने पर, } dt = \sec^2 \theta d\theta$$

$$\therefore \int \frac{1+t^2}{t} dt = \int \left(\frac{1}{t} + t \right) dt$$

$$= \log t + \frac{t^2}{2} + c = \log \tan \theta + \frac{\tan^2 \theta}{2} + c.$$

55. (b) $\cos^{-1} x = t \text{ रखने पर, } -\frac{1}{\sqrt{1-x^2}} dx = dt$

$$\text{तब } \int \frac{1}{\cos^{-1} \sqrt{1-x^2}} dx = -\int \frac{1}{t} dt = -\log t + c = \log \frac{1}{t} + c$$

$$= -\log(\cos^{-1} x) + c.$$

56. (b) $\int x^3 e^{3x^2+5} dx$

$$(3x^2 + 5) = t \text{ सरलतम प्रतिस्थापन है}$$

$$t = 3x^2 + 5 \text{ रखने पर, } dx = \frac{dt}{6x}$$

$$\text{तब } \int x^3 e^{3x^2+5} dx = \frac{1}{6} \int \left(\frac{t-5}{3} \right) e^t dt$$

$$= \frac{1}{18} \int [te^t - 5e^t] dt = \frac{1}{18} \int te^t dt - \frac{5}{18} \int e^t dt$$

$$= \frac{1}{18} \left[te^t - \int e^t dt \right] - \frac{5}{18} \int e^t dt + c = \frac{1}{18} (te^t) - \frac{1}{3} e^t + c$$

$$= \frac{1}{18} (3x^2 + 5) e^{3x^2+5} - \frac{1}{3} e^{3x^2+5} + c.$$

57. (c) सबसे उपयुक्त प्रतिस्थापन $\tan x = t$ है।

58. (b) $1 + \cot x = t$ रखने पर, $\operatorname{cosec}^2 x dx = -dt$

$$\text{तब } \int \frac{\operatorname{cosec}^2 x}{1 + \cot x} dx = -\int \frac{1}{t} dt = -\log t + c = -\log(1 + \cot x) + c$$

59. (b) $\sqrt{x} = t$ रखने पर $\Rightarrow \frac{1}{\sqrt{x}} dx = 2dt,$

$$\therefore \int \sin t dt = -2 \cos t + c = -2 \cos \sqrt{x} + c.$$

60. (d) $e^x = t$ रखने पर, $e^x dx = dt$

$$\text{तब } \int e^x \tan^2(e^x) dx = \int \tan^2 t dt = \int (\sec^2 t - 1) dt$$

$$= \tan t - t + c = \tan(e^x) - e^x + c.$$

61. (a) $\int \frac{dx}{e^{-2x}(e^{2x} + 1)^2} = \int \frac{e^{2x} dx}{(e^{2x} + 1)^2}$

$$t = e^{2x} + 1 \text{ रखने पर, } \frac{dt}{2} = e^{2x} dx$$

$$\therefore \frac{1}{2} \int \frac{1}{t^2} dt = -\frac{1}{2t} + c = \frac{-1}{2(e^{2x} + 1)} + c.$$

62. (b) $\int \tan^4 x dx = \int \tan^2 x (\sec^2 x - 1) dx$

$$= \int \tan^2 x \sec^2 x dx - \int \tan^2 x dx = \frac{\tan^3 x}{3} - \tan x + x + c.$$

63. (c) $\log x = t$ रखने पर, $\frac{1}{x} dx = dt$

$$\therefore \int \frac{dt}{\sqrt{1-t^2}} = \sin^{-1} t = \sin^{-1}(\log x) + c.$$

64. (a) $f(x) = t$ रखने पर, $f'(x) dx = dt$

$$\text{तब } \int \frac{f'(x)}{|f(x)|^2} dx = \int \frac{1}{t^2} dt = -\frac{1}{t} + c = -\frac{1}{f(x)} + c.$$

65. (c) $\int x^3 \cos x^2 dx$ के लिये $x^2 = t$ उपयुक्त प्रतिस्थापन है

$$= \frac{1}{2} \int t \cos t dt = \frac{1}{2} (t \sin t - \int \sin t dt + c)$$

$$= \frac{1}{2} (t \sin t + \cos t + c) = \frac{1}{2} (x^2 \sin x^2 + \cos x^2 + c).$$

66. (a) $\int \tan x \cdot \sec^2 x \sqrt{1 - \tan^2 x} dx$

$$\text{माना } \tan x = t \Rightarrow \sec^2 x dx = dt, \therefore \int t \sqrt{1-t^2} dt$$

$$\text{पुनः } 1 - t^2 = u \text{ रखने पर, } -2t dt = du$$

$$\therefore -\frac{1}{2} \int \sqrt{u} du = -\frac{1}{3} u^{3/2} + c = -\frac{1}{3} (1 - \tan^2 x)^{3/2} + c.$$

67. (c) $\int \frac{\sin 2x}{\sin 5x \sin 3x} dx = \int \frac{\sin(5x - 3x)}{\sin 5x \sin 3x} dx$

$$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x} dx$$

$$= \frac{1}{3} \log \sin 3x - \frac{1}{5} \log \sin 5x + c.$$

68. (b) $e^x = t$ रखने पर $\Rightarrow e^x dx = dt$, तब

$$\int \frac{e^x dx}{\sqrt{1-e^{2x}}} = \int \frac{dt}{\sqrt{1-t^2}} = -\cos^{-1} t + c = -\cos^{-1}(e^x) + c.$$

69. (c) $\int \frac{1}{\log a} (a^x \cos a^x) dx$

अब $a^x = t$ रखने पर $\Rightarrow a^x dx = \frac{dt}{\log a}$,

$$\therefore \int \frac{1}{(\log a)^2} \cos t dt = \frac{1}{(\log a)^2} \sin t + c = \frac{1}{(\log a)^2} \sin a^x + c.$$

70. (b) $a + b \cos x = t$ रखने पर $\Rightarrow dx = -\frac{dt}{b \sin x}$

तब $\int \frac{\sin x}{(a + b \cos x)^2} dx = -\frac{1}{b} \int \frac{1}{t^2} dt = \frac{1}{b} \frac{1}{t} + c$

$$= \frac{1}{b(a + b \cos x)} + c.$$

71. (b) $\int \frac{1}{x^3} [\log x^x]^2 dx = \int \frac{1}{x^3} [x \log x]^2 dx$

$$= \int \frac{1}{x} (\log x)^2 dx = \frac{1}{3} (\log x)^3 + c, \{ \log x = t \text{ रखने पर} \}.$$

72. (a) $t = \log x$ रखने पर $\Rightarrow x dt = dx$

तब $\int \frac{1}{x} \sec^2(\log x) dx = \int \sec^2 t dt = \tan t + c$

$$= \tan(\log x) + c.$$

73. (b) $\int \frac{dx}{x \log x \cdot \log(\log x)}$

अब $\log x = t$ रखने पर, $\int \frac{dt}{t \cdot \log(t)}$

पुनः $\log t = z$ रखने पर,

$$\int \frac{dz}{z} = \log z = \log[\log(\log x)] + c.$$

74. (a) $t = \tan x$ रखने पर $\Rightarrow dt = \sec^2 x dx$

तब $\int \frac{\sec^2 x dx}{\sqrt{\tan^2 x + 4}} = \int \frac{1}{\sqrt{t^2 + 2^2}} dt$

$$= \log[\tan x + \sqrt{\tan^2 x + 4}] + c.$$

75. (b) $t = \tan^{-1} x^2$ रखने पर $\Rightarrow dt = \frac{1}{1+x^4} 2x dx$, तब

$$\int \frac{2x \tan^{-1} x^2}{1+x^4} dx = \int t dt = \frac{t^2}{2} + c = \frac{1}{2} (\tan^{-1} x^2)^2 + c.$$

76. (b) $\sqrt{x} = t$ रखने पर $\Rightarrow \frac{1}{2} \frac{1}{\sqrt{x}} dx = dt$, तब

$$\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int a^t dt = \frac{2a^t}{\log_e a} + c = 2a^{\sqrt{x}} \log_a e + c.$$

77. (c) $\int \frac{x^3}{\sqrt{1-x^8}} dx = \int \frac{x^3}{\sqrt{1-(x^4)^2}} dx$

$$x^4 = t \text{ रखने पर } \Rightarrow 4x^3 dx = dt,$$

$$\therefore \frac{1}{4} \int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{4} [\sin^{-1}(t)] + c = \frac{1}{4} \sin^{-1}(x^4) + c.$$

78. (a) $t = \cos x^2$ रखने पर $\Rightarrow dt = -2x \sin x^2 dx$, तब

$$\int 2x \cos^3 x^2 \sin x^2 dx = -\int t^3 dt = -\frac{t^4}{4} + c$$

$$= -\frac{1}{4} \cos^4 x^2 + c.$$

79. (a) $\int \sec^4 x \tan x dx = \int \sec^3 x \sec x \tan x dx$

$t = \sec x$ रखने पर $\Rightarrow dt = \sec x \tan x dx$, तब

$$\int t^3 dt = \frac{t^4}{4} + c = \frac{1}{4} \sec^4 x + c.$$

80. (a) $2e^{-x} + 5$ रखने पर $= t \Rightarrow -2e^{-x} dx = dt$, तब

$$\int e^{-x} \operatorname{cosec}^2(2e^{-x} + 5) dx = -\frac{1}{2} \int \operatorname{cosec}^2 t dt$$

$$= \frac{1}{2} \cot t = \frac{1}{2} \cot(2e^{-x} + 5) + c.$$

81. (b) $\int \sin^3 x \cdot \cos x dx$

$\sin x = t$ रखने पर, $\cos x dx = dt$

$$\therefore \int t^3 dt = \frac{t^4}{4} = \frac{\sin^4 x}{4} + c.$$

82. (b) $t = 3x + 3$ रखने पर $\Rightarrow dt = 3 dx$, तब

$$\int a^{3x+3} dx = \frac{1}{3} \int a^t dt = \frac{1}{3} \frac{a^t}{\log_e a} + c = \frac{a^{3x+3}}{3 \log_e a} + c.$$

83. (c) माना $x^2 + \sin 2x + 2x = t$

$$\therefore \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \log t + c = \frac{1}{2} \log(x^2 + \sin 2x + 2x) + c.$$

84. (a) $t = x + \log \sec x$ रखने पर $\Rightarrow dt = (1 + \tan x) dx$, तब

$$\int \frac{1 + \tan x}{x + \log \sec x} dx = \int \frac{1}{t} dt = \log t + c$$

$$= \log(x + \log \sec x) + c.$$

85. (c) $t = x + \log x$ रखने पर $\Rightarrow dt = \left(1 + \frac{1}{x}\right) dx$, तब

$$\int \frac{(x+1)(x+\log x)^2}{x} dx = \int t^2 dt = \frac{t^3}{3} + c$$

$$= \frac{1}{3} (x + \log x)^3 + c.$$

86. (a) $x = \sin \theta$ रखने पर $\Rightarrow dx = \cos \theta d\theta$,

$$\therefore \int (1 + \sin^2 \theta) d\theta = \theta + \frac{1}{2} \int (1 - \cos 2\theta) d\theta$$

$$= \frac{3\theta}{2} - \frac{1}{2} \sin \theta \sqrt{1 - \sin^2 \theta} + c$$

$$= \frac{3}{2} \sin^{-1} x - \frac{1}{2} x \sqrt{1 - x^2} + c.$$

87. (a) $\int \frac{\cos x - \sin x}{1 + \sin 2x} dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx$

$$\sin x + \cos x = t \text{ रखने पर } \Rightarrow (\cos x - \sin x)dx = dt$$

$$\text{अभीष्ट समाकलन} = -\frac{1}{\sin x + \cos x} + c.$$

$$88. (c) \text{ माना } 3 + 5x^4 = t \Rightarrow 20x^3 dx = dt, \text{ तब}$$

$$\begin{aligned} \int x^3 \sqrt{3 + 5x^4} dx &= \frac{1}{20} \int t^{1/2} dt \\ &= \frac{2}{3} \times \frac{1}{20} \cdot t^{3/2} + c = \frac{1}{30} (3 + 5x^4)^{3/2} + c. \end{aligned}$$

$$89. (b) x = a(\sin \theta)^{2/3} \text{ रखने पर } \Rightarrow dx = \frac{2}{3} a(\sin \theta)^{-1/3} \cos \theta d\theta$$

$$\begin{aligned} \therefore \int \frac{\sqrt{x}}{a^3 - x^3} dx &= \int \frac{a^{1/2} (\sin \theta)^{1/3} \cdot \frac{2}{3} a(\sin \theta)^{-1/3} \cos \theta}{\sqrt{a^3 - a^3 \sin^2 \theta}} d\theta \\ &= \frac{2}{3} a^{3/2} \int \frac{\cos \theta d\theta}{a^{3/2} \sqrt{1 - \sin^2 \theta}} = \frac{2}{3} \sin^{-1} \left(\frac{x}{a} \right)^{3/2} + c. \end{aligned}$$

$$90. (a) 1 + \log x = t \text{ रखने पर } \Rightarrow \frac{1}{x} dx = dt, \text{ तब}$$

$$\begin{aligned} \int \frac{1}{x \cos^2(1 + \log x)} dx &= \int \frac{dt}{\cos^2 t} = \int \sec^2 t dt \\ &= \tan t + c = \tan(1 + \log x) + c. \end{aligned}$$

$$91. (a) x = \tan \theta \text{ रखने पर } \Rightarrow dx = \sec^2 \theta d\theta, \text{ तब}$$

$$\begin{aligned} \int \frac{1}{x^2 \sqrt{1+x^2}} dx &= \int \frac{\sec^2 \theta d\theta}{\tan^2 \theta \sec \theta} = \int \operatorname{cosec} \theta \cot \theta d\theta \\ &= -\operatorname{cosec} \theta + c = -\frac{\sqrt{x^2+1}}{x} + c. \end{aligned}$$

$$92. (c) x = \tan \theta \text{ रखने पर } \Rightarrow dx = \sec^2 \theta d\theta, \text{ तब}$$

$$\begin{aligned} \int \frac{dx}{(x^2-1)\sqrt{x^2+1}} &= \int \frac{\sec^2 \theta d\theta}{(\tan^2 \theta - 1) \sec \theta} = \int \frac{\cos \theta d\theta}{(2 \sin^2 \theta - 1)} \\ \text{पुनः } t = \sin \theta \text{ रखने पर } &\Rightarrow dt = \cos \theta d\theta, \end{aligned}$$

$$\begin{aligned} \therefore \int \frac{dt}{(2t^2-1)} &= \frac{1}{2} \int \frac{dt}{t^2 - \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{1}{2\sqrt{2}} \log \left(\frac{t - \frac{1}{\sqrt{2}}}{t + \frac{1}{\sqrt{2}}} \right) + c \\ &= \frac{1}{2\sqrt{2}} \log \left(\frac{\sqrt{1+x^2} - x\sqrt{2}}{\sqrt{1+x^2} + x\sqrt{2}} \right) + c. \end{aligned}$$

$$93. (a) \log(x + \sqrt{1+x^2}) = t \text{ रखने पर } \Rightarrow \frac{1}{\sqrt{1+x^2}} dx = dt, \text{ तब}$$

$$\begin{aligned} \int \frac{\log(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx &= \int t dt \\ &= \int \frac{t^2}{2} dt = \frac{1}{2} \left[\log(x + \sqrt{1+x^2}) \right]^2 + c. \end{aligned}$$

$$94. (a) \int e^x \sin(e^x) dx$$

$$\text{अब } e^x = t \text{ रखने पर, } e^x dx = dt$$

$$\therefore \int \sin t dt = -\cos t + c = -\cos(e^x) + c.$$

$$95. (d) \text{ यहाँ } x^5 = x^3 x^2 \text{ और } x^3 \text{ का अवकल गुणांक } 3x^2 \text{ है। भिन्न घातांकों को हटाने के लिये}$$

$$1 + x^3 = t^2 \Rightarrow 3x^2 dx = 2t dt; \text{ साथ ही } x^3 = t^2 - 1$$

$$\begin{aligned} \text{अतः } I &= \int \frac{(t^2-1)}{t} \left(\frac{2}{3} t dt \right) = \frac{2}{3} \int (t^2-1) dt \\ &= \frac{2}{3} \left(\frac{t^3}{3} - t \right) = \frac{2}{9} t(t^2-3) = \frac{2}{9} \sqrt{1+x^3} (x^3-2) \\ &\quad (\because t^2 = 1+x^3). \end{aligned}$$

$$\begin{aligned} 96. (a) \int \frac{(x^4-x)^{1/4}}{x^5} dx &= \int \frac{1}{x^4} \left(1 - \frac{1}{x^3} \right)^{1/4} dx \\ &= \frac{1}{3} \int t^{1/4} dt = \frac{4}{15} t^{5/4} + c = \frac{4}{15} \left(1 - \frac{1}{x^3} \right)^{5/4} + c \\ &\quad \left\{ 1 - \frac{1}{x^3} = t \text{ रखने पर, } \frac{1}{x^4} dx = \frac{1}{3} dt \right\}. \end{aligned}$$

$$\begin{aligned} 97. (a) \int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}} dx &= \int \frac{1}{\left(\frac{x-1}{x+2} \right)^{3/4} (x+2)^2} dx \\ &= \frac{1}{3} \int \frac{1}{t^{3/4}} dt, \quad \left\{ \because \frac{x-1}{x+2} = t \Rightarrow \frac{3}{(x+2)^2} dx = dt \right\} \\ &= \frac{1}{3} \left(\frac{t^{1/4}}{1/4} \right) + c = \frac{4}{3} t^{1/4} + c = \frac{4}{3} \left(\frac{x-1}{x+2} \right)^{1/4} + c. \end{aligned}$$

$$\begin{aligned} 98. (a) I &= \int \frac{1}{1 + \sin^2 x} dx = \int \frac{dx}{2 \sin^2 x + \cos^2 x} \\ &= \int \frac{\sec^2 x dx}{2 \tan^2 x + 1} = \frac{1}{2} \int \frac{\sec^2 x dx}{\tan^2 x + \frac{1}{2}} \end{aligned}$$

$$\tan x = t \text{ रखने पर } \Rightarrow \sec^2 x dx = dt, \text{ तब}$$

$$\begin{aligned} I &= \frac{1}{2} \int \frac{dt}{t^2 + \frac{1}{2}} = \frac{1}{2} \cdot \frac{1}{1/\sqrt{2}} \tan^{-1} \frac{t}{1/\sqrt{2}} \\ &= \frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2} \tan x) + k. \end{aligned}$$

$$\begin{aligned} 99. (c) \text{ दिया है, } I &= \int \frac{\sin x}{\cos^2 x} dx \\ \cos x = t \text{ रखने पर, } &\Rightarrow \sin x dx = -dt \\ \therefore I &= -\int \frac{dt}{t^2} = \frac{1}{t} + k = \sec x + k. \end{aligned}$$

$$\begin{aligned} 100. (a) I &= \int e^x \sec^2(e^x) dx \\ e^x = t \text{ रखने पर } &\Rightarrow e^x dx = dt \\ \therefore I &= \int \sec^2 t dt = \tan t + k = \tan(e^x) + k. \end{aligned}$$

$$\begin{aligned} 101. (a) I &= \int \frac{dx}{x\sqrt{x^4-1}} \\ x^2 = t \text{ रखने पर } &\Rightarrow 2x dx = dt \Rightarrow dx = \frac{dt}{2x} = \frac{dt}{2\sqrt{t}} \\ \therefore I &= \int \frac{dt}{2t\sqrt{t^2-1}} = \frac{1}{2} \sec^{-1} t + k = \frac{1}{2} \sec^{-1} x^2 + k \end{aligned}$$

102. (d) $I = \int t \cdot e^{-3t^2} dt$
 $-3t^2 = z$ रखने पर $\Rightarrow -6t dt = dz \Rightarrow t dt = \frac{-1}{6} dz$

$\therefore I = -\frac{1}{6} \int e^z dz = \frac{-e^z}{6} + c = \frac{-e^{-3t^2}}{6} + c.$

103. (b) $I = \int \frac{dx}{\sqrt{x}(1+(\sqrt{x})^2)}$
 $\sqrt{x} = t$ रखने पर $\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\therefore I = \int \frac{2dt}{1+t^2} = 2 \tan^{-1} t + A$

अतः $I = 2 \tan^{-1} \sqrt{x} + A$; $\therefore f(x) = 2 \tan^{-1} \sqrt{x}.$

104. (d) $x^2 = t$ रखने पर $\Rightarrow dt = 2x dx$
 $\therefore$ दिया गया समाकलन
 $= \frac{1}{2} \int \cos t dt = \frac{1}{2} \sin t = \frac{1}{2} \sin x^2 + c.$

105. (b) $x^3 = t$ रखने पर $\Rightarrow dt = 3x^2 dx$
 अब $\int \frac{x^2 \tan^{-1} x^3 dx}{1+x^6} = \frac{1}{3} \int \frac{\tan^{-1} t}{1+t^2} dt$
 $z = \tan^{-1} t$ रखने पर $\Rightarrow dz = \frac{dt}{1+t^2}$
 $\therefore \frac{1}{3} \int z dz = \frac{1}{3} \cdot \frac{z^2}{2} = \frac{z^2}{6} = \frac{1}{6} (\tan^{-1} x^3)^2 + c.$

106. (a) $I = \int \frac{x^2+1}{x(x^2-1)} dx = \int \frac{1+\left(\frac{1}{x^2}\right)}{x-\left(\frac{1}{x}\right)} dx$
 $x - \frac{1}{x} = t$ रखने पर $\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$
 $\therefore I = \int \frac{dt}{t} = \log t + c = \log \frac{x^2-1}{x} + c.$

107. (a) $I = \int \frac{e^{2x}+1}{e^{2x}-1} dx = \int \frac{e^x+e^{-x}}{e^x-e^{-x}} dx$
 $e^x - e^{-x} = t$ रखने पर $\Rightarrow (e^x + e^{-x}) dx = dt$
 $\therefore I = \int \frac{dt}{t} = \log t + c = \log(e^x - e^{-x}) + c.$

108. (a) $I = \int \frac{\cos x - \sin x}{\sqrt{\sin 2x}} dx = \int \frac{\cos x - \sin x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx$
 $\sin x + \cos x = t$ रखने पर $\Rightarrow (\cos x - \sin x) dx = dt$
 $\therefore I = \int \frac{dt}{\sqrt{t^2-1}} = \cosh^{-1} t + c = \cosh^{-1}(\sin x + \cos x) + c.$

109. (a) $I = \int \left(1 + \frac{1}{x^2}\right) e^{x-\frac{1}{x}} dx$
 $x - \frac{1}{x} = t$ रखने पर $\Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt$
 $\therefore I = \int e^t dt = e^t + c = e^{x-\frac{1}{x}} + c.$

110. (a) $\int (x+3)(x^2+6x+10)^9 dx$

$= \frac{1}{2} \int (2x+6)(x^2+6x+10)^9 dx$
 $= \frac{1}{2} \frac{(x^2+6x+10)^{10}}{10} + c = \frac{1}{20} (x^2+6x+10)^{10} + c.$

111. (c) $f(x) = \frac{x}{1+x^2}, \therefore I = \int f(x) = \int \frac{x}{1+x^2} dx$
 $1+x^2 = t$ रखने पर
 $\Rightarrow 2x dx = dt \Rightarrow x dx = \frac{dt}{2}$
 $\therefore I = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \log t + c; I = \frac{1}{2} \log(1+x^2) + c.$

112. (c) $\int \sin^3 x dx = \int \sin^2 x \cdot \sin x dx$
 $= \int \sin x (1 - \cos^2 x) dx$
 $= \int \sin x dx - \int \cos^2 x \cdot \sin x dx = -\cos x + \frac{\cos^3 x}{3}.$

113. (b) $I = \int \frac{1}{x} \log x dx$
 $\log x = t$ रखने पर $\Rightarrow \frac{1}{x} dx = dt$
 $\therefore I = \int t dt = \frac{t^2}{2} + c = \frac{(\log x)^2}{2} + c.$

114. (c) $I = \int \sin^2 x \cdot \cos x dx$
 $\sin x = t$ रखने पर $\Rightarrow \cos x dx = dt$
 $\therefore I = \int t^2 dt = \frac{t^3}{3} + c = \frac{\sin^3 x}{3} + c.$

115. (b) $\int e^{x^2} \cdot x dx = \frac{1}{2} \int (2x) e^{x^2} dx$
 $x^2 = t$ रखने पर, $dt = 2x dx$
 $= \frac{1}{2} \int e^t dt = \frac{1}{2} e^t = \frac{1}{2} e^{x^2}.$

116. (c) $\int \frac{x^3}{\sqrt{1+x^4}} dx = \frac{1}{4} \int \frac{4x^3}{\sqrt{1+x^4}} dx$
 $1+x^4 = t$ रखने पर
 $\Rightarrow 4x^3 dx = dt$
 $= \frac{1}{4} \int \frac{dt}{t^{1/2}} = \frac{1}{4} \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{1}{2} \sqrt{t} = \frac{1}{2} \sqrt{1+x^4} + c.$

117. (a) $I = \int \frac{dx}{(1+e^x)\left(1+\frac{1}{e^x}\right)} = \int \frac{e^x dx}{(1+e^x)^2}$
 माना $1+e^x = t, \therefore e^x dx = dt$
 $\therefore I = \int \frac{dt}{t^2} = \int t^{-2} dt = \frac{t^{-1}}{-1} = \frac{(1+e^x)^{-1}}{-1} = \frac{-1}{1+e^x}.$

118. (c) $I = \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$
 $\sqrt{x} = t$ रखने पर, $\therefore \frac{1}{2\sqrt{x}} dx = dt$

$$\text{अतः } I = 2 \int e^t dt = 2e^t + C = 2e^{\sqrt{x}} + C.$$

$$\begin{aligned} 119. \quad (d) \quad I &= \int \frac{\sin^3 2x}{\cos^5 2x} dx \\ \Rightarrow I &= \int \frac{\sin^3 2x}{\cos^3 2x} \cdot \frac{1}{\cos^2 2x} dx = \int \tan^3 2x \cdot \sec^2 2x dx. \\ \tan 2x &= t \text{ रखने पर } \Rightarrow 2 \sec^2 2x dx = dt \\ \therefore I &= \int t^3 \frac{dt}{2} = \frac{1}{2} \cdot \frac{t^4}{4} + C = \frac{1}{8} (\tan^4 2x) + C. \end{aligned}$$

$$\begin{aligned} 120. \quad (a) \quad I &= \int x^x (1 + \log x) dx \\ x^x &= t \text{ रखने पर } \Rightarrow x^x (1 + \log x) dx = dt \\ \therefore I &= \int dt \Rightarrow I = t + C \Rightarrow I = x^x + C. \end{aligned}$$

$$\begin{aligned} 121. \quad (b) \quad I &= \int \frac{dx}{(a^2 + x^2)^{3/2}} \\ x &= a \tan \theta \text{ रखने पर } \Rightarrow dx = a \sec^2 \theta d\theta \\ \therefore I &= \int \frac{a \sec^2 \theta}{(a^2 + a^2 \tan^2 \theta)^{3/2}} d\theta = \int \frac{a \sec^2 \theta}{a^3 (\sec^2 \theta)^{3/2}} d\theta \\ \Rightarrow I &= \frac{1}{a^2} \int \frac{d\theta}{\sec \theta} = \frac{1}{a^2} \int \cos \theta d\theta = \frac{1}{a^2} \sin \theta + c \\ \Rightarrow I &= \frac{x}{a^2 (x^2 + a^2)^{1/2}} + c \end{aligned}$$

$$\begin{aligned} 122. \quad (c) \quad I &= \int \frac{e^{m \tan^{-1} x}}{1 + x^2} dx \\ m \tan^{-1} x &= t \text{ रखने पर} \\ \Rightarrow \frac{m}{1 + x^2} dx &= dt \Rightarrow \frac{dx}{1 + x^2} = \frac{dt}{m} \\ \text{अतः } I &= \frac{1}{m} \int e^t dt = \frac{1}{m} e^t + c = \frac{1}{m} e^{m \tan^{-1} x} + c. \end{aligned}$$

$$\begin{aligned} 123. \quad (d) \quad I &= \int \frac{1 + \tan^2 x}{1 - \tan^2 x} dx = \int \frac{\sec^2 x}{1 - \tan^2 x} dx \\ \tan x &= t \text{ रखने पर } \Rightarrow \sec^2 x \cdot dx = dt \Rightarrow I = \int \frac{dt}{1 - t^2} \\ &= \frac{1}{2 \times 1} \log \left| \frac{1+t}{1-t} \right| + c = \frac{1}{2} \log \left| \frac{1 + \tan x}{1 - \tan x} \right| + c. \end{aligned}$$

$$\begin{aligned} 124. \quad (d) \quad I &= \int \frac{2dx}{\sqrt{1-4x^2}} \\ 2x &= \sin \theta \text{ रखने पर } \Rightarrow 2dx = \cos \theta d\theta \\ \Rightarrow I &= \int \frac{\cos \theta d\theta}{\sqrt{1-\sin^2 \theta}} = \int \frac{\cos \theta}{\cos \theta} d\theta = \int d\theta + c = \theta + c. \\ \therefore I &= \sin^{-1}(2x) + c. \end{aligned}$$

$$\begin{aligned} 125. \quad (b) \quad I &= \int e^{3 \log x} (x^4 + 1)^{-1} dx = \int e^{\log x^3} (x^4 + 1)^{-1} dx \\ &= \frac{1}{4} \int \frac{4x^3}{(x^4 + 1)} dx = \frac{1}{4} \log(x^4 + 1) + c. \end{aligned}$$

$$126. \quad (b) \quad I = \int \frac{dx}{2\sqrt{x}(1+x)}$$

$$\sqrt{x} = t \text{ रखने पर } \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\therefore I = \int \frac{dt}{1+t^2} \tan^{-1} t + c = \tan^{-1}(\sqrt{x}) + c.$$

$$\begin{aligned} 127. \quad (c) \quad I &= \int \operatorname{cosec}^4 x dx = \int \operatorname{cosec}^2 x \cdot \operatorname{cosec}^2 x dx \\ &= \int \operatorname{cosec}^2 x (1 + \cot^2 x) dx \\ &= \int \operatorname{cosec}^2 x dx + \int \cot^2 x \cdot \operatorname{cosec}^2 x dx \\ &= -\cot x - \frac{\cot^3 x}{3} + c. \end{aligned}$$

$$\begin{aligned} 128. \quad (b) \quad I &= \int x e^{x^2} dx = \frac{1}{2} \int e^t dt = \frac{e^{x^2}}{2} + c. \\ \{x^2 = t \text{ रखने पर } \Rightarrow 2x dx = dt\} \end{aligned}$$

$$\begin{aligned} 129. \quad (b) \quad \int f(x) dx &= g(x) \\ \int f^{-1}(x) dx &= f^{-1}(x) \int dx - \int \left\{ \frac{d}{dx} f^{-1}(x) \right\} \int dx \Bigg\} dx \\ &= x f^{-1}(x) - \int x \frac{d}{dx} f^{-1}(x) dx \\ &= x f^{-1}(x) - \int x d\{f^{-1}(x)\} \end{aligned}$$

$$\begin{aligned} \text{माना } f^{-1}(x) &= t \Rightarrow x = f(t) \text{ और } d\{f^{-1}(x)\} = dt \\ &= x f^{-1}(x) - \int f(t) dt = x f^{-1}(x) - g(t) = x f^{-1}(x) - g\{f^{-1}(x)\}. \end{aligned}$$

ट्रिक : $f(x) = x^2$ रखने पर, विकल्प (b) सही हैं

$$\begin{aligned} 130. \quad (c) \quad e^x + 1 &= t \text{ रखने पर } \Rightarrow e^x dx = dt \\ \therefore \int \frac{e^x}{e^x + 1} dx &= \int \frac{dt}{t} = \log t + c = \log(e^x + 1) + c. \end{aligned}$$

$$\begin{aligned} 131. \quad (d) \quad \text{माना } \sin x + \cos x &= t \Rightarrow (\cos x - \sin x) dx = dt \\ \Rightarrow -(\sin x - \cos x) dx &= dt \\ \therefore \int \frac{\sin x - \cos x}{\sin x + \cos x} dx &= -\int \frac{dt}{t} = -\log t + c = \log \left(\frac{1}{t} \right) + c \\ \text{अतः } \int \frac{\sin x - \cos x}{\sin x + \cos x} dx &= \log \left(\frac{1}{\sin x + \cos x} \right) + c. \end{aligned}$$

$$\begin{aligned} 132. \quad (b) \quad \tan^{-1} x &= t \text{ रखने पर } \Rightarrow \frac{1}{1+x^2} dx = dt \\ \therefore \int \frac{(\tan^{-1} x)^3}{1+x^2} dx &= \int t^3 dt = \frac{t^4}{4} + c = \frac{(\tan^{-1} x)^4}{4} + c. \end{aligned}$$

$$\begin{aligned} 133. \quad (d) \quad \int \sqrt{\frac{1-x}{1+x}} dx &= \int \frac{1-x}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{x dx}{\sqrt{1-x^2}} \\ \text{हल करने पर अभीष्ट परिणाम प्राप्त होता है।} \end{aligned}$$

$$\begin{aligned} 134. \quad (b) \quad \int \frac{\sqrt{x}}{1+x} dx &= \int \frac{\sqrt{x} \sqrt{x}}{\sqrt{x}(1+x)} dx \\ &= \int \frac{x+1}{\sqrt{x}(x+1)} dx - \int \frac{1}{\sqrt{x}(x+1)} dx \\ &= \int \frac{1}{\sqrt{x}} dx - \int \frac{1}{\sqrt{x}(x+1)} dx \\ &= 2x^{1/2} - 2 \tan^{-1} \sqrt{x} + c = 2(\sqrt{x} - \tan^{-1} \sqrt{x}) + c. \end{aligned}$$

$$135. \quad (b) \quad \int \frac{\sin x dx}{\sin x - \cos x} = \frac{1}{2} \int \frac{2 \sin x}{\sin x - \cos x} dx$$

$$\begin{aligned}
 &= \frac{1}{2} \int \frac{(\sin x - \cos x + \sin x + \cos x)}{\sin x - \cos x} dx \\
 &= \frac{1}{2} \int \left(1 + \frac{\sin x + \cos x}{\sin x - \cos x} \right) dx \\
 &= \frac{1}{2} [x + \log(\sin x - \cos x)] + c.
 \end{aligned}$$

136. (c) $I = \int \sqrt{\frac{1+x}{1-x}} dx = \int \frac{1+x}{\sqrt{1-x^2}} dx$

$$= \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x}{\sqrt{1-x^2}} dx = \sin^{-1} x - \sqrt{1-x^2} + c.$$

137. (d) $\int \frac{x}{\sqrt{4-x^4}} dx = \int \frac{x}{\sqrt{2^2-(x^2)^2}} dx$

$x^2 = t$ रखने पर $\Rightarrow 2x dx = dt$,

अतः अभीष्ट समाकलन $= \frac{1}{2} \sin^{-1} \frac{x^2}{2}$.

138. (c) $I = \int \frac{\sin x}{3+4\cos^2 x} dx$

$\cos x = t$ रखने पर $\Rightarrow -\sin x dx = dt$

$$\therefore I = \int \frac{-dt}{3+4t^2} = \frac{-1}{4} \int \frac{dt}{t^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow I = -\frac{1}{4 \cdot \frac{\sqrt{3}}{2}} \cdot \tan^{-1} \frac{t}{\left(\frac{\sqrt{3}}{2}\right)} + c = \frac{-1}{2\sqrt{3}} \tan^{-1} \frac{2t}{\sqrt{3}} + c$$

$$\Rightarrow I = \frac{-1}{2\sqrt{3}} \tan^{-1} \left(\frac{2\cos x}{\sqrt{3}} \right) + c.$$

139. (d) हमें ज्ञात है कि $I = \int \frac{dx}{\sqrt{x(x+9)}}$

$\sqrt{x} = t$ रखकर दोनों तरफ वर्ग करने पर, $x = t^2$

$\Rightarrow dx = 2t dt$

$$\therefore I = 2 \int \frac{dt}{t^2+3^2} = \frac{2}{3} \tan^{-1} \left(\frac{t}{3} \right) \Rightarrow I = \frac{2}{3} \tan^{-1} \left(\frac{\sqrt{x}}{3} \right).$$

140. (b) $\int \left\{ \frac{\log x - 1}{1 + (\log x)^2} \right\}^2 dx$

$\log x = t$ रखने पर, $\Rightarrow dx = e^t dt$

$$\therefore \text{अभीष्ट समाकलन} = \int e^t \left[\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2} \right] dt$$

$$[\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + c]$$

$$= \frac{e^t}{1+t^2} + C = \frac{x}{1+(\log x)^2} + C.$$

141. (d) $I = \int \frac{\sin 2x}{1+\cos^2 x} dx = \int \frac{2 \sin x \cos x}{1+\cos^2 x} dx$

$1+\cos^2 x = t$ रखने पर $\Rightarrow -2 \sin x \cos x dx = dt$

$$\begin{aligned}
 &\Rightarrow \sin 2x = -dt. \\
 \therefore I &= -\int \left(\frac{dt}{t} \right) = -\log t + c \\
 &= -\log(1+\cos^2 x) + c.
 \end{aligned}$$

142. (b) $\int \frac{1+\cos 4x}{\cot x - \tan x} dx = \int \frac{2\cos^2 2x}{\cos^2 x - \sin^2 x} \cdot \sin x \cos x dx$

$$= \int \cos 2x \sin 2x dx = -\frac{1}{8} \cos 4x + c,$$

$$\therefore k = -\frac{1}{8}.$$

143. (a) $\frac{x^4 dx}{x+x^5} = \int \frac{(x^4+1)dx}{x+x^5} - \int \frac{dx}{x+x^5}$

$$= \int \frac{(x^4+1)dx}{x(1+x^4)} - \int \frac{dx}{x(x^4+1)} = \int \frac{dx}{x} - \int \frac{dx}{x+x^5}$$

$$= \log x - f(x) - c_2 + c_1 = \log x - f(x) + c,$$

जहाँ $c_1 - c_2 = c = \text{एक नया अचर}.$

144. (b) $f(x) = \int \frac{x^2 dx}{(1+x^2)(1+\sqrt{1+x^2})}$

माना $x = \tan \theta, dx = \sec^2 \theta d\theta = (1+x^2) d\theta$

$$f(x) = \int \frac{x^2 dx}{(1+x^2)(1+\sqrt{1+x^2})} = \int \frac{\tan^2 \theta \sec^2 \theta d\theta}{\sec^2 \theta (1+\sec \theta)}$$

$$= \int \frac{\tan^2 \theta d\theta}{1+\sec \theta} = \int \frac{\sin^2 \theta d\theta}{\cos \theta (1+\cos \theta)} = \int \frac{1-\cos^2 \theta d\theta}{\cos \theta (1+\cos \theta)}$$

$$= \int \frac{(1-\cos \theta) d\theta}{\cos \theta} = \int \sec \theta d\theta - \int d\theta$$

$$= \log(x + \sqrt{1+x^2}) - \tan^{-1} x + c$$

$$f(0) = \log(0 + \sqrt{1+0}) - \tan^{-1}(0) + c$$

$$0 = \log 1 - 0 + c \Rightarrow c = 0$$

$$f(1) = \log(1 + \sqrt{1+1^2}) - \tan^{-1}(1) = \log(1 + \sqrt{2}) - \frac{\pi}{4}.$$

145. (a) $A = \int \sqrt{e^x - 1} dx$

माना $e^x - 1 = t^2 \Rightarrow e^x dx = 2t dt$

$$\Rightarrow dx = \frac{2t}{t^2+1} dt$$

$$\therefore A = \int t \frac{2t}{t^2+1} dt = \int \frac{2t^2}{t^2+1} dt$$

$$= \int \frac{2(t^2+1)-2}{t^2+1} dt = \int 2dt - \int \frac{2dt}{t^2+1}$$

$$= 2t - 2 \tan^{-1} t + c = 2\sqrt{e^x - 1} - 2 \tan^{-1} \sqrt{e^x - 1} + c.$$

146. (a) माना $I = \int \frac{dx}{\sin(x-a)\sin(x-b)}$

$$\begin{aligned}
&= \frac{1}{\sin(a-b)} \int \frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)} dx \\
&= \frac{1}{\sin(a-b)} \int \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} dx \\
&= \frac{1}{\sin(a-b)} \left[\int \cot(x-a) dx - \int \cot(x-b) dx \right] \\
&= \frac{1}{\sin(a-b)} [\log \sin(x-a) - \log \sin(x-b)] + c \\
\Rightarrow I &= \frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c .
\end{aligned}$$

147. (c) माना $I = \int \frac{\sin \theta + \cos \theta}{\sqrt{2 \sin \theta \cos \theta}} d\theta$

$$\therefore I = \int \frac{\sin \theta + \cos \theta}{\sqrt{1 - (1 - 2 \sin \theta \cos \theta)}} d\theta$$

$$\Rightarrow I = \int \frac{(\sin \theta + \cos \theta)}{\sqrt{1 - (\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta)}} d\theta$$

$$\Rightarrow I = \int \frac{\sin \theta + \cos \theta}{\sqrt{1 - (\sin \theta - \cos \theta)^2}} d\theta$$

माना $(\sin \theta - \cos \theta) = t \Rightarrow (\cos \theta + \sin \theta) d\theta = dt$

$$\therefore I = \int \frac{dt}{\sqrt{1 - t^2}} = \sin^{-1}(t) + c = \sin^{-1}(\sin \theta - \cos \theta) + c.$$

148. (c) $m + n = -\frac{3}{7} + \left(\frac{-11}{7}\right) = -2$, (ऋणात्मक पूर्णांक)

अब, $I = \int \cos^{-3/7} x (\sin^{(-2+3/7)} x) dx$

$$= \int \cos^{-3/7} x \sin^{-2} x \sin^{3/7} x dx$$

$$I = \int \frac{\operatorname{cosec}^2 x}{\left(\frac{\cos^{3/7} x}{\sin^{3/7} x}\right)} dx = \int \frac{\operatorname{cosec}^2 x}{\cot^{3/7} x} dx$$

$\cot x = t$ रखने पर $\Rightarrow -\operatorname{cosec}^2 x dx = dt$

$$\therefore I = -\int \frac{dt}{t^{3/7}} = -\frac{t^{-\frac{3}{7}+1}}{-\frac{3}{7}+1} + c = -\frac{7}{4} t^{4/7} + c$$

अतः $I = -\frac{7}{4} \cot^{4/7} x + c = -\frac{7}{4} \tan^{-4/7} x + c.$

खण्डशः समाकलन $\left(\int e^x (f(x) + f'(x)) dx \right)$ तथा $\int e^{kx} (kf(x) + f'(x)) dx$ के रूप का समाकलन

1. (d) $\int x \sec^2 x dx = x \tan x - \int \tan x dx$

$$= x \tan x + \log(\cos x) + c.$$

2. (c) माना $I = \int \sin(\log x) dx$

$\log x = t$ रखने पर $\Rightarrow x = e^t \Rightarrow dx = e^t dt$, तब

$$I = \int \sin t \cdot e^t dt = \sin t \cdot e^t - \int e^t \cdot \cos t dt$$

$$= \sin t \cdot e^t - [\cos t \cdot e^t + \int e^t \cdot \sin t dt]$$

$$\Rightarrow 2I = \sin t \cdot e^t - \cos t \cdot e^t$$

$$\Rightarrow I = \int \sin(\log x) dx = \frac{1}{2} x [\sin(\log x) - \cos(\log x)].$$

3. (a) $\therefore \int x \sin x dx = -x \cos x + A$

$$\Rightarrow -x \cos x + \sin x + \text{अचर} = -x \cos x + A$$

तुलना करने पर, $A = \sin x + \text{अचर}.$

4. (b) $\int x \log x dx = \frac{x^2}{2} \log x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx + c = \frac{x^2 \log x}{2} - \frac{x^2}{4} + c.$

5. (a) $\int x \cos x dx = x \sin x - \int \sin x dx + c = x \sin x + \cos x + c.$

6. (b) $\int x \cos^2 x dx = \frac{1}{2} \int x(1 + \cos 2x) dx$

$$= \frac{x^2}{4} + \frac{1}{2} \left[\frac{x \sin 2x}{2} - \int \frac{\sin 2x}{2} dx \right] + c$$

$$= \frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + c.$$

7. (b) $\int \tan^{-1} x dx = x \tan^{-1} x - \int \frac{x}{1+x^2} dx + c$

$$= x \tan^{-1} x - \frac{1}{2} \log(1+x^2) + c.$$

नोट : विद्यार्थी इस प्रश्न को सूत्र मानकर याद रखें।

8. (a) $\int x \cdot \tan^{-1} x dx = \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2 + 1 - 1}{1+x^2} dx$

$$= \frac{x^2}{2} \tan^{-1} x - \frac{1}{2} x + \frac{1}{2} \tan^{-1} x + c$$

$$= \frac{1}{2} \tan^{-1} x \cdot (x^2 + 1) - \frac{1}{2} x + c.$$

9. (b) $\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx = \int \log(\log x) dx + \int \frac{1}{(\log x)^2} dx$

$$= x \log(\log x) - \int \frac{x}{x \log x} dx + \int \frac{1}{(\log x)^2} dx$$

$$= x \log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx$$

$$= x \log(\log x) - \frac{x}{\log x} + c.$$

10. (d) $\int \sin(\log x) dx + \int \cos(\log x) dx$

$$= x \sin(\log x) - \int \frac{x \cos(\log x)}{x} dx + \int \cos(\log x) dx + c$$

$$= x \sin(\log x) + c.$$

11. (b) $\int \frac{\log x}{x^3} dx = \int x^{-3} \log x dx$

$$= -\frac{\log x}{2x^2} + \int \frac{1}{x} \cdot \frac{1}{2x^2} + c = -\frac{\log x}{2x^2} + \frac{1}{2} \cdot \frac{x^{-2}}{-2} + c$$

$$= -\frac{\log x}{2x^2} - \frac{1}{4x^2} + c = -\frac{1}{4x^2} (2 \log x + 1) + c.$$

12. (d) माना $I = \int e^{-2x} \sin 3x dx$

$$= -\frac{e^{-2x} \cos 3x}{3} - \int \frac{2e^{-2x} \cos 3x}{3} dx$$

$$= -\frac{e^{-2x} \cos 3x}{3} - \frac{2}{3} \left[\frac{e^{-2x} \sin 3x}{3} + \int \frac{2e^{-2x} \sin 3x}{3} dx \right]$$

$$\Rightarrow I = -\frac{e^{-2x} \cos 3x}{3} - \frac{2e^{-2x} \sin 3x}{9} - \frac{4}{9} I$$

$$\Rightarrow \frac{13}{9} I = -e^{-2x} \left[\frac{3 \cos 3x + 2 \sin 3x}{9} \right]$$

$$\text{अतः, } I = -\frac{1}{13} e^{-2x} [3 \cos 3x + 2 \sin 3x].$$

$$\begin{aligned} 13. \quad (b) \quad \int x \sin x \sec^3 x \, dx &= \int x \sin x \frac{1}{\cos^3 x} \, dx \\ &= \int x \tan x \cdot \sec^2 x \, dx \end{aligned}$$

$$\tan x = t \text{ रखने पर, } \sec^2 x \, dx = dt \text{ और } x = \tan^{-1} t,$$

$$\begin{aligned} \therefore \int \tan^{-1} t \cdot t \, dt &= \frac{x \tan^2 x}{2} - \frac{1}{2} t + \frac{1}{2} \tan^{-1} t \\ &= \frac{x(\sec^2 x - 1)}{2} - \frac{1}{2} \tan x + \frac{1}{2} x = \frac{1}{2} [x \sec^2 x - \tan x] + c. \end{aligned}$$

$$\begin{aligned} 14. \quad (d) \quad \int x \sin^2 x \, dx &= \int x \cdot \frac{(1 - \cos 2x)}{2} \, dx \\ &= \frac{1}{2} \left[\int x \, dx - \int x \cdot \cos 2x \, dx \right] = \frac{x^2}{4} - \frac{x}{4} \sin 2x - \frac{1}{8} \cos 2x + c. \end{aligned}$$

$$\begin{aligned} 15. \quad (a) \quad \int e^{2x+\log x} \, dx &= \int x e^{2x} \, dx \\ &= \frac{x e^{2x}}{2} - \int \frac{1}{2} e^{2x} \, dx + c = \frac{e^{2x}}{4} (2x - 1) + c. \end{aligned}$$

$$\begin{aligned} 16. \quad (a) \quad \int \log(x+1) \, dx &= x \log(x+1) - \int \frac{x}{x+1} \, dx + c \\ &= x \log(x+1) - x + \log(x+1) + c = (x+1) \log(x+1) - x + c. \end{aligned}$$

$$\begin{aligned} 17. \quad (d) \quad \int \log(x^2 + x) \, dx &= \int \log x \, dx + \int \log(x+1) \, dx \\ &= x \log x - x + x \log(x+1) - x + \log(x+1) \\ &= x \{(\log x + \log(x+1))\} - 2x + \log(x+1) \\ &= x \log(x^2 + x) - 2x + \log(x+1) \\ \text{दिये गये समाकलन से तुलना करने पर,} \\ A &= -2x + \log(x+1). \end{aligned}$$

$$\begin{aligned} 18. \quad (b) \quad \text{माना } I &= \int x^2 \sin 2x \, dx = \frac{-x^2 \cos 2x}{2} + \int \frac{2x \cos 2x}{2} \, dx + c \\ &= -\frac{x^2 \cos 2x}{2} + \frac{x \sin 2x}{2} + \frac{\cos 2x}{4} + c. \end{aligned}$$

$$19. \quad (b) \quad \int \log x \, dx = x \log x - \int x \cdot \frac{1}{x} \, dx + c = x \log x - x + c$$

नोट : विद्यार्थी इस प्रश्न को सूत्र समझकर याद रखें।

$$\begin{aligned} 20. \quad (d) \quad \int \log_{10} x \, dx &= \int \frac{\log x}{\log 10} \, dx \\ &= \frac{1}{\log 10} [x \log x - x] = x [\log_{10} x - \log_{10} e] + c. \end{aligned}$$

$$21. \quad (c) \quad \int \frac{1}{\log_x e} \, dx = \int \log_e x \, dx = x \log x - x + c$$

$$= x(\log_e x - \log_e e) + c = x \log_e \left(\frac{x}{e} \right) + c.$$

$$22. \quad (a) \quad \text{दिया गया है, } \int e^x \sin x \, dx = \frac{1}{2} e^x a + c \quad \dots(i)$$

$$\begin{aligned} \text{माना } I &= \int e^x \sin x \, dx = -e^x \cos x + \int e^x \cos x \, dx + c \\ &= -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx + c \\ \Rightarrow 2I &= e^x (-\cos x + \sin x) + c. \end{aligned}$$

$$\text{अतः (i) से, } \frac{1}{2} e^x a = \frac{1}{2} e^x (\sin x - \cos x) \Rightarrow a = \sin x - \cos x.$$

$$\begin{aligned} 23. \quad (d) \quad \int x^n \log x \, dx &= \log x \cdot \frac{x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1} \cdot \frac{1}{x} \, dx \\ &= \frac{x^{n+1}}{n+1} \log x - \frac{x^{n+1}}{(n+1)^2} + c = \frac{x^{n+1}}{n+1} \left[\log x - \frac{1}{n+1} \right] + c. \end{aligned}$$

$$\begin{aligned} 24. \quad (a) \quad I &= \int \log x (\log x + 2) \, dx \\ \log x = t \text{ रखने पर } &\Rightarrow e^t = x \Rightarrow e^t \, dt = dx \\ \text{तब } I &= \int t(t+2) e^t \, dt = t^2 \cdot e^t + c = x(\log x)^2 + c. \end{aligned}$$

$$\begin{aligned} 25. \quad (b) \quad \int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2} \right] \, dx &= \int \frac{1}{\log x} \, dx - \int \frac{1}{(\log x)^2} \, dx \\ &= \frac{x}{\log x} + \int \frac{1}{(\log x)^2} \cdot \frac{1}{x} \, dx - \int \frac{1}{(\log x)^2} \, dx + c = \frac{x}{\log x} + c. \end{aligned}$$

$$\begin{aligned} 26. \quad (c) \quad \int \frac{\log x}{(1 + \log x)^2} \, dx \\ 1 + \log x = t \text{ रखने पर } &\Rightarrow \frac{1}{x} \, dx = dt \\ \Rightarrow dx &= x \, dt = e^{t-1} \, dt, \\ \therefore \int \frac{(t-1)e^{t-1}}{t^2} \, dt &= \int e^{t-1} \left(\frac{1}{t} - \frac{1}{t^2} \right) \, dt = \frac{e^{t-1}}{t} = \frac{x}{1 + \log x} + c. \end{aligned}$$

$$\begin{aligned} 27. \quad (d) \quad \int \left(\frac{2 + \sin 2x}{1 + \cos 2x} \right) e^x \, dx &= \int \left(\frac{2e^x}{1 + \cos 2x} \right) \, dx + \int \frac{e^x \sin 2x}{1 + \cos 2x} \, dx \\ &= \int e^x \sec^2 x \, dx + \int e^x \tan x \, dx = e^x \tan x + c. \end{aligned}$$

$$\begin{aligned} 28. \quad (b) \quad \text{माना } I &= \int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx + c \\ &= e^x \sin x - e^x \cos x - \int e^x \sin x \, dx + c \\ \Rightarrow 2I &= e^x (\sin x - \cos x) + c \Rightarrow I = \frac{1}{2} e^x (\sin x - \cos x) + c. \end{aligned}$$

$$\begin{aligned} 29. \quad (a) \quad \int (1 - x^2) \log x \, dx &= \int \log x \, dx - \int x^2 \log x \, dx \\ &= x(\log x - 1) - \frac{x^3 \log x}{3} + \frac{x^3}{9} + c \\ &= \left(x - \frac{x^3}{3} \right) \log x - \left(x - \frac{x^3}{9} \right) + c. \end{aligned}$$

$$30. \quad (d) \quad \int e^{2x} (-\sin x + 2 \cos x) \, dx$$

$$= -\int e^{2x} \sin x \, dx + 2 \int e^{2x} \cos x \, dx$$

$$= e^{2x} \cos x - 2 \int e^{2x} \cos x \, dx + 2 \int e^{2x} \cos x \, dx + c$$

$$= e^{2x} \cos x + c.$$

$$\text{वैकल्पिक : } \int e^{2x} (2 \cos x - \sin x) \, dx = e^{2x} \cos x + c$$

$$\left\{ \because \int e^{kx} \{k f(x) + f'(x)\} \, dx = e^{kx} f(x) + c \right\}.$$

$$31. \quad (c) \quad \int [f(x)g''(x) - f''(x)g(x)] \, dx$$

$$= \int f(x)g''(x) \, dx - \int f''(x)g(x) \, dx$$

$$= \left(f(x)g'(x) - \int f'(x)g'(x) \, dx \right) - \left(g(x)f'(x) - \int g'(x)f'(x) \, dx \right)$$

$$= f(x)g'(x) - f'(x)g(x).$$

$$32. \quad (d) \quad I = \int x \sin kx \, dx = \frac{-x \cos kx}{k} + \frac{\sin kx}{k^2} + c.$$

$$33. \quad (b) \quad \int x e^x \, dx = x \cdot e^x - \int 1 \cdot e^x \, dx$$

$$= x e^x - e^x + c = e^x (x - 1) + c.$$

$$34. \quad (c) \quad x^2 = t \text{ रखने पर } \Rightarrow 2x \, dx = dt, \text{ तब}$$

$$\int x^3 e^{x^2} \, dx = \frac{1}{2} \int t e^t \, dt$$

$$= \frac{1}{2} [t e^t - e^t] + c = \frac{1}{2} e^{x^2} (x^2 - 1) + c.$$

$$35. \quad (a) \quad \int \frac{\log x}{(x+1)^2} \, dx = \int \log x (x+1)^{-2} \, dx$$

$$= \log x \cdot \left\{ -(x+1)^{-1} \right\} - \int \frac{1}{x} \cdot \{ -(x+1)^{-1} \} \, dx$$

$$= \frac{-\log x}{(x+1)} + \int \frac{1}{x(x+1)} \, dx = \frac{-\log x}{(x+1)} + \int \left[\frac{1}{x} - \frac{1}{x+1} \right] \, dx$$

$$= \frac{-\log x}{x+1} + \log x - \log(x+1).$$

$$36. \quad (c) \quad \int x e^{2x} \, dx = \frac{x e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} \, dx = \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + c$$

$$= e^{2x} \left(\frac{2x-1}{4} \right) + c \Rightarrow f(x) = \frac{(2x-1)}{4}.$$

$$37. \quad (c) \quad \frac{d}{dx} f(x) = x \cos x + \sin x$$

$$\Rightarrow f(x) = \int (x \cos x + \sin x) \, dx = x \sin x + c$$

$$\because f(0) = 2 \Rightarrow c = 2; \therefore f(x) = x \sin x + 2.$$

$$38. \quad (b) \quad I = \int \cos^{-1} \left(\frac{1}{x} \right) \, dx = \int \sec^{-1} x \cdot 1 \, dx$$

$$= \sec^{-1} x \int dx - \int \left[\frac{d}{dx} \sec^{-1} x \int dx \right] \, dx$$

$$= x \sec^{-1} x - \int \frac{1}{x \sqrt{x^2-1}} \cdot x \, dx$$

$$= x \sec^{-1} x - \int \frac{1}{\sqrt{x^2-1}} \, dx$$

$$= x \sec^{-1} x - \cosh^{-1} x + c.$$

$$39. \quad (b) \quad I = \int x^3 \log x \, dx = \frac{x^4}{4} \log x - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx + c$$

$$= \frac{x^4}{4} \log x - \int \frac{x^3}{4} \, dx = \frac{x^4}{4} \log x - \frac{x^4}{16} + c$$

$$= \frac{1}{16} [4x^4 \log x - x^4] + c$$

$$40. \quad (a) \quad \text{माना } I = \int \cos(\log_e x) \, dx = \int \cos(\log_e x) \cdot 1 \, dx$$

$$I = \cos(\log_e x) \cdot x - \int \frac{-\sin(\log_e x)}{x} \cdot x \, dx$$

$$= x \cos(\log_e x) + \int \sin(\log_e x) \, dx$$

$$= x \cos(\log_e x) + \int \sin(\log_e x) \cdot 1 \, dx$$

$$= x \cos(\log_e x) + \sin(\log_e x) \cdot x - \int \frac{\cos(\log_e x)}{x} \cdot x \, dx$$

$$= x \cos(\log_e x) + x \sin(\log_e x) - I$$

$$\Rightarrow 2I = x [\cos(\log_e x) + \sin(\log_e x)]$$

$$\Rightarrow I = \frac{x}{2} [\cos(\log_e x) + \sin(\log_e x)].$$

$$41. \quad (c) \quad \int e^x (1 + \tan x) \sec x \, dx = \int e^x \sec x \, dx + \int e^x \tan x \sec x \, dx$$

$$= e^x \sec x - \int e^x \sec x \tan x \, dx + \int e^x \sec x \tan x \, dx$$

$$= e^x \sec x + c.$$

$$\text{वैकल्पिक : } \int e^x (\sec x + \sec x \tan x) \, dx = e^x \sec x + c$$

$$\text{स्पष्टतः यह } \int e^x \{f(x) + f'(x)\} \, dx \text{ प्रकार का है।}$$

$$42. \quad (c) \quad \int \frac{x e^x}{(1+x)^2} \, dx = \int \frac{(x+1-1)}{(1+x)^2} e^x \, dx$$

$$= \int e^x \left(\frac{1}{1+x} - \frac{1}{(1+x)^2} \right) \, dx = \frac{e^x}{1+x} + c.$$

$$43. \quad (a) \quad \int e^x [\tan x - \log(\cos x)] \, dx = \int e^x [\tan x + \log(\sec x)] \, dx$$

$$= e^x \log(\sec x) + c \quad \left\{ \because \int e^x \{f(x) + f'(x)\} \, dx = e^x f(x) + c \right\}.$$

$$44. \quad (a) \quad \int e^x \sin x (\sin x + 2 \cos x) \, dx$$

$$= \int e^x \sin^2 x \, dx + \int e^x 2 \sin x \cos x \, dx$$

$$= \int e^x \sin^2 x \, dx + \int e^x \sin 2x \, dx$$

$$= e^x \sin^2 x - \int e^x \sin 2x dx + \int e^x \sin 2x dx + c$$

$$= e^x \sin^2 x + c.$$

$$\text{वैकल्पिक : } \int e^x (\sin^2 x + \sin 2x) dx = e^x \sin^2 x + c.$$

$$45. \quad (c) \quad \int e^{2x} \frac{1+\sin 2x}{1+\cos 2x} dx = \int e^{2x} \left[\frac{1}{1+\cos 2x} + \frac{\sin 2x}{1+\cos 2x} \right] dx$$

$$= \int e^{2x} \left[\frac{\sec^2 x}{2} + \tan x \right] dx$$

$$= \frac{1}{2} \int e^{2x} \sec^2 x dx + \int e^{2x} \tan x dx$$

$$= \frac{e^{2x} \tan x}{2} - \int \frac{e^{2x} \sec^2 x}{2} dx + \int \frac{e^{2x} \sec^2 x}{2} dx + c$$

$$= \frac{e^{2x} \tan x}{2} + c.$$

$$46. \quad (a) \quad \int \frac{e^x(x-1)}{x^2} dx = \int e^x \left[\frac{1}{x} - \frac{1}{x^2} \right] dx = \frac{e^x}{x} + c.$$

$$47. \quad (d) \quad \int e^x \left[\frac{1+x \log x}{x} \right] dx = \int e^x \left(\log x + \frac{1}{x} \right) dx = e^x \log x + c.$$

$$48. \quad (c) \quad \int e^x \left[\sin^{-1} \frac{x}{a} + \frac{1}{\sqrt{a^2 - x^2}} \right] dx$$

$$= \int e^x \sin^{-1} \frac{x}{a} dx + \int \frac{e^x}{\sqrt{a^2 - x^2}} dx$$

$$= e^x \sin^{-1} \frac{x}{a} - \int \frac{e^x}{\sqrt{a^2 - x^2}} dx + \int \frac{e^x}{\sqrt{a^2 - x^2}} dx + c$$

$$= e^x \sin^{-1} \frac{x}{a} + c.$$

$$49. \quad (a) \quad \text{माना } I = \int \frac{e^x(x^2+1)}{(x+1)^2} dx = \int \frac{e^x(x^2-1+2)}{(x+1)^2} dx$$

$$= \int e^x \left[\frac{x-1}{x+1} + \frac{2}{(x+1)^2} \right] dx = \int e^x [f(x) + f'(x)] dx,$$

जहाँ $f(x) = \frac{x-1}{x+1}$ और $f'(x)$

$$\text{अतः } I = \frac{2}{(x+1)^2} = e^x \left(\frac{x-1}{x+1} \right) + c.$$

$$50. \quad (c) \quad \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx = e^x \frac{1}{x} + c$$

$$\left(\because \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c \right).$$

$$51. \quad (c) \quad I = \int e^x (1 + \tan x + \tan^2 x) dx$$

$$\Rightarrow \int e^x (1 + \tan x + \tan^2 x) dx = \int e^x (\tan x + \sec^2 x) dx.$$

$$\Rightarrow I = e^x \tan x + c, \quad \left(\because \int e^x [f(x) + f'(x)] dx = e^x f(x) + c \right).$$

$$52. \quad (b) \quad I = \int e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx = \int e^x \left(\frac{1 - \sin x}{2 \sin^2(x/2)} \right) dx$$

$$\Rightarrow I = \int e^x \left(\frac{1}{2} \operatorname{cosec}^2 \frac{x}{2} - \cot \frac{x}{2} \right) dx,$$

$$\left(\because \int e^x (f(x) + f'(x)) = e^x f(x) + c \right)$$

$$\therefore I = e^x \left(-\cot \frac{x}{2} \right) + c = -e^x \cot \frac{x}{2} + c.$$

$$53. \quad (c) \quad I = \int \frac{(x+3)e^x}{(x+4)^2} dx = \int \frac{(x+4-1)e^x dx}{(x+4)^2}$$

$$\Rightarrow I = \int e^x \left(\frac{1}{x+4} - \frac{1}{(x+4)^2} \right) dx$$

$$\therefore I = \frac{e^x}{x+4} + c.$$

$$54. \quad (a) \quad I = \int \left(\frac{x+2}{x+4} \right)^2 e^x dx = \int e^x \left[\frac{x^2+4x+4}{(x+4)^2} \right] dx$$

$$\Rightarrow I = \int e^x \left[\frac{x(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2} \right] dx$$

$$= e^x \left[\frac{x}{x+4} + \frac{4}{(x+4)^2} \right] dx = e^x \left(\frac{x}{x+4} \right) + c.$$

$$55. \quad (a) \quad \text{यह आधारभूत गुणधर्म है।}$$

$$56. \quad (c) \quad I = \int e^x (1 - \cot x + \cot^2 x) dx = \int e^x (-\cot x + \operatorname{cosec}^2 x) dx$$

$$= e^x (-\cot x) + c = -e^x \cot x + c.$$

$$57. \quad (d) \quad I = \int \sin^{-1} x \cdot 1 dx$$

$$I = \sin^{-1} x \cdot x - \int \frac{1}{\sqrt{1-x^2}} \cdot x dx$$

द्वितीय समाकलन में $1-x^2 = t^2 \Rightarrow -2x dx = 2t dt$ रखकर
हल करने पर, $I = x \sin^{-1} x + \sqrt{1-x^2} + c.$

$$58. \quad (b) \quad \int \frac{x - \sin x}{1 - \cos x} dx = \int \frac{x}{1 - \cos x} dx - \int \frac{\sin x}{1 - \cos x} dx$$

$$= \frac{1}{2} \int x \operatorname{cosec}^2 \left(\frac{x}{2} \right) dx - \int \frac{2 \sin(x/2) \cos(x/2)}{2 \sin^2(x/2)} dx$$

$$= \frac{1}{2} \int x \operatorname{cosec}^2 \left(\frac{x}{2} \right) dx - \int \cot \left(\frac{x}{2} \right) dx = -x \cot \left(\frac{x}{2} \right) + c.$$

$$59. \quad (b) \quad \int \frac{x + \sin x}{1 + \cos x} dx = \frac{1}{2} \int x \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx$$

$$= \frac{1}{2} \int \frac{x \tan \frac{x}{2}}{\frac{1}{2}} dx - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx = x \tan \frac{x}{2} + c.$$

$$\text{ट्रिक : जॉच द्वारा, } \frac{d}{dx} \left\{ x \tan \frac{x}{2} + c \right\}$$

$$= \frac{x}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} = \frac{1}{2} \left[\frac{x}{\cos^2 \frac{x}{2}} + \frac{2 \sin \frac{x}{2}}{\cos \frac{x}{2}} \right] = \frac{x + \sin x}{1 + \cos x}.$$

$$60. \quad (c) \quad I = \int e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx = \int e^x \left[\frac{1 + 2 \sin(x/2) \cos(x/2)}{2 \cos^2(x/2)} \right] dx$$

$$I = \int e^x \left[\frac{1}{2} \sec^2(x/2) + \tan(x/2) \right] dx = e^x \cdot \tan(x/2) + c$$

$$\{ \because \int e^x [f(x) + f'(x)] dx = e^x \cdot f(x) + c \}$$

$$61. \quad (b) \quad I = \int \sqrt{x} \cdot e^{\sqrt{x}} dx. \text{ माना } x = t^2 \Rightarrow dx = 2t dt$$

$$\therefore I = 2 \int t^2 \cdot e^t dt \Rightarrow I = 2(t^2 \cdot e^t - 2te^t + 2e^t) + c$$

$$\Rightarrow I = 2[x \cdot e^{\sqrt{x}} - 2\sqrt{x} \cdot e^{\sqrt{x}} + 2e^{\sqrt{x}}] + c$$

$$\text{अर्थात्, } I = e^{\sqrt{x}} [2x - 4\sqrt{x} + 4] + c.$$

$$62. \quad (a) \quad \text{माना } I = \int 32x^3 (\log x)^2 dx = 32 \int x^3 (\log x)^2 dx$$

$$= 32 \left[(\log x)^2 \int x^3 dx - \int \left(\frac{d}{dx} (\log x)^2 \int x^3 dx \right) dx \right]$$

$$= 32 \left[(\log x)^2 \cdot \frac{x^4}{4} - \int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^4}{4} dx \right]$$

$$= 32 \left[(\log x)^2 \cdot \frac{x^4}{4} - \frac{1}{2} \int x^3 \log x dx \right]$$

$$= 32 \left[\frac{(\log x)^2 x^4}{4} - \frac{1}{2} \left(\frac{\log x \cdot x^4}{4} - \int \frac{1}{x} \cdot \frac{x^4}{4} dx \right) \right]$$

$$= 32 \left[\frac{(\log x)^2 x^4}{4} - \frac{1}{2} \left(\frac{x^4 \log x}{4} - \frac{1}{4} \cdot \frac{x^4}{4} \right) \right] + c$$

$$= 8 \left[(\log x)^2 x^4 - \frac{1}{2} \left(x^4 \log x - \frac{x^4}{4} \right) \right] + c$$

$$= 8x^4 \left[(\log x)^2 - \frac{\log x}{2} + \frac{1}{8} \right] + c$$

$$= x^4 [8(\log x)^2 - 4 \log x + 1] + c.$$

$$63. \quad (d) \quad x = \sin \theta \text{ रखने पर } \Rightarrow dx = \cos \theta d\theta,$$

$$\int \sin^{-1}(3x - 4x^3) dx = \int \sin^{-1}(\sin 3\theta) \cos \theta d\theta$$

$$= \int 3\theta \cos \theta d\theta = 3 \left\{ \theta \sin \theta - \int \sin \theta d\theta \right\}$$

$$= 3 \{ \theta \sin \theta + \cos \theta \} + c = 3 \left\{ x \sin^{-1} x + \sqrt{1 - x^2} \right\} + c.$$

$$64. \quad (a) \quad \sqrt{x} = t \text{ रखने पर } \sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2t dt$$

$$\therefore \int 2t \cdot \cos t dt = 2 \left[t \cdot \sin t - \int \sin t dt \right]$$

$$= 2t \sin t + 2 \cos t = 2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c.$$

$$65. \quad (d) \quad x = \tan \theta \text{ रखने पर } \Rightarrow dx = \sec^2 \theta d\theta, \text{ तब}$$

$$\int \tan^{-1} \frac{2x}{1-x^2} dx = \int \tan^{-1} \frac{2 \tan \theta}{1 - \tan^2 \theta} \sec^2 \theta d\theta$$

$$= \int \tan^{-1}(\tan 2\theta) \sec^2 \theta d\theta = \int 2\theta \sec^2 \theta d\theta$$

$$= 2 \left[\theta \tan \theta - \int \tan \theta d\theta \right] = 2x \tan^{-1} x - \log(x^2 + 1) + c.$$

$$66. \quad (a) \quad \sin^{-1} x = t \text{ रखने पर } \Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\therefore \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = \int t \sin t dt = -t \cos t + \sin t + c$$

$$= -\sin^{-1} x \cos(\sin^{-1} x) + \sin(\sin^{-1} x) + c$$

$$= x - \sin^{-1} x \sqrt{1-x^2} + c.$$

$$67. \quad (a) \quad t = \sin^{-1} x \text{ रखने पर } \Rightarrow \sin t = x \Rightarrow \cos t dt = dx, \text{ तब}$$

$$\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx = \int t \sec^2 t dt = t \tan t + \log \cos t + c$$

$$= \sin^{-1} x \tan(\sin^{-1} x) + \log \cos(\sin^{-1} x) + c$$

$$= \frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \frac{1}{2} \log(1-x^2) + c.$$

$$68. \quad (b) \quad x = \tan \theta \text{ रखने पर } \Rightarrow dx = \sec^2 \theta d\theta, \text{ तब}$$

$$\int \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx = \int \frac{\theta \tan \theta \sec^2 \theta d\theta}{(1 + \tan^2 \theta)^{3/2}}$$

$$= \int \theta \sin \theta d\theta = -\theta \cos \theta + \sin \theta + c$$

$$= \frac{x}{\sqrt{x^2+1}} - \tan^{-1} x \cdot \frac{1}{\sqrt{x^2+1}} = \frac{x - \tan^{-1} x}{\sqrt{1+x^2}} + c.$$

$$69. \quad (a) \quad x^2 = t \text{ रखने पर } \Rightarrow 2x dx = dt, \text{ तब}$$

$$\int x^5 e^{x^2} dx = \frac{1}{2} \int t^2 e^t dt = \frac{1}{2} \left[e^t t^2 - 2 \int t e^t dt \right] + c$$

$$= \frac{t^2 e^t}{2} - [t e^t - e^t] + c = \frac{1}{2} x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} + c.$$

$$70. \quad (a) \quad \tan^{-1} x = t \text{ रखने पर } \Rightarrow \frac{dx}{1+x^2} = dt$$

$$\therefore \int e^{\tan^{-1} x} \left(\frac{1+x+x^2}{1+x^2} \right) dx = \int e^t (\tan t + \sec^2 t) dt$$

$$= e^t \tan t + c = x e^{\tan^{-1} x} + c$$

$$\left[\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C \text{ का प्रयोग करने पर } \right].$$

$$71. \quad (c) \quad I = \int e^{\sqrt{x}} \cdot dx$$

$$\sqrt{x} = t \text{ रखने पर, } \frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2t dt$$

$$\therefore I = \int e^t \cdot 2t dt = 2[t \cdot e^t - e^t] + A = 2[\sqrt{x} \cdot e^{\sqrt{x}} - e^{\sqrt{x}}] + A$$

$$\Rightarrow I = 2(\sqrt{x} - 1) \cdot e^{\sqrt{x}} + A.$$

$$72. \quad (c) \quad I_1 = \int \sin^{-1} x dx = \int \sec x dx - \frac{1}{2} \int \sec^2 \frac{x}{2} dx$$

$$\text{माना } \sin^{-1} x = \theta \Rightarrow x = \sin \theta \Rightarrow dx = \cos \theta d\theta$$

$$I_1 = \int \theta \cos \theta d\theta = \theta \sin \theta - \int \sin \theta d\theta = \theta \sin \theta + \cos \theta$$

$$= x \sin^{-1} x + \sqrt{1-x^2}$$

$$I_2 = \int \sin^{-1} \sqrt{1-x^2} dx = \int \cos^{-1} x dx$$

$$\text{माना } \cos \phi = x, \text{ अतः } -\sin \phi d\phi = dx$$

$$I_2 = -\int \phi \sin \phi d\phi = \phi \cos \phi + \int -\cos \phi d\phi$$

$$= \phi \cos \phi - \sin \phi = x \cos^{-1} x - \sqrt{1-x^2}$$

$$I_1 + I_2 = x(\cos^{-1} x + \sin^{-1} x) = \frac{\pi}{2} x.$$

$$= \log(\sec x + \tan x) - \tan \frac{x}{2} + c.$$

$$5. \quad (c) \quad \int \frac{dx}{(x+1)(x+2)} = \int \left(\frac{1}{x+1} - \frac{1}{x+2} \right) dx$$

$$= \log(x+1) - \log(x+2) + c = \log \frac{x+1}{x+2} + c.$$

$$6. \quad (a) \quad \int \frac{x}{(x-2)(x-1)} dx = -\int \frac{1}{x-1} dx + \int \frac{2}{x-2} dx$$

$$= -\log_e(x-1) + 2 \log_e(x-2) + c = \log_e \frac{(x-2)^2}{(x-1)} + c.$$

$$7. \quad (a) \quad \text{हमें ज्ञात है कि, } \frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+1)}$$

$$\Rightarrow 1 = A(x^2+1) + (Bx+C)(x-1)$$

$$\text{यदि } x=1, \text{ तब } A = \frac{1}{2} \quad \dots(i)$$

$$A-C=1 \Rightarrow C = -\frac{1}{2} \quad \dots(ii)$$

$$A+B=0 \Rightarrow B = -\frac{1}{2} \quad \dots(iii)$$

इन मानों को रखने पर,

$$\therefore \frac{1}{(x-1)(x^2+1)} = \frac{1}{2} \cdot \frac{1}{(x-1)} - \frac{x+1}{2(x^2+1)}$$

$$\therefore \int \frac{1}{(x-1)(x^2+1)} dx = \frac{1}{2} \int \frac{dx}{(x-1)} - \frac{1}{2} \int \frac{x+1}{x^2+1} dx$$

$$= \frac{1}{2} \log(x-1) - \frac{1}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + c.$$

$$8. \quad (b) \quad \int \frac{x^2+x-1}{x^2+x-6} dx = \int \left[1 + \frac{5}{x^2+x-6} \right] dx$$

$$= \int \left[1 + \frac{5}{(x+3)(x-2)} \right] dx = \int dx + \int \frac{dx}{x-2} - \int \frac{dx}{x+3}$$

$$= x + \log(x-2) - \log(x+3) + c.$$

$$9. \quad (b) \quad \int \frac{x^2}{(x^2+2)(x^2+3)} dx = \int \left[\frac{3}{x^2+3} - \frac{2}{x^2+2} \right] dx$$

$$= \frac{3}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - \frac{2}{\sqrt{2}} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) + c$$

$$= \sqrt{3} \tan^{-1} \left(\frac{x}{\sqrt{3}} \right) - \sqrt{2} \tan^{-1} \left(\frac{x}{\sqrt{2}} \right) + c.$$

$$10. \quad (c) \quad \int \frac{dx}{(x^2+1)(x^2+4)} = \frac{1}{3} \left[\int \frac{dx}{x^2+1} - \int \frac{dx}{x^2+4} \right]$$

$$= \frac{1}{3} \left[\tan^{-1} x - \frac{1}{2} \tan^{-1} \frac{x}{2} \right] + c = \frac{1}{3} \tan^{-1} x - \frac{1}{6} \tan^{-1} \frac{x}{2} + c.$$

$$11. \quad (d) \quad \int \frac{1}{x-x^3} dx = \int \frac{1}{x(1+x)(1-x)} dx$$

$$= \frac{1}{2} \int \left(\frac{2}{x} - \frac{1}{1+x} + \frac{1}{1-x} \right) dx$$

$$73. \quad (c) \quad \int f(x) \cos x dx + \int g(x) e^x dx$$

$$= \int e^x \cos x dx + \int (-\sin x) e^x dx$$

$$= \frac{e^x}{2} (\cos x + \sin x) - \frac{e^x}{2} (\sin x - \cos x) + c$$

$$= \frac{e^x}{2} (2 \cos x) + c = e^x \cos x + c.$$

आंशिक भिन्नों द्वारा परिमेय फलनों के समाकलन, समाकलन के विभिन्न रूपों का मान ज्ञात करना

$$1. \quad (a) \quad \int \frac{dx}{(x-x^2)} = \int \left(\frac{1}{x} + \frac{1}{1-x} \right) dx = \log x - \log(1-x) + c.$$

$$2. \quad (a) \quad \int \frac{dx}{1+x+x^2+x^3} = \int \frac{dx}{(1+x)(1+x^2)}$$

$$= \frac{1}{2} \int \frac{1}{1+x^2} dx + \frac{1}{2} \int \frac{1}{1+x} dx - \frac{1}{2} \int \frac{x}{1+x^2} dx$$

$$= \frac{1}{2} \tan^{-1} x + \log \sqrt{1+x} - \frac{1}{2} \log \sqrt{1+x^2} + c.$$

$$3. \quad (b) \quad \int \frac{x-1}{(x-3)(x-2)} dx$$

$$= \int \frac{x-3}{(x-3)(x-2)} dx + \int \frac{2}{(x-3)(x-2)} dx$$

$$= \log \left[\frac{(x-2)(x-3)^2}{(x-2)^2} \right] + c = \log \left[\frac{(x-3)^2}{(x-2)} \right] + c.$$

$$\text{ट्रिक: जॉच द्वारा } \frac{d}{dx} \{ \log(x-3) - \log(x-2) \}$$

$$= \frac{1}{x-3} - \frac{1}{x-2} = \frac{1}{(x-3)(x-2)}$$

$$\Rightarrow \frac{d}{dx} \{ 2 \log(x-3) - \log(x-2) \}$$

$$= \frac{2}{x-3} - \frac{1}{x-2} = \frac{x-1}{(x-3)(x-2)}.$$

$$4. \quad (d) \quad \int \frac{1}{\cos x(1+\cos x)} dx = \int \frac{dx}{\cos x} - \int \frac{dx}{1+\cos x}$$

$$= \frac{1}{2} [2 \log x - \log(1+x) - \log(1-x)] = \frac{1}{2} \log \frac{x^2}{(1-x^2)} + c.$$

$$\begin{aligned} 12. \quad (b) \quad \int \sin 5x \cos 3x \, dx &= \frac{1}{2} \int (\sin 8x + \sin 2x) \, dx \\ &= \frac{-\cos 8x}{16} - \frac{\cos 2x}{4} + c \end{aligned}$$

दिये गये मानों से तुलना करने पर $A = \frac{-\cos 2x}{4} + c.$

$$\begin{aligned} 13. \quad (a) \quad \int \sin^3 x \cos^2 x \, dx &= \int (1 - \cos^2 x) \cos^2 x \cdot \sin x \, dx \\ \cos x = t \text{ रखने पर, } -\sin x \, dx &= dt \\ \therefore -\int (t^2 - t^4) dt &= \frac{t^5}{5} - \frac{t^3}{3} + c = \frac{(\cos x)^5}{5} - \frac{(\cos x)^3}{3} + c \end{aligned}$$

$$\begin{aligned} 14. \quad (b) \quad \int \sin 2x \cos 3x \, dx &= \frac{1}{2} \int 2(\sin 2x \cos 3x) \, dx \\ &= \frac{1}{2} \int (\sin 5x - \sin x) \, dx = \frac{1}{2} \left[-\frac{\cos 5x}{5} + \cos x \right] + c \\ &= \frac{1}{2} \left[\cos x - \frac{\cos 5x}{5} \right] + c. \end{aligned}$$

$$\begin{aligned} 15. \quad (c) \quad \sin x = t \text{ रखने पर, } \cos x \, dx &= dt, \text{ तब} \\ \int \frac{\cos x}{(1 + \sin x)(2 + \sin x)} \, dx &= \int \frac{dt}{(t+1)(t+2)} \\ &= \int \frac{1}{t+1} dt - \int \frac{1}{t+2} dt = \log \left(\frac{t+1}{t+2} \right) + c = \log \left(\frac{\sin x + 1}{\sin x + 2} \right) + c. \end{aligned}$$

$$\begin{aligned} 16. \quad (b) \quad \int \frac{e^x}{(1+e^x)(2+e^x)} \, dx &= \int \left\{ \frac{e^x}{1+e^x} - \frac{e^x}{2+e^x} \right\} dx \\ 1+e^x = t \text{ और } 2+e^x = t, \text{ रखने पर तब अभीष्ट समाकलन} \\ &= \log(1+e^x) - \log(2+e^x) = \log \left(\frac{1+e^x}{2+e^x} \right) + c. \end{aligned}$$

$$\begin{aligned} 17. \quad (c) \quad \int \frac{e^x dx}{e^{2x} + e^x - 2} &= \int \frac{dt}{t^2 + t - 2} \quad \left\{ \because e^x = t \Rightarrow e^x dx = dt \right\} \\ &= \int \frac{dt}{(t+2)(t-1)} = \int \frac{1}{3} \left[\frac{1}{t-1} - \frac{1}{t+2} \right] dt \\ &= \frac{1}{3} \log(e^x - 1) - \frac{1}{3} \log(e^x + 2) + c. \end{aligned}$$

$$\begin{aligned} 18. \quad (a) \quad \int \frac{x}{(x^4 - 1)} \, dx &= \frac{1}{2} \int \left[\frac{x}{x^2 - 1} - \frac{x}{x^2 + 1} \right] dx \\ &= \frac{1}{4} \log(x^2 - 1) - \frac{1}{4} \log(x^2 + 1) = \frac{1}{4} \log \left(\frac{x^2 - 1}{x^2 + 1} \right) + c. \end{aligned}$$

वैकल्पिक : $t = x$ रखने पर, $dt = 2x \, dx$, तब

$$\begin{aligned} \int \frac{x}{x^4 - 1} \, dx &= \frac{1}{2} \int \frac{dt}{t^2 - 1} = \frac{1}{2} \cdot \frac{1}{2} \log \frac{t-1}{t+1} + c \\ &= \frac{1}{4} \log \left[\frac{x^2 - 1}{x^2 + 1} \right] + c. \end{aligned}$$

$$19. \quad (a) \quad \cos x = t \text{ रखने पर, } -\sin x \, dx = dt, \text{ तब}$$

$$\begin{aligned} \int (1 - \cos^2 x)^2 \cdot \cos^4 x \sin x \, dx &= -\int (1 - t^2)^2 \cdot t^4 dt \\ &= -\frac{t^5}{5} + \frac{2}{7} t^7 - \frac{1}{9} t^9 + c = -\frac{\cos^5 x}{5} + \frac{2}{7} \cos^7 x - \frac{1}{9} \cos^9 x + c. \end{aligned}$$

वैकल्पिक : समानयन सूत्र द्वारा

$$\begin{aligned} 20. \quad (c) \quad \int \sqrt{x^2 - 8x + 7} \, dx &= \int \sqrt{(x-4)^2 - (3)^2} \, dx \\ &= \int \sqrt{x^2 - a^2} \, dx. \text{ सूत्र प्रयोग करने पर} \end{aligned}$$

$$21. \quad (b) \quad \int \frac{dx}{\sqrt{2x - x^2}} = \int \frac{dx}{\sqrt{1 - (x-1)^2}} = \sin^{-1}(x-1) + c.$$

$$\begin{aligned} 22. \quad (c) \quad \int \frac{x}{(x^2 - a^2)(x^2 - b^2)} \, dx \\ &= \frac{1}{a^2 - b^2} \left[\int \frac{x}{x^2 - a^2} \, dx - \int \frac{x}{x^2 - b^2} \, dx \right]. \end{aligned}$$

$$\begin{aligned} 23. \quad (c) \quad \int \frac{dx}{5 + 4 \cos x} &= \int \frac{dx}{5 + 4 \left[\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right]} = \int \frac{\sec^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx \end{aligned}$$

$$\tan \frac{x}{2} = t, \text{ रखने पर, } 2 \int \frac{dt}{3^2 + t^2} = \frac{2}{3} \tan^{-1} \left[\frac{1}{3} \tan \frac{x}{2} \right] + c$$

वैकल्पिक : सूत्र का प्रयोग करने पर अर्थात्,

$$\begin{aligned} \int \frac{1}{a + b \cos x} \, dx, \{a > b\} \\ &= \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left[\sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2} \right] + c \end{aligned}$$

$$\therefore \int \frac{dx}{5 + 4 \cos x} = \frac{2}{3} \tan^{-1} \left\{ \frac{1}{3} \tan \frac{x}{2} \right\} + c.$$

$$\begin{aligned} 24. \quad (a) \quad \int \frac{1}{(x^2 + b^2)(x^2 + a^2)} \, dx \\ &= \frac{1}{a^2 - b^2} \int \left[\frac{1}{x^2 + b^2} - \frac{1}{x^2 + a^2} \right] dx \\ &= \frac{1}{(a^2 - b^2)} \left[\frac{1}{b} \tan^{-1} \left(\frac{x}{b} \right) - \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) \right] + c. \end{aligned}$$

नोट : विद्यार्थियों को यह प्रश्न सूत्र की तरह याद रखना चाहिये।

$$\begin{aligned} 25. \quad (c) \quad \int \frac{dx}{1 + \cos^2 x} &= \int \frac{\sec^2 x \, dx}{\sec^2 x + 1} = \int \frac{\sec^2 x}{\tan^2 x + 2} \, dx \\ &= \int \frac{dt}{t^2 + 2} = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{t}{\sqrt{2}} \right) + c \quad \{ \tan x = t \text{ रखने पर} \} \\ &= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{1}{\sqrt{2}} \tan x \right) + c. \end{aligned}$$

ट्रिक : जाँच द्वारा,

$$\begin{aligned} \frac{d}{dx} \left\{ \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{1}{\sqrt{2}} \tan x \right) \right\} &= \frac{1}{\sqrt{2}} \left(\frac{1}{1 + \frac{\tan^2 x}{2}} \right) \frac{1}{\sqrt{2}} \sec^2 x \\ &= \frac{1}{2} \cdot \frac{2 \sec^2 x}{(2 + \tan^2 x)} = \frac{\sec^2 x}{1 + \sec^2 x} = \frac{1}{1 + \cos^2 x}. \end{aligned}$$

$$\begin{aligned} 26. \quad (b) \quad \int \frac{dx}{1 + 3 \sin^2 x} &= \int \frac{dx}{\sin^2 x + \cos^2 x + 3 \sin^2 x} \\ &= \int \frac{dx}{4 \sin^2 x + \cos^2 x} = \int \frac{\sec^2 x dx}{4 \tan^2 x + 1} = \frac{1}{4} \int \frac{\sec^2 x dx}{\tan^2 x + \frac{1}{4}} \end{aligned}$$

$$t = \tan x \text{ रखने पर } \Rightarrow dt = \sec^2 x dx,$$

$$\begin{aligned} \therefore \frac{1}{4} \int \frac{dt}{t^2 + \left(\frac{1}{2}\right)^2} &= \frac{1}{4} 2 \tan^{-1}(2t) + c \\ &= \frac{1}{2} \tan^{-1}(2t) + c = \frac{1}{2} \tan^{-1}(2 \tan x) + c. \end{aligned}$$

$$\begin{aligned} 27. \quad (d) \quad I &= \int \frac{dx}{2x^2 + x + 1} = \int \frac{dx}{2 \left(x^2 + \frac{x}{2} + \frac{1}{2} \right)} \\ &= \frac{1}{2} \int \frac{dx}{x^2 + \frac{x}{2} + \frac{1}{16} - \frac{1}{16} + \frac{1}{2}} = \frac{1}{2} \int \frac{dx}{\left(x + \frac{1}{4} \right)^2 + \left(\frac{\sqrt{7}}{4} \right)^2} \\ &= \frac{1}{2} \cdot \frac{1}{\frac{\sqrt{7}}{4}} \tan^{-1} \left[\frac{x + (1/4)}{\sqrt{7}/4} \right] = \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{4x + 1}{\sqrt{7}} \right) + C. \end{aligned}$$

$$\begin{aligned} 28. \quad (a) \quad I &= \frac{dx}{7 + 5 \cos x} = \int \frac{dx}{7 + 5 \left(\frac{1 - \tan^2(x/2)}{1 + \tan^2(x/2)} \right)} \\ &= \int \frac{\sec^2(x/2) dx}{7 + 7 \tan^2(x/2) + 5 - 5 \tan^2(x/2)} \\ &= \int \frac{\sec^2(x/2) dx}{12 + 2 \tan^2(x/2)} = \int \frac{\frac{1}{2} \sec^2(x/2) dx}{6 + \tan^2(x/2)} \\ \tan \frac{x}{2} = t \text{ रखने पर } &\Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt \\ I &= \int \frac{dt}{t^2 + (\sqrt{6})^2} = \frac{1}{\sqrt{6}} \tan^{-1} \frac{t}{\sqrt{6}} + c \\ &= \frac{1}{\sqrt{6}} \tan^{-1} \left| \frac{\tan(x/2)}{\sqrt{6}} \right| + c. \end{aligned}$$

$$29. \quad (b) \quad \int \frac{dx}{x^2 + 4x + 13} = \int \frac{dx}{(x+2)^2 + 9} = \frac{1}{3} \tan^{-1} \frac{(x+2)}{3} + c.$$

$$30. \quad (a) \quad \text{हमें ज्ञात है कि } I = \int \frac{dx}{\cos x - \sin x} = \frac{1}{\sqrt{2}} \int \frac{dx}{\cos \left(\frac{\pi}{4} + x \right)}$$

$$I = \frac{1}{\sqrt{2}} \int \sec \left(x + \frac{\pi}{4} \right) dx = \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{\pi}{4} + \frac{x}{2} + \frac{\pi}{8} \right) \right| + c$$

$$I = \frac{1}{\sqrt{2}} \log \left| \tan \left(\frac{x}{2} + \frac{3\pi}{8} \right) \right| + c.$$

$$31. \quad (d) \quad \int \frac{dx}{x^2 + 2x + 2} = \int \frac{dx}{(x+1)^2 + 1} = \tan^{-1}(x+1) + c.$$

$$32. \quad (a) \quad \text{अंश को निम्न प्रकार से लिख सकते हैं}$$

$$\text{अंश} = l(\text{हर}) + m \quad (\text{हर का अवकल गुणांक } D').$$

$$\text{माना } 3 \sin x + 2 \cos x = l(3 \cos x + 2 \sin x) + m(-3 \sin x + 2 \cos x)$$

दोनों पक्षों में $\sin x$ व $\cos x$ के गुणांकों की तुलना करने पर $3 = 2l - 3m$ तथा $2 = 3l + 2m$, हल करने पर

$$l = \frac{12}{13}, \quad m = -\frac{5}{13},$$

$$\begin{aligned} \therefore I &= l \int \frac{dx}{3 \cos x + 2 \sin x} + m \int \frac{-3 \sin x + 2 \cos x}{3 \cos x + 2 \sin x} dx \\ &= \frac{12}{13} \log(3 \cos x + 2 \sin x) - \frac{5}{13} \log(3 \cos x + 2 \sin x). \end{aligned}$$

$$33. \quad (a) \quad \log x = t \text{ रखने पर } \Rightarrow \frac{1}{x} dx = dt, \text{ तब}$$

$$\begin{aligned} \int \frac{dx}{x[(\log x)^2 + 4 \log x - 1]} &= \int \frac{dt}{t^2 + 4t - 1} \\ &= \int \frac{dt}{(t+2)^2 - (\sqrt{5})^2} = \frac{1}{2\sqrt{5}} \log \left[\frac{t+2-\sqrt{5}}{t+2+\sqrt{5}} \right] \\ &= \frac{1}{2\sqrt{5}} \log \left[\frac{\log x + 2 - \sqrt{5}}{\log x + 2 + \sqrt{5}} \right] + c. \end{aligned}$$

$$34. \quad (c) \quad x^n = t \text{ रखने पर } \Rightarrow nx^{n-1} dx = dt$$

$$\Rightarrow \frac{nx^n}{x} dx = dt \Rightarrow \frac{1}{x} dx = \frac{dt}{nt},$$

$$\begin{aligned} \therefore \int \frac{dt}{nt(t+1)} &= \frac{1}{n} \left[\int \frac{dt}{t(t+1)} \right] \\ &= \frac{1}{n} \left[\int \frac{1}{t} dt - \int \frac{1}{t+1} dt \right] = \frac{1}{n} \log \frac{x^n}{x^n + 1} + c. \end{aligned}$$

$$35. \quad (b) \quad \text{दिया है } \int \frac{dx}{x(x^7+1)} = \int \frac{dx}{x^8 \left(1 + \frac{1}{x^7} \right)}$$

$$1 + \frac{1}{x^7} = t \text{ रखने पर } \Rightarrow \frac{-7}{x^8} dx = dt$$

$$\therefore I = \frac{-1}{7} \int \frac{dt}{t} = \frac{-1}{7} \log t + c$$

$$\Rightarrow I = -\frac{1}{7} \log \left(\frac{x^7+1}{x^7} \right) + c \Rightarrow I = \frac{1}{7} \log \left(\frac{x^7}{x^7+1} \right) + c.$$

$$36. \quad (d) \quad \text{हमें ज्ञात है कि } I = \int \frac{dx}{x(x^5+1)} = \int \frac{dx}{x^6 \left(1 + \frac{1}{x^5} \right)}$$

$$1 + \frac{1}{x^5} = t \text{ रखने पर } \Rightarrow \frac{-5}{x^6} dx = dt$$

$$\Rightarrow I = -\frac{1}{5} \int \frac{dt}{t} = -\frac{1}{5} \log t + c$$

$$I = -\frac{1}{5} \log \left(1 + \frac{1}{x^5} \right) + c = -\frac{1}{5} \log \left(\frac{x^5 + 1}{x^5} \right) + c$$

$$\therefore I = \frac{1}{5} \log \left(\frac{x^5}{x^5 + 1} \right) + c.$$

$$37. (d) \int \frac{x^2 + 1}{x^4 + 1} dx = \int \frac{\left(1 + \frac{1}{x^2}\right)}{\left(x^2 + \frac{1}{x^2}\right)} dx = \int \frac{\left(1 + \frac{1}{x^2}\right) dx}{\left(x - \frac{1}{x}\right)^2 + 2}$$

$$x - \frac{1}{x} = t \text{ रखने पर } \Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt, \text{ तब अभीष्ट}$$

$$\text{समाकल } \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + c \text{ है।}$$

38. (d) दिये गये समाकल को निम्न प्रकार से लिखा जा सकता है

$$\int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right)^2 - 1} dx$$

$$x + \frac{1}{x} = t \text{ रखने पर } \Rightarrow \left(1 - \frac{1}{x^2}\right) dx = dt,$$

$$\therefore \int \frac{dt}{t^2 - 1} = \frac{1}{2} \log \left| \frac{t-1}{t+1} \right| + c$$

$$= \frac{1}{2} \log \left(\frac{x + \frac{1}{x} - 1}{x + \frac{1}{x} + 1} \right) + c = \frac{1}{2} \log \left(\frac{x^2 - x + 1}{x^2 + x + 1} \right) + c.$$

$$39. (d) I = \int \frac{x dx}{x^2 + 4x + 5} = \int \frac{x + 2 - 2}{(x+2)^2 + 1} dx$$

$$= \frac{1}{2} \int \frac{2(x+2) dx}{(x+2)^2 + 1} - 2 \int \frac{dx}{1 + (x+2)^2}$$

$$= \frac{1}{2} \int \frac{dt}{t} - 2 \int \frac{dx}{1 + (x+2)^2}$$

$$\text{प्रथम व्यंजक में } [1 + (x+2)^2 = t \text{ रखने पर } \Rightarrow 2(x+2)dx = dt]$$

$$= \frac{1}{2} \log t - 2 \tan^{-1}(x+2) + c$$

$$= \frac{1}{2} \log(x^2 + 4x + 5) - 2 \tan^{-1}(x+2) + c.$$

$$40. (a) I = \int \frac{2x^2 + 3}{(x^2 - 1)(x^2 - 4)} dx$$

$$\frac{2x^2 + 3}{(x^2 - 1)(x^2 - 4)} = \frac{A}{x^2 - 1} + \frac{B}{x^2 - 4}$$

$$2x^2 + 3 = A(x^2 - 4) + B(x^2 - 1)$$

$$2x^2 + 3 = x^2(A+B) - 4A - B$$

दोनों पक्षों में x^2 व अचर पदों के गुणांकों की तुलना करने पर

$$A + B = 2 \quad \dots(i)$$

$$4A + B = -3 \quad \dots(ii)$$

दोनों समीकरणों को हल करने पर $A = -\frac{5}{3}, B = \frac{11}{3}$

$$\int \frac{2x^2 + 3}{(x^2 - 1)(x^2 - 4)} dx = \int \frac{-\frac{5}{3} dx}{(x^2 - 1)} + \int \frac{\frac{11}{3} dx}{(x^2 - 4)}$$

$$= -\frac{5}{3} \int \frac{dx}{(x+1)(x-1)} + \frac{11}{3} \int \frac{dx}{(x+2)(x-2)}$$

$$= -\frac{5}{3} \cdot \frac{1}{2} \int \frac{dx}{x-1} + \frac{5}{6} \int \frac{dx}{x+1} + \frac{11}{3} \cdot \frac{1}{4} \int \frac{dx}{x-2} - \frac{11}{12} \int \frac{dx}{x+2} + c$$

$$= -\frac{5}{6} \log(x-1) + \frac{5}{6} \log(x+1) + \frac{11}{12} \log(x-2) - \frac{11}{12} \log(x+2) + c$$

$$= \frac{5}{6} \log \left(\frac{x+1}{x-1} \right) + \frac{11}{12} \log \left(\frac{x-2}{x+2} \right) + c$$

$$= \log \left(\frac{x+1}{x-1} \right)^{5/6} + \log \left(\frac{x-2}{x+2} \right)^{11/12} + c$$

$$\Rightarrow a = \frac{11}{12} \text{ और } b = \frac{5}{6}.$$

Critical Thinking Questions

- (b) $\int \frac{dx}{\cos(x-a)\cos(x-b)}$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)} dx$$

$$= \frac{1}{\sin(a-b)} \int \left\{ \frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)} \right\} dx$$

$$= \operatorname{cosec}(a-b) \log \frac{\cos(x-a)}{\cos(x-b)} + c.$$
- (b) $\int \frac{dx}{\sqrt{x+a} + \sqrt{x+b}} = \int \frac{\sqrt{x+a} - \sqrt{x+b}}{(x+a) - (x+b)} dx$

$$= \frac{1}{(a-b)} \int (x+a)^{1/2} dx - \frac{1}{(a-b)} \int (x+b)^{1/2} dx$$

$$= \frac{2}{3(a-b)} [(x+a)^{3/2} - (x+b)^{3/2}] + c.$$
- (a) स्वयं करें।
- (a) $\int (\sin 2x + \cos 2x) dx = -\frac{\cos 2x}{2} + \frac{\sin 2x}{2} + k$

$$= \frac{1}{\sqrt{2}} \left(\sin 2x \cos \frac{\pi}{4} - \cos 2x \sin \frac{\pi}{4} \right) + k$$

$$= \frac{1}{\sqrt{2}} \sin \left(2x - \frac{\pi}{4} \right) + k$$

$$\Rightarrow c = \frac{\pi}{4} \text{ तथा } a = k, \text{ एक स्वेच्छ अचर}$$
- (d) $\int \frac{x^3 - x - 2}{(1-x^2)} dx = \int \frac{-x(1-x^2)}{(1-x^2)} dx - \int \frac{2}{1-x^2} dx$

$$= -\int x dx - 2 \int \frac{1}{1-x^2} dx = \frac{-x^2}{2} + \log \left(\frac{x-1}{x+1} \right) + c.$$

$$\begin{aligned}
 6. \quad (b) \quad & \int \frac{\sin^8 x - \cos^8 x}{1 - 2 \sin^2 x \cos^2 x} dx \\
 &= \int \frac{(\sin^4 x + \cos^4 x)(\sin^4 x - \cos^4 x)}{(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x} dx \\
 &= \int (\sin^4 x - \cos^4 x) dx \\
 &= \int (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x) dx \\
 &= \int (\sin^2 x - \cos^2 x) dx = \int -\cos 2x dx = -\frac{\sin 2x}{2} + c.
 \end{aligned}$$

$$\begin{aligned}
 7. \quad (d) \quad & a + bx = t \text{ रखने पर } \Rightarrow x = \frac{t-a}{b} \text{ और } dx = \frac{dt}{b} \\
 \therefore I &= \int \left(\frac{t-a}{b} \right)^2 \times \frac{1}{t^2} \frac{dt}{b} \\
 &= \frac{1}{b^2} \int \left(1 - \frac{2a}{t} + a^2 \cdot t^{-2} \right) dt = \frac{1}{b^2} \left[t - 2a \log t - \frac{a^2}{t} \right] \\
 &= \frac{1}{b^2} \left[x + \frac{a}{b} - \frac{2a}{b} \log(a+bx) - \frac{a^2}{b(a+bx)} \right].
 \end{aligned}$$

$$\begin{aligned}
 8. \quad (a) \quad & q \tan^{-1} x = t \text{ रखने पर} \\
 \Rightarrow \frac{q}{1+x^2} dx &= dt \Rightarrow \frac{1}{1+x^2} dx = \frac{dt}{q} \\
 \Rightarrow \int \frac{dx}{(1+x^2)\sqrt{p^2+q^2(\tan^{-1} x)^2}} &= \frac{1}{q} \int \frac{dt}{\sqrt{p^2+t^2}} \\
 &= \frac{1}{q} \log \left[q \tan^{-1} x + \sqrt{p^2+q^2(\tan^{-1} x)^2} \right] + c.
 \end{aligned}$$

$$\begin{aligned}
 9. \quad (c) \quad & 1+x^3 = t^2 \text{ रखने पर } \Rightarrow 3x^2 dx = 2t dt \text{ और } x^3 = t^2 - 1 \\
 \therefore \int \frac{x^5}{\sqrt{1+x^3}} dx &= \int \frac{x^2 \cdot x^3}{\sqrt{1+x^3}} dx \\
 &= \frac{2}{3} \int \frac{(t^2-1) \cdot t dt}{t} = \frac{2}{3} \int (t^2-1) dt = \frac{2}{3} \left[\frac{t^3}{3} - t \right] + c \\
 &= \frac{2}{3} \left[\frac{(1+x^3)^{3/2}}{3} - (1+x^3)^{1/2} \right] + c.
 \end{aligned}$$

$$\begin{aligned}
 10. \quad (d) \quad & I = \int \frac{dx}{\sin x - \cos x + \sqrt{2}} \\
 &= \int \frac{dx}{\sqrt{2}(\sin x \cdot \sin \frac{\pi}{4} - \cos x \cos \frac{\pi}{4} + 1)} \\
 &= \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos(x + \frac{\pi}{4})} = \frac{1}{\sqrt{2}} \int \frac{dx}{1 - \cos 2\left(\frac{x}{2} + \frac{\pi}{8}\right)} \\
 &= \frac{1}{\sqrt{2}} \int \frac{dx}{2 \sin^2\left(\frac{x}{2} + \frac{\pi}{8}\right)} = \frac{1}{2\sqrt{2}} \int \operatorname{cosec}^2\left(\frac{x}{2} + \frac{\pi}{8}\right) dx \\
 &= \frac{1}{2\sqrt{2}} \frac{-\cot\left(\frac{x}{2} + \frac{\pi}{8}\right)}{1/2} + c = \frac{-1}{\sqrt{2}} \cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c.
 \end{aligned}$$

$$\begin{aligned}
 11. \quad (a) \quad & \int \frac{a dx}{b+c e^x} = \int \frac{a e^x}{b e^x + c e^{2x}} dx \\
 e^x = t, \text{ रखने पर } \Rightarrow e^x dx &= dt \\
 a \int \frac{dt}{t(ct+b)} &= a \int -\frac{1}{b} \left\{ \frac{c}{ct+b} - \frac{1}{t} \right\} dt \quad \{\text{आंशिक भिन्न द्वारा}\} \\
 &= \frac{a}{b} \log \left(\frac{e^x}{b+c e^x} \right) + c.
 \end{aligned}$$

$$\begin{aligned}
 12. \quad (b) \quad & \sqrt{x} = t \text{ रखने पर } \Rightarrow \frac{1}{2\sqrt{x}} dx = dt \Rightarrow dx = 2t dt, \text{ तब} \\
 \int \sin \sqrt{x} dx &= 2 \int t \sin t dt = 2(-t \cos t + \sin t) + c \\
 &= 2(\sin \sqrt{x} - \sqrt{x} \cos \sqrt{x}) + c.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad (a) \quad & x = 3 \sin \theta \text{ रखने पर } \Rightarrow dx = 3 \cos \theta d\theta, \\
 \therefore \int \frac{x^2}{(9-x^2)^{3/2}} dx &= \int \frac{9 \sin^2 \theta}{(9-9 \sin^2 \theta)^{3/2} \cdot 3 \cos \theta} d\theta \\
 &= \int \frac{27 \sin^2 \theta \cos \theta}{27 \cos^3 \theta} d\theta = \int \tan^2 \theta d\theta = \int (\sec^2 \theta - 1) d\theta \\
 &= \tan \theta - \theta + c = \tan \left\{ \sin^{-1} \left(\frac{x}{3} \right) \right\} - \sin^{-1} \left(\frac{x}{3} \right) + c \\
 &= \tan \tan^{-1} \left(\frac{\left(\frac{x}{3} \right)}{\sqrt{1-(x^2/9)}} \right) - \sin^{-1} \left(\frac{x}{3} \right) + c \\
 &= \frac{x}{\sqrt{9-x^2}} - \sin^{-1} \left(\frac{x}{3} \right) + c.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad (a) \quad & \int x \sqrt{\frac{1-x^2}{1+x^2}} dx = \int \frac{x(1-x^2)}{\sqrt{1-x^4}} dx \\
 \{\text{अंश व हर को } (1-x^2)^{1/2} \text{ से गुणा करने पर}\} \\
 &= \int \frac{x}{\sqrt{1-x^4}} dx - \int \frac{x^3}{\sqrt{1-x^4}} dx \\
 &= \frac{1}{2} [\sin^{-1}(x^2) + \sqrt{1-x^4}] + c. \\
 (x^2 = t \text{ तथा } \sqrt{1-x^4} = \sqrt{t} \text{ रखने पर})
 \end{aligned}$$

$$\begin{aligned}
 15. \quad (a) \quad & \text{चूँकि } \int f(x) \sin x \cos x dx = \frac{1}{2(b^2-a^2)} \log(f(x)) + c \\
 \therefore f(x) \sin x \cos x &= \frac{1}{2(b^2-a^2)} \cdot \frac{1}{f(x)} f'(x) \\
 \text{दोनों पक्षों का } x \text{ के सापेक्ष अवकलन करने पर,} \\
 \Rightarrow 2(b^2-a^2) \sin x \cos x &= \frac{f'(x)}{f(x)^2} \\
 \Rightarrow \int (2b^2 \sin x \cos x - 2a^2 \sin x \cos x) dx &= \int \frac{f'(x)}{\{f(x)\}^2} dx \\
 \Rightarrow \pm(-b^2 \cos^2 x - a^2 \sin^2 x) &= -\frac{1}{f(x)} \\
 \Rightarrow f(x) &= \pm \frac{1}{(a^2 \sin^2 x + b^2 \cos^2 x)}.
 \end{aligned}$$

$$16. \quad (c) \quad \int \frac{dx}{4 \sin^2 x + 5 \cos^2 x} = \int \frac{\sec^2 x dx}{4 \tan^2 x + 5} = \frac{1}{4} \int \frac{\sec^2 x dx}{\tan^2 x + \frac{5}{4}}$$

$$\tan x = t \text{ रखने पर } \Rightarrow \sec^2 x dx = dt,$$

$$\therefore \frac{1}{4} \int \frac{dt}{t^2 + \left(\frac{\sqrt{5}}{2}\right)^2} = \frac{2}{4\sqrt{5}} \tan^{-1} \left(\frac{2t}{\sqrt{5}} \right) + c$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1} \left(\frac{2 \tan x}{\sqrt{5}} \right) + c.$$

$$17. \quad (c) \quad \int \frac{x^2 + 1}{x^4 - x^2 + 1} dx = \int \frac{\left(1 + \frac{1}{x^2}\right)}{x^2 + \frac{1}{x^2} - 1} dx$$

$$= \int \frac{1 + \frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 + 1} dx = \int \frac{dt}{t^2 + 1} = \tan^{-1} t + c$$

$$\left\{ x - \frac{1}{x} = t \text{ रखने पर } \Rightarrow \left(1 + \frac{1}{x^2}\right) dx = dt \right\}$$

$$= \tan^{-1} \left(x - \frac{1}{x} \right) + c = \tan^{-1} \left(\frac{x^2 - 1}{x} \right) + c.$$

$$18. \quad (c) \quad \int (\log x)^2 dx ; \log x = t \text{ रखने पर } \Rightarrow e^t = x \Rightarrow dx = e^t dt, \text{ तब}$$

$$\int t^2 \cdot e^t dt = t^2 e^t - 2te^t + 2e^t + c$$

$$= x(\log x)^2 - 2x \log x + 2x + c.$$

$$19. \quad (a) \quad \text{माना } \sqrt{(x^2 - a^2)} = t \Rightarrow x^2 - a^2 = t^2$$

$$\Rightarrow x^2 = a^2 + t^2 \Rightarrow x dx = t dt$$

$$\therefore \int \frac{\sqrt{(x^2 - a^2)}}{x} dx = \int \frac{\sqrt{(x^2 - a^2)} x}{x^2} dx$$

$$\Rightarrow I = \int \frac{t}{a^2 + t^2} t dt = \int \frac{t^2}{a^2 + t^2} dt$$

$$\Rightarrow I = \int \left(1 - \frac{a^2}{a^2 + t^2} \right) dt = t - a^2 \frac{1}{a} \tan^{-1} \left(\frac{t}{a} \right)$$

$$\Rightarrow I = \sqrt{(x^2 - a^2)} - a \tan^{-1} \left[\frac{\sqrt{(x^2 - a^2)}}{a} \right] + c$$

$$20. \quad (a) \quad \int \tan^3 2x \sec 2x dx = \int (\sec^2 2x - 1) \sec 2x \tan 2x dx$$

$$= \int (\sec^3 2x \tan 2x - \sec 2x \tan 2x) dx$$

$$= \int \sec^3 2x \tan 2x dx - \int \sec 2x \tan 2x dx \quad \dots (i)$$

$$\int \sec^3 2x \tan 2x dx \text{ लेने पर}$$

$$\sec 2x = t \text{ रखने पर } \Rightarrow \sec 2x \tan 2x dx = \frac{dt}{2}$$

$$\therefore \frac{1}{2} \int t^2 dt = \frac{t^3}{6} = \frac{\sec^3 2x}{6}$$

$$(i) \text{ से, } \int \sec^3 2x \tan 2x dx - \int \sec 2x \tan 2x dx$$

$$= \frac{\sec^3 2x}{6} - \frac{\sec 2x}{2} + c.$$

$$\text{ट्रिक : माना } \sec 2x = t, \text{ तब } \sec 2x \tan 2x dx = \frac{1}{2} dt$$

$$\therefore \frac{1}{2} \int (t^2 - 1) dt = \frac{1}{6} t^3 - \frac{1}{2} t + c = \frac{1}{6} \sec^3 2x - \frac{1}{2} \sec 2x + c.$$

$$21. \quad (a) \quad \int x \sin^{-1} x dx = \frac{x^2}{2} \sin^{-1} x - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx + c$$

$$= \frac{x^2}{2} \sin^{-1} x - \frac{1}{2} \int \frac{(1-x^2)+1}{\sqrt{1-x^2}} dx + c$$

$$= \frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \sqrt{1-x^2} dx - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx + c$$

$$= \frac{x^2}{2} \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + c$$

$$= \frac{x^2}{2} \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} - \frac{1}{4} \sin^{-1} x$$

$$= \left(\frac{x^2}{2} - \frac{1}{4} \right) \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + c.$$

$$22. \quad (a) \quad I = \int \sqrt{\frac{a-x}{x}} dx.$$

$$x = a \sin^2 \theta \text{ रखने पर } \Rightarrow dx = 2a \sin \theta \cos \theta d\theta, \text{ तब}$$

$$I = \int \sqrt{\frac{\cos^2 \theta}{\sin^2 \theta}} \cdot 2a \sin \theta \cos \theta d\theta$$

$$= a \int 2 \cos^2 \theta d\theta = a \int (1 + \cos 2\theta) d\theta$$

$$= a \left[\sin^{-1} \sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a}} \cdot \sqrt{\frac{a-x}{a}} \right] + c.$$

$$23. \quad (a) \quad \int \frac{\sin x - \cos x}{\sqrt{1 - \sin 2x}} e^{\sin x} \cos x dx = \int \frac{\sin x - \cos x}{\sin x - \cos x} e^{\sin x} \cos x dx$$

$$= \int e^{\sin x} \cos x dx = e^{\sin x} + c.$$

$$24. \quad (d) \quad I = \int \frac{4e^x + 6e^{-x}}{9e^{2x} - 4e^{-x}} dx = \frac{4}{9} \int \frac{9e^{2x} dx}{9e^{2x} - 4} + 6 \int \frac{dx}{9e^{2x} - 4}$$

$$\therefore \int \frac{dx}{9e^{2x} - 4} = \frac{1}{8} \log(9e^{2x} - 4) - \frac{1}{4} \log 3 - \frac{1}{4} x + \text{अचर}$$

$$\therefore I = \frac{35}{36} \log(9e^{2x} - 4) - \frac{3}{2} x - \frac{3}{2} \log 3 + \text{अचर}$$

दिये गये समाकल से तुलना करने पर

$$A = -\frac{3}{2}, \quad B = \frac{35}{36}, \quad C = -\frac{3}{2} \log 3 + \text{अचर}$$

25. (a) माना $I = \int \sec^3 x dx = \int \sec x \sec^2 x dx$

$$\Rightarrow I = \sec x \tan x - \int \sec x \tan^2 x dx$$

$$\Rightarrow I = \sec x \tan x - \int \sec x (\sec^2 x - 1) dx$$

$$\Rightarrow I = \sec x \tan x - \int \sec^3 x dx + \int \sec x dx$$

$$\Rightarrow I = \sec x \tan x - I + \log(\sec x + \tan x)$$

$$\Rightarrow 2I = \sec x \tan x + \log(\sec x + \tan x)$$

$$\Rightarrow I = \frac{1}{2} [\sec x \tan x + \log(\sec x + \tan x)].$$

26. (b) $\int \frac{x-1}{(x+1)^3} e^x dx = \int e^x \left(\frac{(x+1)}{(x+1)^3} - \frac{2}{(x+1)^3} \right) dx$

$$= \int e^x \left(\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right) dx = \frac{e^x}{(x+1)^2} + c.$$

$$\left\{ \therefore \frac{d}{dx} \left(\frac{1}{(x+1)^2} \right) = -\frac{2}{(x+1)^3} \right\}$$

27. (c) $I = \int e^x \sin 2x dx = \sin 2x \cdot e^x - 2 \int \cos 2x \cdot e^x dx$

$$= \sin 2x \cdot e^x - 2 \cos 2x \cdot e^x - 4 \int e^x \sin 2x dx$$

$$\Rightarrow 5I = e^x (\sin 2x - 2 \cos 2x) + \text{अचर}$$

दिये गये मान से तुलना करने पर $K = 5$.

28. (a) $\int \frac{dx}{3-2x-x^2} = \int \frac{dx}{4-(x^2+2x+1)}$

$$= \int \frac{dx}{4-(x+1)^2} = \int \frac{dt}{(2)^2 - t^2}$$

जहाँ $x+1 = t, \therefore dx = dt$

$$\therefore I = \frac{1}{2 \cdot 2} \log \left(\frac{2+t}{2-t} \right) = \frac{1}{4} \log \left(\frac{2+x+1}{2-x-1} \right) = \frac{1}{4} \log \left(\frac{3+x}{1-x} \right).$$

29. (a) $\int x(2x+3)^{1/2} dx$

$$= x \frac{(2x+3)^{3/2}}{3/2} \cdot \frac{1}{2} - \int \frac{(2x+3)^{3/2}}{3/2} \cdot \frac{1}{2} dx + c$$

$$= \frac{1}{3} x(2x+3)^{3/2} - \frac{1}{3} \int (2x+3)^{3/2} dx + c$$

$$= \frac{1}{3} x(2x+3)^{3/2} - \frac{1}{15} (2x+3)^{5/2} + c.$$

30. (d) हम जानते हैं कि

$$\log \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) = \log \left(\frac{1 + \tan \theta}{1 - \tan \theta} \right) = \log \tan \left(\frac{\pi}{4} + \theta \right)$$

$$\int \sec \theta d\theta = \log \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right)$$

$$\therefore \int \sec 2\theta d\theta = \frac{1}{2} \log \tan \left(\frac{\pi}{4} + \theta \right)$$

$$\therefore 2 \sec 2\theta = \frac{d}{d\theta} \log \tan \left(\frac{\pi}{4} + \theta \right) \quad \dots(i)$$

दिये गये व्यंजक का खण्डशः समाकलन करने पर,

$$I = \frac{1}{2} \sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta \right) - \frac{1}{2} \int \sin 2\theta \cdot 2 \sec 2\theta d\theta \quad \{(i) \text{ से}\}$$

$$I = \frac{1}{2} \sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta \right) - \int \tan 2\theta d\theta$$

$$I = \frac{1}{2} \sin 2\theta \log \tan \left(\frac{\pi}{4} + \theta \right) - \frac{1}{2} \log \sec 2\theta.$$

31. (c) $x \sin x + \cos x$ का अवकलन $x \cos x$ है, तब

$$I = \int \frac{x^2 dx}{(x \sin x + \cos x)^2} = \int \frac{x \cos x}{(x \sin x + \cos x)^2} \cdot \frac{x}{\cos x} dx$$

खण्डशः समाकलन द्वारा $\left[\int \frac{1}{t^2} dt = -\frac{1}{t} \right]$

$$\therefore I = \frac{-1}{(x \sin x + \cos x)} \cdot \frac{x}{\cos x}$$

$$+ \int \frac{1}{(x \sin x + \cos x)} \cdot \frac{\cos x \cdot 1 - x(-\sin x)}{\cos^2 x} dx$$

$$= -\frac{1}{x \sin x + \cos x} \cdot \frac{x}{\cos x} + \int \sec^2 x dx$$

$$= -\frac{1}{x \sin x + \cos x} \cdot \frac{x}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \frac{-x + x \sin^2 x + \sin x \cos x}{(x \sin x + \cos x) \cos x}$$

$$= \frac{\sin x \cos x - x(1 - \sin^2 x)}{(x \sin x + \cos x) \cos x} = \frac{\sin x - x \cos x}{x \sin x + \cos x}.$$

32. (c) $u = \int e^{ax} \cos bx dx = e^{ax} \frac{\sin bx}{b} - \frac{a}{b} \int e^{ax} \sin bx dx$

$$= \frac{e^{ax} \sin bx}{b} - \frac{a}{b} v \Rightarrow bu + av = e^{ax} \sin bx \quad \dots(i)$$

इसी प्रकार $bv - au = -e^{ax} \cos bx \quad \dots(ii)$

(i) व (ii) को वर्ग करके जोड़ने पर,

$$(a^2 + b^2)(u^2 + v^2) = e^{2ax}.$$

33. (a) $I_n = \int (\log x)^n dx \quad \dots(i)$

$$\therefore I_{n-1} = \int (\log x)^{n-1} dx \quad \dots(ii)$$

अब $I_n = \int (\log x)^n \cdot dx = (\log x)^n x - n \int (\log x)^{n-1} \frac{1}{x} x dx$

$$= x(\log x)^n - n \int (\log x)^{n-1} dx$$

$$I_n = x(\log x)^n - nI_{n-1}; \therefore I_n + nI_{n-1} = x(\log x)^n.$$

34. (d) माना $I = \int e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx$

$$= 2 \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) e^{x/2} - \int \cos\left(\frac{x}{2} + \frac{\pi}{4}\right) \frac{1}{2} 2e^{x/2} dx + c$$

$$= 2 \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) e^{x/2} - 2e^{x/2} \cos\left(\frac{x}{2} + \frac{\pi}{4}\right)$$

$$- \int \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) \frac{1}{2} 2e^{x/2} dx$$

$$\therefore 2I = 2e^{x/2} \left\{ \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) - \cos\left(\frac{x}{2} + \frac{\pi}{4}\right) \right\}$$

$$\Rightarrow I = e^{x/2} \left\{ \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) - \cos\left(\frac{x}{2} + \frac{\pi}{4}\right) \right\}$$

$$= \sqrt{2} e^{x/2} \left(\sin \frac{x}{2} \right) = \sqrt{2} e^{x/2} \sin \frac{x}{2} + c.$$

ट्रिक : जाँच द्वारा,

$$\frac{d}{dx} \left\{ \sqrt{2} e^{x/2} \sin \frac{x}{2} + c \right\} = \sqrt{2} \left[\frac{1}{2} e^{x/2} \cos \frac{x}{2} + \frac{1}{2} e^{x/2} \sin \frac{x}{2} \right]$$

$$= e^{x/2} \left[\frac{1}{\sqrt{2}} \cos \frac{x}{2} + \frac{1}{\sqrt{2}} \sin \frac{x}{2} \right] = e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right).$$

35. (c) $\int \frac{2x+3}{x^2-5x+6} dx = \int \frac{2x-5}{x^2-5x+6} dx + \int \frac{8}{x^2-5x+6} dx$

$$= \log(x^2-5x+6) + 8 \int \frac{1}{(x-2)(x-3)} dx + c.$$

$$= \log[(x-2)(x-3)] + 8 \int \left[\frac{1}{x-3} - \frac{1}{x-2} \right] dx + c.$$

$$= \log(x-2) + \log(x-3) + 8 \log(x-3) - 8 \log(x-2) + c$$

$$= 9 \log(x-3) - 7 \log(x-2) + c \quad \dots(i)$$

दिया गया है

$$\int \frac{2x+3}{x^2-5x+6} dx = 9 \log(x-3) - 7 \log(x-2) + A$$

इसकी (i) से तुलना करने पर $A = \text{अचर}$

36. (b) $\int \frac{dx}{2+\cos x} = \int \frac{dx}{2 \sin^2\left(\frac{x}{2}\right) + 2 \cos^2\left(\frac{x}{2}\right) + \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}$

$$= \int \frac{dx}{\sin^2\left(\frac{x}{2}\right) + 3 \cos^2\left(\frac{x}{2}\right)} = \int \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 3} dx$$

$$\tan\left(\frac{x}{2}\right) = t \text{ रखने पर } \Rightarrow \sec^2\left(\frac{x}{2}\right) dx = 2dt,$$

$$\therefore 2 \int \frac{dt}{t^2+3} = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + c = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + c.$$

37. (b) $I = \int \frac{x}{x^4+x^2+1} dx = \int \frac{xdx}{(x^2+x+1)(x^2-x+1)}$

$$I = \frac{1}{2} \int \frac{dx}{x^2-x+1} - \frac{1}{2} \int \frac{dx}{x^2+x+1}$$

$$I = \frac{1}{2} \int \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} - \frac{1}{2} \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) - \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right).$$

38. (a) $I = \int \frac{dx}{\sin x(1+2\cos x)} = \int \frac{\sin x dx}{\sin^2 x(1+2\cos x)}$

$$= \int \frac{\sin x dx}{(1-\cos x)(1+\cos x)(1+2\cos x)}$$

$\cos x$ का अवकल गुणांक $-\sin x$ है जो कि अंश में दिया गया है और इस कारण हम प्रतिस्थापन $\cos x = t$

$\Rightarrow -\sin x dx = dt$ का प्रयोग करते हैं।

$$\therefore I = - \int \frac{dt}{(1-t)(1+t)(1+2t)}$$

समाकल को आंशिक भिन्न में रखने पर,

$$\therefore I = - \int \left[\frac{1}{6(1-t)} - \frac{1}{2(1+t)} + \frac{4}{3(1+2t)} \right] dt$$

39. (d) $I = \int \frac{2x+3}{(x-1)(x^2+1)} dx$

$$= \int \frac{5dx}{2(x-1)} + \int \frac{-\left(\frac{5}{2}x + \frac{1}{2}\right)}{x^2+1} dx, (\text{आंशिक भिन्न द्वारा})$$

$$I = \frac{5}{2} \log(x-1) - \frac{5}{2} \int \frac{x dx}{1+x^2} - \frac{1}{2} \int \frac{dx}{1+x^2}$$

$$I = \frac{5}{2} \log(x-1) - \frac{5}{4} \log(1+x^2) - \frac{1}{2} \tan^{-1} x + A$$

$$I = \log(x-1)^{5/2} (1+x^2)^{-5/4} - \frac{1}{2} \tan^{-1} x + A$$

तुलना करने पर, $a = -\frac{5}{4}$

40. (a) $I = \int \frac{(2x^2 + 1)}{(x^2 - 4)(x^2 - 1)} dx$

$$\frac{2x^2 + 1}{(x^2 - 4)(x^2 - 1)} = \frac{3}{(x^2 - 4)} - \frac{1}{x^2 - 1}$$

$$\therefore I = \int \left[\frac{3}{(x^2 - 4)} - \frac{1}{x^2 - 1} \right] dx$$

$$= \frac{3}{2 \times 2} \log \left| \frac{x-2}{x+2} \right| - \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + c$$

$$= \frac{3}{4} \log \left| \frac{x-2}{x+2} \right| + \log \left| \frac{x+1}{x-1} \right|^{1/2} + c$$

$$= \log \left| \frac{x-2}{x+2} \right|^{3/4} + \log \left| \frac{x+1}{x-1} \right|^{1/2} + c$$

$$= \log \left[\left(\frac{x+1}{x-1} \right)^{1/2} \left(\frac{x-2}{x+2} \right)^{3/4} \right] + c$$

अनिश्चित समाकलन

Self Evaluation Test - 23

1. निम्न वाक्यों का अवलोकन कीजिये :

प्रथक्कथन (A) : $\frac{1}{x^2 + a^2}$ का समाकलन $x = a \tan \theta$ प्रतिस्थापन द्वारा ज्ञात किया जा सकता है

कारण (R) : क्योंकि सभी समाकलों को प्रतिस्थापन विधि द्वारा ही समाकलित कर सकते हैं।

इन वाक्यों में से [SCRA 1996]

- (a) दोनों A व R सत्य है और R, A का सही कारण है
 (b) दोनों A व R सत्य है पर R, A का सही कारण नहीं है
 (c) A सत्य है पर R असत्य है
 (d) A असत्य है पर R सत्य है

2. $\int \sqrt{1 + \frac{\sin x}{4}} dx =$ [Karnataka CET 2003]

- (a) $8 \left(\sin \frac{x}{8} - \cos \frac{x}{8} \right) + c$ (b) $\left(\sin \frac{x}{8} + \cos \frac{x}{8} \right) + c$
 (c) $\frac{1}{8} \left(\sin \frac{x}{8} - \cos \frac{x}{8} \right) + c$ (d) $8 \left(\cos \frac{x}{8} - \sin \frac{x}{8} \right) + c$

3. $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx =$ [EAMCET 2003]

- (a) $\frac{1}{2} \sqrt{1+x} + c$ (b) $\frac{2}{3} (1+x)^{3/2} + c$
 (c) $\sqrt{1+x} + c$ (d) $2(1+x)^{3/2} + c$

4. $\int (1+x-x^{-1})e^{x+x^{-1}} dx =$ [EAMCET 2003]

- (a) $(x+1)e^{x+x^{-1}} + c$ (b) $(x-1)e^{x+x^{-1}} + c$
 (c) $-xe^{x+x^{-1}} + c$ (d) $xe^{x+x^{-1}} + c$

5. $\int e^{\cos^2 x} \sin 2x dx =$ [AI CBSE 1980]

- (a) $e^{\cos^2 x} + c$ (b) $-e^{\cos^2 x} + c$
 (c) $\frac{1}{2} e^{\cos^2 x} + c$ (d) इनमें से कोई नहीं

6. $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}} =$ [MNR 1985]

- (a) $\frac{1}{\sqrt{2}} \tan^{-1} \left[\frac{\sqrt{1-x^2}}{x\sqrt{2}} \right] + c$ (b) $\frac{1}{\sqrt{2}} \tan^{-1} \left[\frac{x\sqrt{2}}{\sqrt{1-x^2}} \right] + c$
 (c) $\sqrt{2} \tan^{-1} \left[\frac{\sqrt{1-x^2}}{x\sqrt{2}} \right] + c$ (d) $-\sqrt{2} \tan^{-1} \left[\frac{\sqrt{1-x^2}}{x\sqrt{2}} \right] + c$

7. $\int \sqrt{\frac{a+x}{a-x}} dx =$

- (a) $a \cos^{-1} x / a + \sqrt{a^2 - x^2} + c$
 (b) $a \cos^{-1} x / a - \sqrt{a^2 - x^2} + c$
 (c) $-a \cos^{-1} x / a + \sqrt{a^2 - x^2} + c$
 (d) $-a \cos^{-1} x / a - \sqrt{a^2 - x^2} + c$

8. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx =$ [IIT 1985]

- (a) $\cos^{-1} \sqrt{x} + \sqrt{1-x} \cdot (\sqrt{x} - 2) + c$
 (b) $\cos^{-1} \sqrt{x} - \sqrt{1-x} \cdot (\sqrt{x} - 2) + c$
 (c) $\cos^{-1} \sqrt{x} + \sqrt{1-x} \cdot (\sqrt{x} - 2) + c$
 (d) इनमें से कोई नहीं

9. $\int \frac{1}{x^2(x^4+1)^{3/4}} dx =$

[IIT 1984; RPET 2000; UPSEAT 2001]

- (a) $\frac{(x^4+1)^{1/4}}{x} + c$ (b) $-\frac{(x^4+1)^{1/4}}{x} + c$
 (c) $\frac{3(x^4+1)^{3/4}}{4x} + c$ (d) $\frac{4(x^4+1)^{3/4}}{3x} + c$

10. $\int \sqrt{2+\sin 3x} \cdot \cos 3x dx =$ [IIT 1976]

- (a) $\frac{2}{9} (2+\sin 3x)^{1/2} + c$ (b) $\frac{2}{3} (2+\sin 3x)^{2/3} + c$
 (c) $\frac{2}{3} (2+\sin 3x)^{3/2} + c$ (d) $\frac{2}{9} (2+\sin 3x)^{3/2} + c$

11. $\int \frac{dx}{(2 \sin x + \cos x)^2} =$ [MP PET 1994]

- (a) $\frac{1}{2} \left(\frac{1}{2 \tan x + 1} \right) + c$
 (b) $\frac{1}{2} \log(2 \tan x + 1) + c$
 (c) $\frac{1}{2 + \cot x} + c$
 (d) $-\frac{1}{2} \left(\frac{1}{2 \tan x - 1} \right) + c$

12. $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4} dx =$

- (a) $\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{1/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right]+c$
 (b) $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+c$
 (c) $\frac{2}{3}\left(1+\frac{1}{x^2}\right)^{3/2}\left[\log\left(1+\frac{1}{x^2}\right)+\frac{2}{3}\right]+c$
 (d) इनमें से कोई नहीं

13. $\int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}} dx =$

- (a) $\frac{2}{3}\sin^{-1}(\cos^{3/2} x) + c$ (b) $\frac{3}{2}\sin^{-1}(\cos^{3/2} x) + c$
 (c) $\frac{2}{3}\cos^{-1}(\cos^{3/2} x) + c$ (d) इनमें से कोई नहीं

14. यदि $I'(x)$ का अर्थ $\log \log \log \dots \log x$ है, $\log r$ बार पुनरावृत्त होता है, तब $\int \frac{1}{x I(x) I^2(x) I^3(x) \dots I^r(x)} dx =$

- (a) $I^{r+1}(x) + c$ (b) $\frac{I^{r+1}(x)}{r+1} + c$
 (c) $I^r(x) + c$ (d) इनमें से कोई नहीं

15. यदि c कोई स्वेच्छ अचर है, तो $\int 2^{2^{2^x}} 2^{2^x} 2^x dx =$

[J & K 2005]

- (a) $\frac{2^{2^x}}{(\ln 2)^3} + c$ (b) $\frac{2^{2^{2^x}}}{(\ln 2)^3} + c$
 (c) $2^{2^{2^x}} (\ln 2)^3 + c$ (d) इनमें से कोई नहीं

16. यदि $\int \frac{1}{(\sin x + 4)(\sin x - 1)} dx = A \frac{1}{\tan \frac{x}{2} - 1} + B \tan^{-1}(f(x)) + C$,

तब

- (a) $A = \frac{1}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4 \tan x + 3}{\sqrt{15}}$
 (b) $A = -\frac{1}{5}, B = \frac{1}{\sqrt{15}}, f(x) = \frac{4 \tan\left(\frac{x}{2}\right) + 1}{\sqrt{15}}$
 (c) $A = \frac{2}{5}, B = \frac{-2}{5}, f(x) = \frac{4 \tan x + 1}{5}$
 (d) $A = \frac{2}{5}, B = \frac{-2}{5\sqrt{15}}, f(x) = \frac{4 \tan \frac{x}{2} + 1}{\sqrt{15}}$

17. $\int \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}} =$

[AMU 2001]

- (a) $\sqrt{\tan x} + \frac{\tan^{5/2} x}{5} + c$ (b) $\sqrt{\tan x} + \frac{2}{5} \tan^{5/2} x + c$
 (c) $2\sqrt{\tan x} + \frac{2}{5} \tan^{5/2} x + c$ (d) इनमें से कोई नहीं

18. $\int \frac{dx}{1 - \cos x - \sin x} =$

[EAMCET 2002]

- (a) $\log |1 + \cot x / 2| + c$ (b) $\log |1 - \tan x / 2| + c$
 (c) $\log |1 - \cot x / 2| + c$ (d) $\log |1 + \tan x / 2| + c$

19. $\int \frac{3^x}{\sqrt{9^x - 1}} dx =$

[EAMCET 2002]

- (a) $\frac{1}{\log 3} \log |3^x + \sqrt{9^x - 1}| + c$
 (b) $\frac{1}{\log 3} \log |9^x + \sqrt{9^x - 1}| + c$
 (c) $\frac{1}{\log 9} \log |3^x + \sqrt{9^x - 1}| + c$
 (d) $\frac{1}{\log 9} \log |3^x - \sqrt{9^x - 1}| + c$

20. $\int \frac{e^x dx}{\sqrt{a + be^x}} =$

- (a) $\frac{2}{b} \sqrt{a + be^x} + c$ (b) $2b \sqrt{a + be^x} + c$
 (c) $\frac{1}{2b} \sqrt{a + be^x}$ (d) $\frac{a}{b} \sqrt{a + be^x} + c$

21. $\int u \frac{d^2 v}{dx^2} dx - \int v \frac{d^2 u}{dx^2} dx =$

[MP PET 1990]

- (a) $u \frac{dv}{dx} - v \frac{du}{dx} + c$ (b) $2 \frac{du}{dx} \frac{dv}{dx} + c$
 (c) $uv + c$ (d) c

22. यदि $f(x) = g(x)$, तब $\int f'(x) \cdot g(x) dx$ का मान है

[BIT Ranchi 1991]

- (a) $\{f(x)\}^2 + c$ (b) $\{g(x)\}^2 + c$
 (c) $\frac{1}{2} \{f(x)\}^2 + c$ (d) $\frac{1}{2} \{g(x)\}^2 + c$

23. $\int e^{2x} \left(\frac{\sin 4x - 2}{1 - \cos 4x} \right) dx =$

[Mathematics Olympiad 1986]

- (a) $\frac{1}{2} e^{2x} \cot 2x + c$ (b) $-\frac{1}{2} e^{2x} \cot 2x + c$
 (c) $-2e^{2x} \cot 2x + c$ (d) $2e^{2x} \cot 2x + c$

24. $\int e^{2x}(2 \sin 3x + 3 \cos 3x) dx =$ [MP PET 2003]

- (a) $e^{2x} \sin 3x$ (b) $e^{2x} \cos 3x$
(c) e^{2x} (d) $e^{2x}(2 \sin 3x)$

25. $\int \frac{1}{x^2} \log(x^2 + a^2) dx =$ [MNR 1980]

- (a) $\frac{1}{x} \log(x^2 + a^2) + \frac{2}{a} \tan^{-1} \frac{x}{a} + c$
(b) $-\frac{1}{x} \log(x^2 + a^2) + \frac{2}{a} \tan^{-1} \frac{x}{a} + c$
(c) $-\frac{1}{x} \log(x^2 + a^2) - \frac{2}{a} \tan^{-1} \frac{x}{a} + c$
(d) इनमें से कोई नहीं

26. $\int \frac{x^2 - 1}{x^4 + x^2 + 1} dx =$ [MP PET 2004]

- (a) $\log(x^4 + x^2 + 1) + c$ (b) $\frac{1}{2} \log \frac{x^2 - x + 1}{x^2 + x + 1} + c$
(c) $\frac{1}{2} \log \frac{x^2 + x + 1}{x^2 - x + 1} + c$ (d) $\log \frac{x^2 - x + 1}{x + x + 1} + c$

27. $\int \frac{x^4}{(x-1)(x^2+1)} dx =$ [Roorkee 1986]

- (a) $\frac{x(x+2)}{2} + \frac{\log(x-1)}{2} - \frac{\log(x^2+1)}{4} - \frac{\tan^{-1} x}{2} + c$
(b) $\frac{x(x+2)}{2} + \frac{\log(x-1)}{2} + \frac{\log(x^2+1)}{4} - \frac{\tan^{-1} x}{2} + c$
(c) $\frac{x(x+2)}{2} + \frac{\log(x-1)}{2} + \frac{\log(x^2+1)}{4} + \frac{\tan^{-1} x}{2} + c$
(d) इनमें से कोई नहीं

28. $\int \frac{x^3 - 1}{x^3 + x} dx =$ [Roorkee 1988; MP PET 2001]

- (a) $x - \log x + \frac{1}{2} \log(x^2 + 1) + \tan^{-1} x + c$
(b) $x - \log x + \log \sqrt{x^2 + 1} - \tan^{-1} x + c$
(c) $x + \log x + \log \sqrt{x^2 + 1} + \tan^{-1} x + c$
(d) इनमें से कोई नहीं

29. $x > 1$ के लिये, $\int \frac{1}{x(x^4 - 1)} dx =$ [RPET 1997]

- (a) $\log \frac{x^4 - 1}{x^4} + K$
(b) $\frac{1}{4} \log \frac{x^4 - 1}{x^4} + K$
(c) $\log \frac{x^4 - 1}{x} + K$
(d) $\frac{1}{4} \log \frac{x^4 - 1}{x} + K$

30. यदि $\int \frac{2x^2 + 3}{(x^2 - 1)(x^2 + 4)} dx = a \log \left(\frac{x-1}{x+1} \right) + b \tan^{-1} \frac{x}{2} + c$, तब a व b के मान हैं [RPET 2000]

- (a) (1, -1)
(b) (-1, 1)
(c) $\left(\frac{1}{2}, -\frac{1}{2} \right)$
(d) $\left(\frac{1}{2}, \frac{1}{2} \right)$

1. (c) A सत्य है किन्तु R असत्य है

$$\begin{aligned} 2. (a) \int \sqrt{1 + \sin\left(\frac{x}{4}\right)} dx \\ &= \int \sqrt{\left(\sin^2 \frac{x}{8} + \cos^2 \frac{x}{8}\right) + \left(2 \sin \frac{x}{8} \cos \frac{x}{8}\right)} dx \\ &= \int \sqrt{\left(\sin \frac{x}{8} + \cos \frac{x}{8}\right)^2} dx = \int \left(\sin \frac{x}{8} + \cos \frac{x}{8}\right) dx \\ &= \frac{-\cos \frac{x}{8}}{\left(\frac{1}{8}\right)} + \frac{\sin \frac{x}{8}}{\left(\frac{1}{8}\right)} + c = 8 \left(\sin \frac{x}{8} - \cos \frac{x}{8}\right) + c. \end{aligned}$$

$$\begin{aligned} 3. (b) \int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}} dx &= \int \frac{\sqrt{1+x}[\sqrt{1+x}+\sqrt{x}]}{(\sqrt{x}+\sqrt{1+x})} dx \\ &= \int \sqrt{1+x} dx = \frac{2}{3}(1+x)^{3/2} + c. \end{aligned}$$

$$\begin{aligned} 4. (d) \int (1+x-x^{-1})e^{x+x^{-1}} dx \\ &= \int x e^{x+x^{-1}} \left(1 - \frac{1}{x^2}\right) + e^{x+x^{-1}} dx \\ &\quad \left(\because \int [x f'(x) + f(x)] dx = x f(x) + c\right) \\ \therefore \int (1+x-x^{-1})e^{x+x^{-1}} dx &= x e^{x+x^{-1}} + c. \end{aligned}$$

$$\begin{aligned} 5. (b) t = \cos^2 x \text{ रखने पर } \Rightarrow dt &= -\sin 2x dx, \text{ तब} \\ \int e^{\cos^2 x} \sin 2x dx &= -\int e^t dt = -e^t + c = -e^{\cos^2 x} + c. \end{aligned}$$

$$\begin{aligned} 6. (b) x = \sin \theta \text{ रखने पर } \Rightarrow dx &= \cos \theta d\theta, \text{ तब} \\ \int \frac{dx}{(1+x^2)\sqrt{1-x^2}} &= \int \frac{1}{1+\sin^2 \theta} d\theta = \int \frac{\sec^2 \theta}{1+2\tan^2 \theta} d\theta \\ \text{पुनः } t = \tan \theta \text{ रखने पर } \Rightarrow dt &= \sec^2 \theta d\theta, \\ \therefore \int \frac{1}{1+2t^2} dt &= \frac{1}{2} \int \frac{1}{t^2 + \left(\frac{1}{\sqrt{2}}\right)^2} dt \\ &= \frac{1}{2} \left(\frac{1}{(1/\sqrt{2})}\right) \tan^{-1} \left(\frac{t}{(1/\sqrt{2})}\right) + c \\ &= \frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2} \tan \theta) + c = \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{x\sqrt{2}}{\sqrt{1-x^2}}\right) + c. \end{aligned}$$

वैकल्पिक : पहले $x = \frac{1}{t}$ रखने पर तथा तब $t^2 - 1 = z^2$.

$$\begin{aligned} 7. (d) \int \sqrt{\frac{a+x}{a-x}} dx; x = a \cos \theta \text{ रखने पर} \\ \Rightarrow dx = -a \sin \theta d\theta, \\ \therefore -a \int \sqrt{\frac{1+\cos \theta}{1-\cos \theta}} (\sin \theta) d\theta \end{aligned}$$

$$\begin{aligned} &= -2a \int \sqrt{\frac{2\cos^2(\theta/2)}{2\sin^2(\theta/2)}} \cdot \sin \frac{\theta}{2} \cos \frac{\theta}{2} d\theta \\ &= -a \int (1 + \cos \theta) d\theta = -a \left[\cos^{-1} \frac{x}{a} + \sqrt{\frac{a^2 - x^2}{a}} \right] + c \\ &= -a \cos^{-1} \frac{x}{a} - \sqrt{a^2 - x^2} + c. \end{aligned}$$

8. (a) $x = \cos^2 \theta$ रखने पर $\Rightarrow dx = -2 \cos \theta \sin \theta d\theta$, तब

$$\begin{aligned} \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx &= -4 \int \sin^2 \frac{\theta}{2} \cos \theta d\theta \\ &= -2 \int (1 - \cos \theta) \cos \theta d\theta = \theta + \frac{1}{2} \sin 2\theta - 2 \sin \theta \\ &= \theta + \sin \theta \cos \theta - 2 \sin \theta \\ &= \cos^{-1} \sqrt{x} + (\sqrt{1-x})(\sqrt{x} - 2) + c \end{aligned}$$

$$\begin{aligned} 9. (b) \int \frac{1}{x^2(x^4+1)^{3/4}} dx &= \int \frac{dx}{x^5 \left(1 + \frac{1}{x^4}\right)^{3/4}} \\ 1 + \frac{1}{x^4} = t \text{ रखने पर } \Rightarrow \frac{-4}{x^5} dx &= dt, \\ \therefore -\frac{1}{4} \int \frac{dt}{t^{3/4}} &= -\frac{1}{4} \cdot \frac{4}{1} t^{1/4} + c = -t^{1/4} + c \\ &= -\left(1 + \frac{1}{x^4}\right)^{1/4} + c = -\frac{(x^4+1)^{1/4}}{x} + c. \end{aligned}$$

$$\begin{aligned} 10. (d) 2 + \sin 3x = t \text{ रखने पर } \Rightarrow 3 \cos 3x dx &= dt, \text{ तब} \\ \int \sqrt{2 + \sin 3x} \cos 3x dx &= \frac{1}{3} \int t^{1/2} dt = \frac{2}{9} t^{3/2} + c \\ &= \frac{2}{9} (2 + \sin 3x)^{3/2} + c. \end{aligned}$$

$$\begin{aligned} 11. (c) \int \frac{dx}{(2 \sin x + \cos x)^2} &= \int \frac{dx}{\sin^2 x (2 + \cot x)^2} = \int \frac{\operatorname{cosec}^2 x dx}{(2 + \cot x)^2} \\ (2 + \cot x) = t \text{ रखने पर } \Rightarrow -\operatorname{cosec}^2 x dx &= dt \\ &= \int \frac{-dt}{t^2} = \frac{1}{t} + c = \frac{1}{2 + \cot x} + c. \end{aligned}$$

$$\begin{aligned} 12. (b) \int \frac{\sqrt{x^2+1} [\log(x^2+1) - 2 \log x]}{x^4} dx \\ &= \int \sqrt{1 + \frac{1}{x^2}} \log \left(1 + \frac{1}{x^2}\right) \cdot \frac{1}{x^3} dx \\ &\quad \left\{ 1 + \frac{1}{x^2} = t \text{ रखने पर, } -\frac{2}{x^3} dx = dt \right\} \\ &= -\frac{1}{2} \int \sqrt{t} \log t dt = -\frac{1}{2} \left\{ \log t \left(\frac{t^{3/2}}{3/2}\right) - \frac{2}{3} \int \frac{1}{t} t^{3/2} dt \right\} \\ &= -\frac{1}{2} \left\{ \frac{2}{3} \log t \cdot t^{3/2} - \frac{2}{3} \cdot \left(\frac{2}{3} t^{3/2}\right) \right\} + c \end{aligned}$$

$$= -\frac{1}{3}t^{3/2} \log t + \frac{2}{9}t^{3/2} + c$$

$$= -\frac{1}{3}\left(1 + \frac{1}{x^2}\right)^{3/2} \left[\log\left(1 + \frac{1}{x^2}\right) - \frac{2}{3} \right] + c.$$

$$\begin{aligned} 13. \quad (c) \quad \int \sqrt{\frac{\cos x - \cos^3 x}{1 - \cos^3 x}} dx &= \int \sqrt{\frac{\cos x}{1 - \cos^3 x}} \sin x dx \\ &= -\int \sqrt{\frac{t}{1-t^3}} dt = -\int \frac{\sqrt{t}}{\sqrt{1-(t^{3/2})^2}} dt = -\frac{2}{3} \int \frac{\frac{3}{2}\sqrt{t}}{\sqrt{1-(t^{3/2})^2}} dt \\ &= \frac{2}{3} \cos^{-1}(t^{3/2}) + c = \frac{2}{3} \cos^{-1}(\cos^{3/2} x) + c. \end{aligned}$$

$$\begin{aligned} 14. \quad (a) \quad l^{r+1}(x) = t \text{ रखने पर } &\Rightarrow \frac{1}{xl(x)l^2(x) \dots l^r(x)} dx = dt, \\ \therefore \int \frac{1}{xl(x)l^2(x) \dots l^r(x)} dx &= \int 1 \cdot dt = t + c = l^{r+1}(x) + c. \end{aligned}$$

$$\begin{aligned} 15. \quad (b) \quad 2^{2^{2^x}} = t \text{ रखने पर } &\Rightarrow 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x (\log 2)^3 dx = dt, \\ \int 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x dx &= \frac{1}{(\log 2)^3} \int 1 \cdot dt = \frac{t}{(\log 2)^3} + c \\ &= \frac{2^{2^{2^x}}}{(\log 2)^3} + c. \end{aligned}$$

$$\begin{aligned} 16. \quad (d) \quad \int \frac{1}{(\sin x + 4)(\sin x - 1)} dx &= \frac{1}{5} \int \frac{(\sin x + 4) - (\sin x - 1)}{(\sin x + 4)(\sin x - 1)} dx \\ &= \frac{1}{5} \int \frac{1}{\sin x - 1} dx - \frac{1}{5} \int \frac{1}{\sin x + 4} dx \\ &= \frac{1}{5} \int \frac{2 dt}{2t - 1 - t^2} - \frac{1}{5} \int \frac{2 dt}{2t + 4(1 + t^2)} \\ &\quad \left(\tan \frac{x}{2} = t \text{ रखने पर} \right) \\ &= -\frac{2}{5} \int \frac{dt}{t^2 - 2t + 1} - \frac{1}{10} \int \frac{dt}{t^2 + \frac{1}{2}t + 1} \\ &= -\frac{2}{5} \int \frac{1}{(t-1)^2} dt - \frac{1}{10} \int \frac{dt}{\left(t + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{15}}{4}\right)^2} \\ &= \frac{2}{5} \frac{1}{(t-1)} - \frac{2}{5\sqrt{15}} \tan^{-1} \left(\frac{4t+1}{\sqrt{15}} \right) + c \\ &= \frac{2}{5} \cdot \frac{1}{\tan \frac{x}{2} - 1} - \frac{2}{5\sqrt{15}} \tan^{-1} \left(\frac{4 \tan \frac{x}{2} + 1}{\sqrt{15}} \right) + c \\ \therefore A &= \frac{2}{5}, \quad B = \frac{-2}{5\sqrt{15}}, \quad f(x) = \frac{4 \tan \frac{x}{2} + 1}{\sqrt{15}}. \end{aligned}$$

$$17. \quad (a) \quad \int \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}} = \int \frac{dx}{\cos^3 x \sqrt{4 \sin x \cos x}}$$

$$\begin{aligned} &= \frac{1}{2} \int \frac{dx}{\cos^{7/2} x \sin^{1/2} x} \\ &= \frac{1}{2} \int \frac{\sec^4 x}{\sqrt{\tan x}} dx = \frac{1}{2} \int \frac{(1 + \tan^2 x) \sec^2 x}{\sqrt{\tan x}} dx \\ &= \frac{1}{2} \int \frac{1+t^2}{\sqrt{t}} dt \quad (\tan x = t \text{ रखने पर, } \therefore \sec^2 x dx = dt) \\ &= \frac{1}{2} \int t^{-1/2} dt + \frac{1}{2} \int t^{3/2} dt = t^{1/2} + \frac{t^{5/2}}{5} + c \\ &= \sqrt{\tan x} + \frac{1}{5} \tan^{5/2} x + c. \end{aligned}$$

$$\begin{aligned} 18. \quad (c) \quad I &= \int \frac{dx}{1 - \cos x - \sin x} \\ &= \int \frac{dx}{1 - \frac{[(1 - \tan^2(x/2)) - 2 \tan(x/2)]}{[1 + \tan^2(x/2)]} - 1 + \tan^2(x/2)} \\ &= \int \frac{\sec^2(x/2) dx}{1 + \tan^2(x/2) - 1 + \tan^2(x/2) - 2 \tan(x/2)} \\ &= \int \frac{\sec^2(x/2) dx}{2 \tan^2(x/2) - 2 \tan(x/2)} = \int \frac{\frac{1}{2} \cdot \sec^2\left(\frac{x}{2}\right) dx}{\tan^2\left(\frac{x}{2}\right) - \tan\left(\frac{x}{2}\right)} \end{aligned}$$

$$\tan(x/2) = t \text{ रखने पर } \Rightarrow \frac{1}{2} \sec^2(x/2) dx = dt$$

$$\begin{aligned} I &= \int \frac{dt}{t^2 - t} = \int \frac{dt}{t(t-1)} = \int \left[\frac{1}{t-1} - \frac{1}{t} \right] dt \\ &= \int \frac{dt}{t-1} - \int \frac{dt}{t} = \log(t-1) - \log t + c = \log \left| \frac{t-1}{t} \right| + c \\ &= \log \left| \frac{\tan(x/2) - 1}{\tan(x/2)} \right| + c = \log \left| 1 - \cot \frac{x}{2} \right| + c. \end{aligned}$$

$$\begin{aligned} 19. \quad (a) \quad I &= \int \frac{3^x}{\sqrt{9^x - 1}} dx = \int \frac{3^x dx}{\sqrt{(3^x)^2 - 1}} \\ 3^x = t \text{ रखने पर } &\Rightarrow 3^x \log 3 dx = dt \Rightarrow 3^x dx = dt / \log 3 \\ \Rightarrow I &= \frac{1}{\log 3} \int \frac{dt}{\sqrt{t^2 - 1}} = \frac{1}{\log 3} \log \left[t + \sqrt{t^2 - 1} \right] + c \\ &= \frac{1}{\log_e 3} \log \left[3^x + \sqrt{9^x - 1} \right] + c. \end{aligned}$$

$$\begin{aligned} 20. \quad (a) \quad a + be^x = t \text{ रखने पर } &\Rightarrow be^x dx = dt, \text{ तब } \\ \int \frac{e^x dx}{\sqrt{a + be^x}} &= \frac{1}{b} \int \frac{1}{\sqrt{t}} dt = \frac{2}{b} \sqrt{t} + c = \frac{2\sqrt{a + be^x}}{b} + c. \end{aligned}$$

$$\begin{aligned} 21. \quad (a) \quad \int u \frac{d^2 v}{dx^2} dx - \int v \frac{d^2 u}{dx^2} dx \\ &= u \frac{dv}{dx} - \int \frac{du}{dx} \cdot \frac{dv}{dx} dx - v \frac{du}{dx} + \int \frac{dv}{dx} \cdot \frac{du}{dx} dx + c \\ &= u \frac{dv}{dx} - v \frac{du}{dx} + c. \end{aligned}$$

22. (c, d) दिया गया है $f(x) = g(x)$ अब $\int f'(x)g(x)dx$

$$= g(x) \int f'(x)dx - \int \left[g'(x) \int f'(x)dx \right] dx + c$$

$$= g(x)f(x) - \int g'(x)f(x)dx + c$$

$$\Rightarrow \int f'(x)g(x)dx = g(x)f(x) - \int f'(x)g(x)dx + c$$

$$\therefore f(x) = g(x) \Rightarrow f'(x) = g'(x) \quad \dots(i)$$

$$(i) \text{ से, } 2 \int f'(x)g(x)dx = g(x)f(x) + c$$

$$\therefore \int f'(x)g(x)dx = \frac{1}{2} \{g(x)\}^2 + c$$

$$\text{साथ ही } \int f'(x) \cdot g(x)dx = \int f'(x) \cdot f(x)dx$$

$$= \frac{1}{2} \{f(x)\}^2 + c.$$

$$\text{वैकल्पिक : } \int f'(x) \cdot g(x)dx = \int f'(x) \cdot f(x)dx$$

$$= \int t dt, \{f(x) = t \Rightarrow f'(x)dx = dt\}$$

$$= \frac{t^2}{2} + c = \frac{\{f(x)\}^2}{2} + c = \frac{\{g(x)\}^2}{2} + c.$$

23. (a) $\int e^{2x} \left(\frac{\sin 4x - 2}{1 - \cos 4x} \right) dx = \int \frac{e^{2x} \sin 4x}{1 - \cos 4x} dx - 2 \int \frac{e^{2x}}{1 - \cos 4x} dx$

$$= \int e^{2x} \cot 2x dx - \int e^{2x} \operatorname{cosec}^2 2x dx$$

$$= \frac{e^{2x} \cot 2x}{2} + \int 2 \frac{e^{2x}}{2} \operatorname{cosec}^2 2x dx - \int e^{2x} \operatorname{cosec}^2 x dx$$

$$= \frac{1}{2} (e^{2x} \cot 2x) + c.$$

24. (a) $\int e^{2x} (2 \sin 3x + 3 \cos 3x) dx$

$$= e^{2x} \sin 3x + c \left\{ \because \int e^{mx} [mf(x) + f'(x)] dx = e^{mx} f(x) + c \right\}.$$

25. (b) $\int \frac{1}{x^2} \log(x^2 + a^2) dx = \int x^{-2} \log(x^2 + a^2) dx$

$$= \frac{-\log(x^2 + a^2)}{x} + \int \frac{2x}{(x^2 + a^2)} \cdot \frac{1}{x} + c$$

$$= \frac{-\log(x^2 + a^2)}{x} + \frac{2}{a} \tan^{-1} \frac{x}{a} + c.$$

26. (b) $I = \int \frac{x^2 - 1}{x^4 + x^2 + 1} dx = \int \frac{x^2 \left(1 - \frac{1}{x^2} \right)}{x^2 \left[\left(x + \frac{1}{x} \right)^2 - 1 \right]} dx$

$$\left(x + \frac{1}{x} \right) = t \text{ रखने पर } \Rightarrow \left(1 - \frac{1}{x^2} \right) dx = dt$$

$$I = \int \frac{dt}{t^2 - 1} = \frac{1}{2} \log \left| \frac{t-1}{t+1} \right| + c$$

$$\therefore I = \frac{1}{2} \log \left| \frac{x^2 - x + 1}{x^2 + x + 1} \right| + c \Rightarrow a = \frac{1}{2}, b = \frac{1}{2}.$$

27. (a) $\int \frac{x^4}{(x-1)(x^2+1)} dx = \int \frac{x^4 - 1}{(x-1)(x^2+1)} + \int \frac{1}{(x-1)(x^2+1)} dx$

$$= \int \frac{(x+1)(x-1)(x^2+1)}{(x-1)(x^2+1)} dx + \int \frac{dx}{(x-1)(x^2+1)}$$

$$= \int (x+1) dx + \int \frac{dx}{(x-1)(x^2+1)}$$

$$= \frac{x^2}{2} + x + \left[\frac{1}{2} \log(x-1) - \frac{1}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x \right] + c.$$

28. (b) $\int \frac{x^3 - 1}{x^3 + x} dx = \int \frac{x^3}{x(x^2+1)} dx - \int \frac{1}{x(x^2+1)} dx$

$$= \int \frac{x^2}{x^2+1} dx - \int \left(\frac{1}{x} - \frac{x}{x^2+1} \right) dx$$

$$= \int \left(1 - \frac{1}{x^2+1} \right) dx - \int \frac{1}{x} dx + \int \frac{x}{x^2+1} dx$$

$$= x - \tan^{-1} x - \log x + \log \sqrt{x^2+1} + c.$$

29. (b) $\int \frac{1}{x(x^4-1)} dx = \frac{1}{4} \int \left[\frac{4x^3}{(x^4-1)} - \frac{4}{x} \right] dx$

$$= \frac{1}{4} [\log(x^4-1) - 4 \log x] + c = \frac{1}{4} \log \frac{x^4-1}{x^4} + c.$$

30. (d) $I = \int \frac{2x^2 + 3}{(x^2-1)(x^2+4)} dx$

$$\therefore \frac{2x^2 + 3}{(x^2-1)(x^2+4)} = \frac{1}{(x^2-1)} + \frac{1}{(x^2+4)}$$

$$\therefore I = \int \frac{dx}{(x^2-1)} + \int \frac{dx}{x^2+4}$$

$$I = \frac{1}{2 \times 1} \log \left| \frac{x-1}{x+1} \right| + \frac{1}{2} \tan^{-1} \frac{x}{2} + c$$

$$\Rightarrow I = \frac{1}{2} \log \left(\frac{x-1}{x+1} \right) + \frac{1}{2} \tan^{-1} \frac{x}{2} + c$$

$$\Rightarrow a = \frac{1}{2}, b = \frac{1}{2}.$$

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