Sample Question Paper - 7
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study-based questions.

Section A

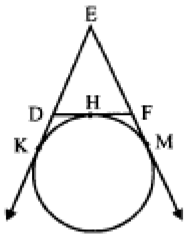
1. Is the given sequence: $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots$ form an AP? If it forms an AP, then find the common difference d and write the next three terms. **[2]**

OR

Find the n^{th} term of the AP: 5, 11, 17, 23, ….

2. If x = 2 and x = 3 are roots of the equation $3x^2 - 2kx + 2m = 0$, find the value of k and m. **[2]**

3. In the adjoining figure, a circle touches the side DF of $\triangle$EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then what is perimeter of $\triangle$EDF? **[2]**



4. A toy is in the form of a cone mounted on a hemisphere with the same radius. The diameter of the base of the conical portion is 6 cm and its height is 4 cm. Determine the surface area of the toy. (Use π = 3.14) **[2]**

5. If the class mark of a continuous frequency distribution are 12, 14, 16, 18, ..., then find the class intervals corresponding to the class marks 16 and 22. **[2]**

6. Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank? **[2]**

OR

Find the values of k for which the given equation has real roots:

$5x^2 - kx + 1 = 0$

Section B

7. Find the median of the following frequency distribution: **[3]**

Weekly wages (in ₹)	60-69	70-79	80-89	90-99	100-109	110-119
No. of days	5	15	20	30	20	8

8. Let PQR be a right triangle in which PQ = 3 cm, QR = 4 cm and $\angle$Q = 90°. QS is the perpendicular from Q on PR. The circle through Q, R, S is drawn. Construct the tangents from P to this circle. **[3]**

9. The arithmetic mean of the following frequency distribution is 50. **[3]**

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	16	p	30	32	14

Find the value of p.

10. From a window (60 metres high above the ground) of a house in street the angles of elevation and depression of the top and the foot of another house on opposite side of street are 60° and 45° respectively. Show that the height of the opposite house is $60(1+\sqrt{3})$metres. **[3]**

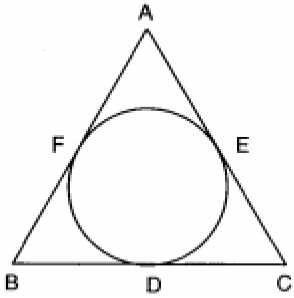
OR

Two boats approach a light house in mid-sea from opposite directions. The angles of elevations of the top of the lighthouse from two boats are 30° and 45° respectively. If the distance between two boats is 100 m, find the height of the lighthouse.

Section C

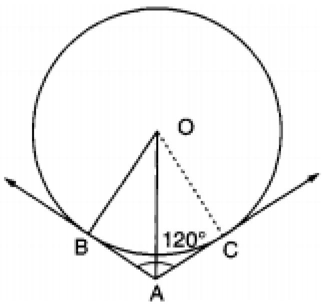
11. The interior of a building is in the form of cylinder of diameter 4.3 m and height 3.8 m, surmounted by a cone whose vertical angle is a right angle. Find the area of the surface and the volume of the building. (Use π = 3.14). **[4]**

12. In figure the incircle of ΔABC touches the sides BC, CA and AB at D, E and F respectively. Show that AF + BD + CE = AE + BF + CD = $\frac{1}{2}$(Perimeter of ΔABC) **[4]**



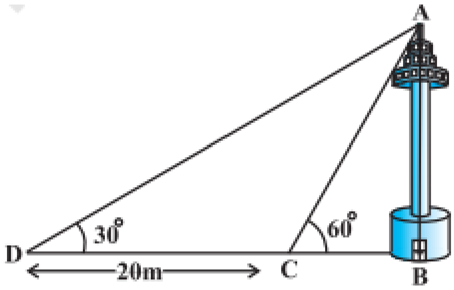
OR

In fig., two tangents AB and AC are drawn to a circle with centre O such that $\angle BAC = 120^\circ$. Prove that OA = 2AB.



13. A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of **[4]**

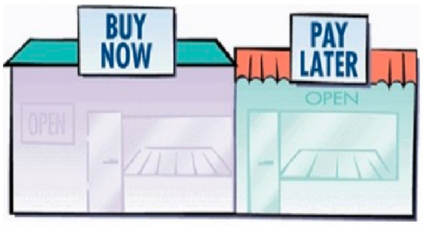
the tower is 60^{o} from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30^{o}.



i. Find the height of the tower
ii. Find the width of the canal.

14. Deepa has to buy a scooty. She can buy scooty either making cashdown payment of Rs. 25,000 or by making 15 monthly instalments as below. **[4]**

Ist month - Rs. 3425, IInd month - Rs. 3225, IIIrd month - Rs. 3025, IVth month - Rs. 2825 and so on.



i. Find the amount of 6th instalment.
ii. Total amount paid in 15 instalments.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

Section A

1. From the given information, we can have
$a_2 - a_1 = \sqrt{6} - \sqrt{3}$
$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$
$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - 3\ (since\sqrt{12} = \sqrt{2} \times 2 \times 3 = 2\sqrt{3})$
since $a_{k+1} - a_k$ is not the same for all values of k.
Hence, it is not an AP.

OR

The given AP is:
5, 11, 17, 23 ....
a = 5,
d = 11 - 5 = 6
So n^{th} term a_n = a + (n - 1)d
= 5 + (n - 1) × 6
= 5 + 6n - 6
= 6n - 1
Hence n^{th} term = 6n - 1

2. It is given that x = 2 and x = 3 are roots of the equation $3x^2 - 2kx + 2m = 0$.
∴ $3 \times 2^2 - 2k \times 2 + 2m = 0$ and $3 \times 3^2 - 2k \times 3 + 2m = 0$
⇒ 12 - 4k + 2m = 0 and 27 - 6k + 2m = 0
⇒ 12 = 4k - 2m...(i) and 27 = 6k - 2m...(ii)
Solving i and ii equation, we get k = $\frac{15}{2}$ and m = 9

3. We know that tangent segments to a circle from the same external point are Equal. Therefore, we have
EK = EM = 9 cm
Now, EK + EM = 18 cm
$\Rightarrow ED + DK + EF + FM = 18cm$
$\Rightarrow ED + DH + EF + HF = 18cm$
$\Rightarrow ED + DF + EF = 18cm$
⇒ Perimeter of $\triangle$EDF $= 18cm$

4. Height of cone = 4 cm
Radius of cone = 3 cm
Slant height of cone (l) = $\sqrt{r^2 + h^2}$ = $\sqrt{3^2 + 4^2}$ = $\sqrt{9 + 16}$ = $\sqrt{25}$ = 5 cm
∴ Surface area of toy = Curved surface area of cone + Curved surface area of hemisphere
$= \pi rl + 2\pi r^2$ = 3.14 ×3 × 5 + 2 × 3.14 × 3 × 3
= 3.14 × 15 + 3.14 × 18 = 3.14 × 33 = 103.62 cm^2

5. Class marks are $12, 14, 16$.......
Class size$= 2$
Class Intervals are $11 - 13, 13 - 15, 15 - 17, 17 - 19, 19 - 21, 21 - 23, \ldots.$.so on.

6. Two tap running together fill the tank in $3\frac{1}{13}$ hr.
$= \frac{40}{13}$ hours
If first tap alone fills the tank in x hrs.
Then second tap alone fills it in (x + 3) hr
Now $\frac{1}{x} + \frac{1}{x+3} = \frac{13}{40}$
$\frac{x+3+x}{x(x+3)} = \frac{13}{40}$
$\frac{2x+3}{x^2+3x} = \frac{13}{40}$
$80x + 120 = 13x^2 + 39x$

or, $13x^2 - 41x - 120 = 0$
$13x^2 - (65 - 24)x + 120 = 0$
(x - 5)(13x + 24) = 0
$x = 5, x = -\frac{24}{13}$
time can't be negative
Hence, 1st tap takes 5 hours and IInd tap
takes = 5 + 3 = 8 hours

OR

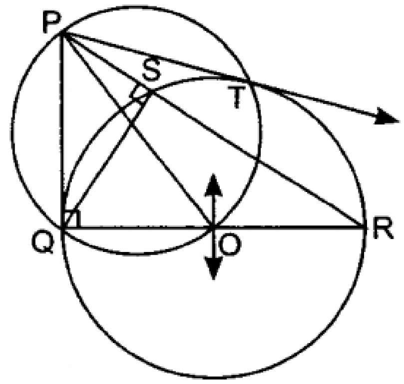
We have, $5x^2 - kx + 1 = 0$.
a = 5, b = -k and c = 1
$\therefore$ D = $b^2 - 4ac = (-k)^2 - 4 \times 5 \times 1 = k^2 - 20$
To have a real roots,
$D \geq 0$
$\Rightarrow \quad k^2 - 20 \geq 0$
$\Rightarrow \quad k \leq -\sqrt{20}$ or , $k \geq \sqrt{20}$

Section B

7. Here, the frequency table is given in inclusive form. So, we first transform it into exclusive form by subtracting and adding $\frac{h}{2}$ to the lower and upper limits respectively of each class, where h denotes the difference of lower limit of a class and the upper limit of the previous class.
Here, h = 1 So, $\frac{h}{2}$ = 0.5
Transforming the above table into exclusive form and preparing the cumulative frequency table, we get:-

Weekly wages (in ₹)	**No of workers**	**Cumulative frequency**
59.5-69.5	5	5
69.5-79.5	15	20
79.5-89.5	20	40
89.5-99.5	30	70
99.5-109.5	20	90
109.5-119.5	8	98
		N = Σf_i = 98

We have, N(Total frequency) = 98 Or, $\frac{h}{2}$ = 49
The cumulative frequency just greater than $\frac{h}{2}$ is 70 and the corresponding class is 89.5-99.5. So, 89.5-99.5 is the median class.
Now,
l = 89.5 (lower limit of median class),
h = 10 (length of interval of median class),
f = 30 (frequency of median class)
F = 40 (cumulative frequency of the class just preceding the median class)
Now, Median is given by:-
= l + $\frac{\frac{N}{2} - f}{F} \times$ h
= 89.5 + $\frac{49-40}{30} \times 10$
= 89.5 + 3 = 92.5

8. 

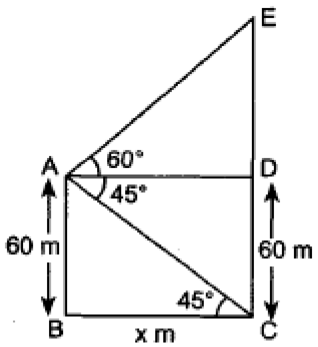
Steps of Construction:

i. Draw perpendicular bisector of QR intersecting at O.
ii. Draw a circle with O as centre and OR as radius.
iii. Join OP.
iv. Draw a circle with OP as diameter intersecting the given circle at Q and T. Join PT.
$\therefore$PQ and PT are required tangents.

9. We have,

Class Interval	**Frequency f_i**	**Mid- value x_i**	**$f_i x_i$**
0 - 10	16	5	80
10 - 20	p	15	15p
20 - 30	30	25	750
30 - 40	32	35	1120
40 - 50	14	45	630
	$\Sigma f_i = 92 + p$		$\sum f_i x_i = 2580 + 15p$

Now, mean = $\frac{\sum f_i x_i}{\sum f_i}$
$\Rightarrow 25 = \frac{2580+15p}{92+p}$
$\Rightarrow$ 25(92 + p) = 2580 + 15p
$\Rightarrow$ 2300 + 25p = 2580 + 15p
$\Rightarrow$ 10p = 280
$\Rightarrow$p = 28

10. 

Let A be the window and CE be the opposite house
CD = AB = 60 m.....(i)
In rt. $\triangle$ABC, $\tan 45^\circ = \frac{60}{\text{BC}}$
$\Rightarrow \quad 1 = \frac{60}{\text{BC}}$
$\Rightarrow \quad \text{BC} = 60\text{m}$ .... (ii)
AD = BC
$\therefore \quad AD = 60\text{m}$ [From(ii)] ....(iii)
In rt. $\triangle$ADE, $\tan 60^\circ = \frac{\text{DE}}{\text{AD}}$
$\Rightarrow \quad \sqrt{3} = \frac{DE}{60}$ [From (iii)]

$\Rightarrow \quad DE = 60\sqrt{3}\text{m}$

$\therefore$ Height of the opposite house

$CE = CD + DE = 60 + 60\sqrt{3}$

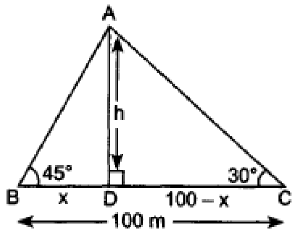
$= 60(1 + \sqrt{3})\text{m}$

OR

In right $\triangle$ADB,

h = x

$\Rightarrow \quad \frac{h}{x} = \tan 45^\circ$...(i)



Now in rt. $\triangle$ADC

$\frac{h}{100-x} = \tan 30^\circ$

Solve for h and x.

$\Rightarrow \frac{h}{100-x} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}h = 100 - x$

$\Rightarrow \quad \sqrt{3}x = 100 - x$ [Using eq.(i)]

$\Rightarrow (\sqrt{3} + 1)x = 100 \Rightarrow x = \frac{100}{\sqrt{3}+1}$

$\Rightarrow \quad x = \frac{100(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

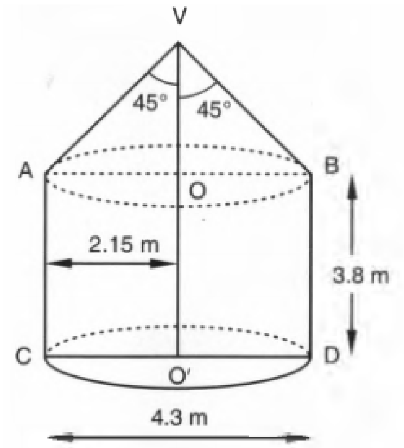
$\Rightarrow \quad x = \frac{100(\sqrt{3}-1)}{2} = 50(\sqrt{3} - 1)\text{m}$

$\therefore$ h = height of lighthouse = $50(\sqrt{3} - 1)\text{m}$

Section C

11. r_1 = Radius of the base of the cylinder = $\frac{4.3}{2}$ m = 2.15 m

$\therefore$ r_2 = Radius of the base of the cone = 2.15 m, h_1 = Height of the cylinder = 3.8 m



In $\triangle$VOA, we have

$\sin 45^\circ = \frac{OA}{VA} \Rightarrow \frac{1}{\sqrt{2}} = \frac{2.15}{VA} \Rightarrow VA = (\sqrt{2} \times 2.15)\text{m} = (1.414 \times 2.15)\text{m} = 3.04\text{m}$

Clearly, $\triangle$VOA is an isosceles triangle. Therefore, VO = OA = 2.15 m

Thus, we have

h_2 = Height of the cone = VO = 2.15 m, l_2 = Slant height of the cone = VA = 3.04 m

Let S be the Surface area of the building. Then,

$\Rightarrow$ S = Surface area of the cylinder + Surface area of cone

$\Rightarrow$ S = $(2\pi r_1 h_1 + \pi r_2 l_2)\ m^2$

$\Rightarrow$ S = $(2\pi r_1 h_1 + \pi r_1 l_2)\ m^2$ [$\because r_1 = r_2$ - 2.15 m]

$\Rightarrow$ S = $\pi r_1(2h_1 + l_2)\ m^2$

$\Rightarrow$ S = $3.14 \times 2.15 \times (2 \times 3.8 + 3.04)\ m^2 = 3.14 \times 2.15 \times 10.64\ m^2 = 71.83\ m^2$

Let U be the volume of the building. Then,
V = Volume of the cylinder + Volume of the cone

$\Rightarrow \quad V=\left(\pi r_1^2 h_1+\frac{1}{3} \pi r_2^2 h_2\right) \mathrm{m}^3$

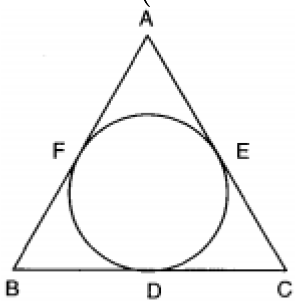
$\Rightarrow \quad V=\left(\pi r_1^2 h_1+\frac{1}{3} \pi r_1^2 h_2\right) \mathrm{m}^3 \quad\left[\because r_2=r_1\right]$

$\Rightarrow \quad V=\pi r_1^2\left(h_1+\frac{1}{3} h_2\right) \mathrm{m}^3$

$\Rightarrow \quad V=3.14 \times 2.15 \times 2.15 \times\left(3.8+\frac{2.15}{3}\right) \mathrm{m}^3$

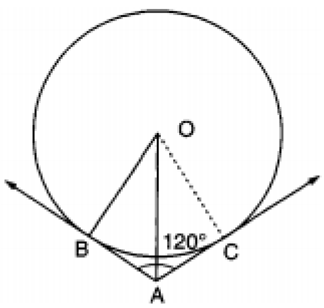
$\Rightarrow \quad V=[3.14 \times 2.15 \times 2.15 \times(3.8+0.7166)] \mathrm{m}^3$

$\Rightarrow \quad V=(3.14 \times 2.15 \times 2.15 \times 4.5166) \mathrm{m}^3=65.55 \mathrm{m}^3$

12. 

Since lengths of the tangents from an exterior point to a circle are equal. Therefore,
AF = AE [From A] ...(i)
BD = BF [From B] ...(ii)
and, CE = CD [From C] ...(iii)
Therefore, Adding equations (i), (ii) and (iii), we get,
AF + BD + CE = AE + BF + CD
Now,
Perimeter of ΔABC = AB + BC + AC
$\Rightarrow$ Perimeter of ΔABC = (AF + FB)+(BD + CD)+(AE + EC)
$\Rightarrow$ Perimeter of ΔABC = (AF + AE)+(BF + BD)+(CD + CE)
$\Rightarrow$ Perimeter of ΔABC = 2 AF + 2 BD + 2 CE
$\Rightarrow$ Perimeter of ΔABC = 2(AF + BD + CE) [From (i), (ii) and (iii), we get AE = AF, BD = BF and CD = CE]
$\Rightarrow AF+BD+CE=\frac{1}{2}(\text{ Perimeter of } \Delta ABC)$
Hence, AF + BD + CE = $AE+BF+CD=\frac{1}{2}(\text{ Perimeter of } \Delta ABC)$

OR

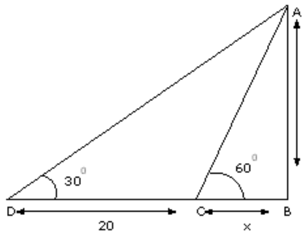


In Δ's OAB and OAC, we have,
$\angle OBA=\angle OCA=90^{\circ}$
OA = OA [Common]
AB = AC [$\because$ Tangents from an external point are equal in length]
Therefore, by RHS congruence criterion, we have,
$\Delta OBA \cong \Delta OCA$
$\Rightarrow \angle OAB=\angle OAC$ [By c.p.c.t.]
$\therefore \angle OAB=\angle OAC=\frac{1}{2} \angle BAC$
$=\frac{1}{2} \times 120^{\circ}=60^{\circ}$
$\Rightarrow \angle OAB=\angle OAC=60^{\circ}$
In ΔOBA, we have,
$\cos \mathrm{B}=\frac{AB}{OA}$
$\Rightarrow \cos 60^{\circ}=\frac{AB}{OA}$

$\Rightarrow \frac{1}{2} = \frac{AB}{OA}$

$\Rightarrow$ OA = 2AB

Hence proved.

13. 

Let 'h' (AB) be the height of tower and x be the width of the river

In $\triangle ABC, \frac{h}{x} = \tan 60^\circ$

$\Rightarrow h = \sqrt{3}x$ ....(i)

In $\Delta ABD, \frac{h}{x+20} = \tan 30^\circ$

$\Rightarrow h = \frac{x+20}{\sqrt{3}}$ ....(ii)

Equating (i) and (ii),

$\sqrt{3}x = \frac{x+20}{\sqrt{3}}$

$\Rightarrow$ 3x = x + 20

$\Rightarrow$ 2x = 20

$\Rightarrow$ x = 10 m

Put x = 10 in (i), $h = \sqrt{3}x$

$\Rightarrow h = 10\sqrt{3}\text{m}$

14. i. 1st installment = Rs. 3425

2nd installment = Rs. 3225

3rd installment = Rs. 3025

and so on

Now, 3425, 3225, 3025, ... are in AP, with

a = 3425 , d = 3225 - 3425 = -200

Now 6th installment = a_n = a + 5d = 3425 + 5(-200) = Rs. 2425

ii. Total amount paid = $\frac{15}{2}$(2a + 14d)

= $\frac{15}{2}$[2(3425) + 14(-200)] = $\frac{15}{2}$(6850 - 2800)

= $\frac{15}{2}$(4050) = Rs.30375.