Hyperbola

Exercise 24

Q. 1. Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Answer : Given Equation:

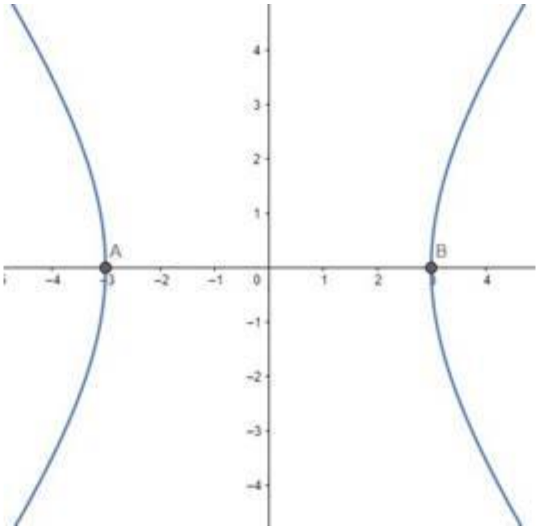
$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Comparing with the equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

we get,

a = 3 and b = 4



(i) Length of Transverse axis = 2a = 6 units.

Length of Conjugate axis = 2b = 8 units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 3, 0)$

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm 5, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{32}{3} = 10.67$ units.

Q. 2. Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :

$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$

Answer : Given Equation:

$$\frac{x^2}{25} - \frac{y^2}{4} = 1$$

Comparing with the equation of hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

a = 5 and b = 2

(i) Length of Transverse axis = 2a = 10 units.

Length of Conjugate axis = 2b = 4 units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 5, 0)$

(iv) Here, eccentricity, $e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{4}{25}} = \sqrt{\frac{29}{25}} = \frac{\sqrt{29}}{5}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm\sqrt{29}, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{8}{5} = 1.6$ units.

Q. 3. Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :

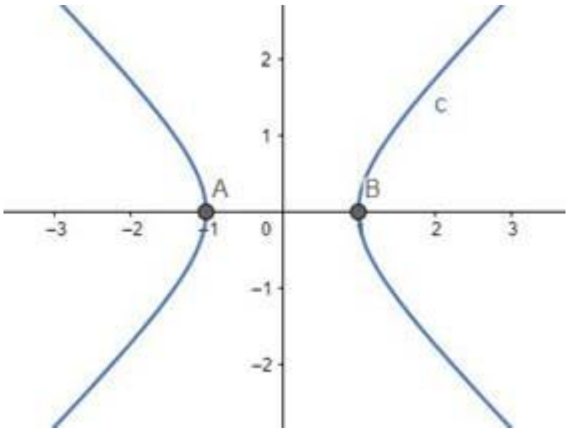
$x^2 – y^2 = 1$

Answer : Given Equation: $x^2 – y^2 = 1$

Comparing with the equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ we get,

a = 1 and b = 1



(i) Length of Transverse axis = 2a = 2 units.

Length of Conjugate axis = 2b = 2 units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 1, 0)$

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{1}{1}} = \sqrt{\frac{2}{1}} = \sqrt{2}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm\sqrt{2}, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = 2$ units.

Q. 4 Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :

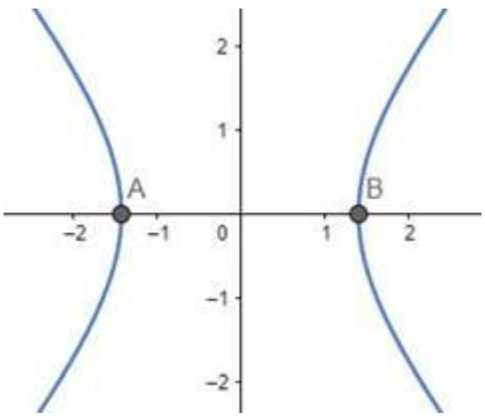
$3x^2 - 2y^2 = 6$

Answer :

Given Equation: $3x^2 - 2y^2 = 6 \Rightarrow \frac{x^2}{2} - \frac{y^2}{3} = 1$

Comparing with the equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ we get,

a = $\sqrt{2}$ and b = $\sqrt{3}$



(i) Length of Transverse axis = 2a = $2\sqrt{2}$ units.

Length of Conjugate axis = 2b = $2\sqrt{3}$ units.

(ii) Coordinates of the vertices = (±a, 0) = ($\pm\sqrt{2}$, 0)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{3}{2}} = \sqrt{\frac{5}{2}}$

(iii) Coordinates of the foci = ($\pm$ae, 0) = ($\pm\sqrt{5}$, 0)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$ units.

Q. 5. Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :

$25x^2 - 9y^2 = 225$

Answer : Given Equation: $25x^2 - 9y^2 = 225 \Rightarrow$

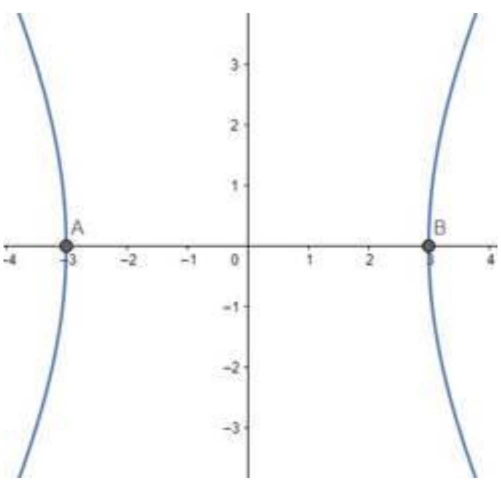
$$\frac{x^2}{9} - \frac{y^2}{25} = 1$$

Comparing with the equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

we get,

a = 3 and b = 5



(i) Length of Transverse axis = 2a = 6 units.

Length of Conjugate axis = 2b = 10 units.

(ii) Coordinates of the vertices = $(\pm a, 0) = (\pm 3, 0)$

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{25}{9}} = \sqrt{\frac{34}{9}} = \frac{\sqrt{34}}{3}$

(iii) Coordinates of the foci = $(\pm ae, 0) = (\pm\sqrt{34}, 0)$

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{50}{3} = 16.67$ units.

Q. 6. Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :

$24x^2 - 25y^2 = 600$

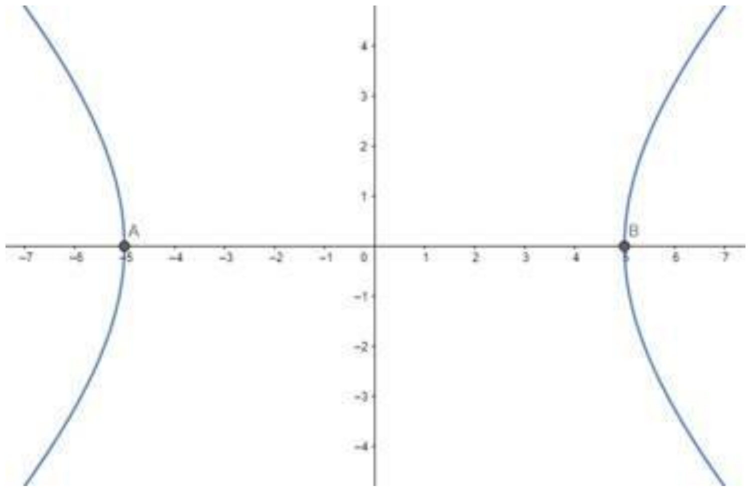
Answer : Given Equation: $24x^2 - 25y^2 = 600 \Rightarrow$

$$\frac{x^2}{25} - \frac{y^2}{24} = 1$$

Comparing with the equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ we get,

a = 5 and b = $\sqrt{24} = 2\sqrt{6}$



(i) Length of Transverse axis = 2a = 10 units.

Length of Conjugate axis = 2b = $4\sqrt{6}$ units.

(ii) Coordinates of the vertices = ($\pm$a, 0) = ($\pm$5, 0)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{24}{25}} = \sqrt{\frac{49}{25}} = \frac{7}{5}$

(iii) Coordinates of the foci = ($\pm$ae, 0) = ($\pm$7, 0)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{48}{5} = 9.6$ units.

Q. 7. Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :

$$\frac{y^2}{16} - \frac{x^2}{49} = 1$$

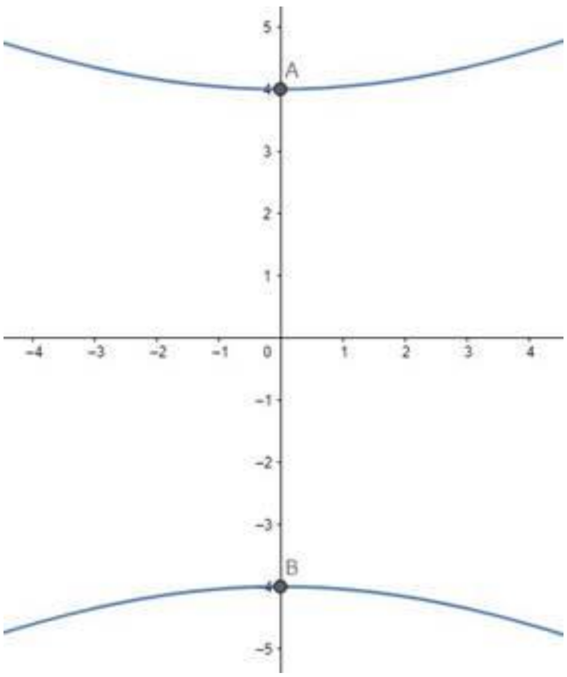
Answer : Given Equation:

$$\frac{y^2}{16} - \frac{x^2}{49} = 1$$

Comparing with the equation of hyperbola

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$ we get,

a = 4 and b = 7



(i) Length of Transverse axis = 2a = 8 units.

Length of Conjugate axis = 2b = 14 units.

(ii) Coordinates of the vertices = (0, $\pm$a) = (0, $\pm$4)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{49}{16}} = \sqrt{\frac{65}{16}} = \frac{\sqrt{65}}{4}$

(iii) Coordinates of the foci = (0, ±ae) = (0, $\pm\sqrt{65}$)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{98}{4} = 24.5$ units.

Q. 8 Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :

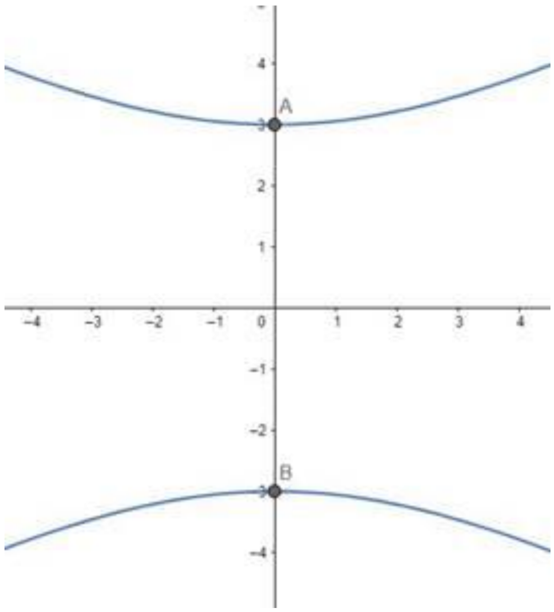
$$\frac{y^2}{9} - \frac{x^2}{27} = 1$$

Answer :

Given Equation: $\frac{y^2}{9} - \frac{x^2}{27} = 1$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

a = 3 and b = $\sqrt{27} = 3\sqrt{3}$



(i) Length of Transverse axis = 2a = 6 units.

Length of Conjugate axis = 2b = $6\sqrt{3}$ units.

(ii) Coordinates of the vertices = (0, $\pm$a) = (0, $\pm$3)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{27}{9}} = \sqrt{\frac{36}{9}} = \frac{6}{3} = 2$

(iii) Coordinates of the foci = (0, $\pm$ae) = (0, $\pm$6)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{54}{3} = 18$ units.

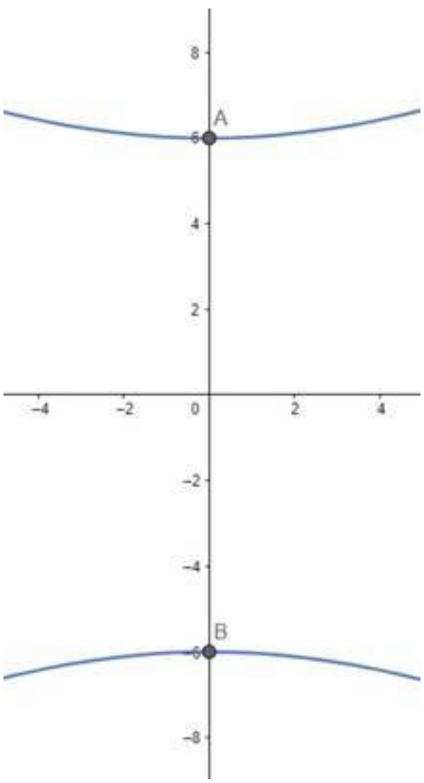
Q. 9. Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :
$3y^2 - x^2 = 108$

Answer :

Given Equation: $3y^2 - x^2 = 108 \Rightarrow \frac{y^2}{36} - \frac{x^2}{108} = 1$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

a = 6 and b = $\sqrt{108} = 6\sqrt{3}$



(i) Length of Transverse axis = 2a = 12 units.

Length of Conjugate axis = 2b = $12\sqrt{3}$ units.

(ii) Coordinates of the vertices = $(0, \pm a) = (0, \pm 6)$

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{108}{36}} = \sqrt{1+3} = 2$

(iii) Coordinates of the foci = (0, $\pm$ae) = (0, $\pm$12)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{216}{6} = 36$ units.

Q. 10. Find the (i) lengths of the axes, (ii) coordinates of the vertices, (iii) coordinates of the foci, (iv) eccentricity and (v) length of the rectum of each of the following the hyperbola :

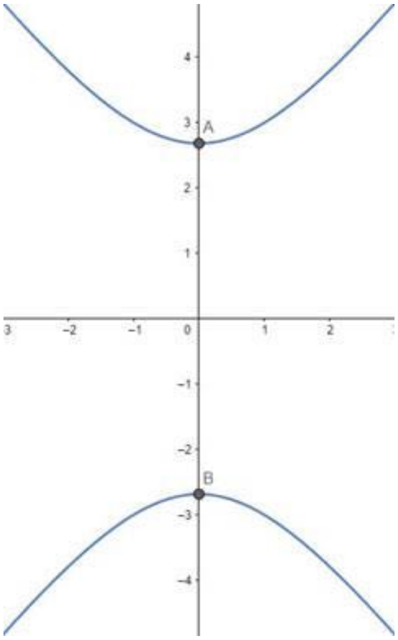
$5y^2 - 9x^2 = 36$

Answer :

Given Equation: $5y^2 - 9x^2 = 36 \Rightarrow \frac{y^2}{36/5} - \frac{x^2}{4} = 1$

Comparing with the equation of hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ we get,

a = $\sqrt{\frac{36}{5}} = \frac{6}{\sqrt{5}}$ and b = 2



(i) Length of Transverse axis = 2a = $\frac{12}{\sqrt{5}}$ units.

Length of Conjugate axis = 2b = 4 units.

(ii) Coordinates of the vertices = (0, ±a) = (0, ± $\frac{6}{\sqrt{5}}$)

(iv) Here, eccentricity, e = $\sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{4}{36/5}} = \sqrt{1+\frac{20}{36}} = \frac{\sqrt{56}}{6} = \frac{\sqrt{14}}{3}$

(iii) Coordinates of the foci = (0, ±ae) = (0, ± $\frac{6}{\sqrt{5}} \cdot \frac{\sqrt{14}}{3}$) = (0, $\frac{2\sqrt{14}}{\sqrt{5}}$)

(v) Length of the rectum = $\frac{2b^2}{a} = \frac{8}{6/\sqrt{5}} = \frac{8\sqrt{5}}{6} = \frac{4\sqrt{5}}{3}$ units.

Q. 11. Find the equation of the hyperbola with vertices at (±6, 0) and foci at (±8, 0).

Answer : Given: Vertices at (±6, 0) and foci at (±8, 0)

Need to find: The equation of the hyperbola.

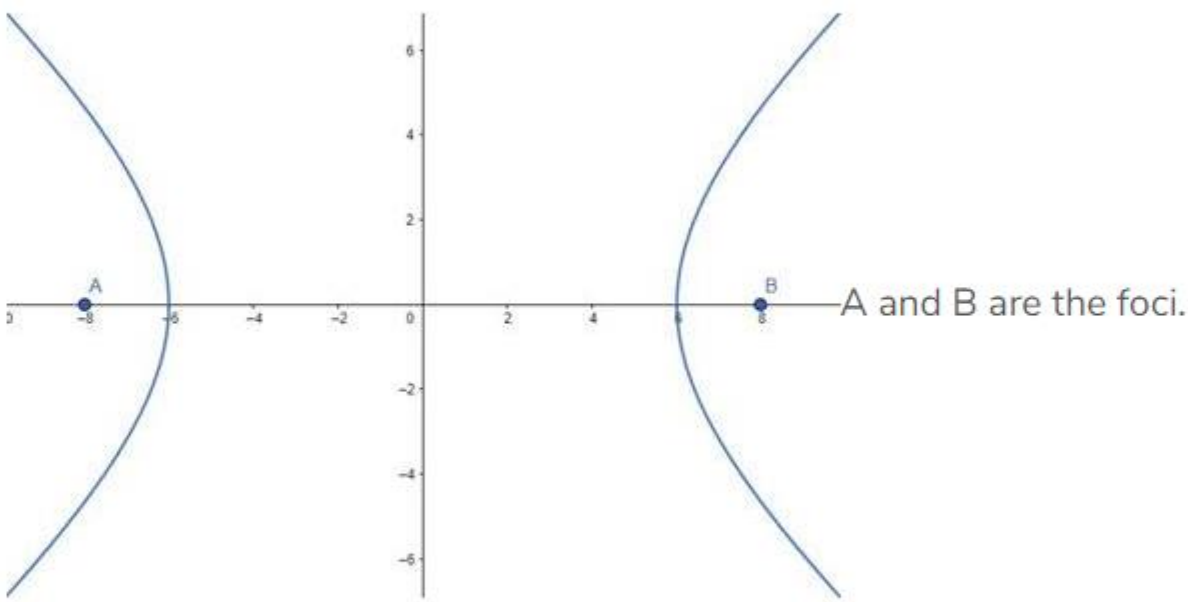
Let, the equation of the parabola be:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices of the parabola is at (±6, 0)

That means a = 6

The foci are given at (±8, 0)



That means, ae = 8, where e is the eccentricity.

$\Rightarrow 6e = 8$ [As a = 6]

$\Rightarrow e = \frac{8}{6} = \frac{4}{3}$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \frac{4}{3}$$

$$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{16}{9} \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{16}{9} - 1 = \frac{7}{9}$$

$$\Rightarrow b^2 = a^2 \frac{7}{9}$$

$$\Rightarrow b^2 = 36 \times \frac{7}{9} = 4 \times 7 = 28 \text{ [As a = 6]}$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{36} - \frac{y^2}{28} = 1 \text{ [Answer]}$$

Q. 12. Find the equation of the hyperbola with vertices at (0, ±5) and foci at (0, ±8).

Answer : Given: Vertices at (0, ±5) and foci at (0, ±8)

Need to find: The equation of the hyperbola.

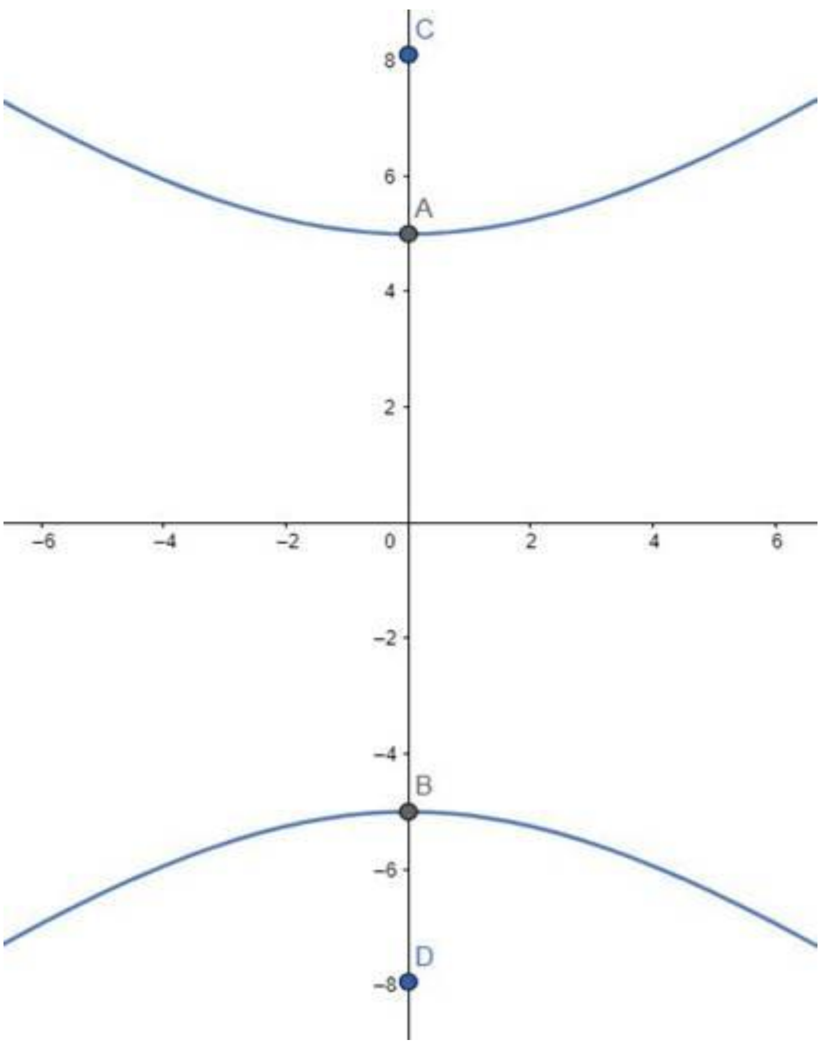
Let, the equation of the parabola be:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Vertices of the parabola are at (0, ±5)

That means a = 5

The foci are given at (0, ±8)



A and B are the vertices. C and D are the foci.

That means, ae = 8, where e is the eccentricity.

$\Rightarrow 5e = 8$ [As a = 5]

$\Rightarrow e = \frac{8}{5}$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \frac{8}{5}$

$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{64}{25}$ [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2} = \frac{64}{25} - 1 = \frac{39}{25}$

$\Rightarrow b^2 = a^2 \frac{39}{25}$

$\Rightarrow b^2 = 25 \times \frac{39}{25} = 39$ [As a = 5]

So, the equation of the hyperbola is,

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{39} = 1$$ [Answer]

Q. 13. Find the equation of the hyperbola whose foci are $(\pm\sqrt{29}, 0)$ and the transverse axis is of the length 10.

Answer : Given: Foci are $(\pm\sqrt{29}, 0)$, the transverse axis is of the length 10

Need to find: The equation of the hyperbola.

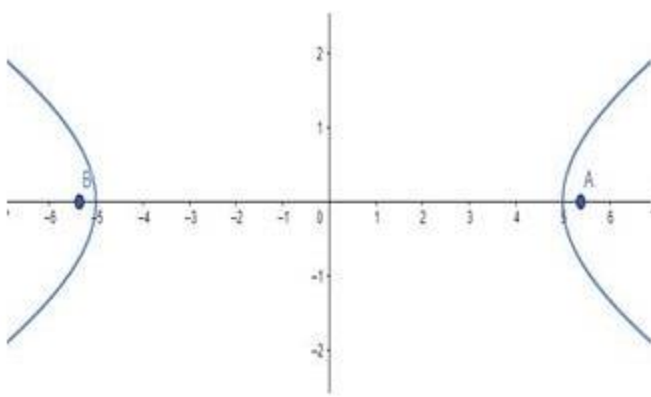
Let, the equation of the hyperbola be:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The transverse axis is of the length 10, i.e., 2a = 10

Therefore, a = 5

The foci are given at $(\pm\sqrt{29}, 0)$



That means, $ae = \pm\sqrt{29}$, where e is the eccentricity.

$\Rightarrow 5e = \sqrt{29}$ [As a = 5]

$\Rightarrow e = \frac{\sqrt{29}}{5}$

We know that,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Therefore,

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \frac{\sqrt{29}}{5}$$

$\Rightarrow 1 + \frac{b^2}{a^2} = \frac{29}{25}$ [Squaring both sides]

$$\Rightarrow \frac{b^2}{a^2} = \frac{29}{25} - 1 = \frac{4}{25}$$

$$\Rightarrow b^2 = a^2 \frac{4}{25}$$

$\Rightarrow b^2 = 25 \times \frac{4}{25} = 4$ [As a = 5]

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{25} - \frac{y^2}{4} = 1$ [Answer]

Q. 14. Find the equation of the hyperbola whose foci are (±5, 0) and the conjugate axis is of the length 8. Also, find its eccentricity.

Answer : Given: Foci are (±5, 0), the conjugate axis is of the length 8

Need to find: The equation of the hyperbola and eccentricity.

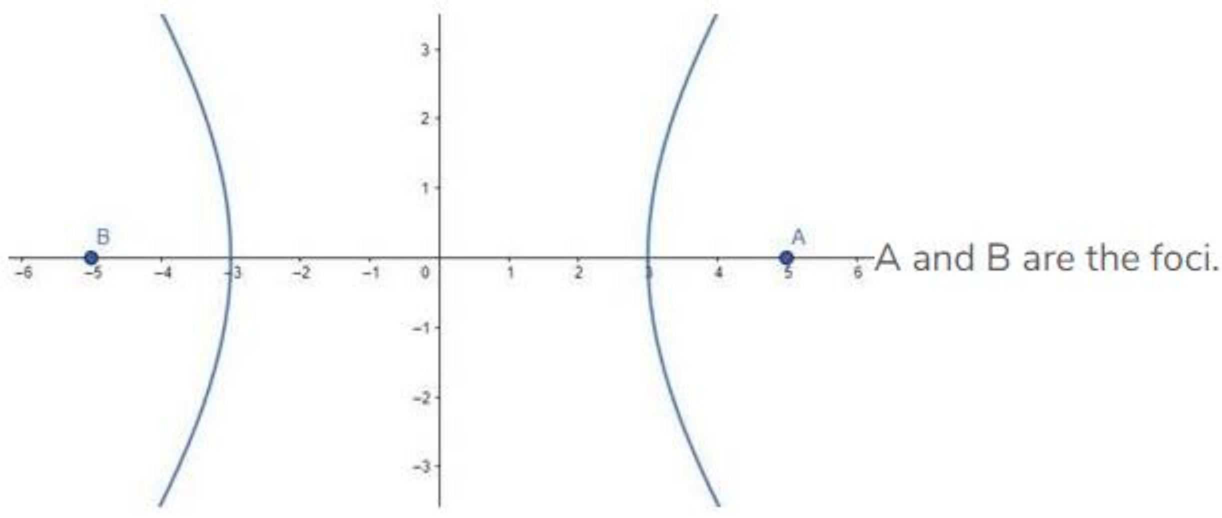
Let, the equation of the hyperbola be:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The conjugate axis is of the length 8, i.e., 2b = 8

Therefore, b = 4

The foci are given at (±5, 0)



That means, ae = 5, where e is the eccentricity.

We know that,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Therefore,

$\Rightarrow a\sqrt{1+\frac{b^2}{a^2}}=5$

$\Rightarrow \sqrt{1+\frac{b^2}{a^2}}=\frac{5}{a}$

$\Rightarrow 1+\frac{b^2}{a^2}=\frac{25}{a^2}$ [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2}=\frac{25}{a^2}-1=\frac{25-a^2}{a^2}$

$\Rightarrow b^2=25-a^2$

$\Rightarrow a^2=25-b^2=25-16=9$

So, the equation of the hyperbola is,

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{9}-\frac{y^2}{16}=1$

Eccentricity, $e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{16}{9}}=\sqrt{\frac{25}{9}}=\frac{5}{3}$ [Answer]

Q. 15. Find the equation of the hyperbola whose foci are $(\pm 3\sqrt{5},0)$ and the length of the latus rectum is 8 units.

Answer : Given: Foci are $(\pm 3\sqrt{5}, 0)$ the length of the latus rectum is 8 units

Need to find: The equation of the hyperbola.

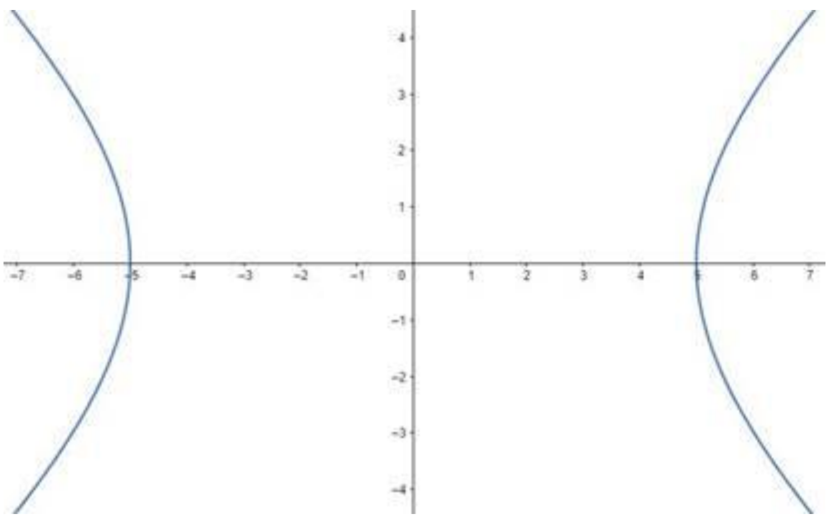
Let, the equation of the hyperbola be:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The length of the latus rectum is 8 units.

Therefore, $\frac{2b^2}{a} = 8 \Rightarrow b^2 = 4a$ ---- (1)

The foci are given at $(\pm 3\sqrt{5}, 0)$



That means, ae $= 3\sqrt{5}$, where e is the eccentricity.

We know that,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Therefore,

$\Rightarrow a\sqrt{1+\frac{b^2}{a^2}} = 3\sqrt{5}$

$\Rightarrow a\frac{\sqrt{a^2+b^2}}{a} = 3\sqrt{5}$

$\Rightarrow a^2 + b^2 = 45$ [Squaring both sides]

$\Rightarrow a^2 + 4a = 45$ [From (1)]

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow a^2 + 9a - 5a - 45 = 0$

$\Rightarrow (a+9)(a-5) = 0$

So, either a = 5 or, a = -9

That means, either b = $2\sqrt{5}$ or, b = $\sqrt{-36}$

The value of b = $\sqrt{-36}$ is not a valid one. So, the b value and its corresponding a value is not acceptable.

Hence, the acceptable value of a is 5 and b is

$2\sqrt{5}$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{25} - \frac{y^2}{20} = 1$$ [Answer]

Q. 16. Find the equation of the hyperbola whose vertices are (±2, 0) and the eccentricity is 2.

Answer : Given: Vertices are (±2, 0) and the eccentricity is 2

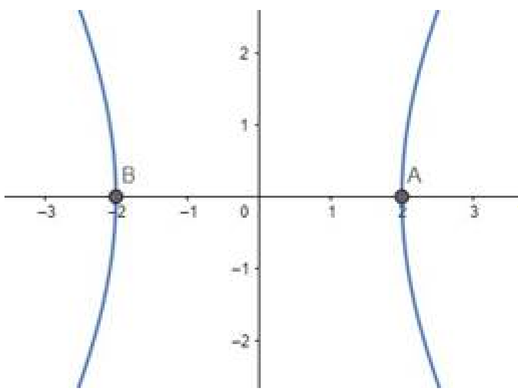
Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices are (±2, 0), that means, a = 2

And also given, the eccentricity, e = 2



A and B are the foci.

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = 2$

$\Rightarrow 1+\frac{b^2}{a^2}=4$ [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2}=3$

$\Rightarrow b^2=3a^2=3\times4=12$ [As a = 2]

So, the equation of the hyperbola is,

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{4}-\frac{y^2}{12}=1$ [Answer]

Q. 17. Find the equation of the hyperbola whose foci are ($\pm\sqrt{5}$ 0) and the eccentricity is $\sqrt{\frac{5}{3}}$.

Answer :

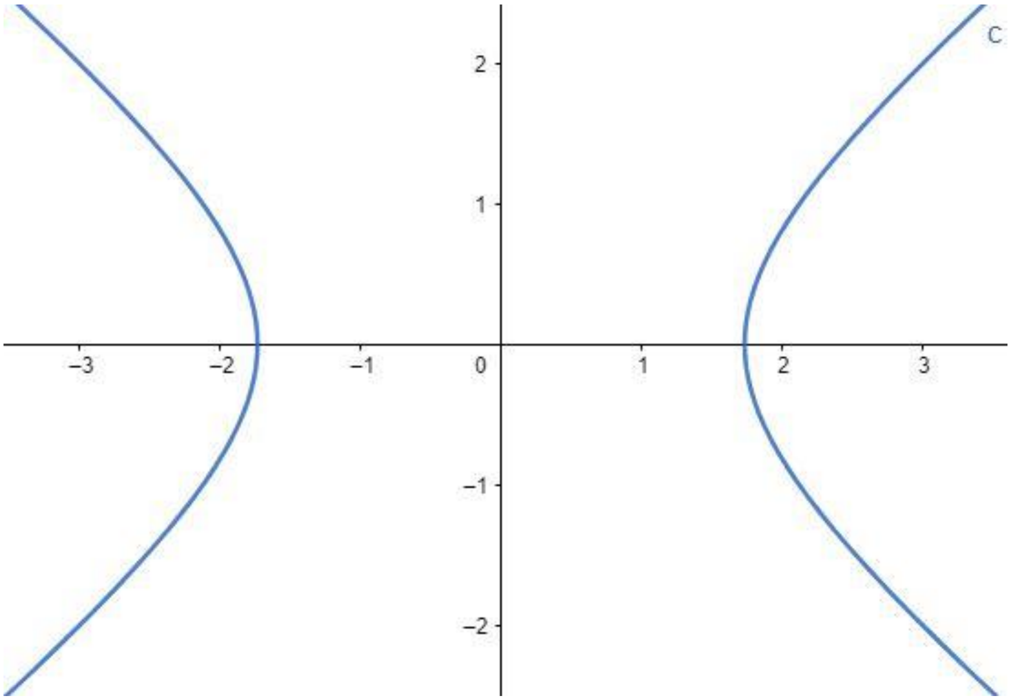
Given: Foci are $(\pm\sqrt{5},0)$, and the eccentricity is $\sqrt{\frac{5}{3}}$

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

The eccentricity, e = $\sqrt{\frac{5}{3}}$

And also given, foci are $(\pm\sqrt{5},0)$



That means, ae = $\sqrt{5}$

$$\Rightarrow a = \frac{\sqrt{5}}{e}$$

$$\Rightarrow a = \frac{\sqrt{5}}{\sqrt{\frac{5}{3}}}$$ [As e = $\sqrt{\frac{5}{3}}$]

$$\Rightarrow a = \sqrt{3}$$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

Q. 18. Find the equation of the hyperbola, the length of whose latus rectum is 4 and the eccentricity is 3.

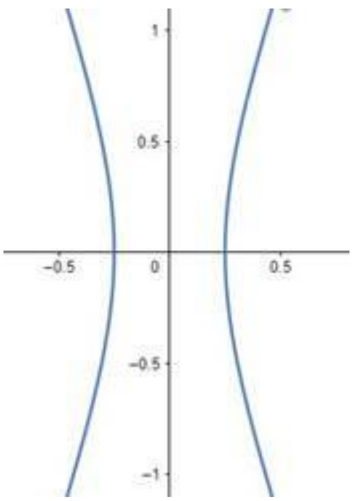
Answer : Given: The length of latus rectum is 4, and the eccentricity is 3

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The length of the latus rectum is 4 units.



Therefore, $\frac{2b^2}{a} = 4 \Rightarrow b^2 = 2a$ ---- (1)

And also given, the eccentricity, e = 3

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$\Rightarrow \sqrt{1+\frac{b^2}{a^2}}=3$$

$$\Rightarrow 1+\frac{b^2}{a^2}=9 \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2}=8$$

$$\Rightarrow b^2=8a^2$$

$$\Rightarrow 2a=8a^2 \text{ [From (1)]}$$

$$\Rightarrow a=\frac{1}{4}$$

Therefore,

$$b^2=2a=2\times\frac{1}{4}=\frac{1}{2}$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{1/16}-\frac{y^2}{1/2}=1 \Rightarrow 16x^2-2y^2=1 \text{ [Answer]}$$

Q. 19. Find the equation of the hyperbola with eccentricity $\sqrt{2}$ and the distance between whose foci is 16.

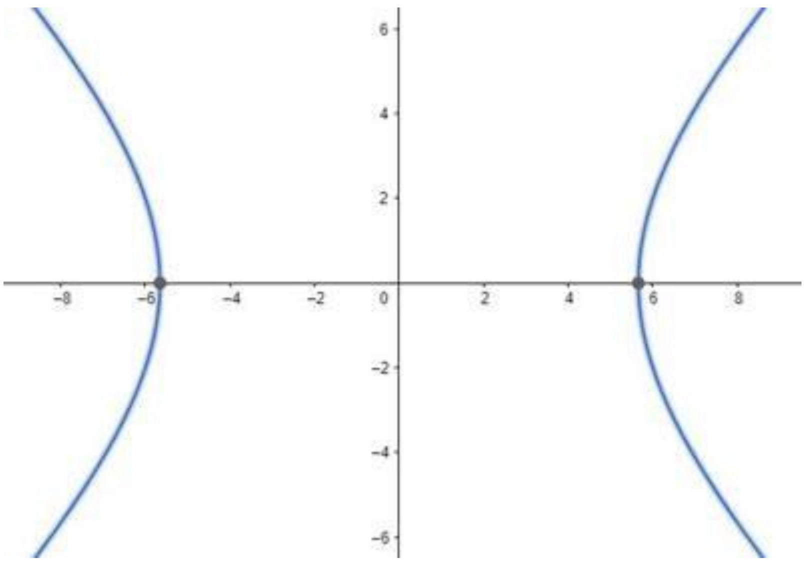
Answer : Given: Eccentricity is $\sqrt{2}$, and the distance between foci is 16

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Distance between the foci is 16, i.e., 2ae = 16



And also given, the eccentricity, e =

$$\sqrt{2}$$

Therefore,

$$2a\sqrt{2} = 16$$

$a = \frac{16}{2\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$ ---- (1)

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{2}$

$\Rightarrow 1 + \frac{b^2}{a^2} = 2$ [Squaring both sides]

$\Rightarrow \frac{b^2}{a^2} = 1$

$\Rightarrow b^2 = a^2 = 32$ [From (1)]

So, the equation of the hyperbola is,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{32} - \frac{y^2}{32} = 1 \Rightarrow x^2 - y^2 = 32$ [Answer]

Q. 20. Find the equation of the hyperbola whose vertices are (0, ±3) and the eccentricity is $\frac{4}{3}$. Also, find the coordinates of its foci.

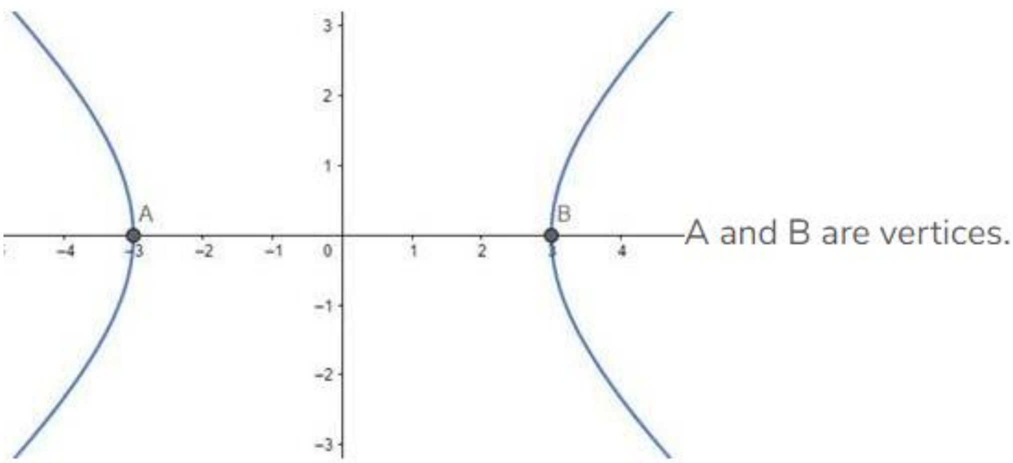
Answer : Given: Vertices are (0, ±3) and the eccentricity is $\frac{4}{3}$

Need to find: The equation of the hyperbola and coordinates of foci.

Let, the equation of the hyperbola be:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertices are (±3, 0), that means, a = 3



And also given, the eccentricity, $e = \frac{4}{3}$

We know that, $e = \sqrt{1 + \frac{b^2}{a^2}}$

Therefore,

$$\Rightarrow \sqrt{1 + \frac{b^2}{a^2}} = \frac{4}{3}$$

$$\Rightarrow 1+\frac{b^2}{a^2}=\frac{16}{9} \text{ [Squaring both sides]}$$

$$\Rightarrow \frac{b^2}{a^2}=\frac{16}{9}-1=\frac{7}{9}$$

$$\Rightarrow b^2=\frac{7}{9}a^2=\frac{7}{9}\times 9=7 \text{ [As a = 3]}$$

So, the equation of the hyperbola is,

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{9}-\frac{y^2}{7}=1$$

Coordinates of the foci = $(\pm ae, 0) = (\pm 4, 0)$ [Answer]

Q. 21. Find the equation of the hyperbola whose foci are (0, ±13) and the length of whose conjugate axis is 24.

Answer : Given: Foci are (0, ±13), the conjugate axis is of the length 24

Need to find: The equation of the hyperbola.

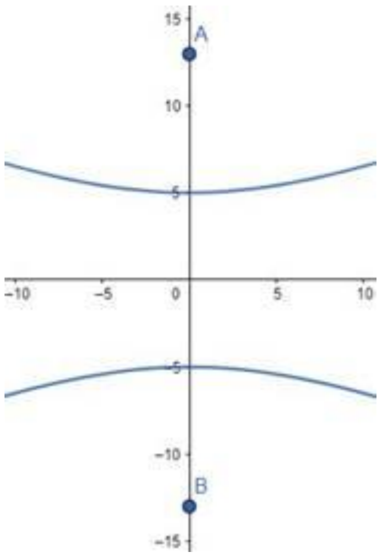
Let, the equation of the hyperbola be:

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$$

The conjugate axis is of the length 24, i.e., 2b = 24

Therefore, b = 12

The foci are given at (0, ±13)



A and B are the foci.

That means, ae = 13, where e is the eccentricity.

We know that,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Therefore,

$$\Rightarrow a\sqrt{1 + \frac{b^2}{a^2}} = 13$$

$$\Rightarrow a\frac{\sqrt{a^2 + b^2}}{a} = 13$$

$\Rightarrow a^2 + b^2 = 169$ [Squaring both sides]

$\Rightarrow a^2 = 169 - b^2 = 169 - 144 = 25$ [As b = 12]

So, the equation of the hyperbola is,

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{144} = 1 \text{ [Answer]}$$

Q. 22. Find the equation of the hyperbola whose foci are (0, ±10) and the length of whose latus rectum is 9 units.

Answer : Given: Foci are (0, ±10) and the length of latus rectum is 9 units

Need to find: The equation of the hyperbola.

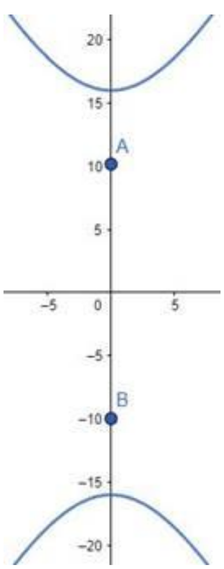
Let, the equation of the hyperbola be:

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

The length of the latus rectum is 9 units.

Therefore, $\frac{2b^2}{a} = 9 \Rightarrow b^2 = \frac{9}{2}a$ ---- (1)

The foci are given at (0, ±10)



That means, ae = 10, where e is the eccentricity.

We know that,

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Therefore,

$$\Rightarrow a\sqrt{1 + \frac{b^2}{a^2}} = 10$$

$$\Rightarrow a\frac{\sqrt{a^2 + b^2}}{a} = 10$$

$$\Rightarrow a^2 + b^2 = 100 \text{ [Squaring both sides]}$$

$$\Rightarrow a^2 + \frac{9}{2}a = 100 \text{ [From (1)]}$$

$$\Rightarrow 2a^2 + 9a - 200 = 0$$

$$\Rightarrow 2a^2 + 25a - 16a - 200 = 0$$

$$\Rightarrow (2a + 25)(a - 16) = 0$$

So, either a = 16 or, a = $-\frac{25}{2}$

That means, either b = $\sqrt{\frac{9}{2} \times 16} = 6\sqrt{2}$ or, b = $\sqrt{-\frac{9 \times 25}{2 \times 2}}$

The value of b = $\sqrt{-\frac{9\times 25}{2\times 2}}$ is not a valid one. So, the b value and its corresponding a value is not acceptable.

Hence, the acceptable value of a is 16 and b is $6\sqrt{2}$

So, the equation of the hyperbola is,

$$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1 \Rightarrow \frac{y^2}{256}-\frac{x^2}{72}=1 \text{ [Answer]}$$

Q. 23. Find the equation of the hyperbola having its foci at $(0,\pm\sqrt{14})$ and passing through the point P(3, 4).

Answer : Given: Foci at $(0,\pm\sqrt{14})$ and passing through the point P(3, 4)

Need to find: The equation of the hyperbola.

Let, the equation of the hyperbola be:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

It passes through the point P(3, 4)

So putting the values of (x, y) we get,

$$\frac{3^2}{a^2}-\frac{4^2}{b^2}=1 \Rightarrow \frac{9}{a^2}-\frac{16}{b^2}=1 \text{ ---- (1)}$$

Foci at $(0,\pm\sqrt{14})$

So, $ae=\sqrt{14}$

We know, $e = \sqrt{1 + \frac{b^2}{a^2}}$

$$\Rightarrow a\sqrt{1 + \frac{b^2}{a^2}} = \sqrt{14}$$

$$\Rightarrow \sqrt{a^2 + b^2} = \sqrt{14}$$

$\Rightarrow a^2 + b^2 = 14$ [Squaring on both sides]

$\Rightarrow a^2 = 14 - b^2$ ---- (2)

Comparing (1) and (2) we get,

$$\frac{9}{14 - b^2} - \frac{16}{b^2} = 1$$

$$\frac{9}{14 - b^2} = 1 + \frac{16}{b^2} = \frac{b^2 + 16}{b^2}$$

$$9b^2 = 14b^2 - b^4 + 224 - 16b^2$$

$$b^4 + 11b^2 - 224 = 0$$

Solving the equations we get,

$$b_1 = \sqrt{\frac{1}{2}(-11 + 3\sqrt{113})}$$

$$b_2 = -\sqrt{\frac{1}{2}(-11+3\sqrt{113})}$$

$$b_3 = (-i)\sqrt{\frac{1}{2}(11+3\sqrt{113})}$$

$$b_4 = i\sqrt{\frac{1}{2}(11+3\sqrt{113})}$$

With the help of any of these values of b we can't find out the equation of the hyperbola.

* This is the only process we can apply in this standard.