CBSE Class 09

Mathematics

Sample Paper 11 (2019-20)

Maximum Marks: 80

Time Allowed: 3 hours

General Instructions:

i. All the questions are compulsory.
ii. The question paper consists of 40 questions divided into 4 sections A, B, C, and D.
iii. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises of 6 questions of 4 marks each.
iv. There is no overall choice. However, an internal choice has been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each, and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
v. Use of calculators is not permitted.

Section A

1. The decimal form of $\frac{1}{999}$ is

   a. 0.999

   b. $0.\overline{001}$

   c. $0.00\overline{1}$

   d. $0.0\overline{01}$

2. A symbol having a fixed value is called a _____.

   a. coefficient

   b. none of these

c. constant

d. variable

3. If two angles are supplementary and the larger is 20^0 less then three times the smaller, then the angles are :-
   a. $72\frac{1}{2}\ ^0, 17\frac{1}{2}\ ^0$
   b. $140^0, 40^0$
   c. $130^0, 50^0$
   d. $62\frac{1}{2}\ ^0, 27\frac{1}{2}\ ^0$

4. In $\triangle$ ABC if $\angle B = \angle C = 30^0$, which of the following is the longest side?

   a. BC

   b. none

   c. AC

   d. AB

5. The remainder when the polynomial $x^4 + 2x^3 - 3x^2 + x - 1$ is divided by $(x - 2)$ is

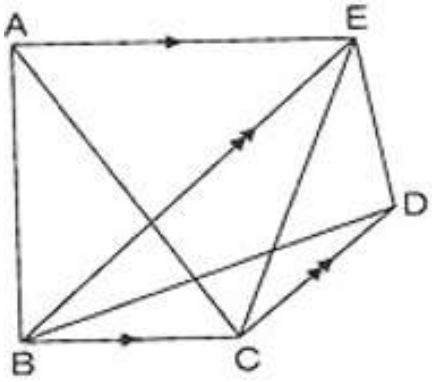
   a. 19

   b. 21

   c. 20

   d. -21

6. In the given figure, if $BC \parallel AE,\ CD \parallel BE$, and $ar\ (\triangle BED) = 6\ cm^2$, than $ar\ (\triangle ABC)$ is



a. $10\ cm^2$.

b. $8\,cm^2$.

c. $12\ cm^2$.

d. $6\ cm^2$.

7. When $p(x) = x^3 + ax^2 + 2x + a$ is divided by $x + a$, the remainder is

   a. a

   b. 0

   c. 1

   d. -a

8. The area of one triangular part of a rhombus ABCD is given as 125 cm^2. The area of rhombus ABCD is

   a. 250 cm^2

   b. 625 cm^2

   c. 500 cm^2

   d. 2500 cm^2

9. Volume of a sphere whose radius is 7 cm

   a. $1437\frac{1}{3}cm^3$

b. $1447cm^3$

c. $1430cm^3$

d. $1337\frac{1}{3}cm^3$

10. The probability of getting a rotten egg in a lot of 400 is 0.035. The number of rotten eggs in the lot is

    a. 21

    b. 14

    c. 28

    d. 7

11. Fill in the blanks:

    The value of $\frac{1+\sqrt{2}}{1-\sqrt{2}} =$ _____.

12. Fill in the blanks:

    The graph of the linear equation 2x + 3y = 6 is a line which meets the X-axis at the point is _____.

OR

Fill in the blanks:

If the point (3, 4) lies on the graph of the equation 3y - ax - 7 = 0, then the value of a is _____.

13. Fill in the blanks:

    The point of intersection of X and Y axes is called _____.

14. Fill in the blanks:

    A circle divides the plane, on which it lies in _____ parts.

15. Fill in the blanks:

    If dimensions of a cuboid are in the ratio are 1 : 2 : 3 and its total surface area is 88 m^2., then its volume is _____ m^3.

16. Prove that $2 - \sqrt{5}$ is an irrational number.

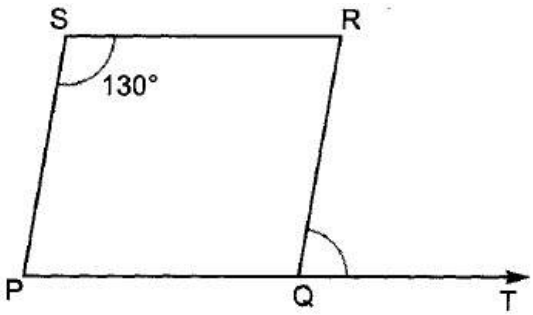
17. Find the value of the polynomial $5x - 4x^2 + 3$ at $x = -1$

18. The outer measurements of a closed wooden box are 42 cm, 30 cm and 27 cm. If the box is made of 1 cm thick wood, determine the capacity of the box.

    OR

    Find the surface area of a sphere of radius 14 cm.

19. In Fig. PQRS is a parallelogram in which ∠PSR = 130°. Find ∠RQT



20. In some countries, temperature is measured in Fahrenheit, whereas in countries like India it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:
    $\mathbf{F} = \left(\frac{9}{5}\right)\mathbf{C} + 32$ If the temperature is 30°C, then what is the temperature in Fahrenheit?

21. Write the decimal form and state its kind of decimal expansion. $\frac{3}{13}$

22. Draw the graph of each of the line a equations in two variables:
    x-y=2

23. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by 5 + 2x.

    OR

Factorize: $27y^3 + 125z^3$

24. A craft mela is organised by welfare Association to promote the art and culture of tribal people. The pandal is to be decorated by using triangular flags around the field. Each flag has dimension 25 cm, 25 cm and 22 cm. Find the area of cloth required for making 200 such flags.

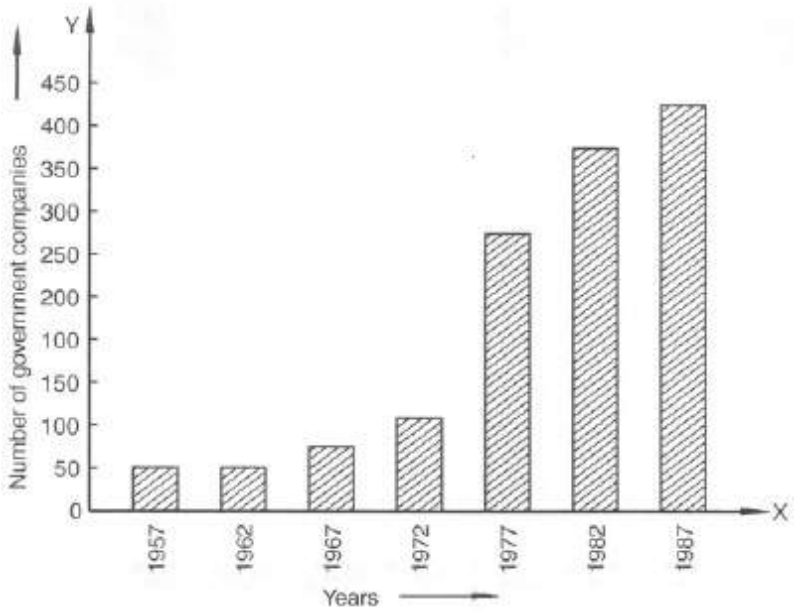
25. The value of up to 15 decimal places is: 3. 419078023195679

    i. List the digits from 0 to 9 & make frequency distributions of the digit after the decimal points.
    ii. What are the most * the least frequently occurring digits?

OR

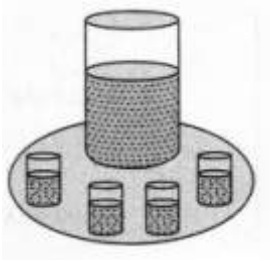
Read the following bar graph in the figure and answer the following questions:

i. What is the information given by the bar graph?
ii. State each of the following whether true or false.
    a. The number of government companies in 1957 is that of 1982 is 1 : 9.
    b. The number of government companies have decreased over the year 1957



26. At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of the base radius 15 cm filled upto a height of 32 cm with the orange juice. The juice is filled in small cylindrical glasses **(see figure)** of radius 3 cm upto a height of 8 cm

and sold for Rs.3 each. How much money does the stall keeper receive by selling the juice completely?



27. Find the value of the following correct to three places of decimals, $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$

OR

Prove that 7.478478... is a rational number.

28. Draw the graph of y – 3x = 9.

29. Write two solutions for the following equation: $\frac{2}{3}$ x - y = 4.

OR

Draw the graph of the following equation. Read a few solutions from the graph and verify the same by actual substitution and find the points where the line meets the two axes. y – 3x = 9

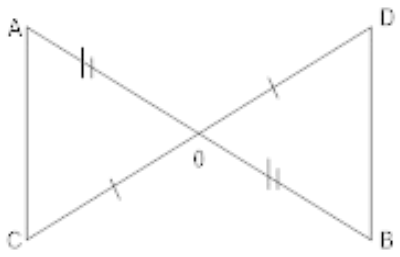
30. Construct perpendicular bisector of line segment 8cm.

31. If the diagonals of a parallelogram are equal, then show that it is a rectangle.

32. In the external bisector of the vertical angle of a triangle is parallel to its base, then prove that the triangle is isosceles.

OR

O is the mid-point of AB and CD. Prove that AC = BD and AC $\parallel$ BD.



33. A parallelogram, the length of whose side is 60 m and 25 m has one diagonal 65 m long. Find the area of the parallelogram.

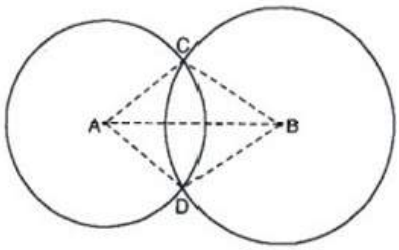
34. The following table gives the lifetime of 400 neon lamps:

Lifetime (in hours)	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Number of lamps	14	56	60	86	74	62	48

A bulb is selected at random. Find the probability that the lifetime of the selected bulb is:

i. less than 400
ii. between 300 to 800 hours
iii. at least 700 hours

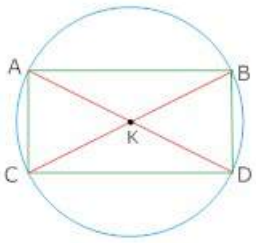
35. If two circles intersect at two points, prove that their centres lie on a perpendicular bisector of the common chord.



OR

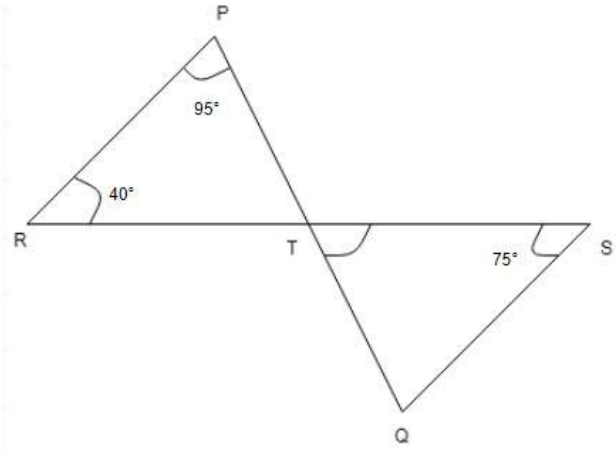
The two chords bisect each other AD and BC show that

i. AD and BC are diameter
ii. ABCD is a rectangle



36. In figure if lines PQ and RS intersect at point T. Such that ∠PRT = 40° ∠RPT = 95° and

$\angle TSQ = 75°$, find $\angle SQT$.



37. If $a + b + c = 6$ and $ab + bc + ca = 11$, find the value of $a^3 + b^3 + c^3 - 3abc$.

OR

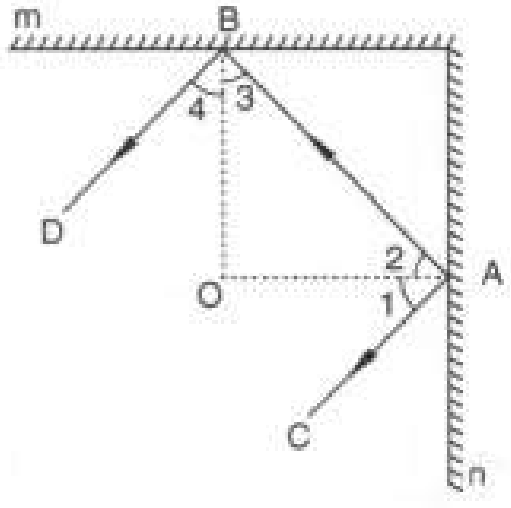
Using factor theorem, factorize the polynomial $x^4 + x^3 - 7x^2 - x + 6$

38. A cylindrical container of radius 6 cm and height 15 cm is filled with ice-cream. The whole ice-cream has to be distributed to 10 children in equal cones with hemispherical tops. If the height of the conical portion is four times the radius of its base, find the radius of the ice-cream cone.

OR

The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost of painting one sq. cm of the surface is 7 paise. Find the total cost to paint the vessel all over, (ignore the area of edge).

39. In a given figure, m and n are two plane mirrors perpendicular each other. Show that the incident ray $\overrightarrow{CA}$ is parallel to the reflected ray BD.



40. In a mathematics test given to 10 students, the following marks [out of 100] are recorded as: 82, 41, 39, 52, 53, 45, 96, 47, 50, 60.
    Find out the mean & median of the above marks.

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Solution

Section A

1. (b) $0.\overline{001}$

   Explanation: When we divide 1 by 999 it result is 0.001001001001001.....

   So,

   $0.\overline{001}=\frac{1}{999}$

2. (c) constant

   Explanation: A symbol having a fixed value is called a constant. Ex. Any natural number, whole number, integers, rational number.

   A symbol having the variable values is called variable.

3. (c) 130^0 , 50^0 **Explanation:** Let the two supplementary angles be x^0 and 180^0 - x

   Let 180^0 - x be the larger angle

   180^0 - x = 3x - 20^0

   4x = 200^0

   x = 50^0

   So the angles are 50^0 and 130^0

4. (a) BC

   Explanation: As, $\angle B = \angle C = 30^0$ (Given)

   Therefore, AB = AC (opposite sides of two equal angles are equal)

   Since, two sides are equal, and the triangle is an abtuse angle isoceles triangle, therefore, the base BC will be the longest side of the triangle.

5. (b) 21

   Explanation: $x^4 + 2x^3 - 3x^2 + x - 1$

   Using remainder theorem,

   = $(2)^4 + 2(2)^3 - 3(2)^2 + 2 - 1$

   = $16 + 16 - 12 + 2 - 1$

   = $34 - 13$

   = 21

6. (d) $6\ cm^2$.

   Explanation: Since triangles BDE and and BEC are on the same base BE and between the same parallels, then

   $\text{area}\,(\triangle \text{BDE}) = \text{area}\,(\triangle \text{BEC}) = 6$ sq. cm
   Since triangles BEC and and ABC are on the same base BE and between the same parallels, then
   $\text{area}\,(\triangle \text{BEC}) = \text{area}\,(\triangle \text{ABC}) = 6$ sq. cm

7. (d) -a

   Explanation:

   $p(x) = x^3 + ax^2 + 2x + a$

   Using remainder theorem,

   $p\,(-a) = (-a)^3 + a(-a)^2 + 2\,(-a) + a$

   $\Rightarrow p\,(-a) = -a^3 + a^3 - 2a + a$

   $\Rightarrow p\,(-a) = -a$

8. (c) 500 cm^2

   Explanation: Since diagonals of a rhombus divide it into 4 triangles of equal area.

Therefore,

Area of rhombus = 4 x Area of triangle

= 4 x 125 = 500 sq. cm

9. (a) $1437\frac{1}{3}cm^3$

Explanation:

Volume of sphere = $\frac{4}{3}\pi r^3$ (Given r = 7cm)

= $\frac{4}{3} \times \frac{22}{7} \times 7^3$

= $\frac{88 \times 343}{7 \times 3}$

= $1437\frac{1}{3}cm^3$

10. (b) 14

Explanation: Let the number of rotten eggs be x.

Total number of eggs = 400

The probability of getting a rotten egg = 0.035 = $\frac{x}{400}$

Therefore , x = 0.035 x 400 = 14

So,the number of rotten eggs =14

11. $-\left(3+2\sqrt{2}\right)$

12. (3, 0)

OR

$\frac{5}{3}$

13. origin

14. three

15. 48

16. $2-\sqrt{5}$

We know that $\sqrt{5} = 2.236...., \text{ which is an irrational number.}$

$2 - \sqrt{5} = 2 - 2.236....$

$= -0.236....,$which is also an irrational number.

Therefore, we conclude that $2 - \sqrt{5}$ is an irrational number.

17. Let f(x) = $5x - 4x^2 + 3$

    The value of f(x) at x = – 1

    $f(-1) = 5(-1) - 4(-1)^2 + 3$

    $= -5 - 4 + 3 = -6$

18. Outer dimensions : l = 42 cm, b = 30 cm, h = 27 cm

    Thickness of wood = 1 cm.

    Inner dimensions :

    l = 42 – (1 + 1) = 40 cm

    b = 30 – (1 + 1) = 28 cm

    h = 27 – (1 + 1) = 25 cm

    ∴ Capacity of the box = l × b × h

    = 40 × 28 × 25 = 28000 cm^3

OR

r = 14 cm

Surface area of a sphere =$4\pi r^2$

$= 4 \times \frac{22}{7} \times (14)^2$

$= 4 \times 22 \times 17 \times 2$

= 2464 cm^2

19. We have $\angle S = \angle Q = 130^\circ$ (Opposite angles of a parallelogram)

    Also, 130^o + ∠RQT = 180^o (Linear pair)

    ∠RQT = 180^o - 130^o

    = 50^o

20. Let F = x and C = y

$\therefore\ x = \frac{9}{5}y + 32 \Rightarrow \therefore\ x = \frac{9}{5}(30) + 32$

= 54 + 32 = 86 F

21.

13) 3.00000000000 (0.230769230769.....
26
40
39
100
91
90
78
120
117
30
26
40
39
100
91
90
78
120
117
3

$\therefore \frac{3}{13}$ = 0.230769230769 ... $= 0.\overline{230769}$

The decimal expansion is non-terminating repeating.

22. x-y=2

if $x = 0 \Rightarrow y = -2$

$x = 1 \Rightarrow y = -1$

$x = 2 \Rightarrow y = 0$

are the solutions of the linear equation x - y=2.

We can optionally consider the given below table for plotting the linear equation x-y=2 on the graph.

X	0	1	2

y	-2	-1	0

23. Let p(x) = $x^3 + 3x^2 + 3x + 1$

    5 + 2x = 0

    $\Rightarrow$ 2x = -5

    $\Rightarrow$ x = $-\frac{5}{2}$

    = $(-\frac{5}{2})^3 + 3(-\frac{5}{2})^2 + 3(-\frac{5}{2}) + 1$

    = $-\frac{125}{8} + \frac{75}{4} - \frac{15}{2} + 1 = -\frac{27}{8}$

OR

The given expression $27y^3 + 125z^3$ can be re written as $(3y)^3 + (5z)^3$

As we know $x^3 + y^3 = (x + y)\left(x^2 - xy + y^2\right)$.

$(3y)^3 + (5z)^3 = (3y + 5z)\left[(3y)^2 - 3y \times 5z + (5z)^2\right]$

$= (3y + 5z)\left(9y^2 - 15yz + 25z^2\right)$

24. Are of cloth required for one flag

    $= \sqrt{s(s-25)(s-25)(s-22)}$, where $s = \frac{25+25+22}{2} = 36cm$

    Area of cloth required $= \sqrt{36(36-25)(36-25)(36-22)}$

    $= \sqrt{36 \times 11 \times 11 \times 14}$

    $= 6 \times 11\sqrt{14} = 66\sqrt{14}cm^2$

    Area of cloth required for 200 such flags $= 66\sqrt{14} \times 200 = 13200\sqrt{14}cm^2$

25. i. Frequency distribution table

Digits	**Tally Marks**	**Frequency**
0	II	2
1	II	2
3	I	1
4	I	1
5	I	1
6	I	1

7	II	2
8	I	1
9	III	3

ii. Most frequency occurring digits = 9 & least frequently occurring digits = 2, 3, 4, 5, 6, 8

OR

i. The given bar graph represents the number of government companies in India during some years.

ii.

a. The number of government companies in 1957 = 50

The number of government companies in 1982 = 375

∴ The number of government companies in 1957 is that of 1982 = $\frac{50}{375}$

= $\frac{2}{15} \neq \frac{1}{9}$

So, the given statement is false.

b. The height of the bars increases over one year, hence, the statement is false.

26. Given, radius of large cylindrical vessel (r) = 15 cm and height of orange juice in large vessel (h) = 32 cm

∴ Volume of juice in the large vessel = $\pi r^2 h = \pi \times 15 \times 15 \times 32$ cm^3

For small cylindrical glasses, radius (r_1) = 3 cm and height of juice (h_1) = 8 cm

∴ Volume of juice in each glass = $\pi r_1^2 h_1 = \pi \times 3 \times 3 \times 8$ cm^3

Since, small cylindrical glasses are filled with juice from large vessel.

So, Volume of juice in vessel = Number of glasses $\times$ Volume of juice in each glass

∴ Number of glasses of juice = $\frac{\text{Volume of juice in vessel}}{\text{Volume of juice in each glass}}$

= $\frac{\pi \times 15 \times 15 \times 32}{\pi \times 3 \times 3 \times 8}$ = 100

Now, cost of one glass juice = Rs.3

∴ Cost of 100 glass juice = 100 $\times$ 3 = Rs.300

Hence, the stall keeper receive Rs.300 by selling the juice completely.

27. To find the value of the given expression, we will first rationalise it.

$\frac{3-\sqrt{5}}{3+2\sqrt{5}} = \frac{3-\sqrt{5}}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$= \frac{(3-\sqrt{5})(3-2\sqrt{5})}{(3+2\sqrt{5})(3-2\sqrt{5})}$

$= \frac{3(3-2\sqrt{5})-\sqrt{5}(3-2\sqrt{5})}{(3)^2-(2\sqrt{5})^2}$

$= \frac{9-6\sqrt{5}-3\sqrt{5}+10}{9-20}$

$= \frac{19-9\sqrt{5}}{-11}$

$= \frac{9\sqrt{5}-19}{11}$

$= \frac{9\times 2.2360-19}{11}$ [$\because \sqrt{5}$ = 2.2360]

$= \frac{20.1240-19}{11}$

$= \frac{1.1240}{11}$

= 0.102

$\therefore \frac{3-\sqrt{5}}{3+2\sqrt{5}}$ = 0.102

OR

We know that 7.478478.... is a non-terminating recurring decimal, which can be converted into $\frac{p}{q}$ form.

While, converting 7.478478.... into $\frac{p}{q}$ form, we get

$x = 7.478478 \ldots\ldots\ldots (a)$

$1000x = 7478.478478 \ldots\ldots (b)$

While, subtracting (a) from (b), we get

$$\begin{array}{r} 1000x = 7478.478478.... \\ -x = \quad 7.478478.... \\ \hline 999x = 7471 \end{array}$$

We know that can also be written as $x = \frac{7471}{999}$

Therefore, we conclude that 7.478478.... is a rational number.

28. The given equation is y - 3x = 9

$\Rightarrow$ y = 3x + 9

Let x = 0 : y = 3(0) + 9 = 0 + 9 = 9

Let x = -1 : y = 3(-1) + 9 = -3 + 9 = 6

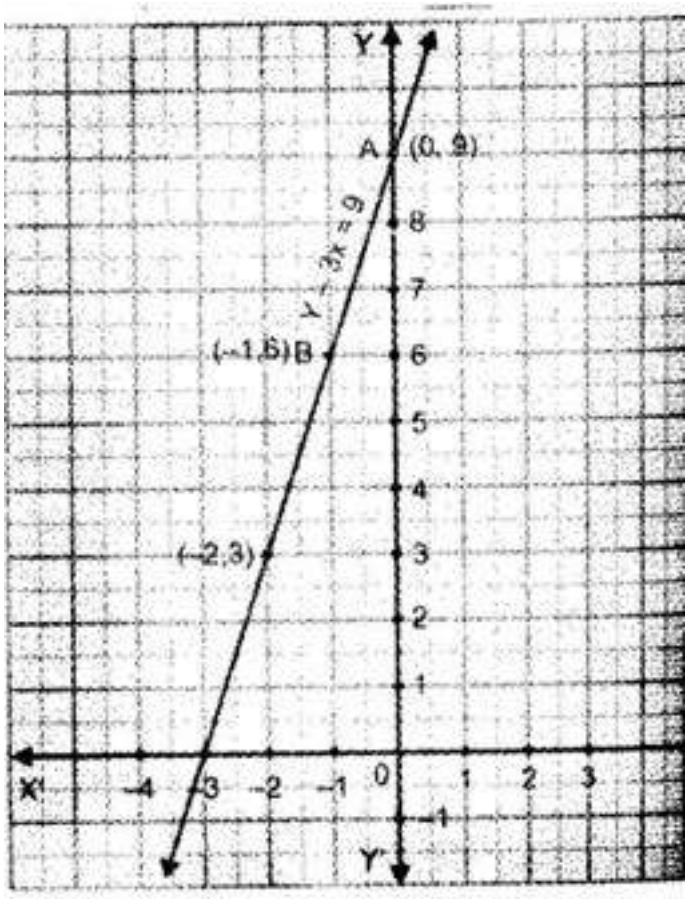
Let x = -2 : y = 3(-2) + 9 = -6 + 9 = 3

We can write these solutions in the form of a table

x	0	-1	-2
y	9	6	3

Now, plot the points A(0, 9), B(-1, 6) and C(-2, 3) on a graph. Join AB and extend it in both the directions.

Then, the line AB is the required graph.



29. We have,

$\frac{2}{3}x - y = 4$

Substituting y = 0 in this equation, we get

$\frac{2}{3}x - 0 = 4$

$\Rightarrow x = 4 \times \frac{3}{2}$

$\Rightarrow x = 6$

So, (6, 0) is a solution of the given equation.

Substituting y = 1, in the given equation, we get

$\frac{2}{3}x - 1 = 4$

$\Rightarrow \frac{2}{3}x = 5$

$\Rightarrow x = \frac{15}{2}$

So, $(\frac{15}{2}, 1)$ is a solution of the given equation.

$\therefore$ We obtain (6, 0) and $(\frac{15}{2}, 1)$ as two solutions of the given equation.

OR

$y - 3x = 9$

$\Rightarrow y = 3x + 9$

x	-2	-3
y	3	0

We plot the points(-2, 3) and (-3, 0) on the graph paper and join the same by a ruler to get the line which is the graph of the equation $y - 3x = 9$.

Few solutions read from the graph are

(0, 9), (-1, 6) and (-4, -3)

For (0, 9)

L.H.S. = 9 - 3(0) = 9- 0 = 9 = R.H.S.

$\therefore$The solution (0, 9) is verified.

For (-1, 6)

L.H.S. = 6 - 3(-1) = 6 +3 = 9

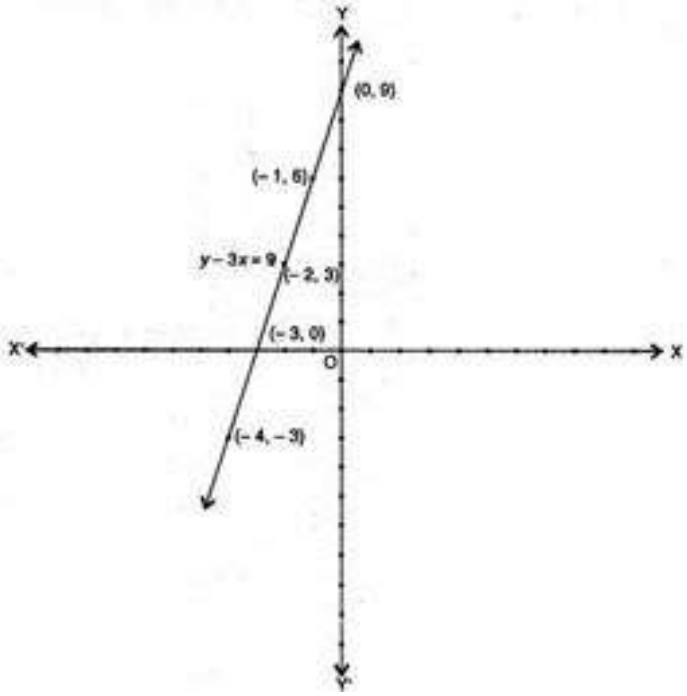
$\therefore$ The solution (-1, 6) is verified.

For (-4, -3)

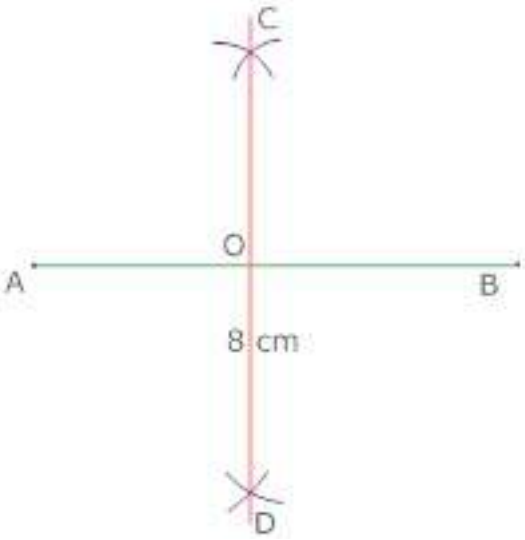
L.H.S. = -3 - 3(-4) = -3 + 12 = 9 = R.H.S.

$\therefore$ The solution (-4, -3) is verified.

The points where the given line meets the x-axis and the y-axis are respectively (-3, 0) and (0, 9) respectively.

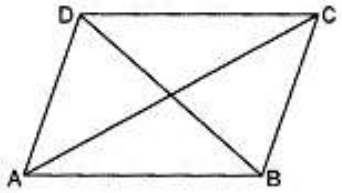


30. Steps of construction



i. Draw a line segment AB = 8cm
ii. Taking A as a centre draw arcs of radius more $\frac{1}{2}AB$ on both side of AB.
iii. Taking B as a centre draw arcs of same radius on both sides of AB which intersect previous arcs at point C and D.
iv. Join CD which intersect AB at point O
v. OA = OB = 4cm

31. Given: The diagonals of a parallelogram are equal.



To prove: Parallelogram is a rectangle.

Proof : In $\triangle ACB$ and $\triangle BDA$,

AC = BD . . . [Given]

AB = BA . . . [Common]

BC = AD . . . [Opposite sides of parallelogram]

$\therefore \triangle ACB \cong \triangle BDA$ . . .[By SSS property]

$\therefore \angle ABC = \angle BAD$ . . . [c.p.c.t.] . . . .(1)

As AD || BC . . . [Opposite sides of parallelogram]

transversal AB intersects them.

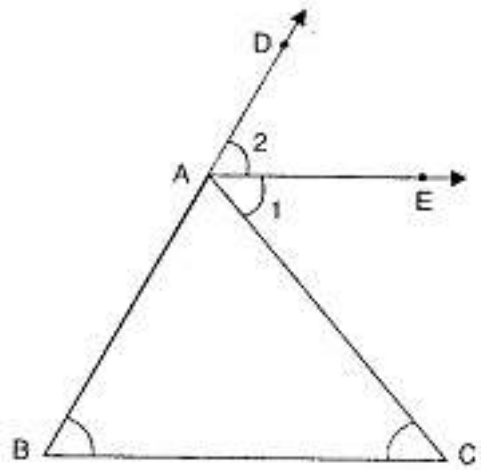
$\therefore \angle BAD + \angle ABC = 180^o$ . . . [Sum of interior angle on the same side of a transversal] . . . .(2)

$\angle BAD = \angle ABC = 90^o$ . . . [From (1) and (2)]

$\therefore \angle A = 90^o$

$\therefore$ Parallelogram ABCD is a rectangle.

32.



As AE is the bisector of $\angle CAD$

$\therefore \angle 1 = \angle 2$ . . . . (1)

As AE || BC and AC cuts them

$\therefore \angle 1 = \angle C$ . . .[Alternate angles] . . . (2)

As AE || BC and AB cuts them

$\therefore \angle 2 = \angle B$ . . . [Corresponding angles] . . . .(3)

$\angle B = \angle C$ . . . [From (1), (2) and (3)]

$\therefore$ AC = AB . . . [Sides opposite to equal angles of $\triangle$ABC]

$\therefore \triangle$ABC is isosceles.

OR

In $\triangle$AOC and $\triangle$BOD

AO = OB [O is the midpoint of AB]

$\angle AOC = \angle BOD$ [vertically opposite angles]

CO = OD [O is the mid-point of CD]

$\triangle AOC \cong \triangle BOD$ [By SAS] AC = BD [CPCT]

$\Rightarrow \angle CAO = \angle DBO$ [CPCT]

Now, AC and BD are two lines intersected by a transversal AB such that $\angle$CAO = $\angle$DBO i.e. alternate angle are equal SO AC $\parallel$ BD.

33. Let, AB=DC=60cm, BC=AD= 25m and AC=65m

Area of parallelogram ABCD= Area ofΔ ABC + area of Δ ACD

= 2 Area of Δ ABC [$\because$ ar Δ ABC =ar Δ ABD, as diagonal of a parallelogram divides it into two congruent triangles and congruent triangles have equal area] - (i)

Now

$S = \frac{60+65+25}{2} m = 75m$

$\therefore$ area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$ $= \sqrt{75(15)(10)(50)}$

$= \sqrt{75(75-60)(75-65)(75-25)} = \sqrt{3 \times 5 \times 5 \times 3 \times 5 \times 2 \times 5 \times 2 \times 5 \times 5}$

$= (5 \times 3 \times 5 \times 2 \times 5)$ sq m

=750 sq m ....(ii)

From (i) and (ii), we get

Area of

$||^{gm}$ ABCD $= 2 \times 750$=1500 sq m.

34. It is given that the total number of light bulbs = 400 Now, let's find the probability of the given events,

    i. The probability that the life time of the selected bulb is less than 400 hours

    = $\frac{Favourable\ outcome}{Total\ outcome}$ = $\frac{14}{400}$ = $\frac{7}{200}$

ii. The probability that the life time of the selected bulb is between 300 to 800 hours

$= \frac{Favourable\ outcome}{Total\ outcome} = \frac{14+56+60+86+74}{400} = \frac{290}{400} = \frac{29}{40}$

iii. The probability that the life time of the selected bulb is at least 700 hours

$= \frac{Favourable\ outcome}{Total\ outcome} = \frac{74+62+48}{400} = \frac{184}{400} = \frac{23}{50}$

35. Given: Two circles with centres O and P intersecting at A and B.

Prove: OP is the perpendicular bisector of AB.

Construction: Join OA, OB, PA and PB. Let OP intersect AB at M.

Proof : In $\triangle$OAP and $\triangle$OBP

OA = OB |Radii of a circle

PA = PB |Radii of a circle

OP = OP |Common

$\therefore \triangle OAP \cong \triangle OBP$ [SSS Rule]

$\therefore$ $\angle$AOP = $\angle$BOP [c.p.c.t.]

$\Rightarrow$ $\angle$AOM = $\angle$BOM ...(1)

In $\triangle$AOM and $\triangle$BOM

OA = OB [Radii of a circle]

$\angle$AOM = $\angle$BOM [From (1)]

OM = OM [Common]

$\therefore \triangle AOM \cong \triangle BOM$ [SAS Rule]

$\therefore$ AM = BM ...(2) |c.p.c.t

and $\angle$AMO = $\angle$BMO ...(3) [c.p.c.t]

But $\angle$AMO + $\angle$BMO = 180° [Linear Pair Axiom]

$\therefore$ $\angle$AMO = $\angle$BMO = 90° ...(4)

$\therefore$ OM, i.e. OP is the perpendicular bisector of AB [From (2) and (4)]

OR

Given that the two chords AD, BC of the circle bisect each other.

Let these cords bisect at K

In $\triangle$AKB and $\triangle$DKC

AK = DK [AB, CD bisect each other at K]

BK = CK

$\angle$AKB = $\angle$DKC [Vertically opposite]

$\triangle$ AKB $\cong$ $\triangle$DKC (by SAS congruency)

Therefore, AB = CD (By CPCT)

$\overset{\frown}{AB} = \overset{\frown}{CD}$

$\overset{\frown}{AB} + \overset{\frown}{BC} = \overset{\frown}{CD} + \overset{\frown}{BC}$

$\overset{\frown}{AC} = \overset{\frown}{BD}$

AC=BD

Also, in quadrilateral ABCD

AB = CD

$\angle A = \angle B = \angle C = \angle D = 90^\circ$ [AC, BD is diameter so angle is semicircle)

36. In $\triangle$PRT

$\angle \mathrm{P} + \angle \mathrm{R} + \angle 1 = 180^0$ [By angle sum property]

$95^0 + 40^0 + \angle 1 = 180^0$

$\angle 1 = 180^0 - 135^0$

$\angle 1 = 45^0$

$\angle 1 = \angle 2$ [vertically opposite angle]

$\angle 2 = \angle 45^0$

In $\triangle$TQS $\angle 2 + \angle \mathrm{Q} + \angle \mathrm{S} = 180°$

$45^0 + \angle Q + 75^0 = 180^0$

$\angle \mathrm{Q} + 120^0 = 180^0$

$\angle \mathrm{Q} = 180^0 - 120^0$

$\angle \mathrm{Q} = 60^0$

$\angle \mathrm{SQT} = 60^0$

37. We know that,

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow a^3 + b^3 + c^3 - 3abc = (a + b + c)\{(a^2 + b^2 + c^2) - (ab + bc + ca)\}$ ....(i)

Clearly, we require the values of $a + b + c$, $a^2 + b^2 + c^2$ and $ab + bc + ca$ to obtain the value of $a^3 + b^3 + c^3 - 3abc$.

We are given the values of $a + b + c$ and $ab + bc + ca$. So, let us first obtain the value of $a^2 + b^2 + c^2$.

We know that,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow (a + b + c)^2 = (a^2 + b^2 + c^2) + 2(ab + bc + ca)$

$\Rightarrow 6^2 = a^2 + b^2 + c^2 + 2 \times 11$ [Putting the values of a + b + c = 6 and ab + bc + ca = 11]

$\Rightarrow 36 = a^2 + b^2 + c^2 + 22$

$\Rightarrow a^2 + b^2 + c^2 = 36 - 22$

$\Rightarrow a^2 + b^2 + c^2 = 14$

Now, putting a + b + c = 6, ab + bc + ca = 11 and $a^2 + b^2 + c^2 = 14$ in (i), we get

$a^3 + b^3 + c^3 - 3abc = 6 \times (14 - 11) = 6 \times 3 = 18$.

OR

Let $f(x) = x^4 + x^3 - 7x^2 - x + 6$

The factors of constant term in f(x) are $\pm 1, \pm 2, \pm 3$ and ± 6.

We have,

$f(1) = 1 + 1 - 7 - 1 + 6 = 8 - 8 = 0$

$\Rightarrow$ (x - 1) is a factor of f(x).

Also, we have,

$f(-1) = 1 - 1 - 7 + 1 + 6 = 8 - 8 = 0$

$\therefore$ (x + 1) is a factor of f(x)

$\because f(2) = 2^4 + 2^3 - 7 \times 2^2 - 2 + 6 = 16 + 8 - 28 - 2 + 6 = 0$

$\therefore$ (x - 2) is a factor of f(x)

$\because f(-2) = (-2)^4 + (-2)^3 - 7(-2)^2 - (-2) + 6 = 16 - 8 - 28 + 2 + 6 - 12 \neq 0$

$\therefore$ (x + 2) is not a factor of f(x)

$\because f(-3) = (-3)^4 + (-3)^3 - 7(-3)^2 - (-3) + 6 = 81 - 27 - 63 + 3 + 6 = 90 - 90 = 0$

$\therefore$ (x + 3) is a factor of f(x)

Since f(x) is a polynomial of degree 4. So, it cannot have more than 4 linear factors.

Thus, the factors of f(x) are (x - 1), (x + 1), (x - 2) and (x + 3).

$\therefore f(x) = k(x - 1)(x + 1)(x - 2)(x + 3)$

$\Rightarrow x^4 + x^3 - 7x^2 - x + 6 = k(x - 1)(x + 1)(x - 2)(x + 3)$ ...(i)

Putting x = 0 on both sides, we get

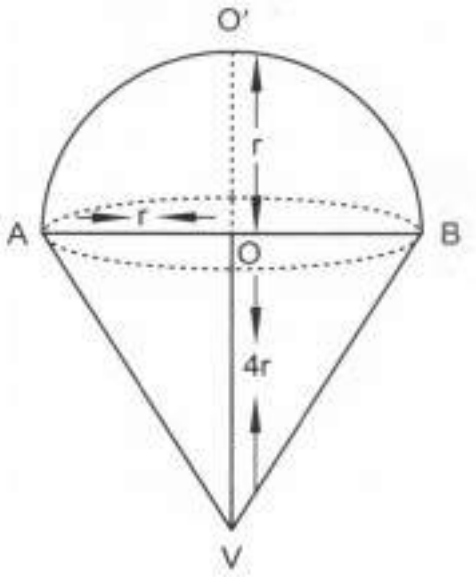
$6 = k(-1)(1)(-2)(3) \Rightarrow 6 = 6k \Rightarrow k = 1$

Substituting k = 1 in (i), we get

$x^4 + x^3 - 7x^2 - x + 6 = (x - 1)(x + 1)(x - 2)(x + 3)$

38. Let the radius of the base of the conical portion be r cm. Then,

Height of the conical portion = 4 r cm.



∴ Volume of cone with hemispherical top

= Volume of the cone + Volume of the hemispherical top

$= \frac{1}{3}\ \pi r^2 \times 4r + \frac{2}{3}\pi r^3$ cm3

$= \frac{6}{3}\ \pi r^3\ cm^3 = 2\pi r^3\ cm^3$

Volume of 10 cones with hemispherical tops = $(10 \times 2\ \pi r^3)\ cm^3 = 20\pi\ r^3\ cm^3$

Volume of the cylindrical container = $(\pi \times 6^2 \times 15)\ cm^3 = 540\ \pi cm^3$

Clearly,

Volume of 10 cones with hemispherical tops = Volume of the cylindrical container

$\Rightarrow 20\ \pi r^3 = 540\pi\ \Rightarrow r^3 = 27 \Rightarrow$ r = 3 cm

Hence, radius of the ice-cream cone is 3 cm.

OR

Let R cm and r cm be respectively the external and internal radii of the hemispherical vessel. Then, R = 12.5 cm, r = 12 cm.

Now,

External surface area of the vessel = $2\pi R^2 = 2 \times \frac{22}{7} \times (12.5)^2\ cm^2$

Internal surface area of the vessel = $2\pi r^2 = 2 \times \frac{22}{7} \times (12)^2\ cm^2$

∴ Total area to be painted = $2 \times \frac{22}{7} \times (12.5)^2 + 2 \times \frac{22}{7} \times 12^2\ cm^2$

$\Rightarrow$ Total area to be painted = $2 \times \frac{22}{7} \times \{(\frac{25}{2})^2 + 12^2\}\ cm^2$

$\Rightarrow$ Total area to be painted = $2 \times \frac{22}{7} \times (\frac{625}{4} + 144)\ cm^2 = \frac{13211}{7}\ cm^2$

Cost of painting at the rate of 7 paise per sq. cm = Rs. $\frac{13211}{7} \times \frac{7}{100}$ = Rs. 132.11

39. In order to prove that CA $\|$ BD. It is sufficient to show that

$\angle$CAB + $\angle$ABD = 180^o

Since mirrors m and n are perpendicular and OB and OA are perpendicular to m and n respectively,

$\therefore$ OA $\perp$ OB $\Rightarrow$ $\angle$BOA = 90^o

In $\triangle$BOA, we have,

$\Rightarrow$ $\angle$2 + $\angle$3 + $\angle$BOA = 180^o

$\Rightarrow$ $\angle$2 + $\angle$3 + 90^o = 180^o [$\because$ $\angle$BOA = 90^o]

$\Rightarrow$ $\angle$2 + $\angle$3 = 90^o

$\Rightarrow$ 2($\angle$2 + $\angle$3) = 180^o [Multiplying both sides by 2]

$\Rightarrow$ 2($\angle$2) + 2($\angle$3) = 180^o

$\Rightarrow$ $\angle$CAB + $\angle$ABD [Angle of incidence = Angle of reflection $\because$ $\angle$1 = $\angle$2 and $\angle$3 = $\angle$4

$\Rightarrow$ 2$\angle$2 = $\angle$CAB and 2$\angle$3 = $\angle$BAD]

Thus, CA and BD are two lines intersected by a transversal AB such that $\angle$CAB + $\angle$ABD = 180^o i.e., the sum of the interior angles on the same side of AB is 180^o. Hence, CA $\|$ BD.

40. The given observation are

82, 41, 39, 52, 53, 45, 96, 47, 50, 60

Sum of 10 observations = 82+ 41+39+52+53+ 45+ 96+ 47+ 50 +60

$\Rightarrow \overline{x} = \frac{x_1+x_2+x_3----------+x_n}{n} - - - - - - - (1)$

$\therefore Mean = \frac{Z}{10} = \frac{565}{10} = 56.5$

Arranging the given observation in ascending order:

39, 41, 45, 47, 50, 52, 53, 60, 82, 96

Here, n = 10 [even no.]

$\therefore Median = \frac{\left(\frac{n}{2}\right)^{th} observation + \left(\frac{n}{2}+1\right)^{th} observation}{2}$

$= \frac{5^{th}\ observation + 6^{th} observation}{2}$

$= \frac{50+52}{2} = \frac{102}{2} = 51$