CAT 2023 Slot 1 Question Paper

Quant

- **45.** Let n be the least positive integer such that 168 is a factor of 1134^n . If m is the least positive integer such that 1134^n is a factor of 168^m , then m + n equals
 - A 9
 - **B** 15
 - C 12
 - D 24
- **46.** If x and y are positive real numbers such that $\log_x(x^2+12)=4$ and $3\log_yx=1$, then x+y equals
 - A 20
 - **B** 68
 - C 10
 - D 11
- **47.** If $\sqrt{5x+9} + \sqrt{5x-9} = 3(2+\sqrt{2})$, then $\sqrt{10x+9}$ is equal to
 - **A** $3\sqrt{31}$
 - B $4\sqrt{5}$
 - c $3\sqrt{7}$
 - D $2\sqrt{7}$
- **48.** If x and y are real numbers such that $x^2+(x-2y-1)^2=-4y(x+y)$, then the value x-2y is
 - **A** 0
 - B 1
 - C -1
 - **D** 2
- **49.** The number of integer solutions of equation $2|x|(x^2+1)=5x^2$ is
- **50.** The equation $x^3 + (2r+1)x^2 + (4r-1)x + 2 = 0$ has -2 as one of the roots. If the other two roots are real, then the minimum possible non-negative integer value of r is

51. Let $lpha$ and eta be the two distinct roots of the equation $2x^2-6x+k=0$, such that ($lpha+eta$) and $lphaeta$ are
the distinct roots of the equation $x^2+px+p=0.$ Then, the value of 8(k - p) is

- **52.** A mixture P is formed by removing a certain amount of coffee from a coffee jar and replacing the same amount with cocoa powder. The same amount is again removed from mixture P and replaced with same amount of cocoa powder to form a new mixture Q. If the ratio of coffee and cocoa in the mixture Q is 16:9, then the ratio of cocoa in mixture P to that in mixture O is
 - A 1:3
 - B 1:2
 - C 5:9
 - D 4:9
- **53.** The minor angle between the hours hand and minutes hand of a clock was observed at 8:48 am. The minimum duration, in minutes, after 8.48 am when this angle increases by 50% is
 - A $\frac{36}{11}$
 - B 2
 - C 4
 - **D** $\frac{24}{11}$
- **54.** Gita sells two objects A and B at the same price such that she makes a profit of 20% on object A and a loss of 10% on object B. If she increases the selling price such that objects A and B are still sold at an equal price and a profit of 10% is made on object B, then the profit made on object A will be nearest to
 - A 42%
 - B 45%
 - C 47%
 - D 49%
- **55.** Brishti went on an 8-hour trip in a car. Before the trip, the car had travelled a total of x km till then, where x is a whole number and is palindromic, i.e., x remains unchanged when its digits are reversed. At the end of the trip, the car had travelled a total of 26862 km till then, this number again being palindromic. If Brishti never drove at more than 110 km/h, then the greatest possible average speed at which she drove during the trip, in km/h, was
 - A 110
 - **B** 90
 - C 100
 - **D** 80

56. In an examination, the average marks of 4 girls and 6 boys is 24. Each of the girls has the same marks while each of the boys has the same marks. If the marks of any girl is at most double the marks of any boy, but not less than the marks of any boy, then the number of possible distinct integer values of the total marks of 2 girls and 6 boys is									
	Α	21							
	В	20							
	С	22							
	D	19							
57.	. The salaries of three friends Sita, Gita and Mita are initially in the ratio 5:6:7, respectively. In the first year, they get salary hikes of 20%, 25% and 20%, respectively. In the second year, Sita and Mita get salary hikes of 40% and 25%, respectively, and the salary of Gita becomes equal to the mean salary of the three friends. The salary hike of Gita in the second year is								
	Α	25%							
	В	28%							
	С	26%							
	D	30%							
58.	8. Arvind travels from town A to town B, and Surbhi from town B to town A, both starting at the same time along the same route. After meeting each other, Arvind takes 6 hours to reach town B while Surbhi takes 24 hours to reach town A. If Arvind travelled at a speed of 54 km/h, then the distance, in km, between town A and town B is								
59.	59. Anil invests Rs. 22000 for 6 years in a certain scheme with 4% interest per annum, compounded half-yearly. Sunil invests in the same scheme for 5 years, and then reinvests the entire amount received at the end of 5 years for one year at 10% simple interest. If the amounts received by both at the end of 6 years are same, then the initial investment made by Sunil, in rupees, is								
	60. The amount of job that Amal, Sunil and Kamal can individually do in a day, are in harmonic progression. Kamal takes twice as much time as Amal to do the same amount of job. If Amal and Sunil work for 4 days and 9 days, respectively, Kamal needs to work for 16 days to finish the remaining job. Then the number of days Sunil will take to finish the job working alone, is								
61. Let C be the circle $x^2+y^2+4x-6y-3=0$ and L be the locus of the point of intersection of a pair of tangents to C with the angle between the two tangents equal to 60° . Then, the point at which L touches the line x = 6 is A (6, 6)									
		(6, 3)							
		(6, 8)							
	D	(6, 4)							

62. A quadrilateral ABCD is inscribed in a circle such that AB : CD = 2 : 1 and BC : AD = 5 : 4. If AC and BD intersect at the point E, then AE : CE equals
A 2:1
B 1:2
C 8:5
D 5:8
63. In a right-angled triangle \triangle ABC, the altitude AB is 5 cm, and the base BC is 12 cm. P and Q are two points on BC such that the areas of \triangle ABP , \triangle ABQ and \triangle ABC are in arithmetic progression. If the area \triangle \triangle ABC is 1.5 times the area of \triangle ABP , the length of PQ, in cm, is
64. For some positive and distinct real numbers x,y and z, if $\frac{1}{\sqrt{y}+\sqrt{z}}$ is the arithmetic mean of $\frac{1}{\sqrt{x}+\sqrt{z}}$ and $\frac{1}{\sqrt{x}+\sqrt{y}}$, then the relationship which will always hold true, is
A \sqrt{x}, \sqrt{z} and \sqrt{y} are in arithmetic progression
B y, x and z are in arithmetic progression
C x, y and z are in arithmetic progression
${f D} = \sqrt{x}, \sqrt{y}$ and \sqrt{z} are in arithmetic progression
65. The number of all natural numbers up to 1000 with non-repeating digits is
A 504
B 648
C 738
D 585
66. A lab experiment measures the number of organisms at 8 am every day. Starting with 2 organisms on the first day, the number of organisms on any day is equal to 3 more than twice the number on the previous day. If the number of organisms on the nth day exceeds one million, then the lowest possible value of n is

Answers

45. B	46. C	47. C	48. B	49. 3	50. 2	51. 6	52. C
53. D	54. C	55. C	56. A	57. C	58. 972	59. 20808	60. 27
61. B	62. C	63. 2	64. B	65. C	66. 19		

Explanations

45.B

Prime Factorising 1134, we get 1134 = $2 \times 3^4 \times 7$ and 168 = $2^3 \times 3 \times 7$

 1134^n is a factor of 168 => the factor of 2 should be atleast 3, for 168 to be a factor => n = 3.

Now, 1134^n = $1134^3 = 2^3 \times 3^{12} \times 7^3$ is a factor of $168^m = \left(2^3 \times 3 \times 7\right)^m$ => m = 12 as power of 3 should be atleast 12.

46.C

Given, $\log_x(x^2+12)=4$

$$\Rightarrow x^2 + 12 = x^4$$

$$\Rightarrow x^4 - x^2 - 12 = 0$$

$$\Rightarrow x^4 - 4x^2 + 3x^2 - 12 = 0$$

$$\Rightarrow x^2(x^2-4)+3(x^2-4)=0$$

 \Rightarrow $(x^2-4)(x^2+3)=0$ \Rightarrow since, x is a positive real number (given) \Rightarrow x = 2.

Now, Given $3\log_u x = 1$

$$\Rightarrow \log_y x = \frac{1}{3}$$

$$=> x = y^{\frac{1}{3}}$$

$$=> y = x^3 => y = 8.$$

$$=> x + y = 2 + 8 = 10.$$

47.C

Given,
$$\sqrt{5}x + 9 + \sqrt{5}x - 9 = 3(2 + \sqrt{2})$$

$$\Rightarrow \sqrt{5x+9} + \sqrt{5x-9} = 6 + 3\sqrt{2}$$

$$\Rightarrow \sqrt{5x+9} + \sqrt{5x-9} = \sqrt{36} + \sqrt{18}$$

Comparing the L.H.S. and R.H.S.

=>
$$5x+9=36$$
 => $5x=27$ => $x=\frac{27}{5}$ (can be verified using the second term as well).

$$\Rightarrow \sqrt{10x+9} = \sqrt{\left(10 \times \frac{27}{5}\right) + 9} = \sqrt{63} = 3\sqrt{7}$$

48 F

Given,
$$x^2 + (x - 2y - 1)^2 = -4y(x + y)$$

$$\Rightarrow x^2 + 4xy + 4y^2 + (x - 2y - 1)^2 = 0$$

$$=> (x+2y)^2 + (x-2y-1)^2 = 0$$

For the L.H.S. of the equation to be 0, each of the square terms should be 0 (as squares cannot be negative)

$$=> x - 2y - 1 = 0 => x - 2y = 1$$

49.3

Let us consider 3 cases:

1) x = 0, This is a solution, as both L.H.S and R.H.S will be equal (0) when x = 0. (1 solution)

$$\Rightarrow 2x(x^2+1)=5x^2$$

$$\Rightarrow 2(x^2+1)=5x$$

$$\Rightarrow 2x^2 - 5x + 2 = 0 \Rightarrow 2x^2 - 4x - x - 2 = 0$$

$$\Rightarrow 2x(x-2)-1(x-2)=0$$

$$=> (x-2)(2x-1) = 0 => x = 2 \text{ or } 1/2 => (1 \text{ integer solution})$$

$$\Rightarrow -2x(x^2+1) = 5x^2$$

$$\Rightarrow 2x^2 + 5x + 2 = 0$$

$$\Rightarrow 2x^2 + 4x + x + 2 = 0$$

$$\Rightarrow 2x(x+2)+1(x+2)=0$$

$$=> (x + 2) (2x + 1) = 0 => x = -2 \text{ or } -1/2 => (1 \text{ integer solution})$$

So, the total number of integer solutions are $0, 2, -2 \Rightarrow 3$.

50.2

Given that -2 is a root of the given cubic equation.

=> Dividing the given equation by (x + 2), Using the Horners method of synthetic division:

coefficient of x^2 is 1, and coefficient of x is (2r+1)-2 = 2r-1 and the constant term = (4r-1)-2(2r-1) = 1.

=> The quadratic obtained by dividing the cubic = $x^2 + (2r - 1)x + 1 = 0$, Since, this equation has 2 real roots => Discriminant should be greater than 0

$$=> (2r-1)^2 > 4 => 2r-1 > 2 \text{ or } 2r-1 < -2 => r > 3/2 \text{ or } r < -1/2.$$

=> Minimum possible non-negative integer value of r is 2.

51.6

Given a and b are the distinct roots of the equation $2x^2-6x+k=0$

$$=> a + b = -(-6/2) = 3$$
 (Sum of the roots)

Now, (a+b) and ab are the roots of the quadratic equation $x^2+px+p=0$

$$=> a + b + ab = -p => 3 + k/2 = -p ---(1)$$

$$=> (a + b)(ab) = p => 3(k/2) = p --(2)$$

$$3 + \frac{k}{2} = -\frac{3k}{2} \Rightarrow 2k = -3 \Rightarrow k = -\frac{3}{2}$$

$$\mathrm{p} = \frac{3k}{2} = \frac{3}{2} \left(-\frac{3}{2} \right) = -\frac{9}{4}$$

=> 8(k-p) = 8
$$\left(-\frac{3}{2} + \frac{9}{4}\right) = -12 + 18 = 6$$

52.C

Given that in the final mixture, the ratio of coffee and cocoa is 16:9

Let us assume coffee is 16 units and cocoa is 9 units.

=> Initially, there are 25 units of coffee and 0 units of cocoa

Let's say x units of the mixture is removed and replaced with cocoa

=> Now, we have (25-x) coffee and x units of cocoa. => Mixture P

Now, if x units of the mixture is removed:

Amount of coffee present = (25-x) - $\frac{(25-x)}{25} \times x$

$$\Rightarrow$$
 $(25-x)\left(1-\frac{x}{25}\right)=16$

$$\Rightarrow (25 - x)^2 = 16 \times 25$$

$$=> 25 - x = 20 => x = 5.$$

In mixture P, cocoa = x = 5

In mixture Q, cocoa = 9 units.

=> Required ratio = 5:9

53.**D**

The given time is 8:48 AM.

Angle made by hours hand w.r.t 12 is 8 * 30 (30 degrees in 1 hour) + 0.5 * 48 (0.5 degree in 1 minute) = 240 + 24 = 264 degrees.

Angle made by minutes hands w.r.t 12 is 48 * 6 = 288 degrees.

=> The angle between them is 288 - 264 = 24 degrees.

This should further increase by 12 degrees (50% of 24)

After m minutes, the further increase in angle = (6 - 0.5)*m = $\frac{11}{2}m = 12$ => m = $\frac{24}{11}$

54.C

Let us assume the initial selling prices of A and B is p.

Given, she made profit of 20% on A => 1.2 * c = p => c = 5p/6 => cost of A is $\frac{5}{6}p$

Given, she made a loss of 10% on B => 0.9 * c = p => c = 10p/9 => cost of B is $\frac{10}{9}p$

Now, she sold them at a price such that a 10% profit is made on B

=> Selling price = s = 11/10 * 10/9 p =>
$$\frac{11}{9}p$$

=> Profit % on A =
$$\frac{\left(\frac{11}{9} - \frac{5}{6}\right)}{\left(\frac{5}{6}\right)} \times 100$$
 = 46.66% = nearly 47%

55.C

Given the total number of kilometres travelled, including the trip = is 26862 Km, and the duration of the trip is 8 hrs

If avg. speed of the car during the trip is 's' => the km travelled till just before the trip is 26862 - 8s, which should also be a palindrome.

- => From the options if s = 110 => The reading will be 26862 110*8 = 25982 (Not a palindrome)
- => If s = 100 => The reading will be 26862 100*8 = 26062 => It is a palindrome.
- => s = 100 is the correct option.

56.A

Given that the average marks of 4 girls and 6 boys is 24.

Let us assume 'b' is the marks scored by a boy and 'g' is the marks scored by a girl.

$$=> 4g + 6b = 10*24 = 240 ---(1)$$

Given that, $b \leq g \leq 2b$

We need to find the distinct possible values of 2g + 6b = 2g + 240 - 4g = 240 - 2g.

From (1)

when
$$b = g \Rightarrow 10g = 240 \Rightarrow g = 24$$

when
$$b = g/2 \Rightarrow 7g = 240 \Rightarrow g = 240/7$$

- => 240 2g varies from 240 2*24 to 240 2*240/7
- => 171.42 to 192 => Integer values of 172 to 192 => 21 values.

57.C

Given, the salaries of Sita, Gita and Mita are initially in the ratio 5:6:7, respectively, Let us assume their salaries are 5p, 6p and 7p.

They get salary hikes of 20%, 25% and 20%, respectively.

=> Their salaries are 6/5 * 5p, 5/4 * 6p and 6/5 * 7p => 6p, 7.5p, 8.4p

Now, Sita and Mita get salary hikes of 40% and 25%, respectively

=> Sita's salary = 1.4 * 6p = 8.4p and Mita's salary = 1.25 * 8.4p = 10.5p

Let Gita's salary be 'g' after hike

$$\Rightarrow$$
 3g = 8.4p + g + 10.5p \Rightarrow 2g = 18.9p \Rightarrow g = 9.45p

=> Hike % =
$$\frac{(9.45-7.5)}{7.5} \times 100$$
 = 26%

58.972

Let us assume the speeds of Arvind and Surbhi are 'a' and 's', respectively.

Let us say they meet after 't' hours

=> Arvind travelled s*t distance in 6 hrs and Surbhi travelled a*t in 24 hrs

=> s*t = a*6 and a*t = s*24 =>
$$t^2 = 6 \times 24$$
 => t = 12

Given
$$a = 54 \Rightarrow s*12 = 54*6 \Rightarrow s = 27$$
.

=> Total distance between A and B is (s+a)*t = (54+27)*12 = 81*12 = 972 Kms.

59.20808

Anil invested 22000 for 6 years at 4% interest compounded half-yearly

$$\Rightarrow$$
 Amount = $22000 (1.02)^{1} 2$

Let Sunil invest 'S' rupees for 5 years at 4% C.I. half-yearly and 10% S.I. for 1 additional year

$$\Rightarrow$$
 Amount = $S(1.02)^{10}(1.1)$

Given that the both amounts are equal

$$\Rightarrow 22000 (1.02)^{12} = S (1.02)^{10} (1.1)$$

$$\Rightarrow S = \frac{22000 (1.02)^2}{1.1} = 20808$$

60.27

Let us assume the efficiencies of Amal, Sunil, and Kamal are a, s, and k, respectively.

Given that they are in H.P.

$$\Rightarrow \frac{2}{s} = \frac{1}{a} + \frac{1}{k}$$
 --(1)

Also, given that Kamal takes twice as much time as Amal to do the same amount of job

Given that when Amal and Sunil work for 4 days and 9 days, respectively, Kamal needs to work for 16 days to finish the remaining job.

=> If W is the total work => 4a + 9s + 16k = W.

from (1)
$$\frac{2}{s}=\frac{1}{a}+\frac{2}{a}$$
 => $a=\frac{3}{2}s$ and $k=\frac{3}{4}s$

$$\Rightarrow 4\left(\frac{3s}{2}\right) + 9s + 16\left(\frac{3s}{4}\right) = W$$

$$=>6s+9s+12s=W$$

$$\Rightarrow 27s = W \implies s = \frac{W}{27}$$

=> Sunil will take 27 days to finish the work when working alone.

61.B

Given equation of circle = $x^2+y^2+4x-6y-3=0$

Center of the circle is (-2,3) and radius of the circle = $\sqrt{g^2+f^2-c}=\sqrt{4+9+3}=4$

Let us assume the point of the intersection of the tangents is (h,k)

The angle made by the line joining (h,k) to the centre makes an angle of 30 degrees with the tangent, and sin(30) will be the ratio of the radius and the distance between the center and (h,k)

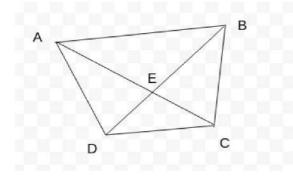
$$\Rightarrow$$
 sin (30) = $\frac{4}{\sqrt{(h+2)^2 + (k-3)^2}}$

Squaring on both sides:

$$\frac{1}{4} = \frac{16}{(h+2)^2 + (k-3)^2}$$
=> $(h+2)^2 + (k-3)^2 = 64$
When x = 6 => h = 6 => $64 + (k-3)^2 = 64$ => k = 3.

=> required point is (6,3)

62.C



Given ABCD is a cyclic quadrilateral.

Angle ADB = Angle ACB (Angle subtended by chord on the same side of arc)

Angle DAC = Angle DBC (Angle subtended by chord on the same side of arc)

=> Triangles AED and BEC are similar triangles

Similarly triangles AEB and DEC are also similar using AA similarity property.

Now, given that AB: CD = 2:1 and BC: AD = 5:4

AE/BE = AD/BC = 4/5 (Similar Triangles AED and BEC)

BE/CE = AB/CD = 2/1 (Similar Triangles AEB and DEC)

Multiplying both, we get AE/CE = 8/5.

63.2

Given that ABC is a right-angled triangle with AB = 5 and BC = 12 => Area of the triangle = 0.5 * 5 * 12 = 30.

Let us assume BP = p, BQ = q

Given the area of ABC is 1.5 times that of ABP

$$=> 30 = 1.5 * 2.5p => 20 = 2.5p => p = 8.$$

Given Areas of ABP, ABQ and ABC are in A.P. => 2 * 2.5q = 2.5 * 8 + 30 => 5q = 50 => q = 10.

$$PQ = BQ - BP = q - p = 10 - 8 = 2$$
.

64.B

Given that $\dfrac{1}{\sqrt{y}+\sqrt{z}}$ is the arithmetic mean of $\dfrac{1}{\sqrt{x}+\sqrt{z}}$ and $\dfrac{1}{\sqrt{x}+\sqrt{y}}$

$$\Rightarrow \frac{2}{\sqrt{y} + \sqrt{z}} = \frac{1}{\sqrt{x} + \sqrt{z}} + \frac{1}{\sqrt{x} + \sqrt{y}}$$

$$\Rightarrow 2(\sqrt{x} + \sqrt{z})(\sqrt{x} + \sqrt{y}) = (\sqrt{y} + \sqrt{z})(\sqrt{x} + \sqrt{y} + \sqrt{x} + \sqrt{z})$$

$$\Rightarrow$$
 2 $\left(x+\sqrt{xy}+\sqrt{xz}+\sqrt{yz}\right)=2\sqrt{xy}+y+\sqrt{yz}+2\sqrt{xz}+\sqrt{yz}+z$

$$=> 2x = y + z$$

=> y, x, z are in A.P. as x is the arithmetic mean of y and z.

65.C

1-digit numbers => We have 1 to 9 => 9

2-digit numbers => x y, we have 9 ways to choose x from 1 to 9 => 9 ways and 9 ways to choose y (0 to 9 except x) => 9*9 = 81

3-digit numbers => x y z, we have 9 ways to choose x, 9 ways to choose y and 8 ways to choose z => 9*9*8 = 648.

Total numbers till 1000 without digits repeated in them is 9 + 81 + 648 = 738.

66.19

Given on day-1, there are 2 organisms.

On day-2, there are 2*2 + 3 = 7 and on day-3, there are 2*7 + 3 = 17...

Let us try to form a pattern:

$$2 = 2 + 0 (n = 1)$$

$$7 = 4 + 3 (n = 2)$$

$$17 = 8 + 9 [8 + 3*3] (n = 3)$$

$$37 = 16 + 21[16 + 3*7] n = 4$$

$$T(n) = 2^n + 3(2^{n-1} - 1)$$

We know that $2^{20}=2^{10} imes 2^{10}=1024 imes 1024$, which is more than 1 million.

Let us check for n = 19

$$2^{19}+3\left(2^{18}-1
ight)=2^{19}\ +\ 3\cdot 2^{18}-3=2\cdot 2^{19}+2^{18}-3=2^{20}+2^{18}-3$$
 , which is more than 1 million.

Let us check for n = 18

=>
$$2^{18} + 3(2^{17} - 1) = 2^{18} + 3 \cdot 2^{17} - 3 = 2 \cdot 2^{18} + 2^{17} - 3 = 2^{19} + 2^{17} - 3$$
 which is not more than a million.