Chapter 6: Permutations and Combinations

EXERCISE 6.1 [PAGES 72 - 73]

Exercise 6.1 | Q 1 | Page 72

A teacher wants to select the class monitor in a class of 30 boys and 20 girls. In how many ways can he select a student if the monitor can be a boy or a girl?

SOLUTION

There are 30 boys and 20 girls in a class.
The teacher wants to select a class monitor from these boys and girls.
A boy can be selected in 30 ways and a girl can be selected in 20 ways.
∴ By using the fundamental principle of addition, the number of ways either a boy or a girl is selected as a class monitor = 30 + 20 = 50.

Exercise 6.1 | Q 2 | Page 72

A teacher wants to select the class monitor in a class of 30 boys and 20 girls, in how many ways can the monitor be selected if the monitor must be a boy? What is the answer if the monitor must be a girl?

SOLUTION

Since the teacher wants to select a class monitor that must be a boy and there are 30 boys in a class.

∴ Total number of ways of selecting boy monitor

= 30 ways.

Since the teacher wants to select a class monitor that must be a girl and there are 20 girls in a class.

∴ Total number of ways of selecting girl monitor

= 20 ways.

Exercise 6.1 | Q 3 | Page 72

A Signal is generated from 2 flags by putting one flag above the other. If 4 flags of different colours are available, how many different signals can be generated?

SOLUTION

A signal is generated from 2 flags and there are 4 flags of different colours available.
∴ 1^{st} flag can be any one of the available 4 flags.
∴ It can be selected in 4 ways.

Now, the 2nd flag is to be selected for which 3 flags are available for a different signal.
∴ 2nd flag can be anyone from these 3 flags.
∴ It can be selected in 3 ways.
∴ By using the fundamental principle of multiplication,
Total number of ways in which a signal can be generated = $4 \times 3 = 12$
∴ 12 different signals can be generated.

Exercise 6.1 | Q 4. (i) | Page 73

How many two-letter words can be formed using letters from the word SPACE, when repetition of letters is allowed?

SOLUTION

Two-letter word is to be formed out of the letters of the word SPACE.
When repetition of the letters is allowed
1st letter can be selected in 5 ways
2nd letter can be selected in 5 ways
∴ By using the fundamental principle of multiplication, the total number of 2-letter words
= $5 \times 5 = 25$

Exercise 6.1 | Q 4. (ii) | Page 73

How many two-letter words can be formed using letters from the word SPACE, when repetition of letters is not allowed?

SOLUTION

Two-letter word is to be formed out of the letters of the word SPACE.
When repetition of the letters is not allowed
1st letter can be selected in 5 ways
2nd letter can be selected in 4 ways
∴ By using the fundamental principle of multiplication, the total number of 2-letter words
= $5 \times 4 = 20$

Exercise 6.1 | Q 5. (i) | Page 73

How many three-digit numbers can be formed from the digits 0, 1, 3, 5, 6 if repetitions of digits are allowed?

SOLUTION

Three-digit number is to be formed from the digits 0, 1, 3, 5, 6
When repetition of digits is allowed:
100’s place digit should be a non zero number.
Hence, it can be anyone from digits 1, 3, 5, 6
∴ 100's place digit can be selected in 4 ways.
0 can appear in 10's and the unit’s place and digits can be repeated.

∴ 10's place digit can be selected in 5 ways and the unit's place digit can be selected in 5 ways
∴ By using the fundamental principle of multiplication, the total number of three-digit numbers = 4 × 5 × 5 = 100

Exercise 6.1 | Q 5. (ii) | Page 73

How many three-digit numbers can be formed from the digits 0, 1, 3, 5, 6 if repetitions of digits are not allowed?

SOLUTION

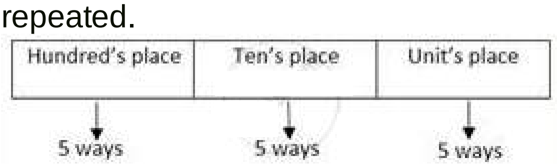
Three-digit number is to be formed from the digits 0, 1, 3, 5, 6
When repetition of digits is not allowed:
100’s place digit should be a non zero number.
Hence, it can be anyone from digits 1, 3, 5, 6
∴ 100’s place digit can be selected in 4 ways.
0 can appear in 10’s and the unit’s place and digits can’t be repeated.
∴ 10’s place digit can be selected in 4 ways and the unit’s place digit can be selected in 3 ways
∴ By using the fundamental principle of multiplication, the total number of three-digit numbers = 4 × 4 × 3 = 48

Exercise 6.1 | Q 6 | Page 73

How many three-digit numbers can be formed using the digits 2, 3, 4, 5, 6 if digits can be repeated?

SOLUTION

A three-digit number is to be formed from the digits 2, 3, 4, 5, 6 where digits can be repeated.



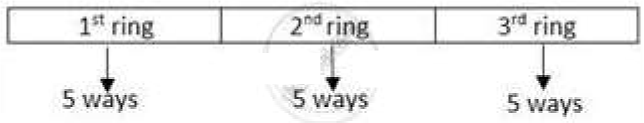
Here, all the places can be filled in 5 ways each.
∴ By using the fundamental principle of multiplication, the total number of three-digit numbers = 5 × 5 × 5 = 125

Exercise 6.1 | Q 7 | Page 73

A letter lock has 3 rings and each ring has 5 letters. Determine the maximum number of trials that may be required to open the lock.

SOLUTION

A letter lock has 3 rings, each ring containing 5 different letters.
∴ Each ring can be adjusted in 5 different ways.



∴ By the principle of multiplication, the 3 rings can be arranged in 5 × 5 × 5 = 125 ways.
Out of these 124 wrong attempts are made and in 125th attempt, the lock gets opened, for maximum number of trials.
∴ Maximum number of trials required to open the lock is 125

Exercise 6.1 | Q 8 | Page 73

In a test that has 5 true/false questions, no student has got all correct answers and no sequence of answers is repeated. What is the maximum number of students for this to be possible?

SOLUTION

For a set of 5 true/false questions, each question can be answered in 2 ways.
∴ By using the fundamental principle of multiplication, the total number of possible sequences of answers = 2 × 2 × 2 × 2 × 2 = 32
Since no student has written all the correct answers
∴ Total number of sequences of answers given by the students in the class = 32 − 1 = 31
Also, no student has given the same sequence of answers.
∴ Maximum number of students in the class = Number of sequences of answers given by the students = 31

Exercise 6.1 | Q 9 | Page 73

How many numbers between 100 and 1000 have 4 in the units place?

SOLUTION

The numbers between 100 and 1000 have 3-digits numbers.
A 3-digit number is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 where the units place digit is 4.
Since Unit's place digit is 4.
∴ it can be selected in 1 way only.
10's place digit can be selected in 10 ways.
For 3-digit number 100's place digit should be a non-zero number.
∴ 100's place digit can be selected in 9 ways.
∴ By using fundamental principle of multiplication, total number of numbers between 100 and 1000 which have 4 in the units place = 1 × 10 × 9 = 90

Exercise 6.1 | Q 10 | Page 73

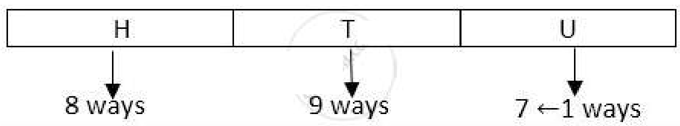
How many numbers between 100 and 1000 have the digit 7 exactly once?

SOLUTION

A number between 100 and 1000 are 3-digit numbers.
A 3-digit number is to be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where exactly one of the digits is 7.

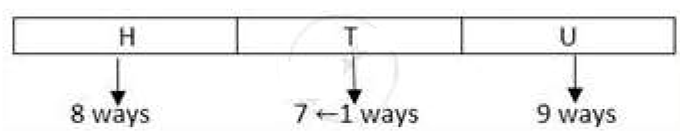
Let us consider the three cases separately.

Case (I): the digit 7 is in the unit's place.



The ten's place is filled by one digit from 0 to excluding 7 in 9 ways.
Here, there are 8 × 9 × 1 = 72 three digit numbers with the required condition.

Case (II): The digit 7 is in the ten's place.

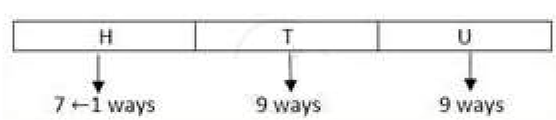


Unit's place can be filled by digit from 0 to 9 excluding 7 in 9 ways.
Zero is not allowed at a hundred's place.

Hundred's place can be filled by digit from 1 to 9 excluding 7 in 8 ways.
The hundred's place can be filled in by any digit from 1 to 9 excluding 7 in 8 ways.

Here, there will be 8 × 1 × 9 = 72
3-digit numbers with the required condition

Case (III): The digit 7 is in the hundred's place.



Then, there are 1 × 9 × 9 = 81
3-digit numbers with the required condition.

Hence, the numbers between 100 and 1000 having the digit 7 exactly once are 72 + 72 + 81 = 225.

Exercise 6.1 | Q 11 | Page 73

How many four-digit numbers Will not exceed 7432 if they are formed using the digits 2, 3, 4, 7 without repetition?

SOLUTION

Among many sets of digits, the greatest number is possible when digits are arranged in descending order.

∴ 7432 is the greatest number, formed from the digits 2, 3, 4, 7.
∴ Since a 4-digit number is to be formed from the digits 2, 3, 4, 7, where repetition of digit is not allowed.

∴ 1000's place digit can be selected in 4 ways.
100's place digit can be selected in 3 ways.
10's place digit can be selected in 2 ways.

Unit's place digit can be selected in 1 way.
∴ Total number of numbers not exceeding 7432 that can be formed from the digits 2, 3, 4, 7
= Total number of four-digit numbers formed from the digits 2, 3, 4, 7
= $4 \times 3 \times 2 \times 1 = 24$

Exercise 6.1 | Q 12 | Page 73

If numbers are formed using digits 2, 3, 4, 5, 6 without repetition, how many of them will exceed 400?

SOLUTION

Case I: Three-digit numbers with 4 occurring in hundred's place:
100's place digit can be selected in 1 way.
Ten's place can be filled by anyone of the number 2, 3, 5, 6.
∴ 10's place digit can be selected in 4 ways.
Unit's place digit can be selected in 3 ways.
∴ Total number of numbers which have 4 in 100's place = $1 \times 4 \times 3 = 12$

Case II: Three digit numbers more than 500
100's place digit can be selected in 2 ways.
10's place digit can be selected in 4 ways.
Unit's place digit can be selected in 3 ways.
∴ Total number of three digit numbers more than 500 = $2 \times 4 \times 3 = 24$

Case III: Number of four-digit numbers formed from 2, 3, 4, 5, 6
Since, repetition of digits is not allowed
∴ Total four digit numbers formed = $5 \times 4 \times 3 \times 2 = 120$

Case IV: Number of five digit numbers formed from 2, 3, 4, 5, 6
Since, repetition of digits is not allowed

∴ Total five digit numbers formed = 5 × 4 × 3 × 2 × 1 = 120
∴ Total number of numbers that exceed 400 = 12 + 24 + 120 + 120 = 276

Exercise 6.1 | Q 13 | Page 73
How many numbers formed with digits 0, 1, 2, 5, 7, 8 will fall between 13 and 1000 if digits can be repeated?

SOLUTION

Case I:
2-digit numbers more than 13, less than 20, formed from the digits 0, 1, 2, 5, 7, 8
Number of such numbers = 3

Case II:
2-digit numbers more than 20 formed from 0, 1, 2, 5, 7, 8
Ten's place digit is selected from 2, 5, 7, 8.
∴ Ten's place digit can be selected in 4 ways.
Unit's place digit is any one from 0, 1, 2, 5, 7, 8
∴ Unit's place digit can be selected in 6 ways.
Using the multiplication principle, the number of such numbers (repetition allowed)
= 4 × 6 = 24

Case III:
3-digit numbers formed from 0, 1, 2, 5, 7, 8
100's place digit is anyone from 1, 2, 5, 7, 8.
∴ 100's place digit can be selected in 5 ways.
As digits can be repeated, the 10's place and unit's place digits are selected from 0, 1, 2, 5, 7, 8

∴ 10's place and unit's place digits can be selected in 6 ways each.
Using multiplication principle, the number of such numbers (repetition allowed) = 5 × 6 × 6 = 180
All cases are mutually exclusive and exhaustive.
∴ Required number = 3 + 24 + 180 = 207

Exercise 6.1 | Q 14 | Page 73
A school has three gates and four staircases from the first floor to the second floor. How many ways does a student have to go from outside the school to his classroom on the second floor?

SOLUTION

A student can go inside the school from outside in 3 ways and from the first floor to the second floor in 4 ways.
∴ Number of ways to choose gates = 3
Number of ways to choose staircase = 4
By using the fundamental principle of multiplication, the number of ways in which a student has to go from outside the school to his classroom = 4 × 3 = 12

Exercise 6.1 | Q 15 | Page 73

How many five-digit numbers formed using the digit 0, 1, 2, 3, 4, 5 are divisible by 3 if digits are not repeated?

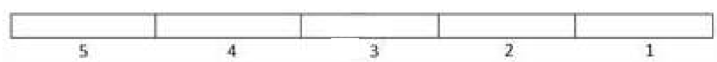
SOLUTION

Five-digits numbers divisible by 3 are to be formed using the digits 0, 1, 2, 3, 4, and 5 without repetition.

For a number to be divisible by 3, the sum of its digits should be divisible by 3,
Consider the digits: 1, 2, 3, 4 and 5
Sum of the digits = 1 + 2 + 3 + 4 + 5 = 15
15 is divisible by 3.

∴ Any 5-digit number formed using the digits 1, 2, 1
Starting with the most significant digit, 5 digits are available for this place.

Since, repetition is not allowed, for the next significant place, 4 digits are available.
Similarly, all the places can be filled as:



Number of 5-digit numbers
= 5 × 4 × 3 × 2 × 1 = 120

Now, consider the digits: 0, 1, 2, 4 and 5
Sum of the digits = 0 + 1 + 2 + 4 + 5 = 12 which is divisible by 3.

∴ Any 5-digit number formed using the digits
0, 1, 2, 4, and 5 will be divisible by 3.

Starting with the most significant digit, 4 digits are available for this place (since 0 cannot be used).

Since, repetition is not allowed, for the next significant place, 4 digits are available (since 0 can now be used).

Similarly, all the places can be filled as:

Number of 5-digit numbers
= 4 × 4 × 3 × 2 × 1 = 96

Next, consider the digits: 0, 1, 2, 3, 4
Sum of the digits = 0 + 1 + 2 + 3 + 4 = 10 which is not divisible by 3.

∴ None of the 5-digit numbers formed using the digits 0, 1, 2, 3, and 4 will not be divisible by 3.

Further, no other selection of 5 digits (out of the given 6)
will give a 5-digit number, which is divisible by 3.

∴ Total number of 5adigit numbers divisible by 3
= 120 + 96 = 216

EXERCISE 6.2 [PAGES 74 - 76]

Exercise 6.2 | Q 1. (i) | Page 75

Evaluate: 8!

SOLUTION

$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
∴ 8! = 40,320

Exercise 6.2 | Q 1. (ii) | Page 75

Evaluate: 6!

SOLUTION

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$
∴ 6! = 720

Exercise 6.2 | Q 1. (iii) | Page 75

Evaluate: 8! – 6!

SOLUTION

$8! - 6!$
$= 8 \times 7 \times 6! - 6!$
$= 6! (8 \times 7 - 1)$
$= 6! (56 - 1)$
$= 6! \times 55$
$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 55$
= 39,600

Exercise 6.2 | Q 1. (iv) | Page 75

Evaluate: (8 – 6)!

SOLUTION

$(8 - 6)!$
$= 2!$

= 2 × 1
= 2

Exercise 6.2 | Q 2. (i) | Page 75

Compute: $\frac{12!}{6!}$

SOLUTION

$$\frac{12!}{6!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

= 12 × 11 × 10 × 9 × 8 × 7

= 665280

Exercise 6.2 | Q 2. (ii) | Page 75

Compute: $\left(\frac{12}{6}\right)!$

SOLUTION

$\left(\frac{12}{6}\right)!$ = 2! = 2 × 1 = 2

Exercise 6.2 | Q 2. (iii) | Page 75

Compute: (3 × 2)!

SOLUTION

(3 × 2)!
= 6!
= 6 × 5 × 4 × 3 × 2 ×1
= 720

Exercise 6.2 | Q 2. (iv) | Page 75

Compute: 3! × 2!

SOLUTION

3! × 2!
= (3 × 2 × 1) × (2 × 1)
= 6 × 2
= 12

Exercise 6.2 | Q 3. (i) | Page 75

Compute: $\frac{9!}{3!6!}$

SOLUTION

$\frac{9!}{3!6!} = \frac{9 \times 8 \times 7 \times 6!}{(3 \times 2 \times 1) \times 6!}$

$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1}$

= 84

Exercise 6.2 | Q 3. (ii) | Page 75

Compute: $\frac{6! - 4!}{4!}$

SOLUTION

$\frac{6! - 4!}{4!}$

$= \frac{6 \times 5 \times 4! - 4!}{4!}$

$= \frac{4!(6 \times 5 - 1)}{4!}$

= 30 – 1

= 29

Exercise 6.2 | Q 3. (iii) | Page 75

Compute: $\frac{8!}{6! - 4!}$

SOLUTION

$$\frac{8!}{6!-4!}$$

$$=\frac{8\times7\times6\times5\times4!}{6\times5\times4!-4!}$$

$$=\frac{4!(8\times7\times6\times5)}{4!(6\times5-1)}$$

$$=\frac{1680}{29}$$

Exercise 6.2 | Q 3. (iv) | Page 75

Compute: $\frac{8!}{(6-4)!}$

SOLUTION

$$\frac{8!}{(6-4)!}$$

$$=\frac{8!}{2!}$$

$$=\frac{8\times7\times6\times5\times4\times3\times2!}{2!}$$

$= 20160$

Exercise 6.2 | Q 4. (i) | Page 75

Write in terms of factorial:

5 × 6 × 7 × 8 × 9 × 10

SOLUTION

$5 \times 6 \times 7 \times 8 \times 9 \times 10 = 10 \times 9 \times 8 \times 7 \times 6 \times 5$

Multiplying and dividing by 4!, we get

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4!}$$

$$= \frac{10!}{4!}$$

Exercise 6.2 | Q 4. (ii) | Page 75

Write in terms of factorial:

$3 \times 6 \times 9 \times 12 \times 15$

SOLUTION

$3 \times 6 \times 9 \times 12 \times 15 = 3 \times (3 \times 2) \times (3 \times 3) \times (3 \times 4) \times (3 \times 5)$

$= (3^5)(5 \times 4 \times 3 \times 2 \times 1)$

$= 3^5 (5!)$

Exercise 6.2 | Q 4. (iii) | Page 75

Write in terms of factorial:

$6 \times 7 \times 8 \times 9$

SOLUTION

$6 \times 7 \times 8 \times 9 = 9 \times 8 \times 7 \times 6$

Multiplying and dividing by 5!, we get

$$= \frac{9 \times 8 \times 7 \times 6 \times 5!}{5!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{5!}$$

$$= \frac{9!}{5!}$$

Exercise 6.2 | Q 4. (iv) | Page 75

Write in terms of factorial:

5 × 10 × 15 × 20 × 25

SOLUTION

5 × 10 × 15 × 20 × 25

= (5 × 1) × (5 × 2) × (5 × 3) × (5 × 4) × (5 × 5)

= (5^5) (5 × 4 × 3 × 2 × 1)

= (5^5) (5!)

Exercise 6.2 | Q 5. (i) | Page 75

Evaluate: $\dfrac{n!}{r!(n-r!)}$ For n = 8, r = 6

SOLUTION

n = 8, r = 6

$$\frac{n!}{r!(n-r!)} = \frac{8!}{6!(8-6!)}$$

$$= \frac{8 \times 7 \times 6!}{2!6!}$$

$$= \frac{8 \times 7}{2!}$$

$$= \frac{8 \times 7}{1 \times 2}$$

= 28

Exercise 6.2 | Q 5. (ii) | Page 75

Evaluate: $\dfrac{n!}{r!(n-r!)}$ For n = 12, r = 12

SOLUTION

n = 12, r = 12

$$\therefore \frac{n!}{r!(n-r!)} = \frac{12!}{12!(12-12)!} = \frac{12!}{12!\ 0!}$$

= 1 ...[∵ 0! = 1]

Exercise 6.2 | Q 6. (iii) | Page 75

Find n, if $\frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$

SOLUTION

$$\frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$$

$$\therefore \frac{1}{n!} = \frac{1}{4!} - \frac{4}{5!}$$

$$\therefore \frac{1}{n!} = \frac{5}{5 \times 4!} - \frac{4}{5!}$$

$$\therefore \frac{1}{n!} = \frac{5}{5!} - \frac{4}{5!}$$

$$\therefore \frac{1}{n!} = \frac{1}{5!}$$

$\therefore$ n! = 5!

$\therefore$ n = 5

Exercise 6.2 | Q 7. (i) | Page 75

Find n, if (n + 1)! = 42 × (n – 1)!

SOLUTION

(n + 1)! = 42 × (n – 1)!
$\therefore$ (n + 1) n (n – 1)! = 42 × (n – 1)!
$\therefore$ n^2 + n = 42
$\therefore$ n (n + 1) = 6 × 7
Comparing on both side, we get
$\therefore$ n = 6

Exercise 6.2 | Q 7. (ii) | Page 74

Find n, if (n + 3)! = 110 × (n + 1)!

SOLUTION

(n + 3)! = 110 × (n + 1)!
$\therefore$ (n + 3) (n + 2) (n + 1)! = 110 × (n + 1)!
$\therefore$ (n + 3) (n + 2) = (11) (10)
Comparing on both sides, we get

n + 3 = 1

∴ n = 8

Exercise 6.2 | Q 8. (i) | Page 76

Find n if: $\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!}$ = 5:3

SOLUTION

$\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!}$ = 5:3

$\therefore \frac{n!}{3!(n-3)!} \times \frac{5!(n-5)!}{n!} = \frac{5}{3}$

$\therefore \frac{n!}{3!(n-3)(n-4)(n-5)!} \times \frac{5 \times 4 \times 3!(n-5)!}{n!} = \frac{5}{3}$

$\therefore \frac{5 \times 4}{(n-3)(n-4)} = \frac{5}{3}$

∴ (n – 3) (n – 4) = $\frac{20 \times 3}{5}$

∴ (n – 3) (n – 4) = 12

∴ (n – 3) (n – 4) = 4 × 3

Comparing on both sides, we get

∴ n – 3 = 4

∴ n = 7

Exercise 6.2 | Q 8. (ii) | Page 76

Find n if: $\frac{n!}{3!(n-5)!} : \frac{n!}{5!(n-7)!}$ = 10:3

SOLUTION

$\dfrac{n!}{3!(n-5)!} : \dfrac{n!}{5!(n-7)!}$ = 10:3

$\therefore \dfrac{n!}{3!(n-5)!} \times \dfrac{5!(n-7)!}{n!} = \dfrac{10}{3}$

$\therefore \dfrac{n!}{3!(n-5)(n-6)(n-7)!} \times \dfrac{5 \times 4 \times 3!(n-7)!}{n!} = \dfrac{10}{3}$

$\therefore \dfrac{5 \times 4}{(n-5)(n-6)} = \dfrac{10}{3}$

$\therefore$ (n – 5) (n – 6) = 3 × 2

Comparing on both sides, we get

$\therefore$ n – 5 = 3

$\therefore$ n = 8

Exercise 6.2 | Q 9. (i) | Page 76

Find n, if: $\dfrac{(17-n)!}{(14-n)!}$ = 5!

SOLUTION

$\dfrac{(17-n)!}{(14-n)!}$ = 5!

$\therefore \dfrac{(17-n)(16-n)(15-n)(14-n)!}{(14-n)!}$ = 5 × 4 × 3 × 2 × 1

$\therefore$ (17 – n) (16 – n) (15 – n) = 6 × 5 × 4

Comparing on both sides, we get

17 – n = 6

$\therefore$ n = 11

Exercise 6.2 | Q 9. (ii) | Page 76

Find n, if: $\dfrac{(15-n)!}{(13-n)!}$ = 12

SOLUTION

$$\frac{(15-n)!}{(13-n)!} = 12$$

$$\therefore \frac{(15-n)(14-n)(13-n)!}{(13-n)!} = 12$$

$\therefore$ (15 – n) (14 – n) = 4 × 3

Comparing on both sides, we get

15 – n = 4

$\therefore$ n = 11

Exercise 6.2 | Q 10 | Page 76

Find n, if: $\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!}$ = 24:1

SOLUTION

$$\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 24:1$$

$$\therefore \frac{(2n)!}{7!(2n-7)!} : \frac{4!(n-4)!}{n!} = 24$$

$$\therefore \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)(2n-6)(2n-7)!}{7 \times 6 \times 5 \times 4!(2n-7)!} \times \frac{4!(n-4)!}{n(n-1)(n-2)(n-3)(n-4)!} = 24$$

$$\therefore \frac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)(2n-5)(2n-6)}{7 \times 6 \times 5} \times \frac{1}{n(n-1)(n-2)(n-3)} = 24$$

$$\therefore \frac{(2n)(2n-1)2(n-2)(2n-3)2(n-2)(2n-5)2(n-3)}{7 \times 6 \times 5} \times \frac{1}{n(n-1)(n-2)(n-3)} = 24$$

$$\therefore \frac{16(2n-1)(2n-3)(2n-5)}{7 \times 6 \times 5} = 24$$

$$\therefore (2n-1)(2n-3)(2n-5) = \frac{24 \times 7 \times 6 \times 5}{16}$$

$\therefore$ (2n – 1) (2n – 3) (2n – 5) = 9 × 7 × 5

Comparing on both sides, we get

$\therefore$ 2n – 1 = 9

$\therefore$ n = 5

Exercise 6.2 | Q 11 | Page 76

Show that

$$\frac{n!}{r!(n-r)!}+\frac{n!}{(r-1)!(n-r+1)!}=\frac{(n+1)!}{r!(n-r+1)!}$$

SOLUTION

L.H.S = $\frac{n!}{r!(n-r)!}+\frac{n!}{(r-1)!(n-r+1)!}$

$$=\frac{n!}{r(r-1)!(n-r)!}+\frac{n!}{(r-1)!\times(n-r+1)(n-r)!}$$

$$=\frac{n!}{(r-1)!(n-r)!}+\left[\frac{1}{r}+\frac{1}{n-r+1}\right]$$

$$=\frac{n!}{(r-1)!(n-r)!}+\left[\frac{n-r+1+r}{r(n-r+1)}\right]$$

$$=\frac{n!.\,(n+1)}{r(r-1)!(n-r+1)(n-r)!}$$

$$=\frac{(n+1)!}{r!(n-r+1)!}$$ = R.H.S.

Exercise 6.2 | Q 12 | Page 76

Show that: $\frac{9!}{3!6!}+\frac{9!}{4!5!}=\frac{10!}{4!6!}$

SOLUTION

L.H.S. = $\frac{9!}{3!6!} + \frac{9!}{4!5!}$

= $\frac{9!}{3! \times 6 \times 5!} + \frac{9!}{4 \times 3! \times 5!}$

= $\frac{9!}{5!3!}\left[\frac{1}{6} + \frac{1}{4}\right]$

= $\frac{9!}{5! \times 3!}\left[\frac{4+6}{6 \times 4}\right]$

= $\frac{9! \times 10}{6 \times 5! \times 4 \times 3!}$

= $\frac{10!}{6!4!}$

= $\frac{10!}{4!6!}$

= R.H.S.

Exercise 6.2 | Q 13. (i) | Page 76

Find the value of: $\frac{8! + 5(4!)}{4! - 12}$

SOLUTION

$\frac{8! + 5(4!)}{4! - 12}$

= $\frac{8! + 5!}{4 \times 3 \times 2 - 12}$

= $\frac{8 \times 7 \times 6 \times 5! + 5!}{4 \times 3 \times (2 - 1)}$

= $\frac{5!(8 \times 7 \times 6 + 1)}{4 \times 3}$

$= \dfrac{5 \times 4 \times 3 \times 2 \times 1(336 + 1)}{4 \times 3}$

= 5 × 2 × 337

= 3370

Exercise 6.2 | Q 13. (ii) | Page 76

Find the value of: $\dfrac{5(26!) + (27!)}{4(27!) - 8(26!)}$

SOLUTION

$\dfrac{5(26!) + (27!)}{4(27!) - 8(26!)}$

$= \dfrac{5(26!) + 27(26!)}{4(27 \times 26!) - 8(26!)}$

$= \dfrac{26!(5 + 27)}{4(26!)(27 - 2)}$

$= \dfrac{32}{(4)(25)}$

$= \dfrac{8}{25}$

Exercise 6.2 | Q 14 | Page 76

Show that: $\dfrac{(2n)!}{n!}$ = $2^n(2n - 1)(2n - 3)....5.3.1$

SOLUTION

L.H.S. = $\dfrac{(2n)!}{n!}$

$= \dfrac{(2n)(2n-1)(2n-2)(2n-3)(2n-4)...6 \times 5 \times 4 \times 3 \times 2 \times 1}{n!}$

$= \dfrac{(2n)(2n-1)[2(n-1)](2n-3)[2(n-2)]...(2 \times 3) \times 5 \times (2 \times 2) \times 3 \times (2 \times 1) \times 1}{n!}$

$= \dfrac{2^n[n(n-1)(n-2)....3.2.1][(2n-1)(2n-3)...5.3.1]}{n!}$

$= \dfrac{2^n(n!)(2n-1)(2n-3)...5.3.1}{n!}$

$= 2^n(2n-1)(2n-3)...5.3.1$

= R.H.S.

EXERCISE 6.3 [PAGE 81]

Exercise 6.3 | Q 1 | Page 81

Find n if ${}^nP_6 : {}^nP_3$ = 120:1

SOLUTION

$$\frac{{}^nP_6}{{}^nP_5} = \frac{120}{1}$$

$$\therefore \frac{n!}{(n-6!)} \div \frac{n!}{(n-3)!} = \frac{120}{1}$$

$$\therefore \frac{n!}{(n-6)!} \times \frac{(n-3)!}{n!} = 120$$

$$\therefore \frac{n!}{(n-6)!} \times \frac{(n-3)(n-4)(n-5)(n-6)!}{n!} = 120$$

$\therefore$ (n – 3) (n – 4) (n – 5) = 120

$\therefore$ (n – 3) (n – 4) (n – 5) = 6 × 5 × 4

Comparing on both sides, we get

n – 3 = 6

∴ n = 9

Exercise 6.3 | Q 2 | Page 81

Find m and n if $^{(m+n)}P_2$ = 56 and $^{(m-n)}P_2$ = 12

SOLUTION

$^{(m+n)}P_2$ = 56

∴ $\frac{(m+n)!}{(m+n-2)!} = 56$

∴ $\frac{(m+n)(m+n-1)(m+n-2)!}{(m+n-2)!}$ = 56

∴ (m + n) (m + n – 1) = 8 × 7

Comparing on both sides, we get

(m + n) = 8 ......(i)

Also $^{(m-n)}P_2$ = 12

∴ $\frac{(m-n)!}{(m-n-2)!}$ = 12

∴ $\frac{(m-n)(m-n-1)(m-n-2)!}{(m+n-2)!}$ = 12

∴ (m – n) (m – n – 1) = 4 × 3

Comparing on both sides, we get

∴ m – n = 4 ......(ii)

Adding (i) and (ii), we get

2m = 12

$\therefore$ m = 6

Substituting m = 6 in (ii), we get

6 − n = 4

$\therefore$ n = 2

Exercise 6.3 | Q 3 | Page 81

Find r if $^{12}\mathrm{P}_{\mathrm{r}-2} : {}^{11}\mathrm{P}_{\mathrm{r}-1}$ = 3:14

SOLUTION

$^{12}\mathrm{P}_{\mathrm{r}-2} : {}^{11}\mathrm{P}_{\mathrm{r}-1}$ = 3:14

$$\therefore \frac{12!}{(12-\mathrm{r}+2)!} \div \frac{11!}{(11-\mathrm{r}+1)!} = \frac{3}{14}$$

$$\therefore \frac{12!}{(14-\mathrm{r})!} \times \frac{(12-\mathrm{r})!}{11!} = \frac{3}{14}$$

$$\therefore \frac{12 \times 11!}{(14-\mathrm{r})(13-\mathrm{r})(12-\mathrm{r})!} \times \frac{(12-\mathrm{r})!}{11!} = \frac{3}{14}$$

$$\therefore \frac{12}{(14-\mathrm{r})(13-\mathrm{r})} = \frac{3}{14}$$

$\therefore$ (14 – r) (13 – r) = 8 × 7

Comparing on both sides, we get

14 – r = 8

$\therefore$ r = 6

Exercise 6.3 | Q 4 | Page 81

Show that (n + 1) $^{\mathrm{n}}\mathrm{P}_{\mathrm{r}} = (\mathrm{n}-\mathrm{r}+1)^{(\mathrm{n}+1)}\mathrm{P}_{\mathrm{r}}$

SOLUTION

L.H.S. = (n + 1) ^nP_r

$$= (n+1) \times \frac{n!}{(n-r)!}$$

$$= \frac{(n+1)!}{(n-r)!} \quad .....(I)$$

$$= \text{R.H.S.} = (n-r+1)^{(n+1)}P_r$$

$$= (n-r+1) \times \frac{(n+1)!}{(n-r+1)!}$$

$$= \frac{(n-r+1)(n+1)!}{(n-r+1)(n-r)!}$$

$$= \frac{(n+1)!}{(n-r)!} \quad .....(II)$$

From (I) and (II), L.H.S. = R.H.S.

$\therefore$ (n + 1)$^nP_r = (n-r+1)^{(n+1)}P_r$

Exercise 6.3 | Q 5. (i) | Page 81

How many 4 letter words can be formed using letters in the word MADHURI if letters can be repeated?

SOLUTION

There are 7 letters in the word MADHURI.

A 4 letter word is to be formed from the letters of the word MADHURI and repetition of letters is allowed.
$\therefore$ 1st letter can be filled in 7 ways.
2nd letter can be filled in 7 ways.
3rd letter can be filled in 7 ways.
4th letter can be filled in 7 ways.
$\therefore$ Total no. of ways a 4-letter word can be formed = 7 × 7 × 7 × 7 = 2401
$\therefore$ 2401 four-lettered words can be formed when the repetition of letters is allowed.

Exercise 6.3 | Q 5. (ii) | Page 81

How many 4 letter words can be formed using letters in the word MADHURI if letters cannot be repeated?

SOLUTION

When repetition of letters is not allowed, the number of 4-letter words formed from the letters of the word MADHURI is

$$\therefore {}^7P_4 = \frac{7!}{(7-4)!} = \frac{7 \times 6 \times 5 \times 4 \times 3!}{3!} = 840$$

∴ 840 four-letter words can be formed when the repetition of letters is not allowed.

Exercise 6.3 | Q 6. (i) | Page 81

Determine the number of arrangements of letters of the word ALGORITHM if vowels are always together.

SOLUTION

A word is to be formed using the letters of the word ALGORITHM.
There are 9 letters in the word ALGORITHM.

When vowels are always together:
There are 3 vowels in the word ALGORITHM. (i.e., A, I, O)
Let us consider these 3 vowels as one unit.

This unit with 6 other letters is to be arranged.
∴ It becomes an arrangement of 7 things which can be done in ${}^7P_7$ i.e., 7! ways and

3 vowels can be arranged among themselves in ${}^3P_3$ i.e., 3! ways.
∴ Total number of ways in which the word can be formed= 7! × 3! = 30240
∴ 30240 words can be formed if vowels are always together.

Exercise 6.3 | Q 6. (ii) | Page 81

Determine the number of arrangements of letters of the word ALGORITHM if no two vowels are together.

SOLUTION

A word is to be formed using the letters of the word ALGORITHM.
There are 9 letters in the word ALGORITHM.

When consonants are at even positions:
There are 4 even places and 6 consonants in the word ALGORITHM.
1^{st}, 2^{nd}, 3^{rd}, 4^{th} even places are filled in 6, 5, 4, 3 way respectively.

∴ The number of ways to fill four even places by consonants = 6 × 5 × 4 × 3 = 360
Remaining 5 letters (3 vowels and 2 consonants) can be arranged among themselves in ${}^5P_5$ i.e., 5! ways.

∴ Total number of ways the words can be formed in which even place is occupied by consonants

$= 360 \times 5! = 360 \times 120 = 43200$
∴ 43200 words can be formed if even positions are occupied by consonants.

Exercise 6.3 | Q 6. (iii) | Page 81
Determine the number of arrangements of letters of the word ALGORITHM if consonants are at even positions.

SOLUTION

A word is to be formed using the letters of the word ALGORITHM.
There are 9 letters in the word ALGORITHM.

When consonants are at even positions:
There are 4 even places and 6 consonants in the word ALGORITHM.
1^{st}, 2^{nd}, 3^{rd}, 4^{th} even places are filled in 6, 5, 4, 3 way respectively.

∴ The number of ways to fill four even places by consonants $= 6 \times 5 \times 4 \times 3 = 360$
Remaining 5 letters (3 vowels and 2 consonants) can be arranged among themselves in ${}^5P_5$ i.e., 5! ways.

∴ Total number of ways the words can be formed in which even place is occupied by consonants

$= 360 \times 5! = 360 \times 120 = 43200$
∴ 43200 words can be formed if even positions are occupied by consonants.

Exercise 6.3 | Q 6. (iv) | Page 81

Determine the number of arrangements of letters of the word ALGORITHM if O is the first and T is the last letter.

SOLUTION

A word is to be formed using the letters of the word ALGORITHM.

There are 9 letters in the word ALGORITHM.

When beginning with O and ends with T:
All the letters of the word ALGORITHM are to be arranged among themselves such that arrangement begins with O and ends with T.

7 letters other than O and T can be filled between O and T in ${}^7P_7$ i.e., 7! ways = 5040 ways.
∴ 5040 words beginning with O and ending with T can be formed.

Exercise 6.3 | Q 7 | Page 81

In a group photograph, 6 teachers are in the first row and 18 students are in the second row. There are 12 boys and 6 girls among the students. If the middle position is reserved for the principal and if no two girls are together, find the number of arrangements.

SOLUTION

In the first row, the middle seat is fixed for the principal.
Also first row, 6 teachers can be arranged among themselves in ${}^6P_6$ i.e., 6! ways.
In the second row, 12 boys can be arranged among themselves in ${}^{12}P_{12}$ i.e., 12! ways.
13 gaps are created by 12 boys, in which 6 girls are to be arranged. together which can be done in ${}^{13}P_6$ ways.
∴ total number of arrangements
= 6! × 12! ! ${}^{13}P_6$ .....[using Multiplications Principle]

$$= 6! \times 12! \times \frac{13!}{(13-6)!}$$

$$= 6! \times 12! \times \frac{13!}{7!}$$

$$= \frac{6! \times 12! \times 13!}{7 \times 6!} = \frac{12!13!}{7}$$

Exercise 6.3 | Q 8. (i) | Page 81

Find the number of ways letters of the word HISTORY can be arranged if Y and T are together.

SOLUTION

There are 7 letters in the word HISTORY
When ‘Y’ and ‘T’ are together.
Let us consider ‘Y’ and ‘T’ as one unit
This unit with the other 5 letters is to be arranged.
∴ The number of arrangements of one unit and 5 letters = ${}^6P_6$ = 6!
Also, ‘Y’ and ‘T’ can be arranged among themselves in ${}^2P_2$ i.e., 2! ways.
∴ Total number of arrangements when Y and T are always together = 6! × 2! = 720 × 2 = 1440
∴ 1440 words can be formed if Y and T are together.

Exercise 6.3 | Q 8. (ii) | Page 81

Find the number of ways letters of the word HISTORY can be arranged if Y is next to T.

SOLUTION

There are 7 letters in the word HISTORY
When ‘Y’ is next to ‘T’
Let us take this (‘Y’ next to ‘T’) as one unit.

This unit with 5 other letters is to be arranged.
∴ The number of arrangements of 6 letters and one unit = $^6P6 = 6!$

Also ‘Y’ has to be always next to ‘T’.
So they can be arranged in 1 way.

∴ Total number of arrangements possible when Y is next to T = 6! ×1 = 720
∴ 720 words can be formed if Y is next to T.

Exercise 6.3 | Q 9 | Page 81

Find the number of arrangements of the letters in the word BERMUDA so that consonants and vowels are in the same relative positions.

SOLUTION

There are 7 letters in the word “BERMUDA” out of which 3 are vowels and 4 are consonants.

If relative positions of consonants and vowels are not changed.

3 vowels can be arranged among themselves in $^3P_3$ i.e., 3! ways.

4 consonants can be arranged among themselves in $^4P_4$ i.e., 4! ways.

∴ Total no. of arrangements possible if relative positions of vowels and consonants are not changed = 4! × 3! = 24 × 6 = 144

Exercise 6.3 | Q 10. (i) | Page 81

Find the number of 4-digit numbers that can be formed using the digits 1, 2, 4, 5, 6, 8 if digits cannot be repeated.

SOLUTION

A 4 different digit number is to be made from the digits 1, 2, 4, 5, 6, 8 without repetition of digits.
∴ 4 different digits are to be arranged from 6 given digits which can be done in $^6P_4$

$$= \frac{6!}{(6-4)!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360 \text{ ways}$$

∴ 360 four-digit numbers can be formed if the repetition of digits is not allowed.

Exercise 6.3 | Q 10. (ii) | Page 81
Find the number of 4-digit numbers that can be formed using the digits 1, 2, 4, 5, 6, 8 if digits can be repeated.

SOLUTION

A 4-digit number is to be made from the digits 1, 2, 4, 5, 6, 8 such that digits can be repeated.
∴ Unit's place digit can be filled in 6 ways.
10's place digit can be filled in 6 ways.
100's place digit can be filled in 6 ways.
1000's place digit can be filled in 6 ways.
∴ total number of numbers = $6 \times 6 \times 6 \times 6 = 6^4 = 1296$
∴ 1296 four-digit numbers can be formed if repetition of digits is allowed.

Exercise 6.3 | Q 11 | Page 81
How many numbers can be formed using the digits 0, 1, 2, 3, 4, 5 without repetition so that the resulting numbers are between 100 and 1000?

SOLUTION

A number between 100 and 1000 is a 3 digit number and is to be formed from the digits 0, 1, 2, 3, 4, 5, without repetition of digits.
∴ 100's place digit must be a non-zero number which can be filled in 5 ways.
10's place digits can be filled in 5 ways.
Unit's place digit can be filled in 4 ways.
∴ Total number of ways the number can be formed = $5 \times 5 \times 4 = 100$
∴ 100 numbers between 100 and 1000 can be formed.

Exercise 6.3 | Q 12. (i) | Page 81
Find the number of 6-digit numbers using the digits 3, 4, 5, 6, 7, 8 without repetition. How many of these numbers are divisible by 5?

SOLUTION

We have to form 6 digit numbers using the digits 3, 4, 5, 6, 7, 8 without repetition.
Total number of ways of arranging 6 digits in six places = ${}^6P_6 = 6!$
$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways
Here, the number is divisible by 5. So it will have the digit 5 in the unit's place.
Hence, the unit's place can be filled in 1 way.
The other five places can be filled in by the remaining 5 digits
(Since repetition is not allowed) in ${}^5P_5 = 5!$ Ways.
Total number of ways in which numbers divisible by 5 can be formed = $1 \times 5! = 120$

Exercise 6.3 | Q 12. (ii) | Page 81
Find the number of 6-digit numbers using the digits 3, 4, 5, 6, 7, 8 without repetition. How many of these numbers are not divisible by 5?

SOLUTION

A number of 6 different digits is to be formed from the digits 3, 4, 5, 6, 7, 8 which can be done in ${}^6P_6$
i.e., 6! = 720 ways
If the number is not divisible by 5, then
Unit's place can be any digit from 3, 4, 6, 7, 8 which can be selected in 5 ways.
Other 5 digits can be arranged in ${}^5P_5$ i.e., 5! ways
∴ Total number of ways in which numbers not divisible by 5 can be formed = 5 × 5! = 5 × 120 = 600

Exercise 6.3 | Q 13. (i) | Page 81
A code word is formed by two distinct English letters followed by two non-zero distinct digits. Find the number of such code words.

SOLUTION

There are total of 26 alphabets.

A code word contains 2 English alphabets.

∴ 2 alphabets can be filled in ${}^{26}P_2$

$= \frac{26!}{(26-2)!} = \frac{26 \times 25 \times 24!}{24!} = 650$ way

Also, alphabets to be followed by two distinct non-zero digits from 1 to 9 which can be filled in

${}^9P_2 = \frac{9!}{(9-2)!} = \frac{9 \times 8 \times 7!}{7!} = 72$ ways

∴ Total number of a code words = 650 × 72 = 46800

Exercise 6.3 | Q 13. (ii) | Page 81
A code word is formed by two distinct English letters followed by two non-zero distinct digits. Find the number of such code words that end with an even digit.

SOLUTION

There is a total of 26 alphabets.

A code word contains 2 English alphabets.

∴ 2 alphabets can be filled in ${}^{26}P_2$

$= \frac{26!}{(26-2)!} = \frac{26 \times 25 \times 24!}{24!} = 650$ way

For a code word to end with an even integer, the digit in unit's place should be an even number between 1 to 9 which can be filled in 4 ways.
Also, 10's place can be filled in 8 ways.

∴ Total number of a code words = 650 × 4 × 8 = 20800 ways
∴ 20800 codewords end with an even integer.

Exercise 6.3 | Q 14 | Page 81
Find the number of ways in which 5 letters can be posted in 3 post boxes if any number of letters can be posted in a post box.

SOLUTION

There are 5 letters and 3 post boxes and any number of letters can be posted in all three post boxes.
∴ Each letter can be posted in 3 ways.
∴ Total number of ways in which 5 letters can be posted = 3 × 3 × 3 × 3 × 3 = 243

Exercise 6.3 | Q 15. (i) | Page 81
Find the number of arranging 11 distinct objects taken 4 at a time so that a specified object always occurs.

SOLUTION

There are 11 distinct objects and 4 objects are arranged at a time.
The number of permutations of n distinct objects, taken r at a time, when one specified object will always occur is

$$\mathrm{r} \times {}^{(\mathrm{n}-1)}\mathrm{P}_{(\mathrm{r}-1)}$$

Here, r = 4, n = 11

∴ The number of permutations of 4 out of 11 objects when a specified object occurs.

$$= 4 \times {}^{(11-1)}\mathrm{P}_{(4-1)} = 4 \times {}^{10}\mathrm{P}_3$$

$$= 4 \times \frac{10!}{(10-3)!}$$

$$= 4 \times \frac{10!}{7!}$$

$$= 4 \times \frac{10 \times 9 \times 8 \times 7!}{7!}$$

= 2880

∴ There are 2880 permutations of 11 distinct objects, taken 4 at a time, in which one specified object always occurs.

Exercise 6.3 | Q 15. (ii) | Page 81
Find the number of arranging 11 distinct objects taken 4 at a time so that a specified object never occurs.

SOLUTION

There are 11 distinct objects and 4 objects are arranged at a time.
When one specified object does not occur than 4 things are to be arranged from the remaining 10 things, which can be done in ${}^{10}P_4$ ways
= 10 × 9 × 8 × 7 ways
= 5040 ways
∴ There are 5040 permutations of 11 distinct objects, taken 4 at a time, in which one specified object never occurs.

EXERCISE 6.4 [PAGES 82 - 83]

Exercise 6.4 | Q 1. (i) | Page 82

Find the number of permutations of letters of the following word: DIVYA

SOLUTION

The word DIVYA has 5 letters, all are different.
Hence, the number of distinct permutations of the letters
= n!
= 5!
= 5 × 4 × 3 × 2 × 1
= 120 ways

Exercise 6.4 | Q 1. (ii) | Page 82

Find the number of permutations of letters of the following word: SHANTARAM

SOLUTION

The word SHANTARAM has 9 letters of which 'A' is repeated 3 times and the rest all are different.
Hence, the number of permutations

$$= \frac{n!}{p!}$$

$$= \frac{9!}{3!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3!}$$

$$= 60480$$

Exercise 6.4 | Q 1. (iii) | Page 82

Find the number of permutations of letters of the following word: REPRESENT

SOLUTION

The word REPRESENT has 9 letters of which 'E' is repeated 3 times, 'R' is repeated 2 times, and the rest all are different.

∴ The number of distinct permutations

$$= \frac{n!}{p!q!}$$

$$= \frac{9!}{3!2!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4}{2}$$

$$= 30240$$

Exercise 6.4 | Q 1. (iv) | Page 82

Find the number of permutations of letters of the following word: COMBINE

SOLUTION

There are 7 distinct letters in the word COMBINE which can be arranged among themselves in

= n!

= 7!

= 7 × 6 × 5 × 4 × 3 × 2 × 1

= 5040 ways

Exercise 6.4 | Q 2 | Page 82

You have 2 identical books on English, 3 identical books in Hindi, and 4 identical books on Mathematics. Find the number of distinct ways of arranging them on a shelf.

SOLUTION

There are total of 9 books to be arranged on a shelf.

Out of these 9 books, 2 books on English, 3 books on Hindi, and 4 books on Mathematics are identical.

∴ Total number of distinct arrangements

$$= \frac{9!}{2!3!4!}$$

$$= \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4!}{2 \times 3 \times 2 \times 4!}$$

$= 9 \times 4 \times 7 \times 5$

$= 1260$

$\therefore$ In 1260 distinct ways the books can be arranged on a shelf.

Exercise 6.4 | Q 3. (i) | Page 82

A coin is tossed 8 times. In how many ways can we obtain 4 heads and 4 tails?

SOLUTION

A coin is tossed 8 times. All heads are identical and all tails are identical.

We can obtain 4 heads and 4 tails in $\frac{8!}{4!4!}$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4!}{(4 \times 3 \times 2 \times 1)4!}$$

$= 2 \times 7 \times 5$

$= 70$ ways

$\therefore$ In 70 different ways, we can obtain 4 heads and 4 tails

Exercise 6.4 | Q 3. (ii) | Page 82

A coin is tossed 8 times. In how many ways can we obtain at least 6 heads?

SOLUTION

A coin is tossed 8 times. All heads are identical and all tails are identical.
When at least 6 heads are to be obtained

$\therefore$ Outcome can be (6 heads and 2 tails) or (7 heads and 1 tail) or (8 heads)
$\therefore$ Number of ways in which it can be obtained

$= \frac{8!}{6!2!} + \frac{8!}{7!1!} + \frac{8!}{8!}$

$= \frac{8 \times 7}{2} + 8 + 1$

= 28 + 8 + 1

= 37

∴ In 37 different ways we can obtain at least 6 heads.

Exercise 6.4 | Q 4 | Page 82

A bag has 5 red, 4 blue, and 4 green marbles. If all are drawn one by one and their colours are recorded, how many different arrangements can be found?

SOLUTION

There are total of 13 marbles in the bag.
Out of these 5 are Red, 4 Blue, and 4 are Green marbles. All balls of the same colour are taken to be identical.

∴ Required number of arrangements = $\frac{13!}{5!4!4!}$

Exercise 6.4 | Q 5 | Page 82

Find the number of ways of arranging letters of the word MATHEMATICAL. How many of these arrangements have all vowels together?

SOLUTION

There are 12 letters in the word MATHEMATICAL in which 'M' repeats 2 times, 'A' repeats 3 times and 'T' repeats 2 times.

∴ Total number of arrangements = $\frac{12!}{2!3!2!}$

When all the vowels i.e., 'A', 'A', 'A', 'E', 'I' are to be kept together Number of arrangements of these vowels = $\frac{5!}{3!}$ ways.

Let us consider these vowels together as one unit.

This unit is to be arranged with 7 other letters in which 'M' and 'T' repeated 2 times each.

∴ Number of arrangements = $\frac{8!}{2!2!}$

∴ Total number of arrangements = $\frac{8! \times 5!}{2!2!3!}$

Exercise 6.4 | Q 6. (a) | Page 82

Find the number of different arrangements of letters in the word MAHARASHTRA. How many of these arrangements have letters M and T never together?

SOLUTION

Total number of letters in the word MAHARASHTRA = 11
The letter 'A' is repeated '4' times.
The letter 'H' is repeated twice.
The letter 'R' is repeated twice.

$\therefore$ Number of arrangements = $\dfrac{11!}{4!2!2!}$

Other than M and T. there are 9 letters in which A repeats 4 times, H repeats twice, R repeats twice

The number of arrangements of the letter = $\dfrac{9!}{4!2!2!}$

These 9 letters create 10 gaps in which M and T are to be arranged

The number of the arrangement of M and T = ${}^{10}P_2$

$\therefore$ Total number arrangement having M and T never together = $\dfrac{9! \times {}^{10}P_2}{4!2!2!}$

Exercise 6.4 | Q 6. (b) | Page 82

Find the number of different arrangements of letters in the word MAHARASHTRA. How many of these arrangements have all vowels together?

SOLUTION

Total number of letters in the word MAHARASHTRA = 11
The letter 'A' is repeated '4' times.
The letter 'H' is repeated twice.
The letter 'R' is repeated twice.

$\therefore$ Number of arrangements = $\dfrac{11!}{4!2!2!}$

Here, all vowels are together. The given word has 4 vowels A, A, A, A.

We consider 4 vowels together as a single letter, say G.

We have 8 letters G, M, H, R, S, H, T, R of which R and H are repeated 2 times each.

The number of ways of arranging 8 letters

$= \dfrac{8!}{2!2!}$

After this is done, 4 vowels (in which A is repeated 4 times) can be arranged in $\frac{4!}{4!}$ = 1 way.

∴ The number of arrangements in which the vowels are together = $\frac{8!}{2!2!}$

Exercise 6.4 | Q 7 | Page 82

How many different words are formed if the letter R is used thrice and letters S and T are used twice each?

SOLUTION

When R is used thrice, S is used twice and T is used twice,

∴ Total number of letters available = 7, of which S and T repeat 2 times each, R repeats 3 times.

∴ Required number of arrangements = $\frac{7!}{2!2!3!}$

$= \frac{7 \times 6 \times 5 \times 4 \times 3!}{2 \times 1 \times 2 \times 1 \times 3!}$

= 7 × 6 × 5

= 210

∴ 210 different words can be formed with the letter R is used thrice and letters S and T are used twice each.

Exercise 6.4 | Q 8 | Page 83

Find the number of arrangements of letters in the word MUMBAI so that the letter B is always next to A.

SOLUTION

There are 6 letters in the word MUMBAI.
These letters are to be arranged in such a way that B is always next to A.
Let us consider the AB as one unit. This unit with the other 4 letters in which M repeats twice, is to be arranged.
∴ Total number of arrangements when B is always next to A.

$= \frac{5!}{2!}$

$= \frac{5 \times 4 \times 3 \times 2!}{2!}$

= 60

Exercise 6.4 | Q 9 | Page 83

Find the number of arrangements of letters in the word CONSTITUTION that begin and end with N.

SOLUTION

There are 12 letters in the word CONSTITUTION, in which ‘O’, ‘N’, ‘I’ repeat two times each, ‘T’ repeats 3 times.
The arrangement starts and ends with ‘N’, 10 letters other than N can be arranged between two N, in which ‘O’ and ‘I’ repeat twice each and ‘T’ repeats 3 times.

∴ Total number of arrangements with the letter N at the beginning and at the end =
$$\frac{10!}{2!2!3!}$$

Exercise 6.4 | Q 10 | Page 83

Find the number of different ways of arranging letters in the word ARRANGE. How many of these arrangements the two R’s and two A’s are not together?

SOLUTION

The word ARRANGE has 7 letters of which A and R are repeated 2 times.
∴ The number of ways of arranging letters of the word

$$= \frac{7!}{2!2!}$$

$$= \frac{(7 \times 6 \times 5 \times 4 \times 3 \times 2)!}{2!2!}$$

$= 1260$

= 1260

Here, we have to find the number of arrangements in which two R’s nor A’s are together.

A: set of words having 2A together
B: set of words having 2R together

Number of words having both A and both R not together
= 1260 − n(A ∪ B)
= 1260 − [n(a) + n(B) − n(A ∩ B)] ....(i)

n(A) = number of ways in which (AA) R, R, N, G, E are to be arranged

$\therefore$ n(A) = $\frac{6!}{2!}$ = 360

n(B) = number of ways in which (RR), A, A, N, G, E are to be arranged

$\therefore$ n(B) = $\frac{6!}{2!}$ = 360

n(A ∩ B) = number of ways in which (AA), (RR), N, G, E are to be arranged

$\therefore$ n(A ∩ B) = 5! = 120

Substituting n(A), n(B), n(A ∩ B) in (i), we get

Number of words having both A and both R not together

= 1260 – [360 + 360 – 120]

= 1260 − 600

= 660

Exercise 6.4 | Q 11 | Page 83

How many distinct 5 digit numbers can be formed using the digits 3, 2, 3, 2, 4, 5

SOLUTION

5 digit numbers are to be formed from 2, 3, 2, 3, 4, 5.

Case I: Numbers formed from 2, 2, 3, 4, 5 OR 2, 3, 3, 4, 5

Number of such numbers = $\frac{5!}{2!} \times \mathbf{2}$ = 5! = 120

Case II: Numbers formed from 2, 2, 3, 3 and any one of 4 or 5

Number of such numbers = $\frac{5!}{2!2!} \times \mathbf{2}$ = 60

Required number = 120 + 60 = 180

$\therefore$ 180 distinct 5 digit numbers can be formed using the digit 3, 2, 3, 2, 4, 5.

Exercise 6.4 | Q 12 | Page 83

Find the number of distinct numbers formed using the digits 3, 4, 5, 6, 7, 8, 9, so that odd positions are occupied by odd digits.

A number is to be formed with digits 3, 4, 5, 6, 7, 8, 9 such that odd digits always occupy the odd places.

There are 4 odd digits i.e. 3, 5, 7, 9.
They can be arranged at 4 odd places among themselves in 4! ways = 24 ways

3 even places of the number are occupied by even digits (i.e. 4, 6, 8).
∴ They can be arranged in 3! ways = 6 ways

∴ Total number of arrangements = 24 × 6 = 144
∴ 144 numbers can be formed so that odd digits always occupy the odd positions.

Exercise 6.4 | Q 13 | Page 83
How many different 6-digit numbers can be formed using digits in the number 659942? How many of them are divisible by 2?

SOLUTION

A 6-digit number is to be formed using digits of 659942, in which 9 repeats twice.

∴ Total number of arrangements = $\frac{6!}{2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!}$

= 360

∴ 360 different 6-digit numbers can be formed.

For a number to be divisible by 2,

Last digits can be selected in 3 ways

Remaining 5 digits in which, 9 appears twice are arranged in $\frac{5!}{2!}$ ways

∴ Total number of arrangements

$= \frac{5!}{2!} \times 3 = 180$

∴ 180 numbers are divisible by 2.

Exercise 6.4 | Q 14 | Page 83
Find the number of distinct words formed from letters in the word INDIAN. How many of them have two N's together?

SOLUTION

There are 6 letters in the word INDIAN in which I and N repeat twice.
Number of different words that can be formed using the letters of the word INDIAN

$= \frac{6!}{2!2!}$

$= \frac{6 \times 5 \times 4 \times 3 \times 2!}{2 \times 2!}$

= 180

∴ 180 different words can be formed with the letters of the word INDIAN.

When two N's are together.

Let us consider the two N's as one unit.

They can be arranged with 4 other letters in

$= \frac{5!}{2!}$

$= \frac{5 \times 4 \times 3 \times 2!}{2!}$

= 60 ways

∴ 2 N can be arranged in 1 way

∴ Total number of arrangements = 60 × 1 = 60 ways

∴ 60 words are such that two N's are together

Exercise 6.4 | Q 15. (i) | Page 83

Find the number of different ways of arranging letters in the word PLATOON if the two O's are never together.

SOLUTION

When the two O's are never together:

Let us arrange the other 5 letters first, which can be done in 5! = 120 ways.

The letters P, L, A, T, N create 6 gaps, in which O's are arranged.

∴ Two O's in 6 gaps can be arranged in $\frac{{}^6P_2}{2!}$ ways

$= \frac{\frac{6!}{(6-2)!}}{2!}$ ways

$= \frac{6 \times 5 \times 4!}{4! \times 2 \times 1}$ ways

= 3 × 5 ways

= 15 ways

∴ Total number of arrangements if the two O's are never together = 120 × 15 = 1800

Exercise 6.4 | Q 15. (ii) | Page 83

Find the number of different ways of arranging letters in the word PLATOON if consonants and vowels occupy alternate positions.

SOLUTION

When consonants and vowels occupy alternate positions:
There are 4 consonants and 3 vowels in the word PLATOON.
∴ At odd places, consonants occur and at even places, vowels occur.
4 consonants can be arranged among themselves in 4! ways
3 vowels in which O occurs twice and A occurs once.

∴ They can be arranged in $\frac{3!}{2!}$ ways

∴ Required number of arrangements if the consonants and vowels occupy alternate positions

$= 4! \times \frac{3!}{2!}$

$= 4 \times 3 \times 2 \times \frac{3 \times 2!}{2!}$

= 72

EXERCISE 6.5 [PAGE 85]

Exercise 6.5 | Q 1 | Page 85

In how many different ways can 8 friends sit around a table?

SOLUTION

We know that 'n' persons can sit around a table in (n − 1)! ways
∴ 8 friends can sit around a table in 7! ways
= 7 × 6 × 5 × 4 × 3 × 2 × 1
= 5040 ways
∴ 8 friends can sit around a table in 5040 ways.

Exercise 6.5 | Q 2 | Page 85

A party has 20 participants and a host. Find the number of distinct ways for the host to sit with them around a circular table. How many of these ways have two specified persons on either side of the host?

SOLUTION

A party has 20 participants.
All of them and the host (i.e., 21 persons) can be seated at a circular table in $(21 - 1)! = 20!$ ways.
When two particular participants are seated on either side of the host.
Host takes the chair in 1 way.
These 2 persons can sit on either side of the host in 2! ways
Once the host occupies his chair, it is not a circular permutation any more.
Remaining 18 people occupy their chairs in 18! ways.
∴ Total number of arrangements possible if two particular participants are seated on either side of the host = $2! \times 18!$

Exercise 6.5 | Q 3. (i) | Page 85

Delegates from 24 countries participate in a round table discussion. Find the number of seating arrangements where two specified delegates are always together.

SOLUTION

Delegates of 24 countries are to participate in a round table discussion such that two specified delegates are always together.

Let us consider these 2 delegates as one unit.

They can be arranged among themselves in 2! ways.

Also, these two delegates are to be seated with 22 other delegates (i.e. total 23) which can be done in $(23 - 1)! = 22!$ ways

∴ Total number of arrangement if two specified delegates are always together

$= 22! \times 2!$

Exercise 6.5 | Q 3. (ii) | Page 85

Delegates from 24 countries participate in a round table discussion. Find the number of seating arrangements where two specified delegates are never together.

SOLUTION

When 2 specified delegates are never together then, the other 22 delegates can participate in a round table discussion in $(22 - 1)! = 21!$ ways.
∴ There are 22 places of which any 2 places can be filled by those 2 delegates who are never together.
∴ Two specified delegates can be arranged in $^{22}P_2$ ways.
∴ Total number of arrangements if two specified delegates are never together

$= {}^{22}P_2 \times 21!$

$= \dfrac{22!}{(22-2)!} \times 21!$

$= \dfrac{22!}{20!} \times 21!$

$= 22 \times 21 \times 21!$

$= 21 \times 22 \times 21!$

$= 21 \times 22!$

Exercise 6.5 | Q 4 | Page 85

Find the number of ways for 15 people to sit around the table so that no two arrangements have the same neighbours.

SOLUTION

15 people can sit around a table in (15 – 1)! = 14! ways.
Total number of arrangements = 14!
Now, the number of arrangements in which any person can have the same neighbours on either side by clockwise or anticlockwise arrangements = 14!/2!

∴ The number of arrangements in which no two arrangements have the same neighbours

$= 14! - \dfrac{14!}{2!}$

$= 14!\left(1 - \dfrac{1}{2}\right)$

$= 14! \times \dfrac{1}{2}$

$= \dfrac{14!}{2!}.$

Exercise 6.5 | Q 5 | Page 85

A committee of 20 members sits around a table. Find the number of arrangements that have the president and the vice president together.

SOLUTION

A committee of 20 members sits around a table.
But, President and Vice-president sit together.

Let us consider the President and Vice-president as one unit.
They can be arranged among themselves in 2! ways.
Now, this unit with the other 18 members of the committee is to be arranged around a table, which can be done in $(19 - 1)! = 18!$ ways.
∴ Total number of arrangements possible if President and Vice-president sit together = $18! \times 2!$

Exercise 6.5 | Q 6. (i) | Page 85

Five men, two women, and a child sit around a table. Find the number of arrangements where the child is seated between the two women.

SOLUTION

Five men, two women, and a child sit around a table.
Child is seated between the two women.
∴ The two women can be seated on either side of the child in 2! ways.
Let us consider these 3 (two women and a child) as one unit.
Also, these 3 are to seated with 5 men,
(i.e. total 6 units) which can be done in $(6 - 1)! = 5!$ ways
∴Total number of arrangements if the child is seated between two women = $5! \times 2!$

Exercise 6.5 | Q 6. (ii) | Page 85

Five men, two women, and a child sit around a table. Find the number of arrangements where the child is seated between two men.

SOLUTION

Five men, two women, and a child sit around a table.
Here, the child is seated between 2 men.
Since there are 5 men, such a group can be formed in $^5P_2$

$= 5!/3!$

$= 5 \times 4 = 20$ ways
Thus, there are 20 ways in which the child can be seated between 2 men.
We consider the 2 men and the child between as <u>one unit.</u>
Also, we have 3 more men and 2 women.
Thus, we have $1 + 3 + 2 = 6$ persons.
These 6 persons can be arranged around a table in $(6 – 1)! = 5!$ – ways.
∴ Total number of required arrangements
$= 20 \times 5!$
$= 20 \times 120$
$= 2400$

Exercise 6.5 | Q 7 | Page 85

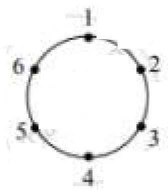
Eight men and six women sit around a table. How many of sitting arrangements will have no two women together?

SOLUTION

8 men can be seated around a table in $(8 - 1)! = 7!$ ways.
There are 8 gaps created by 8 men's seats.
∴ 6 Women can be seated in 8 gaps in ${}^8P_6$ ways
∴ Total number of arrangements so that no two women are together = $7! \times {}^8P_6$

Exercise 6.5 | Q 8 | Page 85
Find the number of sitting arrangements for 3 men and 3 women to sit around a table so that exactly two women are together.

SOLUTION



Two women sit together and one woman sits separately.
Woman sitting separately can be selected in 3 ways.
Other two women occupy two chairs in one way (as it is circular arrangement). They can be seated on those two chairs in 2 ways. Suppose two chairs are chairs 1 and 2 shown in the figure. Then the third woman has only two options viz chairs 4 or 5.
∴ Third woman can be seated in 2 ways. 3 men are seated in 3! ways
Required number = $3 \times 2 \times 2 \times 3!$
$= 12 \times 6$
$= 72$

Exercise 6.5 | Q 9 | Page 85
Four objects in a set of ten objects are alike. Find the number of ways of arranging them in a circular order.

SOLUTION

There are 10 objects.
These 10 objects can be arranged in a circular order in $(10 – 1)! = 9!$ ways.
∴ $n = 9!$

Out of 10 objects, 4 are alike.
$r = 4$
∴ Required number of arrangements

$$= \frac{n!}{r!}$$
$$= \frac{9!}{4!}$$

Exercise 6.5 | Q 10 | Page 85

Fifteen persons sit around a table. Find the number of arrangements that have two specified persons not sitting side by side.

SOLUTION

Since 2 particular persons can't be sitting side by side.
The other 13 persons can be arranged around the table in (13 − 1)! = 12!

13 people around a table create 13 gaps in which 2 person are to be seated
Number of arrangements of 2 person = $^{13}P_2$

∴ Total number of arrangements in which two specified persons not sitting side by side
= 12! × $^{13}P_2$

= 12 × 13 × 12
= 13 × 12! × 12
= 12 × 13!

EXERCISE 6.6 [PAGES 89 - 90]

Exercise 6.6 | Q 1. (i) | Page 89

Find the value of $^{15}C_4$

SOLUTION

$^{15}C_4$

$= \frac{15!}{4!(15-4)!} \quad \ldots\left(\because {}^nC_r = \frac{n!}{r!(n-r)!}\right)$

$= \frac{15!}{4!11!}$

$= \frac{15 \times 14 \times 13 \times 12 \times 11!}{4 \times 3 \times 2 \times 1 \times 11!}$

$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$

= 1365

Exercise 6.6 | Q 1. (ii) | Page 89

Find the value of $^{80}C_2$

SOLUTION

$$^{80}C_2 = \frac{80!}{2!(80-2)!} \ldots\left(\because\ ^nC_r = \frac{n!}{r!(n-r!)!}\right)$$

$$= \frac{80!}{2!78!}$$

$$= \frac{80 \times 79 \times 78!}{2 \times 78!}$$

= 40 × 79

= 3160

Exercise 6.6 | Q 1. (iii) | Page 89

Find the value of $^{15}C_4 + {}^{15}C_5$

$$^{15}C_4 + {}^{15}C_5 = {}^{15}C_5 + {}^{15}C_4 = {}^{15}C_5 + {}^{15}C_{5-1}$$

$$= {}^{16}C_5 \ldots\ldots\left[\because\ ^nC_r + {}^nC_r = {}^{n+1}C_r\right]$$

= 4368

Exercise 6.6 | Q 1. (iv) | Page 89

Find the value of $^{20}C_{16} - {}^{19}C_{16}$

SOLUTION

$$^{20}C_{16} - {}^{19}C_{16}$$

$$= \frac{20!}{16!4!} - \frac{19!}{16!3!}$$

$$= \frac{20 \times 19!}{16! \times 4 \times 3!} - \frac{19!}{16!3!}$$

$$= \frac{19!}{3!16!}\left[\frac{20}{4} - 1\right]$$

$= \dfrac{19!}{3!16!}(4)$

$= \dfrac{19!}{3!(16)(15!)}4$

$= \dfrac{19!}{4(3!)(15!)}$

$= \dfrac{19!}{4!15!}$

$= {}^{19}C_{15}$ or ${}^{19}C_{4} = 3876$

Exercise 6.6 | Q 2. (i) | Page 89

Find n if ${}^{6}\mathrm{P}_{2} = \mathrm{n}\,{}^{6}\mathrm{C}_{2}$

SOLUTION

$${}^{6}\mathrm{P}_{2} = \mathrm{n}\,{}^{6}\mathrm{C}_{2}$$

$$\therefore \frac{6!}{(6-2)!} = \mathrm{n}\frac{6!}{2!(6-2)!}$$

$$\therefore \frac{6!}{4!} = \mathrm{n}\frac{6!}{2!4!}$$

$\therefore$ n = 2! = 2 × 1 = 2

Exercise 6.6 | Q 2. (ii) | Page 89

Find n if ${}^{2\mathrm{n}}\mathrm{C}_{3} : {}^{\mathrm{n}}\mathrm{C}_{2}$ = 52:3

SOLUTION

$$\therefore \frac{{}^{2n}C_3}{{}^{n}C_2} = \frac{52}{3}$$

$$\therefore \frac{(2n)!}{3!(2n-3)!} \div \frac{n!}{2!(n-2)!} = \frac{52}{3}$$

$$\therefore \frac{(2n)!}{3!(2n-3)!} \times \frac{2!(n-2)!}{n!} = \frac{52}{3}$$

$$\therefore \frac{(2n)(2n-1)(2n-2)(2n-3)!}{3 \times 2!(2n-3)!} \times \frac{2!(n-2)!}{n(n-1)(n-2)!} = \frac{52}{3}$$

$$\therefore \frac{2n(2n-1) \times 2(n-1)}{3} \times \frac{1}{n(n-1)} = \frac{52}{3}$$

$$\therefore \frac{4(2n-1)}{3} = \frac{52}{3}$$

$$\therefore 2n - 1 = \frac{52}{3} \times \frac{3}{4}$$

$\therefore$ 2n – 1 = 13

$\therefore$ 2n = 14

$\therefore$ n = 7

Exercise 6.6 | Q 2. (iii) | Page 89

Find n if ${}^{n}C_{n-3}$ = 84

SOLUTION

${}^{n}C_{n-3}$ = 84

$$\therefore \frac{n!}{(n-3)![n-(n-3)]!} = 84$$

$$\therefore \frac{n(n-1)(n-2)(n-3)!}{(n-3)! \times 3!} = 84$$

$\therefore$ n (n – 1) (n – 2) = 84 × 6

$\therefore$ n (n – 1) (n – 2) = 9 × 8 × 7

Comparing on both sides, we get n = 9

Exercise 6.6 | Q 3 | Page 89

Find r if ${}^{14}\mathrm{C}_{2\mathrm{r}} : {}^{10}\mathrm{C}_{2\mathrm{r}-4}$ = 143:10

SOLUTION

${}^{14}\mathrm{C}_{2\mathrm{r}} : {}^{10}\mathrm{C}_{2\mathrm{r}-4}$ = 143:10

$$\therefore \frac{14!}{2\mathrm{r}!(14-2\mathrm{r})!} \div \frac{10!}{(2\mathrm{r}-4)!(14-2\mathrm{r})!} = \frac{143}{10}$$

$$\therefore \frac{14!}{2\mathrm{r}!(14-2\mathrm{r})!} \times \frac{(2\mathrm{r}-4)!(14-2\mathrm{r})!}{10!} = \frac{143}{10}$$

$$\therefore \frac{14 \times 13 \times 12 \times 11 \times 10!}{2\mathrm{r}(2\mathrm{r}-1)(2\mathrm{r}-2)(2\mathrm{r}-3)(2\mathrm{r}-4)!(14-2\mathrm{r})!} \times \frac{(2\mathrm{r}-4)!(14-2\mathrm{r})!}{10!} = \frac{143}{10}$$

$$\therefore \frac{14 \times 13 \times 12 \times 11}{2\mathrm{r}(2\mathrm{r}-1)(2\mathrm{r}-2)(2\mathrm{r}-3)} = \frac{143}{10}$$

$\therefore$ 2r(2r – 1) . (2r – 2) (2r – 3) = 14 × 12 × 10

$\therefore$ 2r(2r – 1) . (2r – 2) (2r – 3) = 8 × 7 × 6 × 5

Comparing on both sides, we get

$\therefore$ r = 4

Exercise 6.6 | Q 4. (i) | Page 89

Find n and r if ${}^{\mathrm{n}}\mathrm{P}_{\mathrm{r}}$ = 720 and ${}^{\mathrm{n}}\mathrm{C}_{\mathrm{n}-\mathrm{r}}$ = 120

SOLUTION

${}^{n}P_{r}$ = 720

$\therefore \frac{n!}{(n-r)!}$ = 720 .....(i)

Also, ${}^{n}C_{n-r}$ = 120

$$\therefore \frac{n!}{(n-r)!(n-n+r)!} = 120$$

$\therefore \frac{n!}{(n-r)!}$ = 120 ......(ii)

Dividing (i) by (ii), we get

$$\therefore \frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!(n-r)!}} = \frac{720}{120}$$

$\therefore$ r! = 6

$\therefore$ r = 3

Substituting r = 3 in (i), we get

$\therefore \frac{n!}{(n-3)!}$ = 720

$\therefore \frac{n(n-1)(n-2)(n-3)!}{(n-3)!}$ = 720

$\therefore$ n(n – 1)(n – 2) = 10 × 9 × 8

$\therefore$ n = 10

Exercise 6.6 | Q 4. (ii) | Page 89

Find n and r if ${}^{n}C_{r-1} : {}^{n}C_{r} : {}^{n}C_{r+1}$ = 20: 35: 42

SOLUTION

${}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}-1} : {}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} : {}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}+1}$ = 20: 35: 42

$\therefore {}^{\mathrm{n}}\mathrm{C}_{r-1} : {}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} = \frac{20}{35}$

$\therefore \frac{\mathrm{n}!}{(\mathrm{r}-1)![\mathrm{n}-(\mathrm{r}-1)]} \div \frac{\mathrm{n}!}{\mathrm{r}!(\mathrm{n}-\mathrm{r})!} = \frac{4}{7}$

$\therefore \frac{\mathrm{n}!}{(\mathrm{r}-1)!(\mathrm{n}+1-\mathrm{r})!} \times \frac{\mathrm{r}!(\mathrm{r}-1)!}{\mathrm{n}!} = \frac{4}{7}$

$\therefore \frac{\mathrm{n}!}{(\mathrm{r}-1)!(\mathrm{n}+1-\mathrm{r})!} \times \frac{\mathrm{r}(\mathrm{r}-1)!(\mathrm{n}-\mathrm{r})!}{\mathrm{n}!} = \frac{4}{7}$

$\therefore \frac{\mathrm{r}(\mathrm{n}-\mathrm{r})!}{(\mathrm{r}-1)!(\mathrm{n}-\mathrm{r}+1)} = \frac{4}{7}$

∴ 7r = 4 (n + 1 − r)

∴ 7r = 4n + 4 − 4r

∴ 11r = 4n + 4 ...(I)

Also, ${}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} : {}^{\mathrm{n}}\mathrm{C}_{\mathrm{n}-\mathrm{r}}$ = 35: 42

$\therefore \frac{{}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}}}{{}^{\mathrm{n}}\mathrm{C}_{\mathrm{n}-r}} = \frac{35}{42}$

$\therefore \frac{\mathrm{n}!}{\mathrm{r}!(\mathrm{n}-\mathrm{r})!} \div \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r}-1)(\mathrm{r}+1)!} = \frac{5}{6}$

$\therefore \frac{\mathrm{n}!}{(\mathrm{n}-\mathrm{r})!\mathrm{r}!} \times \frac{(\mathrm{n}-\mathrm{r}-1)!(\mathrm{r}+1)!}{\mathrm{n}!} = \frac{5}{6}$

$\therefore \frac{(\mathrm{n}-\mathrm{r}-1)!(\mathrm{r}+1)!}{(\mathrm{n}-\mathrm{r})(\mathrm{n}-\mathrm{r}-1)!\mathrm{r}!} = \frac{5}{6}$

$\therefore \frac{\mathrm{r}+1}{\mathrm{n}-\mathrm{r}} = \frac{5}{6}$

∴ 4n + 4 = 5n − 6 .....[From (I)]

∴ n = 10

∴11r = 4(10) + 4 ....(I)

= 44

∴ r = 4

Exercise 6.6 | Q 5 | Page 89

If $^{\mathrm{n}}\mathrm{P}_{\mathrm{r}} = 1814400$ and $^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} = 45$, find r.

SOLUTION

$^{\mathrm{n}}\mathrm{P}_{\mathrm{r}}$ = 1814400

$$^{\mathrm{n}}\mathrm{P}_{\mathrm{r}} = \frac{\mathrm{n}!}{(\mathrm{n}-r)!};\ ^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} = \frac{\mathrm{n}!}{(\mathrm{n}-r)!}$$

$$\therefore \frac{\mathrm{n}!}{(\mathrm{n}-r)!} = 1814400 \qquad \text{...(I)}$$

$$\therefore \frac{\mathrm{n}!}{r!(\mathrm{n}-r)!} = 45 \qquad \text{...(II)}$$

Dividing (I) by (II),

$$\frac{\frac{\mathrm{n}!}{(\mathrm{n}-r)!}}{\frac{\mathrm{n}!}{r!(\mathrm{n}-r)!}} = \frac{1814400}{45}$$

∴ r! = 40320

∴ r! = 8!

∴ r = 8.

Exercise 6.6 | Q 6 | Page 89

If $^{\mathrm{n}}\mathrm{C}_{r-1} = 6435, ^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} = 5005, ^{\mathrm{n}}\mathrm{C}_{r+1} = 3003,\ \text{find}\ ^{r}\mathrm{C}_{5}$

SOLUTION

Given: ${}^{\mathrm{n}}\mathrm{C}_{r-1} = 6435, {}^{\mathrm{n}}\mathrm{C}_{\mathrm{r}} = 5005, {}^{\mathrm{n}}\mathrm{C}_{r+1}$ = 3003

$$\therefore \frac{{}^{\mathrm{n}}\mathrm{C}_{r-1}}{{}^{n}\mathrm{C}_{r}} = \frac{6435}{5005}$$

$$\therefore \frac{\frac{\mathrm{n}!}{[\mathrm{n}-(r-1)]!(r-1)!}}{\frac{\mathrm{n}!}{(\mathrm{n}-r)!r!}} = \frac{13 \times 11 \times 9 \times 5}{13 \times 11 \times 7 \times 5}$$

$$\therefore \frac{(\mathrm{n}-r)!r!}{(\mathrm{n}-r+1)!(r-1)!} = \frac{9}{7}$$

$$\therefore \frac{(\mathrm{n}-r)!r!(r-1)!}{(\mathrm{n}-r+1)(\mathrm{n}-r)!(r-1)!} = \frac{9}{7}$$

$$\therefore \frac{r}{\mathrm{n}-r+1} = \frac{9}{7}$$

∴ 7r = 9n – 9r + 9

∴ 16r – 9n = 9 ...(I)

Also,

$$\therefore \frac{{}^{\mathrm{n}}\mathrm{C}_{r}}{{}^{\mathrm{n}}\mathrm{C}_{r+1}} = \frac{5005}{3003}$$

$$\therefore \frac{\frac{\mathrm{n}!}{(\mathrm{n}-r)!r!}}{\frac{\mathrm{n}!}{[\mathrm{n}-(r+1)]!(r+1)!}} = \frac{5 \times 1001}{3 \times 1001}$$

$$\therefore \frac{(\mathrm{n}-r-1)!(r+1)!}{(\mathrm{n}-r)!r!} = \frac{5}{3}$$

$$\therefore \frac{(\mathrm{n}-r-1)!(r+1)r!}{(\mathrm{n}-r)(\mathrm{n}-r-1)!r!} = \frac{5}{3}$$

$$\therefore \frac{(r+1)}{(\mathrm{n}-r)} = \frac{5}{3}$$

∴ 3(r + 1) = 5(n – r)

$\therefore 3r + 3 = 5n - 5r$

$\therefore 8r - 5n = -3 \quad$ ...(II)

Multiplying equations (II), by 2 and

Subtracting equation (I) from (III)

$16r - 10n = -6 \quad$ ...(III)

$16r - 9n = 9 \quad$ ...(I)

$\quad + \quad -$

$-n = -15$

$\therefore n = 15$

Substituting n = 15 in equation (I)

$16r - 9(15) = 9$

$\therefore 16r - 135 = 9$

$\therefore 16r = 144$

$\therefore r = \dfrac{144}{16}$

$\therefore r = 9$

$$\therefore {}^{r}C_5 = {}^{9}C_5 = \frac{9!}{4!5!} = \frac{9 \times 8 \times 7 \times 6 \times 5!}{4 \times 3 \times 2 \times 1 \times 5!}$$

$\therefore {}^{r}C_5 = {}^{9}C_5 = 126.$

Exercise 6.6 | Q 7 | Page 89

Find the number of ways of drawing 9 balls from a bag that has 6 red balls, 5 green balls, and 7 blue balls so that 3 balls of every colour are drawn.

SOLUTION

Total number of balls = 6 + 5 + 7 = 18
Number of red balls = 6
Number of green balls = 5
Number of blue balls = 7
9 balls are to be drawn, 3 of each Colour,
No. of ways of drawing 3 red balls out of 6 red balls = ${}^{6}C_3$
Similarly, No. of ways of drawing 3 green balls out of
5 green balls = ${}^{5}C_3$
No. of ways of drawing 3 blue balls out of

7 blue balls = $^7C_3$
∴ Total number of ways = $^6C_3 \times {}^5C_3 \times {}^7C_3$

$$= \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{7!}{3!4!}$$

$$= \frac{120}{6} \times \frac{20}{2} \times \frac{210}{6}$$

= 7000.

Exercise 6.6 | Q 8 | Page 89

Find the number 6f selecting a team of 3 boys and 2 girls from 6 boys and 4 girls.

SOLUTION

Number of boys = 6
∴ Number of ways of selecting 3 boys = $^6C_3$
Number of girls = 4
∴ Number of ways of selecting 2 girls = $^4C_2$
Hence, the number of ways of selecting a team of 3 boys and 2 girls
= $^6C_3 \times {}^4C_2$

$$= \frac{6!}{(6-3)!3!} \times \frac{4!}{(4-2)!2!}$$

$$= \frac{6!}{3!3!} \times \frac{4!}{2!2!}$$

$$= \frac{6 \times 5 \times 4 \times 3!}{(3 \times 2 \times 1)3!} \times \frac{4 \times 3 \times 2!}{(2 \times 1)2!}$$

= (5 x 4) x (2 x 3)

= (20) x (6)

= 120.

Exercise 6.6 | Q 9 | Page 89

After a meeting, every participant shakes hands with every other participants. If, the number of handshakes is 66, find the number of participants in the meeting.

SOLUTION

For shaking hands, minimum two persons are needed.
So, the total number of handshakes will be the same as the number of ways of selecting 2 persons from those who are present.
Let n be the number of members present at the meeting.

Then, the number of ways of selecting any 2 persons from them = ${}^nC_2$
Now, in all 66 handshakes were exchanged.
∴ ${}^nC_2 = 66$

$$\therefore \frac{n1}{(n-2)!2!} = 66$$

$$\therefore \frac{n \times (n-1) \times (n-2)!}{(n-2)!} = 66 \times 2$$

∴ $n(n - 1) = 132$

∴ $n(n - 1) = 12 \times 11$

∴ $n = 12$ ...(or $n - 1 = 11$)

∴ The number of participants in the meeting = 12.

Exercise 6.6 | Q 10 | Page 90
If 20 points are marked on a circle, how many chords can be drawn?

SOLUTION

To draw a chord of a circle, we need two points lying on the circle.
There are 20 points on the circle.
∴ We can draw 20C2 chords of the circle.

$$= \frac{20!}{(20-2)!2!}$$

$$= \frac{20!}{18!2!}$$

$$= \frac{20 \times 19}{2}$$

$= 190$

190 chords can be drawn.

Exercise 6.6 | Q 11.1 | Page 90
Find the number of diagonals of an n-shaded polygon. In particular, find the number of diagonals when: n = 10

SOLUTION

Two points are needed to draw a segment.
A polygon of n sides has n vertices.

So, in a polygon of n sides, there will be
${}^{n}C_{2}$ segments, which include its sides and diagonals both.
Since the polygon has n sides, the number of its diagonals is ${}^{n}C_{2} - n$
Here, n = 10
∴ The number of diagonals = ${}^{10}C_{2} - 10$

$$= \frac{10!}{(10-2)!2!} - 10$$

$$= \frac{10 \times 9}{2} - 10$$

= 45 – 10

= 35.

Exercise 6.6 | Q 11.2 | Page 90
Find the number of diagonals of an n-shaded polygon. In particular, find the number of diagonals when: n = 15

SOLUTION

Two points are needed to draw a segment.
A polygon of n sides has n vertices.
So, in a polygon of n sides, there will be
${}^{n}C_{2}$ segments, which include its sides and diagonals both.
Since the polygon has n sides, the number of its diagonals is ${}^{n}C_{2} - n$
Here, n = 15
∴ The number of diagonals = ${}^{15}C_{2} - 15$

$$= \frac{15!}{(15-2)!2!} - 15$$

$$= \frac{15 \times 14}{2} - 15$$

= 105 – 15

= 90.

Exercise 6.6 | Q 11.3 | Page 90
Find the number of diagonals of an n-shaded polygon. In particular, find the number of diagonals when: n = 12

SOLUTION

Two points are needed to draw a segment.
A polygon of n sides has n vertices.

So, in a polygon of n sides, there will be
${}^{n}C_2$ segments, which include its sides and diagonals both.
Since the polygon has n sides, the number of its diagonals is ${}^{n}C_2 - n$
Here, n = 12
∴ The number of diagonals = ${}^{12}C_2 - 12$

$$= \frac{12!}{(12-2)!2!} - 12$$

$$= \frac{12 \times 11}{2} - 12$$

= 66 – 12

= 54.

Exercise 6.6 | Q 12 | Page 90

There are 20 straight lines in a plane so that no two lines are parallel and no three lines are concurrent. Determine the number of points of intersection.

SOLUTION

Two coplanar lines which are not parallel intersect each other in a point.
There are 20 straight lines, no two of them 'are parallel and no three of them are concurrent.
So, the number of points of intersection

= ${}^{20}C_2$

$$= \frac{20!}{(20-2)!2!}$$

$$= \frac{20!}{18!2!}$$

$$= \frac{20 \times 19}{2}$$

= 190

190 points of intersection.

Exercise 6.6 | Q 13.1 | Page 90

Ten points are plotted on a plane. Find the number of straight lines obtained by joining these points if : No three points are collinear

SOLUTION

To draw a line, two points are needed.
There are 10 points in a plane such that no three of them are collinear.
Hence, the number of lines formed $^{10}C_2$

$$= \frac{10!}{(10-2)!2!}$$

$$= \frac{10 \times 9}{2}$$

$= 5 \times 9$

$= 45$

45 straight lines are obtained if no three points are collinear.

Exercise 6.6 | Q 13.2 | Page 90

Ten points are plotted on a plane. Find the number of straight lines obtained by joining these points if : Four points are collinear.

SOLUTION

To draw a line, two points are needed.
There are 10 points in a plane such that four points are collinear.
If no three of the given 10 points are collinear,
we will get $^{10}C_2$ lines. But 4 points are collinear.
So, we will not get $^{4}C_2$ lines from these points. Instead,
we get only one line containing the 4 points.
∴ Number of straight lines formed

$$= {}^{10}C_2 - {}^{4}C_2$$

$$= \frac{10 \times 9}{2} + \frac{4 \times 3}{3} + 1$$

$= 45 - 6 + 1$

$= 40$

40 straight lines are obtained if four points are collinear.

Exercise 6.6 | Q 14.1 | Page 90

Find the number of triangles formed by joining 12 points if: no three points are collinear

SOLUTION

There are 12 points in a plane.

No three of them are collinear.

We need three non-collinear points to 'form a triangle.

∴ The number of triangles formed $^{12}C_3$

$$= \frac{12!}{(12-3)!3!}$$

$$= \frac{12 \times 11 \times 10}{3 \times 2}$$

= 220.

Exercise 6.6 | Q 14.2 | Page 90

Find the number of triangles formed by joining 12 points if: four points are collinear.

SOLUTION

There are 12 points 1n a plane of which four points are collinear.
If no three points are collinear, we will get 1003 triangles. Since four points are collinear, the number of triangles will reduce by 403.
∴ The number of triangles formed

$$= {}^{12}C_3 - {}^{4}C_3$$

$$= 220 - \frac{4 \times 3 \times 2}{3 \times 2}$$

= 220 – 4

= 216.

Exercise 6.6 | Q 15 | Page 90

A word has 8 consonants and 3 vowels, How many distinct words can be formed if 4 consonants and 12 vowels are chosen?

SOLUTION

There are 3 consonants and 3 vowels.
So, 4 consonants and 2 vowels can be selected in $^8C_4$ X $^3C_2$ ways.
Now, $^8C_4$ X $^3C_2$

$= \dfrac{(8 \times 7 \times 6 \times 5)}{(4 \times 3 \times 2)} \times \dfrac{(3 \times 2 \times 1)}{(2 \times 10)}$

= 35 x 2 x 3

= 210

Thus, there are 210 groups consisting of 4 consonants and 2 vowels.

We need to form different words from these 210 groups.

Now, each group has 6 letters.

These 6 letters can be arranged amongst themselves m 6! Ways.

∴ The number of required words

= (210) x 6!

= (210) x 720

= 151,200.

EXERCISE 6.7 [PAGE 90]

Exercise 6.7 | Q 1 | Page 90

Find n if $^{\mathrm{n}}\mathrm{C}_8 = {}^{\mathrm{n}}\mathrm{C}_{12}$

SOLUTION

We have $^{\mathrm{n}}\mathrm{C}_8 = {}^{\mathrm{n}}\mathrm{C}_{12}$

But, $^{\mathrm{n}}\mathrm{C}_{12} = {}^{\mathrm{n}}\mathrm{C}_{\mathrm{n}-12}$ $\ldots(\because\ {}^{\mathrm{n}}\mathrm{C}_r = {}^{\mathrm{n}}\mathrm{C}_{r-1})$

∴ $^{\mathrm{n}}\mathrm{C}_8 = {}^{n}\mathrm{C}_{\mathrm{n}-12}$

∴ 8 = n – 12

∴ 8 + 12 = n

∴ 20 = n

∴ n = 20.

Exercise 6.7 | Q 2 | Page 90

Find n, if $^{23}\mathrm{C}_{3\mathrm{n}} = {}^{23}\mathrm{C}_{2\mathrm{n}+3}$

SOLUTION

We have $^{23}\mathrm{C}_{3\mathrm{n}} = {}^{23}\mathrm{C}_{2\mathrm{n}+3}$

∴ 3n = 2n + 3 ...(∵ $^{n}C_{x}$ then x = y)

∴ 3n – 2n = 3

∴ n = 3.

Exercise 6.7 | Q 3 | Page 90

Find n, if $^{21}\mathrm{C}_{6\mathrm{n}} = {}^{21}\mathrm{C}_{(\mathrm{n}^2+5)}$

SOLUTION

We have $^{21}\mathrm{C}_{6\mathrm{n}} = {}^{21}\mathrm{C}_{(\mathrm{n}^2+5)}$

∴ 6n = n2 + 5

∴ n^2 – 6n + 5 = 0

∴ (n – 5)(n – 1) = 0

Ig n = 5, $^{21}\mathrm{C}_{6\mathrm{n}} = {}^{21}\mathrm{C}_{30}$

Which is possible,

∴ n = 5 is discarded.

Also $^{21}\mathrm{C}_{6\mathrm{n}} = {}^{21}\mathrm{C}_{(\mathrm{n}^2+5)}$

∴ $^{21}\mathrm{C}_{21-6\mathrm{n}} = {}^{21}\mathrm{C}_{(\mathrm{n}^2+5)}$

∴ n^2 + 6n – 16 = 0

∴ n + 8 = 0 or n – 2 = 0

∴ n = – 8 or n = 2

But n ∈ N

∴ n = 2.

Exercise 6.7 | Q 4 | Page 90

Find n, if $^{2\mathrm{n}}\mathrm{C}_{r-1} = {}^{2\mathrm{n}}\mathrm{C}_{r+1}$

SOLUTION

${}^{2n}C_{r-1} = {}^{2n}C_{r+1}$

But ${}^{2n}C_{r-1} = {}^{2n}C_{2n-r+1}$

$\therefore$ ${}^{2n}C_{2n-r+1} = {}^{2n}C_{r+1}$

$\therefore$ 2n – r + 1 = r + 1

$\therefore$ 2n = r + 1 + r – 1

$\therefore$ 2n = 2r

$\therefore$ n = r.

Exercise 6.7 | Q 5 | Page 90

Find n, if ${}^{n}C_{n-2}$ = 15

SOLUTION

${}^{n}C_{n-2}$ = 15

$\therefore \dfrac{n!}{[n-(n-2)]!(n-2)!}$ = 15

$\therefore \dfrac{n!}{2!(n-2)!}$ = 15

$\therefore \dfrac{n(n-1)(n-2)!}{2 \times (n-2)!}$ = 15

$\therefore$ n(n – 1) = 15 x 2

$\therefore$ n(n – 1) = 30

$\therefore$ n(n – 1) = 6 x 5

$\therefore$ n = 6 ...(or n – 1 = 5)

Exercise 6.7 | Q 6 | Page 90

Find x if ${}^{n}P_{r} = {}^{x^n}C_{r}$

$^{\mathrm{n}}\mathrm{P}_r = {}^{x^{\mathrm{n}}}\mathrm{C}_r$

$\therefore \dfrac{\mathrm{n}!}{(\mathrm{n}-r)!} = x\dfrac{\mathrm{n}1}{(\mathrm{n}-r)!r!}$

$\therefore$ x = $\dfrac{\mathrm{n}!}{(\mathrm{n}-r)!} \times \dfrac{(\mathrm{n}-r)!r!}{\mathrm{n}!}$

$\therefore$ x = r!

Exercise 6.7 | Q 7 | Page 90

Find r if ${}^{11}\mathrm{C}_4 + {}^{11}\mathrm{C}_5 + {}^{12}\mathrm{C}_6 + {}^{13}\mathrm{C}_7 = {}^{14}\mathrm{C}_r$.

SOLUTION

Use ${}^{\mathrm{n}}\mathrm{C}_r + {}^{\mathrm{n}}\mathrm{C}_{r-1} = {}^{\mathrm{n}+1}\mathrm{C}_r$

$\therefore {}^{11}\mathrm{C}_4 + {}^{11}\mathrm{C}_5 + {}^{12}\mathrm{C}_6 + {}^{13}\mathrm{C}_7 = {}^{14}\mathrm{C}_r$

$\therefore \left({}^{11}\mathrm{C}_4 + {}^{11}\mathrm{C}_5\right) + {}^{12}\mathrm{C}_6 + {}^{13}\mathrm{C}_7 = {}^{14}\mathrm{C}_r$

$\therefore \left({}^{11}\mathrm{C}_4 + {}^{11}\mathrm{C}_5\right) + {}^{12}\mathrm{C}_6 + {}^{3}\mathrm{C}_7 = {}^{14}\mathrm{C}_r$

$\therefore {}^{13}\mathrm{C}_6 + {}^{13}\mathrm{C}_7 = {}^{14}\mathrm{C}_r$

$\therefore {}^{14}\mathrm{C}_7 = {}^{14}\mathrm{C}_r$

$\therefore$ r = 7.

Exercise 6.7 | Q 8 | Page 90

find the value of $\sum\limits_{r=1}^{4} {}^{21-r}\mathrm{C}_4 + {}^{17}\mathrm{C}_5$

SOLUTION

$$\sum_{r=1}^{4} {}^{21-r}C_4 + {}^{17}C_5$$

$= {}^{20}C_4 + {}^{19}C_4 + {}^{18}C_4 + {}^{17}C_4 + {}^{17}C_5$

$= {}^{20}C_4 + {}^{19}C_4 + {}^{18}C_4 + {}^{18}C_5$

$= {}^{20}C_4 + {}^{19}C_4 + {}^{19}C_5$

$= {}^{20}C_4 + {}^{20}C_5$

$= {}^{21}C_5$

$\because {}^{n}C_r + {}^{n}C_{r+1}$

$= {}^{(n+1)}C_{(r+1)}.$

Exercise 6.7 | Q 9.1 | Page 90

Find the differences between the largest values in the following: ${}^{14}C_r$ and ${}^{12}C_r$

SOLUTION

Use $^{n}C_r$ has maximum value if,

(a) r = $\frac{n}{2}$ when n is even

(b) r = $\frac{n-1}{2}$ or $\frac{n+1}{2}$ when n is odd

$^{14}C_r$ and $^{12}C_r$

Maximum value of $^{14}C_r$ occurs at $r = \frac{14}{2}$ = 7

∴ Maximum value of $^{14}C_r = {}^{14}C_7$

= $\frac{14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

= 3432

Maximum value of $^{12}C_r$ occurs at $r = \frac{12}{2}$ = 6

∴ Maximum value of $^{12}C_r = {}^{12}C_6$

= $\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2}$

= 924

∴ The difference between the maximum values of $^{14}C_r$ and $^{12}C_r$

= 3432 – 924

= 2508.

Exercise 6.7 | Q 9.2 | Page 90

Find the differences between the largest values in the following: $^{13}C_r$ and $^{8}C_r$

SOLUTION

Use $^{\mathrm{n}}\mathrm{C}_r$ has maximum value if,

(a) r = $\frac{\mathrm{n}}{2}$ when n is even

(b) r = $\frac{\mathrm{n}-1}{2}$ or $\frac{\mathrm{n}+1}{2}$ when n is odd

$^{13}\mathrm{C}_r$ and $^{8}\mathrm{C}_r$

Maximum value of $^{13}\mathrm{C}_r$ occurs at $r = \frac{12}{2} = 6$ or $r = \frac{14}{2}$ = 7

∴ Maximum value of $^{13}\mathrm{C}_r = 13\mathrm{C}_6$

= $\frac{13 \times 12 \times 11 \times 10 \times 9 \times 8}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$

= 1716

Maximum value of $^{8}\mathrm{C}_r$ occurs at $r = \frac{8}{2}$ = 6

∴ Maximum value of $^{8}\mathrm{C}_r = {}^{8}\mathrm{C}_4$

= $\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}$

= 70

∴ The difference between the maximum values of $^{13}\mathrm{C}_r$ and $^{8}\mathrm{C}_r$

= 1716 – 70

= 16468.

Exercise 6.7 | Q 9.3 | Page 90

Find the differences between the largest values in the following: $^{15}\mathrm{C}_r$ and $^{11}\mathrm{C}_r$

SOLUTION

Use $^{n}C_r$ has maximum value if,

(a) r = $\frac{n}{2}$ when n is even

(b) r = $\frac{n-1}{2}$ or $\frac{n+1}{2}$ when n is odd

$^{15}C_r$ and $^{11}C_r$

Maximum value of $^{15}C_r$ occurs at $r = \frac{14}{2} = 7$ or $r = \frac{16}{2}$ = 8

∴ Maximum value of $^{15}C_r = {}^{15}C_7 = {}^{15}C_8$

= $\frac{15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9}{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}$

= 6435

Maximum value of $^{11}C_r$ occurs at $r = \frac{10}{2} = 5$ or $r = \frac{12}{2}$ = 6

∴ Maximum value of $^{11}C_r = {}^{11}C_5 = {}^{11}C_6$

$^{11}C_r = {}^{11}C_5 = \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1}$

= 462

∴ The difference between the maximum values of $^{15}C_r$ and $^{11}C_r$

= 6435 – 462

= 5973.

Exercise 6.7 | Q 10 | Page 90

In how many ways can a boy invite his 5 friends to a party so that at least three join the party?

SOLUTION

The number of ways of inviting at least three friends from 5 friends

$= {}^5C_3 + {}^5C_4 + {}^5C_5$

$= \dfrac{5!}{(5-3)!3!} + \dfrac{5!}{(5-4)!4!} + 1$

$= \dfrac{5 \times 4}{2 \times 1} + 5 \ + 1$

= 10 + 16

= 16.

Exercise 6.7 | Q 11 | Page 90

A group consists of 9 men and 6 women. A team of 6 is to be selected. How many of possible selections will have at least 3 women?

SOLUTION

Number of men = 9
Number of women = 6
Number of persons in the team = 6
A team of 6 persons consisting of at least 3 women can be formed as follows:
(I) 3 women and 3 men or
(II) 4 women and 2 men or
(III) 5 women and 1 man or
(IV) 6 women
(I) 3 women and 3 men: The number of ways of forming the team

$= {}^6C_3 \times {}^9C_3$

$= \dfrac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \dfrac{9 \times 8 \times 7}{3 \times 2 \times 1}$

= 20 x 8 4

= 1680

(II) 4 women and 2 men: The number of ways of forming the team

$= {}^6C_4 \times {}^9C_1$

$= \frac{6 \times 5}{2 \times 1} \times \frac{9 \times 8}{2 \times 1}$

= 15 x 36

= 540

(III) 5 women and 1 man: The number of ways of forming the team

$= {}^{6}C_{5} \times {}^{9}C_{1}$

= 6 x 9

= 54

(IV) 6 women: The number of ways of forming the team

$= {}^{6}C_{6}$

= 1

∴ The total number of ways of forming the team

= 1680 + 540 + 54 + 1

= 2275.

Exercise 6.7 | Q 12 | Page 90

A committee of 10 persons is to be formed from a group of 10 women and 8 men. How many possible committees will have at least 5 women? How many possible committees will have men in majority?

SOLUTION

Number of women = 10
Number of men = 8
Number of persons in the team = 10
A committee of 10 persons consisting of at least 5 women can be formed as follows:
(I) 5 women and 5 men or
(II) 6 women and 4 men or
(III) 7 women and 3 man or
(IV) 8 women and 2 man or
(V) 9 women and 1 man or
(IV) 10 women
The number of ways of forming the committee:
(I) 5 women and 5 men

$= {}^{10}C_5 \times {}^8C_5$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4}{5 \times 4 \times 3 \times 2 \times 1} \times \frac{8 \times 7 \times 6}{3 \times 2 \times 1}$$

= (2 x 9 x 2 x 7) x (8 x 7)

= 14112

(II) 6 women and 4 men

$= {}^{10}C_6 \times {}^8C_4$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \times \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}$$

= (5 x 2 x 3 x 7) x (2 x 7 x 5)

= 14700

(III) 7 women and 3 men

$= {}^{10}C_7 \times {}^8C_3$

$$= \frac{10 \times 9 \times 8}{3 \times 2 \times 1} \times \frac{8 \times 7 \times 6}{3 \times 2 \times 1}$$

= (5 x 3 x) x (8 x 7)

= 6720

(IV) 8 women and 2 men

$= {}^{10}C_8 \times {}^8C_2$

$$= \frac{10 \times 9}{2 \times 1} \times \frac{8 \times 7}{2 \times 1}$$

= (5 x 9) x (4 x 7)

= 1260

(V) 9 women and 1 men

$= {}^{10}C_9 \times {}^8C_1$

$$= \frac{10}{1} \times \frac{8}{1}$$

= 80

(VI) 10 women

= $^{10}C_{10}$

= 1

Hence, the number of ways of forming the required committee

= 14112 + 14700 + 6720 + 1260 + 80 + 1

= 36873

For men to be in majority, the committee should have 6 or more men.

Following are the possibilities:

(I) 6 men and 4 women or

(II) 7 men and 3 women or

(III) 8 men and 2 women

The number of ways of forming the committee:

(I) 6 men and 4 women

= $^{8}C_{6} \times {}^{10}C_{4}$

= $\frac{8 \times 7}{1 \times 2} \times \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4}$

= 5880

(II) 7 men and 3 Women

= $^{8}C_{7} \times {}^{10}C_{3}$

= $8 \times \frac{10 \times 9 \times 8}{1 \times 2 \times 3}$

= 960

(III) 8 men and 2 women

= $^{8}C_{8} \times {}^{10}C_{2}$

= $1 \times \frac{10 \times 9}{1 \times 2}$

= 45

Hence, number of ways of forming the required committee

= 5880 + 960 + 45

= 6885.

Exercise 6.7 | Q 13 | Page 90

A question paper has two sections, section I has 5 questions and section II has 6 questions. A student must answer at least two questions from each section among 6 questions he answers. How many different choices does the student have in choosing questions?

SOLUTION

A question paper has two sections.
Number of questions in section I = 5
Number of questions in section II = 6
At least 2 questions have to be selected from each section and in all 6 questions are to be selected.
This can be done as follows:
(I) 2 questions (out of 5) from section I and
4 questions (out of 6) from section II are selected.
or (II) 3 questions from section I and
3 questions from section II are selected.
or (III) 4 questions from section I and
2 questions from section II are selected.
Now, number of selections in (I)

$= {}^5C_2 \times {}^6C_2$

$= \frac{5 \times 4}{2 \times 1} \times \frac{6 \times 5}{2 \times 1}$

= 10 x 15

= 150

Number of selections in (II)

$= = {}^5C_3 \times {}^6C_3$

$= \frac{5 \times 4}{2 \times 1} \times \frac{6 \times 5 \times}{3 \times 2 \times 1}$

= 10 x 20

= 200

Number of selections in (III)

$= = {}^5C_4 \times {}^6C_2$

$= 5 \times \dfrac{6 \times 5}{2 \times 1}$

= 75

By addition principle, total number of required selections

= 150 + 200 + 75

= 425.

Exercise 6.7 | Q 14 | Page 90

There are 3 wicketkeepers and 5 bowlers among 22 cricket players. A team of 11 players is to be selected so that there is exactly one wicketkeeper and at least 4 bowlers in the team. How many different teams can be formed?

SOLUTION

Total number of cricket players = 22
Number of wicketkeepers = 3

Number of bowlers = 5
Remaining players = 14

A team of 11 players consisting of exactly one wicketkeeper and at least 4 bowlers can be selected as follows

(I) 1 wicketkeeper, 4 bowlers, 6 players
or (II) 1 wicketkeeper, 5 bowlers, 5 players

Now, number of selections in (I)

$= {}^{3}C_1 \times {}^{5}C_4 \times {}^{14}C_6$

$= 3 \times 5 \times \dfrac{14 \times 13 \times 12 \times 11 \times 10 \times 9}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$

$= \dfrac{15 \times 14 \times 13 \times 2 \times 11 \times 3}{4}$

= 45045

Number of selections in (II)

$= {}^{3}C_1 \times {}^{5}C_5 \times {}^{14}C_5$

$= 3 \times 1 \times \dfrac{14 \times 13 \times 12 \times 11 \times 10}{5 \times 4 \times 3 \times 2 \times 1}$

= 6006

By addition principle, total number of ways choosing a team of 11 players

= 45045 + 6006

= 51051.

Exercise 6.7 | Q 15.1 | Page 90

Five students are selected from 11. How many ways can these students be selected if: two specified students are selected

SOLUTION

Number of students = 11

Number of students to be selected = 5

Here, 2 specified students are included.

So, we need to select 3 more students from the remaining 9 students.

This can be done in:

$= {}^{9}C_3$

$= \dfrac{9 \times 8 \times 7}{3 \times 2 \times 1}$

= 84ways

∴ Number of required selections

= 84 × 1 × 1

= 84

Thus, 84 selections. can be made such .that 2 specified students are included.

Exercise 6.7 | Q 15.2 | Page 90

Five students are selected from 11. How many ways can these students be selected if: two specified students are not selected.

SOLUTION

Number of students = 11

Number of students to be selected = 5
Here, 2 specified students are not included.

So, we need to select 5 students from the remaining 9 students.

This can be done in:

$= {}^{9}C_5$

$= \dfrac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}$

= 126 ways

Thus, 126 selections can be made such that 2 specified students are not included.

MISCELLANEOUS EXERCISE 6 [PAGES 92 - 93]

Miscellaneous Exercise 6 | Q 1 | Page 92

Find the value of $r\left({}^{56}C_{r+6}\right) : {}^{54}P_{r-1}$ = 30800:1

SOLUTION

$$^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$$

$$\therefore \frac{^{56}P_{r+6}}{^{54}P_{r+3}} = \frac{30800}{1}$$

$$\therefore \frac{\frac{56!}{(56-r-6)!}}{\frac{54!}{(54-r-3)!}} = 30800$$

$$\therefore \frac{56!}{(54-r-6)!} \times \frac{(54-r-3)!}{54!} = 30800$$

$$\therefore \frac{56!}{(50-r)!} \times \frac{(51-r)!}{54!} = 30800$$

$$\therefore \frac{56 \times 55 \times 54!}{(50-r)!} \times \frac{(51-r)(50-r)!}{54!} = 30800$$

$$\therefore 51 - r = \frac{30800}{56 \times 55}$$

$$\therefore 51 - r = 10$$

$$\therefore r = 51 - 10$$

$$\therefore r = 41.$$

Miscellaneous Exercise 6 | Q 2 | Page 92

How many words can be formed by writing letters in the word CROWN in different order?

SOLUTION

Five Letters of the word CROWN are to be permuted.
Number of different words = 5! = 120

Miscellaneous Exercise 6 | Q 3 | Page 92

Find the number of words that can be formed by using all the letters in the word REMAIN. If these words are written in dictionary order, what will be the 40th word?

SOLUTION

There are 6 letters A, E, I, M, N, R
Number of words starting with A = 5!
Number of words starting with E = 5!

Number of words starting with I = 5!
Number of words starting with M = 5!
Number of words starting with N = 5!

Number of words starting with R = 5!
Total number of words = 6 x 5! = 720

Number of words starting with AE = 4! = 24 Number of words starting with AIE = 3! = 6
Number of words starting with AIM = 3! = 6 Number of words starting with AINE = 2!

Total words = 24 + 6 + 6 + 2 = 38
39th word is AINMER
40th word is AINMRE

∴ Required number of numbers
= Total number of arrangements possible among these digits – number of arrangements of 7 digits which begin with 0.

$$= \frac{7!}{2!3!} - \frac{6!}{2!3!}$$

$$= \frac{7 \times 6 \times 5 \times 4 \times 3!}{2 \times 3!} - \frac{6 \times 5 \times 4 \times 3!}{2 \times 3!}$$

= 7 x 6 x 5 x 2 – 6 x 5 x 2

= 6 x 5 x 2(7 – 1)

= 60 x 6

= 360

∴ 360 numbers that exceed one million can be formed with the digits 3, 2, 0, 4, 3, 2, 3.

Miscellaneous Exercise 6 | Q 4.1 | Page 92

Find the number of ways of distributing n balls in n cells. What will be the number of ways if each cell must be occupied?

SOLUTION

There are n balls and n cells
Every ball can be put in any of the n cells.
Number of distributions
= n x n x ... x n
= $(n)^n$

Miscellaneous Exercise 6 | Q 4.2 | Page 92
Find the number of ways of distributing n balls in n cells. What will be the number of ways if each cell must be occupied?

SOLUTION

For filling first cell, n balls are available.
First cell is filled in n ways.
Second cell is filled in (n – 1) ways
Third cell is filled in (n – 2) ways and so on.
n^{th} cell is tilled in one way.
Required number
= n (n – 1) (n – 2) … 1
= n!

Miscellaneous Exercise 6 | Q 5 | Page 92
Thane is the 20^{th} station from C.S.T. If a passenger can purchase a ticket from any station to any other station, how many different tickets must be available at the booking window?

SOLUTION

Taking CST as first station and Thane as 20^{th}, let us name CST as A_0 next station as A1 and so on, Thane is A_{20}
From station A_0, 20 different journeys are possible from station A_1, 20 different journeys are possible.
From station A_{20}, 20 different journeys are possible.
Total number of different tickets of different journeys
= 21 x 20
= 420.

Miscellaneous Exercise 6 | Q 6 | Page 92
English alphabet has 11 symmetric letters that appear same when looked at in a mirror. These letters are A, H, I, M, O, T, U, V, W, X and Y. How many symmetric three letters passwords can be formed using these letters?

SOLUTION

Number of 3 Letter passwords
= $^{11}P_3$
= 11 x 10 x 9
= 990.

Miscellaneous Exercise 6 | Q 7 | Page 92

How many numbers formed using the digits 3, 2, 0, 4, 3, 2, 3 exceed one million?

SOLUTION

A number that exceeds one million is to be formed from the digits 3, 2, 0, 4, 3, 2, 3.
Then the numbers should be any number of 7 digits which can be formed from these digits.
Also among the given numbers 2 repeats twice and 3 repeats thrice.

Miscellaneous Exercise 6 | Q 8 | Page 92

Ten students are to be selected for a project from a class of 30 students. There are 4 students who want to be together either in the project or not in the project. Find the number of possible selections.

SOLUTION

Case I	**Case II**
4 Students are in	4 Students are out
Number of ways = ${}^{26}C_6$	Number of ways = ${}^{26}C_{10}$

Required number = ${}^{26}C_6 + {}^{26}C_{10}$

Miscellaneous Exercise 6 | Q 9 | Page 92

A student finds 7 books of his interest, but can borrow only three books. He wants to borrow Chemistry part II book only if Chemistry Part I can also be borrowed. Find the number of ways he can choose three books that he wants to borrow.

SOLUTION

Chemistry Part I borrowed	**Chemistry Part I not borrowed**
Only one book from remaining 5 books borrowed	All three books borrowed from remaining 5 books
Number of selections $= {}^{5}C_1 = 5$	Number of selections $= {}^{5}C_3 = 10$

Required Number = 5 + 10 = 15

Miscellaneous Exercise 6 | Q 10 | Page 92

30 objects are to be divided in three groups containing 7, 10, 13 objects. Find the number of distinct ways for doing so.

SOLUTION

Required number = ${}^{30}C_7 \times {}^{23}C_{10} \times {}^{13}C_{13}$

Similarly for box II and III.

$\therefore$ n(A) + n(B) + n(C) = 3 x 2^5 ... (iv)

If boxes I and II remain empty

then all balls go to box III

Similarly we would have two more cases.

$\therefore$ n(A ∩ B) + n(B ∩ C) + n(C ∩ A)

= 3 x 15 ...(v)

n(A ∩ B ∩ C) = 0 ...(vi) $\begin{bmatrix} \text{as all boxes} \\ \text{cannot be empty} \end{bmatrix}$

Substitute from (iv), (v), (vi) to (iii) to get

n(A ∪ B ∪ C) = 3 x 2^5 – 3 x 1^5

= 96 – 3

= 93

Substitute n(A ∪ B ∪ C) and from (ii) to (i),

we get

Required number

= 243 – 93

= 150.

Miscellaneous Exercise 6 | Q 11 | Page 92

A student passes an examination if he secures a minimum in each of the 7 subjects. Find the number of ways a student can fail.

SOLUTION

Every subject a student may pass or fail.
$\therefore$ Total number of outcomes
= 2^7
= 128
This number includes one case when the student passes in all subjects.
Required number
= 128 – 1
= 127.

Miscellaneous Exercise 6 | Q 12 | Page 92

Nine friends decide to go for a picnic in two groups. One group decides to go by car and the other group decides to go by train. Find the number of different ways of doing so if there must be at least 3 friends in each group.

SOLUTION

	Train	Car	Number of outcomes
No. of friends	3	6	$^9C_3$
	4	5	$^9C_4$
	5	4	$^9C_5$
	6	3	$^9C_6$

Required number = $^9C_3 + {}^9C_4 + {}^9C_5 + {}^9C_6$

= $\left({}^9C_4 + {}^9C_3\right) + \left({}^9C_6 + {}^9C_5\right) = {}^{10}C_4 + {}^{10}C_6$

= $\dfrac{10!}{6!4!} + \dfrac{10!}{4!6!}$

= 210 + 210

= 420.

Miscellaneous Exercise 6 | Q 13 | Page 93

Five balls are to be placed in three boxes, where each box can contain upto five balls. Find the number of ways if no box is to remain empty.

SOLUTION

Let boxes be named as I, II, III

Let sets A, B, C represent cases in which boxes I, II, III remain empty

Then A ∪ B ∪ C represent the cases in which at least one box remains empty.

Then we use method of indirect counting

Required number = Total number of distributions – n(A ∪ B ∪ C) ...(i)

n(A ∪ B ∪ C) represent the number of undesirable cases

Total number of distributions

= 3 x 3 x 3 x 3

= 3^5

= 243

$n(A \cup B \cup C) = n(A) + n(B) + n(C) – n(A \cap B)$ n $n(B \cap C) – n(C \cap A) + n(A \cap B \cap C)$ ...(iii)
In box I is empty then every ball has two places (boxes) to go.

Miscellaneous Exercise 6 | Q 14 | Page 93

A hall has 12 lamps and every lamp can be switched on independently. Find the number of ways of illuminating the hall.

SOLUTION

Every lamp is either ON or OFF.
There are 12 lamps
Number of instances = 2^{12}
This number includes the case in which all 12
lamps are OFF.
∴ Required Number
= $2^{12} – 1$
= 4095.

Miscellaneous Exercise 6 | Q 15 | Page 93

How many quadratic equations can be formed using numbers from 0, 2, 4, 5 as coefficients if a coefficient can be repeated in an equation.

SOLUTION

Let the quadratic equation be
$ax^2 + bx + c = 0, a \neq 0$

Coefficient	Values	Numbers of ways
a	2, 4, 5	3
b	0, 2, 4, 5	4
c	0, 2, 4, 5	4

Required number
= 3 x 4 x 4
= 48.

Miscellaneous Exercise 6 | Q 16 | Page 93

How many six-digit telephone numbers can be formed if the first two digits are 45 and no digit can appear more than once?

SOLUTION

Let the telephone number be 45abcd

	Number of ways to fill
a	8
b	7
c	6
d	5

Required number = ${}^{8}P_{4}$ = 1680.

Miscellaneous Exercise 6 | Q 17 | Page 93
A question paper has 6 questions. How many ways does a student have if he wants to solve at least one question?

SOLUTION

Every question is 'SOLVED' or 'NOT SOLVED'.
There are 6 question.
Number of outcomes = 26
This number includes the case when the student solves NONE of the question.
Required number
= $2^6 - 1$
= 64 – 1
= 63.

Miscellaneous Exercise 6 | Q 18 | Page 93
Find the number of ways of dividing 20 objects in three groups of sizes 8, 7 and 5.

SOLUTION

Select 8 object out of 20 in ${}^{20}C_{8}$ ways
Select 7 object from remaining 12 in ${}^{12}C_{7}$ ways and 5 objects form remaining 5 in ${}^{5}C_{5}$ ways
Required number is = ${}^{20}C_{8} \times {}^{12}C_{7} \times {}^{5}C_{5}$

Miscellaneous Exercise 6 | Q 19 | Page 93
There are 8 doctors and 4 lawyers in a panel. Find the number of ways for selecting a team of 6 if at least one doctor must be in the team.

SOLUTION

There are 8 doctors and 4 lawyers.
We need to select a team of 6 which contains at least one doctor.
Since, there are only 4 lawyers any team of 6 will contain at least two doctors.
Required number

$= {}^{12}C_6$

$= 924.$

Miscellaneous Exercise 6 | Q 20 | Page 93

Four parallel lines intersect another set of five parallel lines. Find the number of distinct parallelograms that can be formed.

SOLUTION

We need 2 lines from each set.
Required number
$= {}^{4}C_2 \times {}^{5}C_2$
$= 6 \times 10$
$= 60.$