TANGENTS AND NORMALS

SYNOPSIS

SLOPE OF TANGENT:

- If the tangent drawn to the curve $y = f(x)$ at $P(x_1, y_1)$ on it makes an angle θ with $\overline{ox}$ then $\tan\theta$ is defined as the slope of the tangent and it is also called the gradient of the curve.

  $$\tan\theta = \left(\frac{dy}{dx}\right)_{(x_1,y_1)}$$

  For the curve $f(x, y) = 0$ slope of tangent line at

  $$p(x_1, y_1) \text{ is } -\left(\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}\right)_{(x_1,\, y_1)}$$

  For the curve $x = f(t)$; $y = g(t)$. Slope of the tangent at $p(t)$ is $\dfrac{g^1(t)}{f^1(t)}$.

- **SLOPE OF NORMAL:**
  A straight line perpendicular to the tangent to the curve at the point of contact is called the normal to the curve.
  Slope of normal at any point $p(x_1, y_1)$ on a curve is given by $\left(\dfrac{-1}{\frac{dy}{dx}}\right)_{(x_1,y_1)}$

  For the curve $f(x, y) = 0$ the slope of the normal line at $p(x_1, y_1)$ is $\left(\dfrac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}\right)_{(x_1,y_1)}$

  For the curve $x = f(t)$; $y = g(t)$ the slope of the normal line at $p(t)$ is $-\left(\dfrac{f^1(t)}{g^1(t)}\right)$

EQUATIONS OF TANGENT AND NORMAL:

- Equation of the tangent to the curve $y = f(x)$ at (x_1, y_1) is $(y - y_1) = m(x - x_1)$ where

  $$m = \left(\frac{dy}{dx}\right)_{(x_1,y_1)}$$

  Equation of the normal to the curve $y = f(x)$ at (x_1, y_1) is $(y - y_1) = \left(\dfrac{-1}{m}\right)(x - x_1)$.

LENGTHS OF TANGENT, NORMAL, SUB-TANGENT AND SUB-NORMAL:

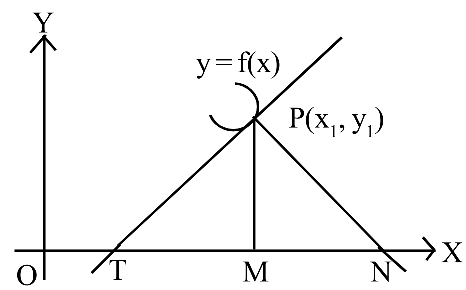
- Let the tangent and Normal drawn to the curve $y = f(x)$ at $p(x_1, y_1)$ meet the X-axis at T and N. Draw the line PM perpendicular to X-axis. If m is the slope of the tangent then

  gPT - Length of Tangent $= \left|\dfrac{y_1 \cdot \sqrt{1+m^2}}{m}\right|$

  gPN - Length of Normal $= \left|y_1 \cdot \sqrt{1+m^2}\right|$

  gTM - Length of sub-tangent $= \left|\dfrac{y_1}{m}\right|$

  gMN - Length of the sub-normal = $|y_1 m|$



ANGLE BETWEEN TWO CURVES:

- The angle between any two curves at the point of intersection is defined as the angle between the tangents to the curves at that point of intersection.
- Let P be a point of intersection of the two curves $y = f(x)$, $y = g(x)$ and m_1, m_2 be the slopes of the tangents to the curves at p. If θ is the angle between the curves then $\tan\theta = \left(\dfrac{m_1 - m_2}{1 + m_1 m_2}\right)$
- The curves $y = f(x)$ and $y = g(x)$ cut each other orthogonally if $m_1\, m_2 = -1$.

4. The curves $f(x, y) = 0$ and $g(x, y) = 0$ cut each other orthogonally if $\dfrac{\partial f}{\partial x}\cdot\dfrac{\partial g}{\partial x} + \dfrac{\partial f}{\partial y}\cdot\dfrac{\partial g}{\partial y} = 0$

- The curves $y = f(x)$; $y = g(x)$ touch each other at $p(x_1, y_1)$ if $f^1(x_1) = g^1(x_1)$
- If the curves $xy = c^2$ and $y^2 = 4ax$ cut each other orthogonally then $c^4 = 32a^4$
- If the curves $a_1x^2 + b_1y^2 = 1$ and $a_2x^2 + b_2y^2 = 1$ cut each other orthogonally then

$$\frac{1}{a_1}-\frac{1}{a_2}=\frac{1}{b_1}-\frac{1}{b_2}$$

- The curves $f(x, y) = 0$; $g(x, y) = 0$ touch each other if $\frac{\partial f}{\partial x}\cdot\frac{\partial g}{\partial y}=\frac{\partial f}{\partial y}\cdot\frac{\partial g}{\partial x}$
- The curves $y^2 = 4ax$; $x^2 = 4ay$ intersect at (0, 0) and (4a, 4a). At (0, 0) the curves cut each other orthogonally and at (4a, 4a) the angle between the curves is $\operatorname{Tan}^{-1}\left(\frac{3}{4}\right)$. The area between the two curves is $16\frac{a^2}{3}$ sq.units

AREAS OF TRIANGLES:

- The area of the triangles formed by any tangent on the curve $xy = c^2$ and the coordinate axes is $2c^2$ sq.units.
- If the area of the triangle formed by any tangent to the curve $x.y^n = a^{n+1}$ and the co-ordinate axes is a constant then $n = 1$.
- The area of the triangle formed by the tangent,normal at a point $P(x_1, y_1)$ on the curve $y = f(x)$ and the line

$x = k$ is $\frac{1}{2}\left|\frac{(x_1-k)^2(m^2+1)}{m}\right|$ sq.units

$y = k$ is $\frac{1}{2}\left|\frac{(y_1-k)^2(m^2+1)}{m}\right|$ sq.units

X-axis is $\frac{1}{2}\left|\frac{y_1^2(m^2+1)}{m}\right|$ sq.units

Y - axis is $\frac{1}{2}\left|\frac{x_1^2(m^2+1)}{m}\right|$ sq.units

- If the area of the triangle formed by any tangent to the curve $x^m y^n = K(m \neq 0, n \neq 0)$ and the coordinate axes is a constant then $m = n$

OTHER POINTS:

- A tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ (or) $x = a\cos^4\theta$; $y = a\sin^4\theta$ cuts the axes in A and B then OA + OB = a.
- A tangent to the curve $x^{2/3}+y^{2/3}=a^{2/3}$ (or) $x = a\cos^3\theta$; $y = a\sin^3\theta$ cuts the co-ordinate axes in A and B then AB = a.
- The tangent at any point 't' of the curve $x = at^3$ and $y = at^4$ divides the abscissa of the point of contact in the ratio 1 : 3.
- If p and q are the lengths of perpendiculars from the origin to tangent and normal to the curve. $x = ae^{\theta}(\sin\theta-\cos\theta)$ and $y = ae^{\theta}(\sin\theta+\cos\theta)$ then p = q.
- The tangent and Normal at a point (x_1, y_1) on the curve meets the x-axis in T and G then $TG=\left|y_1\left(m+\frac{1}{m}\right)\right|$
- The length of the subnormal at any point on the curve $x.y^n = a^{n+1}$ is a constant then $n = -2$.
- At any point on the curve in the curve $by^2 = (x+a)^3$, $\frac{(L.S.T)^2}{L.S.N}=\frac{8b}{27}$
- The equation of the tangent at (a, b) to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ is $\frac{x}{a}+\frac{y}{b}=2$ (which is independent of n)
- If the normal at (x_1, y_1) on the curve $y = f(x)$ makes equal intercepts on the coordinate axes then $\left(\frac{dy}{dx}\right)_{(x_1,y_1)}=1$.
- Point on the curve $ay^2 = x^3$ the normal at which makes equal intercepts on the axes is $\left(\frac{4a}{9},\frac{8a}{27}\right)$.
- If p, q are the lengths of perpendiculars from the origin to tangent and normal at a point respectively on the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ then $4p^2+q^2=a^2$.
- If the length of the subnormal at any point on the curve $x^m y^n = K(m \neq 0, n \neq 0)$ is a constant then $2m + n = 0$.

CONCEPTUAL QUESTIONS

1. If the tangent to the curve y = f(x) is perpendicular to y-axis then

1) $\frac{dy}{dx}=0$ 2) $\frac{dx}{dy}=0$

3) $\frac{dy}{dx}=1$ 4) $\frac{dx}{dy}=-1$

2. If the value of $\frac{dy}{dx}$ at (0, 0) to the given curve is not defined then the angle made by the tangent with the positive x-axis is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{2}$ 4) $\frac{3\pi}{4}$

3. If the tangent line at the point of P(t) to the curve x = f(t), y = g(t) makes an angle θ with $\overline{OX}$. Then $f^1(t):g^1(t)=$

1) $\tan\theta$ 2) $\cos\theta:\sin\theta$

3) $-\cos\theta:\sin\theta$ 4) $-\tan\theta$

4. The slope of the normal to the curve given by $x=a(\theta-\sin\theta), y=a(1-\cos\theta)$ at $\theta=\frac{\pi}{2}$

1) $\frac{-1}{2}$ 2) $\frac{1}{2}$ 3) –1 4) 2

5. The slope of normal to the curve $y=\log_x(\log_e x)$ at x = e is

1) – e 2) e

3) $\frac{1}{e}$ 4) $\frac{-1}{e}$

6. The slope of normal to the curve $\left(\frac{x}{a}\right)^4+\left(\frac{y}{b}\right)^4=2$ at(a, b) on it is

1) –2 2) –1 3) $\frac{-b}{a}$ 4) $\frac{a}{b}$

7. The line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at (a, b) then n =

1) $n\in Z$ 2) $n\in R-Z$ 3) $n\in N$ 4) $n\in R$

8. The equation of the tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at (a, b) on it is

1) $\frac{x}{a}+\frac{y}{b}=1$ 2) $\frac{x}{a}-\frac{y}{b}=0$

3) $\frac{x}{a}+\frac{y}{b}=2$ 4) $\frac{x}{a}+\frac{y}{b}=0$

9. The equation of the tangent to the curve $\sqrt{\frac{x}{a}}+\sqrt{\frac{y}{b}}=2$ at the point (a, b) is

1) $\frac{x}{a}-\frac{y}{b}=0$ 2) $\frac{x}{a}+\frac{y}{b}=2$

3) $\frac{x}{a}-\frac{y}{b}=1$ 4) $\frac{x}{a}+\frac{y}{b}=0$

10. If the straight line $x\cos\alpha+y\sin\alpha=p$ touches the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b) on it, then $\frac{1}{a^2}+\frac{1}{b^2}=$

1) $\frac{1}{p^2}$ 2) $\frac{2}{p^2}$ 3) $\frac{3}{p^2}$ 4) $\frac{4}{p^2}$

11. The equation of the tangent to the curve $x^2=4by$ at the origin is

1) x = 0 2) y = 0 3) x = 2 4) y = 2

12. The equation of the normal to the curve $y^2=4ax$ at the origin is

1) x = 0 2) x = 2 3) y = 0 4) y = 2

13. The equation of the tangent at the point where the curve $y=be^{\frac{-x}{a}}$ cuts the y-axis

1) $\frac{x}{a}+\frac{y}{b}=0$ 2) $\frac{x}{a}+\frac{y}{b}=1$

3) X + Y = a 4) $\frac{x}{a}-\frac{y}{b}=1$

14. The point on the curve $y=be^{\frac{-x}{a}}$ at which the tangent drawn is $\frac{x}{a}+\frac{y}{b}=1$ is

1) (0, b) 2) $\left(a,\frac{1}{e}\right)$ 3) (0, 1) 4) (1, 0)

15. The equation of the normal to the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point (x_1, y_1) on it is

1) $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$ 2) $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2$

3) $a^2xx_1+b^2yy_1=1$ 4) $b^2xx_1-a^2yy_1=0$

16. The equation of the normal to the curve given by $x=at^2, y=2at$ at the point 't' is

1) $xt+y=2at+at^3$ 2) $x+yt=2at+at^3$

3) $xt-y=at+at^3$ 4) $x=0$

17. The equation of the tangent to the curve $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point θ on it is

1) $bx\cos\theta-ay\sin\theta=ab$

2) $bx\sin\theta+ay\cos\theta=ab$

3) $bx\cos\theta+ay\sin\theta=ab$

4) $y=0$

18. The equation of the tangent to the curve $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point θ on it is

1) $bx\sec\theta-ay\tan\theta=ab$

2) $ax\sec\theta-by\tan\theta=ab$

3) $ax\sec\theta+by\tan\theta=ab$

4) $x\sec\theta+y\tan\theta=ab$

19. If the line $ax+by+c=0$ is normal to the curve $xy=1$ then

1) $a>0, b>0$ 2) $a>0, b<0$

3) $a<0, b<0$ 4) $a=0, b=0$

20. If the line $ax-by+c=0$ is a tangent to $xy+1=0$ then

1) $ab>0$ 2) $ab<0$

3) $ab=1$ 4) $a=1, b=-1$

21. If the tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at any point θ on it meets the axes at A and B then OA + OB =

1) $\frac{a}{2}$ 2) a 3) $\frac{3a}{2}$ 4) 2a

22. The sum of the squares of the intercepts on the axes of the tangent at any point on the curve $x^{2/3}+y^{2/3}=a^{2/3}$ is

1) $\frac{a^2}{2}$ 2) a^2 3) 2a 4) $\frac{3a}{2}$

23. The condition that the line $\frac{x}{a}+\frac{y}{b}=1$ a tangent to the curve $x^{2/3}+y^{2/3}=1$ is

1) $a^2+b^2=2$ 2) $a^2+b^2=1$

3) $\frac{1}{a^2}+\frac{1}{b^2}=1$ 4) $a^2+b^2=\frac{2}{3}$

24. If the tangent at any point on the curve $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$ makes intercepts p, q on the axes then $\frac{p^2}{a^2}+\frac{q^2}{b^2}=$

1) 0 2) $\frac{2}{3}$ 3) 1 4) 2

25. The tangent at the point P(x, y) on the curve $x^m.y^n=a^{m+n}$ meets the axes at A and B. The ratio in which P divides $\overline{AB}$ is

1) m : 1 2) 1 : n 3) n : m 4) m : n

26. The tangent at point P(x, y) on the curve $xy=c^2$ meets the axes at A and B. The ratio in which P divides $\overline{AB}$ is

1) 1 : 2 2) 2 : 1 3) 1 : 1 4) 2 : 3

27. The tangent to the curve $2a^2y=x^3-3ax^2$ is parallel to the x-axis at the points

1) (0, 0), (2a, –2a) 2) (0, 0), (–2a, –2a)

3) (0, 0), (–2a, 2a) 4) (2, 2), (0, 0)

28. The points on the curve y = sinx where the tangents are parallel to x-axis are given by

1) $x=n\pi, n\in z$ 2) $x=2n\pi, n\in z$

3) $x=(2n+1)\frac{\pi}{2}, n\in z$ 4) $x=\frac{2n\pi}{3}, n\in z$

29. The points on the curve y = sinx, where the tangent is parallel to the x-axis is

1) $\left(\frac{n\pi}{2},1\right)$ 2) $\left((2n+1)\frac{\pi}{2},(-1)^n\right)$

3) $\left((2n-1)\frac{\pi}{3},(-2)^n\right)$ 4) $\left(n\pi,(-1)^n\right)$

30. The points on the curve $y = x^2 + \sqrt{1-x^2}$ at which the tangent is perpendicular to x-axis are

1) (1, 1) only 2) $(\pm 1, 1)$

3) $(1, \pm 1)$ 4) (–1, 1) only

31. The point at which the tangent line to the curve $x^3 + y^3 = a^3$ is parallel to y-axis is

1) (0, a) 2) (a, 0) 3) (–a, 0) 4) (0, –a)

32. All the points of the curve $y^2 = 4a\left[x + a\sin\frac{x}{a}\right]$ $a \neq 0$ at which tangents are parallel to x-axis lie on

1) Circle 2) Straight line

3) Ellipse 4) Parabola

33. All the points on the curve $y = \sqrt{x + \sin x}$ at which the tangent are parallel to the x-axis lie on

1) a straight line 2) circle

3) a parabola 4) an ellipse

34. The length of the normal to the curve $y = c.\cosh\left(\frac{x}{c}\right)$ at the point (h, k) is

1) h^2c 2) k^2c

3) $\frac{k^2}{c}$ 4) $\frac{h^2}{c}$

35. The length of the sub-tangent at $\theta = \frac{\pi}{3}$ on the curve given by $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$ is

1) $\frac{\sqrt{3}a}{2}$ 2) $\frac{a}{2\sqrt{3}}$ 3) $\frac{2a}{\sqrt{3}}$ 4) 2a

36. The length of sub-normal to the curve $xy = a^2$ at (x, y) on it varies as

1) x^2 2) y^2 3) x^3 4) y^3

37. The value of n for which the sub-normal to the curve $xy^n = a^{n+1}$ is of constant length

1) –2 2) –1 3) 0 4) 2

38. If the subnormal to the curve $x^2.y^n = a^2$ is a constant then n =

1) -4 2) -3 3) -2 4) -1

39. The value of k for which the length of the sub-tangent to the curve $xy^k = c^2$ is constant is

1) 0 2) 1 3) 2 4) –2

40. The length of sub-tangent to the curve $y^n = a^{n-1}.x$ at (x, y) on it is

1) $\frac{n}{x}$ 2) nx 3) n^2x 4) $\frac{n^2}{x}$

41. At any point on the curve y = f(x), if the tangent makes an angle θ with positive x-axis, then $\left(\frac{\text{length of tangent}}{\text{length of normal}}\right)^2$ is

1) y^2 2) $\left(\frac{dy}{dx}\right)^2$ 3) $\frac{1}{y^2}$ 4) $\left(\frac{dx}{dy}\right)^2$

42. The length of normal at θ on the curve given by $x = a\cos^3\theta,\ y = a\sin^3\theta$ is

1) $a\sin^2\theta$ 2) $a\sin^2\theta\,|\cos\theta|$

3) $a\sin^2\theta\,|\tan\theta|$ 4) $a\,|\cos\theta|$

43. The length of the sub-tangent to the curve given by $x = a\left[\cos t + \log\left(\tan\frac{t}{2}\right)\right],\ y = a\sin t$ at $t = \frac{\pi}{3}$ is

1) $\frac{a}{2}$ 2) a 3) $\frac{3a}{2}$ 4) 2a

44. The sub-normal at (x, y) on the curve $y^2x^2 = a^2(x^2 - a^2)$ varies as

1) x^2 2) $\frac{1}{x^2}$ 3) y^3 4) $\frac{1}{x^3}$

45. The length of sub-tangent to the curve $x^m.y^n = a^{m+n}$ at (x, y) on it is

1) |mx| 2) |nx|

3) $\left|\frac{nx}{m}\right|$ 4) $\left|\frac{mx}{n}\right|$

46. At any point on the curve y = f(x), the sub-tangent, the ordinate of the point and the sub-normal are in

1) A.P. 2) G.P. 3) H.P. 4) A.G.P.

47. If the relation between the sub-normal and sub-tangent at any point on the curve $y^2 = (x+a)^3$ is $p(S.N.) = q(S.T.)^2$ then $\frac{p}{q} =$

1) $\frac{8}{27}$ 2) $\frac{27}{8}$ 3) $\frac{4}{9}$ 4) $\frac{9}{4}$

48. If at any point on a curve the subtangent and subnormal are equal, then the length of the tangent is equal to

1) ordinate 2) $\sqrt{2}$ ordinate

3) $\sqrt{2 \text{ ordinate}}$ 4) $2\sqrt{\text{ordinate}}$

49. The length of the tangent, normal, subtangent and subnormal are respectively p, q, s, t then $p^2 : q^2 =$

1) s : t 2) t : s 3) $s^2 : t^2$ 4) $t^2 : s^2$

50. For the curve $y = a^x (a > 1)$ the L.S.N. at any point on the curve varies as the ...... of the ordinate

1) Square 2) Cube 3) Twice 4) Thrice

51. The length of subnormal to the curve $y = be^{x/a}$ at any point (x, y) is proportional to

1) x 2) y 3) x^2 4) y^2

52. The subnormal at any point (x, y) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

1) $\frac{b^2}{a^2}\left|\frac{x}{y}\right|$ 2) $\frac{b^2|x|}{a^2}$ 3) $\frac{a^2|x|}{b^2}$ 4) $\frac{a^2}{b^2}\left|\frac{y}{x}\right|$

53. For the curve y = f(x) at (x_1, y_1) the intercept made by the normal on the y-axis is

1) $\left|x_1 - \frac{y_1}{m}\right|$ 2) $\left|x_1 + \frac{y_1}{m}\right|$

3) $\left|y_1 - \frac{x_1}{m}\right|$ 4) $\left|y_1 + \frac{x_1}{m}\right|$

54. The intercept made by the tangent of the curve y = f(x) at (x_1, y_1) on the x – axis is

1) $\left|x_1 - \frac{y_1}{m}\right|$ 2) $\left|x_1 + \frac{y_1}{m}\right|$

3) $\left|\frac{y_1}{m}\right|$ 4) $|y_1.m|$

55. If the tangent at $\theta = \frac{\pi}{4}$ to the curve $x = a\cos^3\theta, y = a\sin^3\theta$ meets the x and y axes in A and B then the area of the triangle OAB is

1) $\frac{a^2}{4}$ sq.units 2) $\frac{a^2}{2}$ sq.units

3) $\frac{3a^2}{4}$ sq.units 4) $\frac{5a^2}{4}$ sq.units

56. The area of the triangle formed by the tangent to the curve $xy = a^2$ at (x_1, y_1) on it and the axes is

1) a^2 sq.units 2) $\frac{3}{2}a^2$ sq.units

3) $2a^2$ sq.units 4) $4a^2$ sq.units

57. If the area of the triangle formed by a tangent to the curve $x^n y = a^{n+1}$ and the coordinate axes is constant, then n =

1) 2 2) -2 3) –1 4) 1

58. Area of the triangle formed by the tangent, normal at the point $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$ on the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the x-axis is

1) $\frac{ab}{4}\sqrt{a^2 + b^2}$ 2) 4ab

3) $\frac{b(a^2 + b^2)}{4a}$ 4) 1

59. Area of the triangle formed by the tangent and normal at (a, a) on the curve $y(2a-x) = x^2$ and the x-axis is

1) $\frac{a^2}{3}$ sq.units 2) $5a^2$ sq.units

3) $\frac{5a^2}{3}$ sq.units 4) a^2 sq.units

60. The angle that the curve $y = \frac{x}{x+1}$ makes with x-axis is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

61. The angle made by the normal to the curve $y = \log x$ where it crosses the x-axis is

1) $\frac{\pi}{4}$ 2) $\frac{3\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

62. The angle between the curves $xy = 2a^2$ and $y^2 = 4ax$ at (a, 2a) is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\tan^{-1}(3)$ 4) $\tan^{-1}4$

63. The angle between the curves y = sinx and $y = \cos x$ is

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{2}$ 3) $\tan^{-1}(2)$ 4) $\tan^{-1}(2\sqrt{2})$

64. The curves $y = e^{-ax}$. Sinbx, $y = e^{-ax}$ touch each other at the points for which bx =

1) $n\pi + \frac{\pi}{2}, n \in z$ 2) $2n\pi + \frac{\pi}{2}, n \in z$

3) $2n\pi + \frac{\pi}{6}, n \in z$ 4) $2n\pi + \frac{\pi}{3}, n \in z$

65. The angle between the curves $y = \cos^2 x$ and $y = \sin^2 x$ is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

66. If the two curve $y = a^x$ and $y = b^x$ intersect at an angle α. Then $\tan\alpha$ equals

1) $\frac{\log a - \log b}{1 + \log a \log b}$ 2) $\frac{\log a + \log b}{1 - \log a \log b}$

3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

67. The condition for the curves $1x^2 + my^2 = 1$ and $1^1x^2 + m^1y^2 = 1$ to cut orthogonally is

1) $1 - m = 1^1 - m^1$ 2) $1 + m = 1^1 + m^1$

3) $\frac{1}{1} - \frac{1}{m} = \frac{1}{1^1} - \frac{1}{m^1}$ 4) $1 + m = 1^1 - m^1$

68. If the curves $x = y^2$ and $xy = k$ cut orthogonally then k^2 =

1) $\frac{1}{2}$ 2) $\frac{1}{4}$ 3) $\frac{1}{8}$ 4) $\frac{1}{16}$

69. If the curves $y^2 = 4ax$ and $xy = c^2$ cut orthogonally then c^4 =

1) $4a^2$ 2) $8a^2$ 3) $16a^4$ 4) $32a^4$

KEY

01. 1	02. 3	03. 2	04. 3	05. 1	06. 4
07. 4	08. 3	09. 2	10. 4	11. 2	12. 3
13. 2	14. 1	15. 1	16. 1	17. 3	18. 1
19. 2	20. 1	21. 2	22. 2	23. 2	24. 3
25. 3	26. 3	27. 1	28. 3	29. 2	30. 2
31. 2	32. 4	33. 3	34. 3	35. 2	36. 4
37. 1	38. 1	39. 1	40. 2	41. 4	42. 3
43. 1	44. 4	45. 3	46. 2	47. 1	48. 2
49. 1	50. 1	51. 4	52. 2	53. 4	54. 1
55. 1	56. 3	57. 4	58. 3	59. 3	60. 2
61. 2	62. 3	63. 4	64. 2	65. 4	66. 1
67. 3	68. 3	69. 4			

LEVEL-I

1. The slope of the tangent at (1, 6) on the curve $2x^2 + 3y^2 = 5$ is

1) –9 2) $\frac{-1}{9}$ 3) $\frac{1}{9}$ 4) 9

2. The slope of the tangent to the curve $y = \frac{8}{4 + x^2}$ at x = 2 on it is

1) –2 2) $\frac{-1}{2}$ 3) $\frac{1}{2}$ 4) 2

3. The gradient of the curve $x^m . y^n = (x + y)^{m+n}$ is

1) $\frac{x}{y}$ 2) $\frac{y}{x}$ 3) $-\frac{x}{y}$ 4) $-\frac{y}{x}$

4. The slopes of the tangents at the points where the curve $Y = x^2 - 4x$ intersects the x-axis is

1) ± 1 2) ± 2 3) ± 3 4) ± 4

5. The inclination of the tangent at $\theta = \frac{\pi}{3}$ on the curve $x = a\left(\theta + \sin\theta\right), y = a\left(1 + \cos\theta\right)$ is

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{6}$ 3) $\frac{2\pi}{3}$ 4) $\frac{5\pi}{6}$

6. The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, –1) on it is

1) $\frac{6}{7}$ 2) –6 3) $\frac{22}{7}$ 4) $\frac{7}{22}$

7. If the normal to the curve y = f(x) at (3, 4) makes an angle $\frac{3\pi}{4}$ with $\overrightarrow{OX}$ then $f^1(3)$ =

1) 1 2) – 1 3) $\frac{-3}{4}$ 4) $\frac{4}{3}$

8. If the slope of the tangent to the curve $y = x^3$ at a point on it is equal to the ordinate of the point then the point is

1) (27, 3) 2) (3, 27) 3) (3, 3) 4) (1, 1)

9. The point on the curve $y^2 = x$, the tangent at which has inclination $\frac{\pi}{4}$ is

1) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ 2) $\left(\frac{1}{4}, \frac{1}{3}\right)$ 3) $(4, 2)$ 4) $\left(\frac{1}{4}, \frac{1}{2}\right)$

10. The point on the curve $y = x^2 - 4x + 5$ at which the tangent drawn is parallel to x-axis is

1) (0, 5) 2) (1, 2) 3) (2, 1) 4) (3, 2)

11. A point on the curve $y = x^4 - 4x^3 + 4x^2 + 1$ the tangent at which is parallel to x-axis is

1) (1, 1) 2) (2, 1) 3) (3, 1) 4) (1, 3)

12. The point on the curve $y = \frac{x^3 - 3a\,x^2}{2a^2}$ at which tangent is parallel to the x-axis is

1) (2a, 2a) 2) (–2a, –2a)
3) (2a, –2a) 4) (–2a, 2a)

13. The curve $y - e^{xy} + x = 0$ has a vertical tangent line at the point

1) (1, 1) 2) (0, 1) 3) (1, 0) 4) (–1, –1)

14. If V is the set of points on the curve $y^3 - 3xy + 2 = 0$ where the tangent is vertical then V =

1) ϕ 2) {(1, 0)}
3) {(1,1)} 4) {(0,0), (1,1)}

15. For the curve $x = t^2 - 1, y = t^2 - t$, the tangent is perpendicular to x-axis then

1) t = 0 2) $t = \frac{1}{2}$ 3) t = 1 4) $t = \frac{1}{\sqrt{3}}$

16. The point ont he curve $y = 3x^2 + 2x + 5$ at which the tangent is perpendicular to the line $x + 2y + 3 = 0$

1) (0, –5) 2) (0, 5) 3) (–5, 0) 4) (5, 0)

17. The point on the curve $y = x^2 + 4$ at which the tangent is perpendicular to the line $x + 2y = 3$ is

1) (–1, 5) 2) (5, 1) 3) (1, 5) 4) (1, –5)

18. The point on the curve $y = 5x - x^2$ at which the normal is perpendicular to the line $x + y = 0$ is

1) (3, –6) 2) (3, 6) 3) (–3, –6) 4) (6, 3)

19. The point at which the tangent to the curve $y = x^2 + 3x$ passes through the origin is

1) (0, 3) 2) (0, 0) 3) (2, 0) 4) (2, 3)

20. A point on the curve $y = 2x^3 + 13x^2 + 5x + 9$ the tangent at which passes through the origin is

1) (1, 15) 2) (1, –15) 3) (15, 1) 4) (–1, 15)

21. The equation of the tangent to the curve $y = \log(\sin x)$ at $\left(\frac{\pi}{2}, 0\right)$ on it is

1) x = 0 2) x – 1 = 0 3) y = 0 4) Y = 1

22. The equation of the tangent to the curve $y = 2\sin x + \sin 2x$ at $x = \frac{\pi}{3}$ on it is

1) $y - 3 = 0$ 2) $y + \sqrt{3} = 0$
3) $2y - 3 = 0$ 4) $2y - 3\sqrt{3} = 0$

23. The equation of the normal to the curve $y = x + \sin x \,.\, \cos x$ at $x = \frac{\pi}{2}$ on it is

1) $x - \pi = 0$ 2) $x + \pi = 0$
3) $2x - \pi = 0$ 4) $2x + \pi = 0$

24. The tangent to the curve $y = e^{2x}$ at the point (0,1) meets the x-axis at the point

1) $\left(0, \frac{1}{2}\right)$ 2) $\left(\frac{1}{2}, 0\right)$ 3) $\left(\frac{-1}{2}, 0\right)$ 4) (1, 0)

25. The tangent to the curve $y = 2x^3 + 13x^2 + 5x + 9$ at (–1, 15) on it

1) is parallel to x-axis 2) is parallel to y-axis
3) passes through origin
4) passing through (0,1)

26. P(1, 1) is a point on the parabola $y = x^2$ whose vertex is A. The point on the curve at which the tangent drawn is parallel to the chord $\overline{AP}$ is

1) $\left(\frac{1}{2}, \frac{1}{4}\right)$ 2) $\left(\frac{-1}{2}, \frac{1}{4}\right)$ 3) (2, 4) 4) (4, 2)

27. Equation of the tangent to the curve $y(x-2)(x-3) - x + 7 = 0$ at the point where the curve cuts x-axis is

1) x – 20y = 7 2) x – 20y + 7 = 0
3) x + 20y – 7 = 0 4) x + 20y + 7 = 0

28. Equation of the tangent to the curve $y = 1 - e^{\frac{x}{2}}$ at the point where the curve cuts y-axis is

1) x + y = 0 2) x + 2y = 0
3) 2x + y = 0 4) 2x – y = 0

29. The tangent to the curve $y = 2 + bx + 3x^2$ at the point where the curve meets y-axis has the equation $4x - y + 2 = 0$ then b is

1) 7 2) 27 3) 3 4) 4

30. Equation of the normal to the curve $x = 1 + 2\log(\cot(t))$ and $y = \tan(t) + \cot(t)$ at $t = \frac{\pi}{4}$ on it is

1) $x = \frac{\pi}{4}$ 2) $y = 2$ 3) $x = 2$ 4) $x = 1$

31. The equation of the normal at $t = \frac{\pi}{2}$ to the curve $x = 2\sin t$, $y = 2\cos t$ is

1) $x = 0$ 2) $y = 0$ 3) $y = 2x+3$ 4) $y = 3$

32. Equation of the tangent at $(1, -1)$ to the curve $x^3 - xy^2 - 4x^2 - xy + 5x + 3y + 1 = 0$ is

1) $x - 1 = 0$ 2) $x + 1 = 0$
3) $y - 1 = 0$ 4) $y + 1 = 0$

33. Equation of a tangent to the curve $y = \cos(x + y)$, $0 \le x \le 2\pi$ that is parallel to the line $x + 2y = 0$ is

1) $x + 2y = \pi/2$ 2) $x + 2y = \pi/4$
3) $x + 2y = \pi$ 4) $x + y = \pi$

34. Equation of a normal to the curve $y = x\log x$, parallel to $2x - 2y + 3 = 0$ is

1) $x + y = 3e^{-2}$ 2) $x - y = 3e^{-2}$
3) $x - y = 3e^{2}$ 4) $x + y = 3e^{2}$

35. The equation of the normal to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ at $x = 0$ is

1) $x + y = 1$ 2) $x - y + 1 = 0$
3) $2x + y = 2$ 4) $2x - y + 1 = 0$

36. If the normal line at $(1, -2)$ on the curve $y^2 = 5x - 1$ is $ax - 5y + b = 0$ then the values of a and b are

1) $-14, 4$ 2) $4, -14$ 3) $4, 6$ 4) $4, 10$

37. If $y = 4x - 5$ is a tangent to the curve $y^2 = px^3 + q$ at (2, 3) on it then $(p,q) =$

1) (2, 7) 2) (–2, 7) 3) (–2, –7) 4) (2, –7)

38. If the curve $y = ax^2 + bx$ passes through (–1,0) and $y = x$ is the tangent line at $x = 1$ then (a, b)

1) (1, 1) 2) (1/2, 1/2)
3) (1/3, 1/3) 4) (3, 3)

39. The length of sub-normal at (–1, 4) on the curve $y = 4x^2$ is

1) – 32 2) $\frac{-1}{2}$ 3) $\frac{1}{2}$ 4) 32

40. The length of normal at (2, 1) on the curve $xy + 2x - y = 5$ is

1) $\sqrt{5}$ 2) $\sqrt{\frac{5}{3}}$ 3) $\sqrt{\frac{10}{3}}$ 4) $\sqrt{10}$

41. If the length of the subtangent is 9 and the length of the subnormal is 4 at (x, y) on $y = f(x)$ then $y =$

1) 36 2) ± 9 3) ± 4 4) ± 6

42. The tangent at A (2, 4) on the curve $y = x^3 - 2x^2 + 4$ cuts the x-axis at T then the length of AT =

1) $\sqrt{10}$ 2) $\sqrt{12}$
3) $\sqrt{15}$ 4) $\sqrt{17}$

43. If the curves $y = x^2 - 1$, $y = 8x - x^2 - 9$ touch each other at (2, 3) then equation of the common tangent is

1) $4x - y = 5$ 2) $4x + y = 5$
3) $x - 4y = 5$ 4) $x + 4y = 14$

44. The two curves $y = x^2 - 1$ and $y = 8x - x^2 - 9$ at the point (2, 3)

1) have common tangent $4x - y - 5 = 0$
2) have common tangent $x + 4y - 14 = 0$
3) have common normal $4x + y = 11$
4) have common normal $x - 4y = 10$

45. Point where $y = x^3 + x + 1$ and $2y = x^3 + 5x$ touch each other is

1) (1, 3) 2) (–1, –1) 3) (0, 1) 4) (–2, –9)

46. $xy = 4;\ x^2 - y^2 = 15$

1) touch each other
2) cut orthogonally
3) intersect at an angle $\pi/3$
4) intersect at an angle $\pi/4$

47. The angle between the curves $y = x^3$ and $y = e^{3(x-1)}$ at (1, 1) is

1) 0 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

48. The angle between the curves $x^3 - 3xy^2 = 2$ and $3x^2y - y^3 = 2$ is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

49. If the curves $ay + x^2 = 7$ and $x^3 = y$ cut orthogonally at (1, 1) then a =

1) 1 2) – 6 3) 6 4) $\frac{1}{6}$

KEY

01. 2	02. 2	03. 2	04. 4	05. 4	06. 1
07. 1	08. 2	09. 4	10. 3	11. 2	12. 3
13. 3	14. 3	15. 1	16. 2	17. 3	18. 2
19. 2	20. 4	21. 3	22. 4	23. 3	24. 3
25. 3	26. 1	27. 1	28. 2	29. 4	30. 4
31. 2	32. 4	33. 1	34. 2	35. 1	36. 2
37. 4	38. 3	39. 4	40. 4	41. 4	42. 4
43. 1	44. 1	45. 1	46. 2	47. 1	48. 4
49. 3					

HINTS

1. Find $\frac{dy}{dx}$ and substitute (1, 6)

2. Find $\frac{dy}{dx}$ and substitute x = 2

4. Put y = 0 and find points and substitute in $\frac{dy}{dx}$

6. At (2, –1) find value of t. At that t find slope $\frac{dy}{dx}$.

10. Find $\frac{dy}{dx}$ and equate to zero

11. $\frac{dy}{dx} = 0$ and find point.

12. $\frac{dy}{dx} = 0$

13. $\frac{dy}{dx}$ is not defined.

16. Find $\frac{dy}{dx}$ and equate to $2(\because \perp r)$

17. $\frac{dy}{dx} = 2\ (\because \perp \gamma)$

18. Find $\frac{dy}{dx}$ and equate to –1 ($\because$ parallel)

19. $mx_1 - y_1 = 0$ verify the options $\left(m = \frac{dy}{dx}\right)$

20. $m = \frac{dy}{dx}$ which option is having $mx_1 - y_1$ as zero that option is correct.

21. Find $\frac{dy}{dx}$ and find equation

22. Find $\frac{dy}{dx}$ and substitute $x = \frac{\pi}{3}$ and find equation

23. Find $-\left(\frac{dx}{dy}\right)$ substitute $x = \frac{\pi}{2}$ and find equation

24. Find Tangent equation at put y = 0 in that.

25. Find tangent equation at (–1, 15) and observe options.

26. Vertex A(0, 0) slope of Tangent = Slope of $\overline{AP}$

27. Put y = 0 in the equation and find the point and find equation of Tangent at that point.

28. Put x = 0 in curve and find point. At that point find the tangent.

29. Put x = 0 in the curve find the point at that point find tangent and compare.

30. Find slope as $-\frac{dx}{dy}$ where t is the parameter and find the normal.

31. Find $\frac{dy}{dx}$ at $t = \frac{\pi}{2}$ and find normal.

34. $-\frac{dx}{dy} = 1$ then find normal.

35. Find $-\frac{dx}{dy}$ at x = 0, and find normal.

36. Find normal at (1, –2) on the curve and compare the given line.

37. Find tangent at (2, 3) and compae.

38. at x = 1, $\frac{dy}{dx} = 1$ and substitute (–1, 0) in the curve and solve the two equations for 'a' and 'b'.

39. Find $\left|y.\frac{dy}{dx}\right|$ at (–1, 4)

40. Find $\left|y.\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right|$ at (2, 1)

43. Find $\frac{dy}{dx}$ to any of the curve at (2, 3) and find tangent.

44. Find $\frac{dy}{dx}$ at (2, 3) to any of the curve and tangent.

45. Slope are equal.

46. Product of the slopes is equal to –1.

47. Find $\frac{dy}{dx}$ to the two curves at (1, 1) they are m_1 and m_2. Then $\text{Tan}\theta = \left|\frac{m_1 - m_2}{1+m_1m_2}\right|$.

48. $m_1 \cdot m_2 = -1$

LEVEL-II

1. If the slope of the tangent to the curve $xy + ax + by = 0$ at the point (1, 1) on it is 2 then values of a and b are
1) 1, 2 2) 1, –2
3) –1, 2 4) –1, –2

2. The distance of the origin from the normal to the curve $y = e^{2x} + x^2$ at $x = 0$ is
1) $\frac{2}{5}$ 2) $\frac{2}{\sqrt{5}}$ 3) $2\sqrt{5}$ 4) $5\sqrt{2}$

3. Equation of the tangent to the parabola $y^2 = 4x + 5$ which is parallel to the line $y = 2x + 7$ is
1) y = 2x + 3 2) y = 2x – 3
3) y = 2x + 5 4) y = 2x – 5

4. If y = x + c is a tangent to the curve $9x^2 + 16y^2 = 144$ then value of c is
1) – 5 2) 5 3) ± 5 4) 2

5. The equation of the normal at x = 2a for the curve $y = \frac{8a^3}{4a^2 + x^2}$ is
1) 2x – y = 3a 2) 2x + y = 2a
3) x + 2y = 6a 4) x + y = a

6. The point on the curve $y^2 = 4ax$ at which the normal makes equal intercepts on the coordinate axes is
1) (a, a) 2) (a, 0)
3) (a, 2a) 4) (–a, 2a)

7. The normal to the curve $x = a(1+\cos\theta), y = a\sin\theta$ at θ always passes through the fixed point
1) (a, a) 2) (a, 0) 3) (0, a) 4) (0, 0)

8. The equation of the tangent to the curve $y = e^{-|x|}$ at the point where the curve cuts the line x = 1 is
1) x + y = e 2) e(x + y) = 1
3) y + ex = 1 4) x + ey = 2

9. The slope of the tangent to the curve at a point (x, y) on it is proportional to (x-2). If the slope of the tangent to the curve at (10, –9) on it is -3. The equation of the curve is
1) $Y = k(x-2)^2$ 2) $y = \frac{-3}{16}(x-2)^2 + 1$
3) $y = \frac{-3}{16}(x-2)^2 + 3$ 4) $y = K(x+2)^2$

10. The values of ‘a’ for which $y = x^2 + ax + 25$ touches x-axis are
1) ± 10 2) ± 2 3) ± 1 4) 0

11. The area of the triangle formed by the tangent to the curve $y = \frac{8}{4+x^2}$ at x = 2 on it and the axes is
1) 2 sq.units 2) 4 sq.units
3) 8 sq.units 4) 16 sq.units

12. The area of the triangle formed by the positive x-axis, the normal and the tangent to the curve $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ in sq.units is
1) $2\sqrt{3}$ 2) $\sqrt{3}$ 3) $4\sqrt{3}$ 4) 6

13. The angle between the curves $x^2 = 4y$ and $y^2 = 4x$ at (4, 4) is
1) $\frac{\pi}{2}$ 2) $\text{Tan}^{-1}(3)$
3) $\text{Tan}^{-1}\left(\frac{3}{4}\right)$ 4) $\text{Tan}^{-1}\left(\frac{4}{3}\right)$

14. The curves $y^2 = 2x$ and $y = 3x^2$ intersect at the origin at an angle

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{3}$

15. The angle between the curves $2x^2 + y^2 = 20$ and $4y^2 - x^2 = 8$ at the point $(2\sqrt{2}, 2)$

1) $\frac{\pi}{2}$ 2) $\mathrm{Tan}^{-1}\left(\frac{1}{2}\right)$

3) $\mathrm{Tan}^{-1}(2)$ 4) $\mathrm{Tan}^{-1}\left(\frac{2}{3}\right)$

16. The angle between the curves $y = x^2$ and $6y = 7 - x^3$ at the point (1, 1) is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\mathrm{Tan}^{-1}\left(\frac{1}{2}\right)$

17. The acute angle between curves $x^2 + y^2 = 4$ and $x^2 = 3y$ is

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{6}$

3) $\mathrm{Tan}^{-1}(2)$ 4) $\mathrm{Tan}^{-1}\left(\frac{5}{\sqrt{3}}\right)$

18. The angle between the curves $x^3 + y^3 + x + 2y = 0$ and $xy + 2x = y$ at the origin is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

19. The angle between the curves $y = 2x$ and $x^3 + y^3 = 3xy$ at the point other than origin is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\mathrm{Tan}^{-1}\left(\frac{6}{13}\right)$

20. The angle between the curves $y = \cos 2x$ and $y = 2\sin^2 x$ is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

21. If α is the angle between the curves $y^2 = 2x$ and $x^2 + y^2 = 8$, then $\tan\alpha$ is

1) 1 2) 2 3) 3 4) 4

22. If the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{l^2} - \frac{y^2}{m^2} = 1$ cut each other orthogonally then ......

1) $a^2 + b^2 = l^2 + m^2$ 2) $a^2 - b^2 = l^2 - m^2$

3) $a^2 - b^2 = l^2 + m^2$ 4) $a^2 + b^2 = l^2 - m^2$

23. The angle between the curves $\frac{x^2}{a^2 + k_1} + \frac{y^2}{b^2 + k_1} = 1$ and $\frac{x^2}{a^2 + k_2} + \frac{y^2}{b^2 + k_2} = 1$ is

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\mathrm{Tan}^{-1}\left(\frac{k_1}{k_2}\right)$

24. The curves $x^2 - y^2 = 5$ and $\frac{x^2}{18} + \frac{y^2}{8} = 1$ cut each other at the common point at an angle

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) π

25. If the curves $2x^2 + 3y^2 = 1$ and $ax^2 + 4y^2 = 1$ cut orthogonally, the value of a is

1) $\frac{4}{5}$ 2) $\frac{12}{5}$ 3) $\frac{5}{12}$ 4) 8

26. If the curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^2 = 16x$ intersect at right angles then value of a^2 is

1) 2 2) 4 3) $\frac{8}{3}$ 4) $\frac{16}{3}$

27. If the curves $y^2 = 16x$ and $9x^2 + by^2 = 16$ cut each other at right angles then the value of b is

1) 2 2) 4 3) $\frac{9}{2}$ 4) $-\frac{9}{2}$

28. If the curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ cut orthogonally the value of a^2 is

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) 1 4) $\frac{4}{3}$

29. If the curves $y = 1 - ax^2$ and $y = x^2$ intersect orthogonally then 'a' value is

1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) 2 4) 3

KEY

01. 2	02. 2	03. 1	04. 3	05. 1	06. 3
07. 2	08. 4	09. 3	10. 1	11. 2	12. 1
13. 3	14. 1	15. 1	16. 3	17. 4	18. 4
19. 4	20. 3	21. 3	22. 3	23. 3	24. 3
25. 2	26. 1	27. 3	28. 4	29. 2	

HINTS

1. $\frac{dy}{dx}$ at (1, 1) is 2 and obtain equation in a and b (1, 1) also lies on curve and obtain another equation and solve.
2. Find normal at x = 0 and apply $\frac{|c|}{\sqrt{a^2+b^2}}$
3. $\frac{dy}{dx}=2$ and find point and at that point find tangent.
4. Apply $c^2=a^2m^2+b^2$ where $a^2=16; b^2=9$ and m = 1.
5. (2a, a) is a point on the curve. Substitute in the options.
9. Substitute the point in the options.
   Hint: Equate y-coordinate of the point of contact to zero.
11. Find the tangent and apply $\frac{c^2}{2|a.b|}$ sq.units
12. Find slope m. Apply formula $\frac{y^2(1+m^2)}{2|m|}$
13. Angle between the curves $x^2=4ay$ and $y^2=4ax$ at (4a, 4a) is $\text{Tan}^{-1}\left(\frac{3}{4}\right)$.
15. Find $\frac{dy}{dx}$ for the two curves m_1 and m_2 at $\left(2\sqrt{2},2\right)$ then $\text{Tan}\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$
16. Same in problem 3
20. Find m_1 and m_2 and apply $\text{Tan}\theta$
21. Point of contact is (2, 2) $\text{Tan}\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$
26. Find the point of intersections and find m_1 and m_2 then $m_1 . m_2=-1$
27. Apply $m_1 m_2=-1$ and put $y^2=16x$.
28. Find point of intersection and $m_1 m_2=-1$ and find a^2.

LEVEL-III

1. If the tangent to the curve $2y^3=ax^2+x^3$ at the point (a, a) cuts off intercepts α and β on the coordinate axes such that $\alpha^2+\beta^2=61$ then a =
   1) ± 30 2) ± 5 3) ± 6 4) ± 61
2. The equations of the tangents at the origin to the curve $y^2=x^2(1+x)$ are
   1) $y=\pm x$ 2) $y=\pm 2x$
   3) $y=\pm 3x$ 4) $x=\pm 2y$
3. If the chord joining the points where x = p, x = q on the curve y = ax² + bx +c is parallel to the tangent drawn to the curve at (α,β) then $\alpha=$
   1) 2pq 2) $\sqrt{pq}$ 3) $\frac{p+q}{2}$ 4) $\frac{p-q}{2}$
4. The equation of the common normal at the point of contact of the curves $x^2=y$ and $x^2+y^2-8y=0$
   1) x = y 2) x = 0
   3) y = 0 4) x + y = 0
5. The length of perpendicular drawn from the origin to the normal at any point on the curve given by $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$ is
   1) $\frac{a}{2}$ 2) a 3) $\frac{3a}{2}$ 4) 2a
6. If the normal at the point $p(\theta)$ of the curve $x=3\cos\theta-\cos^3\theta,\ y=3\sin\theta-\sin^3\theta$ passes through the origin then
   1) $\theta=\pi/3$ 2) $\theta=\pi/6$
   3) $\theta=\pi/4$ 4) $\theta=\pi/2$
7. Area of the triangle formed by the tangent, normal at (1, 1) on the curve $\sqrt{x}+\sqrt{y}=2$ and the y-axis is (in sq. units)
   1) 1 2) 2 3) $\frac{1}{2}$ 4) 4
8. The sum of the lengths of the sub-tangent and tangent drawn at the point (x, y) on the curve $y=a\log(x^2-a^2)$ varies as
   1) x^2 2) y^2 3) xy 4) y/x

9. At origin the curve $y^2 = x^3 + x$
1) Touches the x-axis
*2) Touches the y-axis
3) Bisects the angle between the axes
4) touches both the axes

10. The angle between the curves $y^2 = 4ax$ and $ay = 2x^2$ is

1) $Tan^{-1}\left(\frac{3}{4}\right)$ 2) $Tan^{-1}\left(\frac{3}{5}\right)$

3) $Tan^{-1}\left(\frac{4}{3}\right)$ 4) $Tan^{-1}\left(\frac{5}{3}\right)$

11. The angle between the curves $x^2 + y^2 = \sqrt{2}a^2$ and $x^2 - y^2 = a^2$ is

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

KEY

01. 1	02. 1	03. 3	04. 2	05. 2	06. 3
07. 1	08. 3	09. 2	10. 2	11. 1	

HINTS

2. Hint: The equations of the tangents at the origin can be obtained by equating the lowest degree terms to zero.

LEVEL-IV

NEW MODEL QUESTIONS

I. 1. Observe the following statements

I: If p and q are the lengths of perpendiculars from the origin on the tangent and normal at any point on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ then $4p^2 + q^2 = 1$.

II: If the tangent at any point P on the curve $x^3.y^2 = a^5$ cuts the coordinate axes at A and B then
AP : PB = 3 : 2

which of the above statement is correct.
1) only I 2) only II
3) both I and II 4) neither I nor II

2. Observe the following statements for the curve $y = 2.e^{\frac{-x}{3}}$.

I : The slope of the tangent to the curve where it meets y-axis is $\frac{-2}{3}$.

II: The equation of normal to the curve where it meets y-axis is $3x + 2y + 4 = 0$.

Which of the above statement is correct
1) only I 2) only II
3) both I and II 4) neither I nor II

3. Observe the following statements for the curve $x = at^3$, $y = at^4$ at $t = 1$.

I : The equation of the tangent to the curve is $4x - 3y - a = 0$

II : The equation of the normal to the curve is $3x + 4y - 7a = 0$

III: Angle between tangent and normal at any point on the curve is $\frac{\pi}{2}$.

Which of the above statements are correct.
1) I and II 2) II and III
3) I and III 4) I, II, III

4. I: The angle between the curves $y = x^3$, $6y = 7 - x^2$ at (1, 1) is $\frac{\pi}{2}$.

II : The angle between the curves $y = x$, $y = \frac{1}{x}$ at (1, 1) is $\frac{\pi}{2}$

Which of the above statements is correct.
1) I only 2) II only
3) both I and II 4) neither I nor II

5. I. The straight line $x + y = a$ will be a tangent to the Ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ if $a = \pm 5$.

II. The straight line $x + y = a$ will be a tangent to the Hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ if $a = \pm 7$.

Which of the above statements is correct ?
1) only II 2) only I
3) both I and II 4) Neither I nor II

6. I. If the subnormal to the curve $x.\,y^n = a^{n+1}$ is constant then the value of n is –2.

II. The length of the subtangent, ordinate of a point, (not the origin) length of the subnormal on $y^2 = 4ax$ are in G.P.

Which of the above statements is correct
1) only I 2) only II
3) both I and II 4) Neither I nor II

7. I. If the curve $y = x^2 + bx + c$ touches the straight line y = x at the point (1, 1) then b and c are given by 1, 1.

II. If the line $lx + my + n = 0$ is a normal to the curve xy = 1, then $l > 0, m < 0$.

Which of the above statements is correct

1) only I 2) only II
3) both I and II 4) Neither I nor II

8. In the curve $y = ce^{x/a}$ the

a) Subtangent is constant
b) subnormal varies as the square of the ordinate

1) Both a, b are correct 2) Only b is correct
3) only a is correct
4) both a, b are wrong

II. 9. Observe the following lists:

List-I	List-II
I. Angle between the curves $y^2 = x$ and $x^2 = y$ at the point (1, 1)	a) 0
II. Angle between the curves $x^2y = 1$ and $y = e^{2(1-x)}$ at the point (1, 1)	b) $\text{Tan}^{-1}\left(\frac{4}{3}\right)$
III. Angle between the curves $xy = 4, x^2 - y^2 = 15$ at (– 4, –1)	c) $\frac{\pi}{2}$
	d) $\text{Tan}^{-1}(2\sqrt{2})$
	e) $\text{Tan}^{-1}\left(\frac{3}{4}\right)$

Choose the correct answer

1) I - a, II - b, III - c 2) I - b, II - a, III - c
3) I - e, II - a, III - c 4) I - e, II - c, III - a

10. Observe the following lists for the curve $y = x^3 + 1$ at the point (1, 2)

List-I	List-II
I. Length of tangent	a) $2\sqrt{10}$
II. Length of normal	b) $\frac{3}{2}$
III. Length of sub-tangent	c) 6
IV. Length of sub-normal	d) $\frac{2}{3}\sqrt{10}$
	e) $\frac{2}{3}$

Match the List-I from List-II and choose the correct answer.

1) d a c e 2) d a e c
3) a d e c 4) c a d e

11. Observe the following lists for the curve $y = 6 + x - x^2$ with the slopes of tangents at the given points

Point	Tangent slope
I : (1, 6)	a) 3
II : (2, 4)	b) 5
III : (–1, – 4)	c) – 1
IV : (–2, 0)	d) – 3

Correct match is ....

1) a b c d 2) b c d a
3) c d b a 4) c d a b

12. Observe the following lists:

If the line y = mx + c is a tangent to the following curves then

List-I	List-II
A. $x^2 + y^2 = a^2$	1) $c = \frac{a}{m}$
B. $y^2 = 4ax$	2) $c^2 = a^2m^2 + b^2$
C. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	3) $c^2 = \frac{a}{m}$
D. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	4) $c^2 = a^2m^2 - b^2$
	5) $c^2 = a^2(1 + m^2)$

The correct match for list-I from list-II is

	A	B	C	D
	A	B	C	D
1)	5	4	3	2
2)	1	2	3	4
3)	3	4	1	2
4)	5	1	2	4

13. Match List-I with List-II and select the correct answer using the code given below.

List-I	List-II
a) Equation of tangent to the curve $y = be^{-x/a}$ at x = 0	1) $x - 2y = 2$

b) Equation of tangent to the curve $y = x^2 + 1$ at (1, 2) — 2) $y = 2x$

c) Equation of normal to the curve $y = 2x - x^2$ at (2, 0) — 3) $x - y = \pi$

d) Equation of normal to the curve y= sinx at $x = \pi$ — 4) $\frac{x}{a} + \frac{y}{b} = 1$

	A	B	C	D
1)	4	1	2	3
2)	4	2	1	3
3)	1	2	3	4
4)	1	4	3	2

III. 14. Assertion (A) : The normal to the curve $ay^2 = x^3$ $(a \neq 0, x \neq 0)$ at a point (x, y) on it makes equal intercepts on the axes then $x = \frac{4a}{9}$.

Reason (R): The normal at (x_1, y_1) on the curve y = f(x) makes equal intercepts on the coordinate axes then $\left(\frac{dy}{dx}\right)_{(x_1, y_1)} = 1$

Choose the correct answer.

1) Both A and R are true and R is the correct explanation for A

2) Both A and R are true but R is not the correct explanation for A

3) A is true but R is false

4) A is false but R is true

15. Assertion (A):- The points on the curve $y = x^3 - 3x$ at which the tangent is parallel to x-axis are (1, –2) and (–1, 2).

Reason (R): The tangent at (x_1, y_1) on the curve y = f(x) is vertical then $\frac{dy}{dx}$ at (x_1, y_1) is not defined.

Choose the correct answer.

1) Both A and R are true and R is the correct explanation for A

2) Both A and R are true but R is not the correct explanation for A

3) A is true but R is false

4) A is false but R is true

16. Assertion A: The angle between the curves y = sinx, y = cosx is $\mathrm{Tan}^{-1}\sqrt{2}$.

Reason (R) : The slopes of their tangents at $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ are $\frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$ respectively.

1) Both A and R are true R is the correct explanation of A

2) Both A and R are true but R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true

17. Assertion A: The curves $x^2 = y, x^2 = -y$ touch each other at (0, 0).

Reason R: The slopes of the tangents at (0, 0) for both the curves are equal.

1) Both A and R are true R is the correct explanation of A

2) Both A and R are true but R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true

18. Assertion A : The value of 'a' is $\frac{12}{5}$ when the two curves $2x^2 + 3y^2 = 1$ and $ax^2 + 4y^2 = 1$ to cut orthogonally.

Reason R: If the curves $a_1x^2 + b_1y^2 = 1, a_2x^2 + b_2y^2 = 1$ cut each other orthogonally then $\frac{1}{a_1} - \frac{1}{a_2} = \frac{1}{b_1} - \frac{1}{b_2}$

1) Both A and R are true R is the correct explanation of A

2) Both A and R are true but R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true

19. Assertion(A): If the tangent at any point P on the curve $xy = a^2$ meets the axes at A and B then AP : PB = 1 : 1

Reason(R): The tangent at P(x, y) on the curve $x^m.y^n = a^{m+n}$ meets the axes at A and B. Then the ratio of P divides $\overline{AB}$ is n : m.

1) Both A and R are true R is the correct explanation of A

2) Both A and R are true but R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true

20. Assertion(A): The tangent to the curve $y = x^3 - x^2 - x + 2$ at (1, 1) is parallel to the x-axis.

Reason(R): The slope of the tangent to the above curve at (1, 1) is zero.

1) Both A and R are true R is the correct explanation of A

2) Both A and R are true but R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true

IV. 21. A : Length of sub-normal to $y = 4x^2$ at $(-1, 4)$

B : Length of sub-normal to $y = x^3 + 1$ at (1, 2)

C : Length of sub-tangent to $xy = 12$ at (3, 4)

D: Length of normal to $x^3 + y^3 = 6xy$ at (3, 3)

Then the ascending order of the above values are

1) DCBA 2) ABCD 3) CDBA 4) DBCA

22. A : Angle between the curves $y^2 = x, y^2 = -x$ at (0, 0).

B : Angle between the curves $y = 3x^2; y^2 = 2x$ at (0, 0)

C : Angle between the curves $y^2 = 4x, x^2 = 4y$ at (4, 4)

Then the descending order of the above values are

1) ABC 2) CBA 3) BCA 4) ACB

23. The arrangement of the following curves in the ascending order of slopes of their tangents at the given points.

A) $y = \frac{1}{1+x^2}$ at $x = 0$

B) $y = 2e^{\frac{-x}{4}}$ where it cuts the y-axis

C) $y = \cos x$ at $x = \frac{-\pi}{4}$

D) $y = 4x^2$ at $x = -1$

1) DCBA 2) ACBD

3) ABCD 4) DBAC

24. The arrangement of following values for $y = x^2$ at (1, 1) in the descending order.

A) Length of the tangent

B) Length of the normal

C) Length of the subtangent

D) Length of the sub normal

1) A, C, D, B 2) B, D, A, C

3) C, A, D, B 4) B, A, D, C

25. The arr7angement of the slopes of the normals to the curve $y = e^{\log(\cos x)}$ in the ascending order at the points given below.

A) $x = \frac{\pi}{6}$ B) $x = \frac{7\pi}{4}$

C) $x = \frac{11\pi}{6}$ D) $x = \frac{\pi}{3}$

1) C, B, D, A 2) B, C, A, D

3) A, D, C, B 4) D, A, C, B

KEY

01. 1	02. 1	03. 4	04. 3	05. 2
06. 3	07. 2	08. 1	09. 3	10. 2
11. 4	12. 4	13. 2	14. 1	15. 2
16. 4	17. 1	18. 1	19. 1	20. 1
21. 3	22. 3	23. 4	24. 2	25. 1

PREVIOUS EAMCET PAPERS

2004

1. Match the points on the curve $2y^2 = x + 1$ with the slope of normals at those points and choose the correct answer

Point	Slope of normal
I : (7, 2)	a) $-4\sqrt{2}$
II : $\left(0, \frac{1}{\sqrt{2}}\right)$	b) – 8
III : (1, –1)	c) 4
IV : $(3, \sqrt{2})$	d) 0
	e) $-2\sqrt{2}$

1) b d c a 2) b e c a

3) b c e a 4) b e a c

2003

2. The angle between the curves y = sinx and y = cosx is......

1) $\mathrm{Tan}^{-1}(2\sqrt{2})$ 2) $\mathrm{Tan}^{-1}(3\sqrt{2})$

3) $\mathrm{Tan}^{-1}(3\sqrt{3})$ 4) $\mathrm{Tan}^{-1}(5\sqrt{2})$

2002

3. The two curves $x = y^2$, $xy = a^3$ cut orthogonally at a point then $a^2 =$

1) $\frac{1}{3}$ 2) $\frac{1}{2}$ 3) 2 4) 3

2001

4. The equation of the tangent to the curve $6y = 7 - x^3$ at (1, 1) is

1) $2x + y = 3$ 2) $x + 2y = 3$

3) $x + y = -1$ 4) $x + y + 2 = 0$

21.

2000

5. Area of the triangle formed by the normal to the curve $x = e^{\sin y}$ at (1, 0) with the coordinate axes is

1) $\frac{1}{4}$ 2) $\frac{1}{2}$ 3) $\frac{3}{4}$ 4) 1

6. The angle between the curves $y^2 = 4x, x^2 = 4y$ at (4, 4) is

1) $\mathrm{Tan}^{-1}\frac{1}{2}$ 2) $\mathrm{Tan}^{-1}\frac{3}{4}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$

1999

7. If the acute angle between the curves xy = 2 and $y^2 = 4x$ is θ, then tan θ =

1) $\frac{1}{3}$ 2) 3

3) 2 4) $\frac{2}{3}$

8. The equation of the tangent to the curve $y = x^3 - 2x + 7$ at the point (1, 6) is

1) y = x + 5 2) x + y = 7

3) 2x + y = 8 4) x + 2y = 13

1998

9. At any point on the curve y = f(x), the length of the sub-normal is constant, then the curve is

1) circle 2) ellipse

3) parabola 4) straight line

1997

10. Acute angle between the curves xy = 2 and $y^2 = 4x$ is

1) $\mathrm{Tan}^{-1}\left(\frac{1}{3}\right)$ 2) $\mathrm{Tan}^{-1}(3)$

3) $\mathrm{Tan}^{-1}\left(\frac{1}{2}\right)$ 4) $\mathrm{Tan}^{-1}\left(\frac{2}{3}\right)$

11. The length of sub-tangent at any point θ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

1) $a|\sin\theta|\sec^2\theta$ 2) $a\sin\theta|\sec\theta|$

3) $a|\sin\theta.\cos\theta|$ 4) $a\sin^2\theta|\sec\theta|$

12. The length of the sub-normal at (–1, 4) on $Y = 4X^2$ is

1) 4 2) 16 3) 32 4) 8

1996

13. Acute angle between the curves $y^2 = x$ and $X^2 = Y$ at (1, 1) is

1) $\frac{\pi}{2}$ 2) $Tan^{-1}\left(\frac{4}{3}\right)$

3) $Tan^{-1}\left(\frac{1}{2}\right)$ 4) $Tan^{-1}\left(\frac{3}{4}\right)$

14. The length of the subtangent to the curve $x^2 + xy + y^2 = 7$ at (1, –3) is

1) 15 2) 7 3) 12 4) 10

15. The portion of the tangent to $xy = a^2$ at any point on it between the axes is

1) Trisected at that point

2) bisected at that point 3) constant

4) with ratio 1 : 4 at the point

1995

16. The sum of the lengths of subtangent and tangent to the curve $x = c\left[2\cos\theta - \log\left(\cos ec\theta + \cot\theta\right)\right]$, $y = c\sin 2\theta$ at $\theta = \frac{\pi}{3}$ is

1) $\frac{c}{2}$ 2) 2c 3) $\frac{3c}{2}$ 4) $\frac{5c}{2}$

17. Sub-normal to $xy = c^2$ at any point on it varies directly as

1) cube of ordinate 2) square of ordinate

3) ordinate 4) cube of abscissa

18. Equation of the tangent line to $y = be^{\frac{-x}{a}}$ where it crosses y-axis is

1) ax + by = 1 2) $\frac{x}{a}+\frac{y}{b}=1$

3) $\frac{x}{b}+\frac{y}{a}=1$ 4) ax – by = 1

1994

19. An angle at which the two curves $x^3 - 3xy^2 = 4$ and $3x^2y - y^3 = 4$ intersect is

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) 0

20. For the parabola $y^2 = 4ax$, the ratio of the length of the sub-tangent to the abscissa at any point on the curve is

1) 1 : 1 2) 2 : 1 3) x : y 4) x^2 : y

1993

21. The point on the hyperbola $y = \frac{x-1}{x+1}$ at which the tangents are parallel to y = 2x + 1 are

1) (0, –1) only 2) (–2, 3) only

3) (0, –1), (–2, 3) 4) (2, 3) (5, 4)

22. The length of the sub-tangent, ordinate of a point and length of the sub-normal at a point (not origin) on $y^2 = 4ax$ are in

1) A.P 2) G.P 3) H.P 4) A.G.P

1991

23. If the relation between length of sub-normal and length of sub-tangent at any point on the curve $by^2 = (x + a)^3$ is $p(S.N) = q(S.T)^2$ then p:q =

1) b : a 2) a : b 3) 8b : 27a 4) 8b : 27

1990

24. The curve $x^4-2xy^2+y^2+3x-3y=0$ cuts the x-axis at (0, 0) at an angle

1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{2}$

1986

25. Equation of the normal to the curve $x^2=4y$ at (1, 2) on it is

1) x – 2y = 3 2) 2x + y = 4
3) 2x – y = 4 4) x + 2y = 3s

1985

26. The length of the sub-normal at any point on the curve $y^2=2px$ is

1) Constant 2) Varies as abscissa
3) Varies as ordinate 4) Varies as P

1984

27. The length of sub-normal at x = a on the parabola $y^2=16x$ is

1) 8 2) $\frac{2}{\sqrt{a}}$ 3) 2a 4) a

1983

28. If the sub-normal to the curve $xy^n=a^{n+1}$ is constant then value of n is

1) – 2 2) – 1 3) 0 4) 2

1982

29. Equation of the tangent to the curve $y=3x^3+6x^2-9$ at the point where the curve crosses y-axis is

1) y – 9 = 0 2) y + 9 = 0
3) 2y – 9 = 0 4) 2y + 9 = 0

30. The curves $y=x^2$ and $6y=7-x^3$ intersect at (1, 1) at an angle

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) π

KEY

1. 2	2.1	3. 2	4. 2
5. 2	6.2	7. 2	8. 1
9. 3	10. 2	11. 4	12. 3
13. 4	14. 1	15. 2	16. 3
17. 1	18. 2	19. 1	20. 2
21. 3	22. 2	23. 4	24. 2
25. 2	26. 1	27. 1	28. 1
29. 2	30. 3		

LEVEL -V
COMPREHENSIVE QUESTIONS

1. The portion of the tangent at any point to the curve $x^m y^n = K\left(m \neq 0 \,\&\, n \neq 0\right)$ intercepted between the coordinate axes is divded by the point of contact in the constant ratio and the triangle formed by the tangent with the axes is constant if m = n

I. The area of triangle formed by the tangent at any point on the curve $xy=c^2$ with the coordenate axes is ________ sq.units

1) c^2 2) $2c^2$
3) 2c 4) c

II. If the area of triangle formed by the tangent at any point on the curve $xy^K=a^{n+2}$ with the coordinate axes is constant then K = ___

1) 1 2) 3
3) 2 4) -2

III. A point P on the curve divides the portion of the tangent between the axes which is drawn at P on the curve $xy^2=K$, in the ratio ______

1) 1:2 2) 1:3
3) 2:1 4) 1:1

IV. If the tangent at P on the curve $xy=c^2$ meets the coordinate axes in A & B respectively then

1) $\overline{AB}$ is trisected by P
2) $\overline{AB}$ is bisected by P
3) PA:PB = 3:2 4) PA:PB = 2:3

KEY

I. 2 II. 1 III. 3 IV. 2

2. If the tangent and normal at P $\left(x_1, y_1\right)$ to the curve $y=f\left(x\right)$ meet the X-axis in T & N respectively & PM be the perpendicular drawn to X-axis then
i) PT is the length of the tangent
ii) PN is the length of the normal
iii) TM is the length of sub-tangent
iv) MN is the length of sub-normal
For the curve $y=4x^2$;

I) Length of the tangent to the curve at (-1,4)

1) $\sqrt{65}$ 2) $2\sqrt{65}$
3) $4\sqrt{65}$ 4) $\frac{\sqrt{65}}{2}$

II. Length of the normal to the curve at (-1,4)

1) $\frac{1}{2}$ 2) 32 3) $4\sqrt{65}$ 4) $\frac{\sqrt{65}}{2}$

III. Length of the sub tangent to the curve at (-1,4)

1) $\frac{1}{3}$ 2) $\frac{1}{2}$ 3) $\frac{1}{4}$ 4) $\frac{2}{3}$

IV. Length of the sub normal to the curve at (-1,4)

1) 32 2) 31 3) 30 4) 15

KEY

I. 4 II. 3 III. 2 IV. 1