PRACTICE SET -1

1. The domain of definition of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is:

   a. $R-\{-1,-2\}$ b. $(-2,\infty)$

   c. $R-\{-1,-2,-3\}$ d. $(-3,\infty)-\{-1,-2\}$

2. How many roots the equation $x-\frac{2}{x-1}=1-\frac{2}{x-1}$ have

   a. One b. Two

   c. Infinite d. None of these

3. From the following find the correct relation

   a. $(AB)' = A'B'$ b. $(AB)' = B'A'$

   c. $A^{-1} = \frac{\text{adj } A}{A}$ d. $(AB)^{-1} = A^{-1}B^{-1}$

4. If the roots of the cubic equation $ax^3+bx^2+cx+d=0$ are in G.P., then

   a. $c^3a = b^3d$ b. $ca^3 = bd^3$

   c. $a^3b = c^3d$ d. $ab^3 = cd^3$

5. $C_1 + 2C_2 + 3C_3 + 4C_4 + \ldots + nC_n =$

   a. 2^n b. $n.2^n$

   c. $n.2^{n-1}$ d. $n.2^{n+1}$

6. $1+\frac{(\log_e n)^2}{2!}+\frac{(\log_e n)^4}{4!}+\ldots =$

   a. n b. $1/n$

   c. $\frac{1}{2}(n+n^{-1})$ d. $\frac{1}{2}(e^n+e^{-n})$

7. How many words can be made from the letters of the word INSURANCE, if all vowels come together.

   a. 18270 b. 17280

   c. 12780 d. None of these

8. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. Then, the probability that only two tests are needed, is

   a. $\frac{1}{3}$ b. $\frac{1}{6}$

   c. $\frac{1}{2}$ d. $\frac{1}{4}$

9. Find real part of $\cosh^{-1}(1)$

   a. -1 b. 1

   c. 0 d. None of these

10. From a 60 meter high tower angles of depression of the top and bottom of a house are α and β respectively. If the height of the house is $\frac{60\sin(\beta-\alpha)}{x}$, then $x=$

    a. $\sin\alpha\sin\beta$ b. $\cos\alpha\cos\beta$

    c. $\sin\alpha\cos\beta$ d. $\cos\alpha\sin\beta$

11. The function $f(x) = \frac{\log(1+ax)-\log(1-bx)}{x}$ is not defined at $x=0$. The value which should be assigned to f at $x=0$ so that it is continuous at $x=0$, is

    a. $a-b$ b. $a+b$

    c. $\log a+\log b$ d. $\log a-\log b$

12. If $x = a\cos^4\theta, y = a\sin^4\theta$, then $\frac{dy}{dx}$, at $\theta=\frac{3\pi}{4}$, is

    a. -1 b. 1

    c. $-a^2$ d. a^2

13. $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx =$

    a. $\log(1+x^2)+c$ b. $\log e^{\tan^{-1}x}+c$

    c. $e^{\tan^{-1}x}+c$ d. $\tan^{-1}e^{\tan^{-1}x}+c$

14. The solution of the equation $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$ is

    a. $e^y=e^x+\frac{x^3}{3}+c$ b. $e^y=e^x+2x+c$

    c. $e^y=e^x+x^3+c$ d. $y=e^x+c$

15. The distance between $4x+3y=11$ and $8x+6y=15$, is

    a. $\frac{7}{2}$ b. 4

    c. $\frac{7}{10}$ d. None of these

16. The area of a circle whose centre is (h, k) and radius a is

    a. $\pi(h^2+k^2-a^2)$ b. πa^2hk

    c. πa^2 d. None of these

17. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix

a. $x = -a$ b. $x = -\frac{a}{2}$

c. $x = 0$ d. $x = \frac{a}{2}$

18. If $\vec{a} = (2, 5)$ and $\vec{b} = (1, 4)$, then the vector parallel to $(\vec{a} + \vec{b})$ is

a. (3, 5) b. (1, 1) c. (1, 3) d. (8, 5)

19. The acute angle between the line joining the points (2, 1,–3), (–3,1,7) and a line parallel to $\frac{x-1}{3} = \frac{y}{4} = \frac{z+3}{5}$ through the point (–1, 0, 4) is

a. $\cos^{-1}\left(\frac{7}{5\sqrt{10}}\right)$ b. $\cos^{-1}\left(\frac{1}{\sqrt{10}}\right)$

c. $\cos^{-1}\left(\frac{3}{5\sqrt{10}}\right)$ d. $\cos^{-1}\left(\frac{1}{5\sqrt{10}}\right)$

20. $\sin\left(\frac{\pi}{10}\right)\sin\left(\frac{3\pi}{10}\right) =$

a. 1/2 b. – ½ c. 1/4 d. 1

21. The minimum value of $[(5+x)(2+x)]/[1+x]$ for non-negative real x is

a. 12 b. 1

c. 9 d. 8

22. If ω is the cube root of unity, then $(3+5\omega+3\omega^2)^2 + (3+3\omega+5\omega^2)^2 =$

a. 4 b. 0

c. – 4 d. None of these

23. The measurement of the area bounded by the co-ordinate axes and the curve $y = \log_e x$ is

a. 1 b. 2

c. 3 d. ∞

24. The number of solutions of the system of equations $2x+y-z=7, x-3y+2z=1, x+4y-3z=5$ is

a. 3 b. 2

c. 1 d. 0

25. The n^th term of the series 3 + 10 + 17 + ….. and 63 + 65 + 67 + …… are equal, then the value of n is

a. 11 b. 12

c. 13 d. 15

26. Let $f'(x) = \frac{192x^3}{2+\sin^4 \pi x}$ for all $x \in R$ with $f\left(\frac{1}{2}\right) = 0.$ If $m \le \int_{1/2}^{1} f(x)\,dx \le M$, then the possible values of m and M are

a. $m = 13, M = 24$

b. $m = \frac{1}{4}, M = \frac{1}{2}$

c. $m = -11, M = 0$

d. $m = 1, M = 12$

27. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ?

a. $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$ b. $\left(-\frac{1}{\sqrt{5}}, 0\right)$

c. $\left(0, \frac{1}{\sqrt{5}}\right)$ d. $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

28. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is (are)

a. $\cos\beta > 0$ b. $\sin\beta < 0$

c. $\cos(\alpha+\beta) > 0$ d. $\cos\alpha < 0$

29. Let E_1 and E_2 be two ellipse whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line $x + y = 3$ touches the curves S, E_1 and E_2 at P, Q and R, respectively suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricities of E_1 and E_2, respectively, then the correct expression(s) is(are)

a. $e_1^2 + e_2^2 = \frac{43}{40}$ b. $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

c. $|e_1^2 - e_2^2| = \frac{5}{8}$ d. $e_1 e_2 = \frac{\sqrt{3}}{4}$

30. Consider the hyperbola $H : x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroix of the triangle ΔPMN, then the correct expression(s) is (are)

a. $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x > 1$

b. $\frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 > 1$

c. $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$ for $x_1 > 1$

d. $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

31. The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} (\sin^6 at + \cos^4 at)\, dt}{\int_0^{\pi} e^1 (\sin^6 at + \cos^4 at)\, dt} = L?$$

a. $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$
b. $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$
c. $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$
d. $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

32. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has

a. only purely imaginary roots
b. all real roots
c. two real and two purely imaginary roots
d. neither real nor purely imaginary roots

33. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to:

a. 6
b. –6
c. 3
d. –3

34. Let S be the set of all non -zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S?

a. $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$
b. $\left(-\frac{1}{\sqrt{5}}, 0\right)$
c. $\left(0, \frac{1}{\sqrt{5}}\right)$
d. $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

35. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is:

a. 3
b. – 4
c. 6
d. 5

36. The least value of $\alpha \in R$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

a. $\frac{1}{64}$
b. $\frac{1}{32}$
c. $\frac{1}{27}$
d. $\frac{1}{25}$

37. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

a. $2(\sec\theta - \tan\theta)$
b. $2\sec\theta$
c. $-2\tan\theta$
d. 0

38. Let $a, \lambda, \mu \in R$. Consider the system of linear equations $ax + 2y = \lambda$, $3x - 2y = \mu$. Which of the following statement(s) is (are) correct?

a. If $a = -3$, then the system has infinitely many solutions for all values of λ and μ

b. If $a \neq -3$, then the system has unique solutions for all values of λ and μ

c. If $\lambda + \mu = 0$, then the system has infinitely many solutions for $a = -3$

d. If $\lambda + \mu \neq 0$, then the system has no solutions for $a = -3$

39. If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + \ldots + (x + \overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to

a. 10
b. 11
c. 12
d. 9

40. A line $L: y = mx + 3$ meets y-axis at E(0, 3) and the arc of the parabola $y^2 = 16x$, $0 \leq y \leq 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the

y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the ΔEFG has a local maximum. Match Column I with Column II and select the correct answer using the code given below the Columns.

Column I	Column II
(A) $m =$	1. 1/2
(B) Maximum area of ΔEFG is	2. 4
(C) $y_0 =$	3. 2
(D) $y_1 =$	4. 1

a. A→4; B→1;C→2; D→3 b. A→1; B→4;C→2; D→3
c. A→1; B→2;C→3; D→4 d. A→4; B→1;C→3; D→2

41. Observe the following columns:

Column I	Column II
(A) $y = \sin^{-1}(3x - 4x^3)$, then $\frac{dy}{dx}$ is	1. $\frac{3}{1+x^2}$, $x \in \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$
(B) $y = \cos^{-1}(4x^3 - 3x)$, then $\frac{dy}{dx}$ is	2. $\frac{3}{1+x^2}$, $x \in \left(\frac{1}{\sqrt{3}}, \infty\right)$
(C) $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$, then $\frac{dy}{dx}$ is	3. $\frac{3}{\sqrt{(1-x^2)}}$, $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$
	4. $\frac{3}{1+x^2}$, $x \in \left(-\infty, -\frac{1}{\sqrt{3}}\right)$
	5. $\frac{3}{\sqrt{1-x^2}}$, $x \in \left(-1, \frac{1}{2} \cup \left(\frac{1}{2}, 1\right)\right)$

a. A→3,5; B→3,5; C→1,2 b. A→3,5; B→3,5; C→1,2,4
c. A→3,5; B→2,3; C→1,4 d. A→1,3; B→3,5; C→1,2

42. Match the statements given in Column I with the interval/union of intervals given in Column II

Column I	Column II
(A) The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right)\right\}$: z is a complex number, $\lvert z\rvert = 1, z \neq \pm 1$ is	1. $(-\infty, -1) \cup (1, \infty)$
(B) The domain of the function $f(x) = \sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	2. $(-\infty, 0) \cup (0, \infty)$
$f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$ (C) If then the set $\left\{f(\theta) : 0 \le \theta < \frac{\pi}{2}\right\}$ is	3. $[2, \infty)$
(D) If $f(x) = x^{3/2}(3x - 10)$, $x \ge 0$, then $f(x)$ is increasing in	4. $(-\infty, -1] \cup [1, \infty)$
	5. $(-\infty, 0] \cup [2, \infty)$

a. A→4; B→2, 4; C→4; D→2
b. A→4; B→5; C→3; D→3
c. A→2; B→1, 4; C→3; D→4
d. A→1; B→2; C→5; D→4

43. Match the statements/expressions in Column I with the values given in Column II.

Column I	Column II
(A) The number of solutions of the equation $xe^{\sin x} - \cos x = 0$ in the interval $\left(0, \frac{\pi}{2}\right)$	1. 1
(B) Value (s) of k for which the plane $kx + 4y + z = 0$, $4x + ky + 2z = 0$ and $2x + 2y + z = 0$ intersect in a straight line	2. 2
(C) Value (s) of k for which $\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$ has integer solution (s)	3. 3
(D) If $y'y + 1$ and $y(0) = 1$ then value (s) of $y(\ln 2)$	4. 4
	5. 5

a. A→1; B→2,3; C→2,3,4,5; D→3
b. A→4; B→5; C→3; D→3
c. A→2; B→1, 4; C→3; D→2,3,4,5
d. A→1; B→2; C→5; D→4

44. Let ΔPQR be a triangle. Let $\vec{a} = \overrightarrow{QR}, \vec{b} = \overrightarrow{RP}$, and $\vec{c} = \overrightarrow{PQ}$. If $|\vec{a}| = 12, |\vec{b}| = 4\sqrt{3}$ and $\vec{b}.\vec{c} = 24$, then which of the following is/are true?

a. $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$ b. $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 30$

c. $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$ d. $\vec{a}\vec{b} = -72$

45. Let X and Y be two arbitrary, 3 × 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 × 3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

a. $Y^3Z^4 - Z^4Y^3$ b. $X^{44} + Y^{44}$

c. $X^4Z^3 - Z^3X^4$ d. $X^{23} + Y^{23}$

46. Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha\ ?$$

a. -4 b. 9 c. -9 d. 4

47. In R^3, consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be a plane, different from P_1 and P_2 which passes through the intersection of P_1 and P_2. If the distance of the point (0, 1, 0) from P_3 is 1 and the distance of a point (α, β, γ) from P_3 is 2, then which of the following relations is (are) true?

a. $2\alpha + \beta + 2\gamma + 2 = 0$ b. $2\alpha - \beta + 2\gamma + 4 = 0$

c. $2\alpha + \beta - 2\gamma - 10 = 0$ d. $2\alpha - \beta + 2\gamma - 8 = 0$

48. In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M ?

a. $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$ b. $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

c. $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$ d. $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

49. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P?

a. $(4, 2\sqrt{2})$ b. $(9, 3\sqrt{2})$ c. $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ d. $(1, \sqrt{2})$

50. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

a. 2 b. –1

c. 0 d. 1

Answers and Solutions

1. (d) $x + 3 > 0$ and $x^2 + 3x + 2 \neq 0$

2. (d) If $x \neq 1$, multiplying each term by $(x-1)$, the given equation reduces to $x(x-1) = (x-1)$ or $(x-1)^2 = 0$ or $x = 1$, which is not possible as considering $x \neq 1$. Thus given equation has no roots.

3. (b) It is understandable.

4. (a) Let $\frac{A}{R}, A, AR$ be the roots of the equation

$ax^3 + bx^2 + cx + d = 0$

then A^3 = Product of the roots $= -\frac{d}{a}$

$$\Rightarrow A = -\left(\frac{d}{a}\right)^{1/3}$$

Since A is a root of the equation.

$$\therefore \quad aA^3 + bA^2 + cA + d = 0$$

$$\Rightarrow a\left(-\frac{d}{a}\right) + b\left(-\frac{d}{a}\right)^{2/3} + c\left(-\frac{d}{a}\right)^{1/3} + d = 0$$

$$\Rightarrow b\left(\frac{d}{a}\right)^{2/3} = c\left(\frac{d}{a}\right)^{1/3} \Rightarrow b^3\frac{d^2}{a^2} = c^3\frac{d}{a} \Rightarrow b^3 d = c^3 a.$$

5. (c) Trick: Put $n = 1, 2, 3, \ldots.\ S_1 = 1, S_2 = 2 + 2 = 4$

Now by alternate (c), put $n = 1, 2$

$S_1 = 1.2^0 = 1, S_2 = 2.2^1 = 4$

6. (c) $1 + \frac{(\log_e n)^2}{2!} + \frac{(\log_e n)^4}{4!} + \ldots = \frac{e^{\log_e n} + e^{-\log_e n}}{2} = \frac{n + n^{-1}}{2}$.

7. (d) IUAENSRNC Obviously required number of words are $\frac{6!}{2!} \times 4! = 8640$

8. (b) The probability that only two tests are needed = probability that the first machine tested is faulty ×

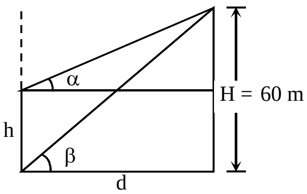
probability that the second machine tested is faulty

$= \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$

9. (c) We know that $\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$

$\therefore \cosh^{-1}(1) = \log\left(1 + \sqrt{1^2 - 1}\right) = \log 1 = 0.$

10. (d)



$H = d \tan\beta$ and $H - h = d \tan\alpha$

$\Rightarrow \frac{60}{60-h} = \frac{\tan\beta}{\tan\alpha} \Rightarrow -h = \frac{60\tan\alpha - 60\tan\beta}{\tan\beta}$

$\Rightarrow h = \frac{60\sin(\beta-\alpha)}{\cos\alpha\cos\beta\frac{\sin\beta}{\cos\beta}} \Rightarrow x = \cos\alpha\sin\beta.$

11. (b) Since limit of a function is $a+b$ as $x \to 0$, therefore to be continuous at a function, its value must be $a+b$ at $x = 0$

$\Rightarrow f(0) = a + b.$

12. (a) $y = a\sin^4\theta \Rightarrow \frac{dy}{d\theta} = 4a\sin^3\theta\cos\theta$

and $x = a\cos^4\theta$

$\Rightarrow \frac{dx}{d\theta} = -4a\cos^3\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-\sin^2\theta}{\cos^2\theta} = -\tan^2\theta$

$\therefore \left(\frac{dy}{dx}\right)_{\theta=\frac{3\pi}{4}} = -\tan^2\left(\frac{3\pi}{4}\right) = -1.$

13. (c) Putting $t = \tan^{-1}x \Rightarrow dt = \frac{1}{1+x^2}dx$, we get

$\int \frac{e^{\tan^{-1}x}}{1+x^2}dx = \int e^t dt = e^t + c = e^{\tan^{-1}x} + c.$

14. (a) $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y} = e^{-y}(e^x + x^2)$

$\Rightarrow e^y dy = (x^2 + e^x)dx$

Now integrating both sides, we get $e^y = \frac{x^3}{3} + e^x + c.$

15. (c) $4x + 3y = 11$ and $4x + 3y = \frac{15}{2}$

$$D = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}} = \frac{\left|11 - \frac{15}{2}\right|}{\sqrt{4^2 + 3^2}}$$

Therefore, $D = \left|\frac{11 - \frac{15}{2}}{5}\right| = \frac{7}{10}$.

16. (c) Since area $= \pi r^2$, where $r = a$

$\Rightarrow$ Area $= \pi a^2$.

17. (c) Let $P(h,k)$ be the mid-point of the line segment joining the focus $(a,0)$ and a general point $Q(x,y)$ on the parabola.

Then $h = \frac{x+a}{2}, k = \frac{y}{2} \Rightarrow x = 2h - a, y = 2k.$

Put these values of x and y in $y^2 = 4ax$, we get

$4k^2 = 4a(2h - a)$

$\Rightarrow 4k^2 = 8ah - 4a^2 \Rightarrow k^2 = 2ah - a^2$

So, locus of $P(h,k)$ is $y^2 = 2ax - a^2$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

Its directrix is

$x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0.$

18. (c) $\vec{a} + \vec{b} = 3\hat{i} + 9\hat{j} = 3(\hat{i} + 3\hat{j})$. Hence it is parallel to $(1, 3)$.

19. (a) Direction ratio of the line joining the point $(2, 1, -3), (-3, 1, 7)$ are (a_1, b_1, c_1)

$\Rightarrow (-3 - 2, 1 - 1, 7 - (-3))$

$\Rightarrow (-5, 0, 10)$

Direction ratio of the line parallel to line

$\frac{x-1}{3} = \frac{y}{4} = \frac{z+3}{5}$ are

(a_2, b_2, c_2)

$\Rightarrow (3, 4, 5)$

Angle between two lines,

$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$\cos\theta = \frac{(-5 \times 3) + (0 \times 4) + (10 \times 5)}{\sqrt{25 + 0 + 100}\sqrt{9 + 16 + 25}}$

$\Rightarrow \cos\theta = \frac{35}{25\sqrt{10}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{7}{5\sqrt{10}}\right)$

20. (c) $\sin\frac{\pi}{10}\sin\frac{3\pi}{10} = \sin 18°.\sin 54°$

$= \sin 18°.\cos 36° = \frac{\sqrt{5}-1}{4}.\frac{\sqrt{5}+1}{4} = \frac{1}{4}.$

21. (c) Given $f(x) = \frac{[(5+x)(2+x)]}{[1+x]}$

$f(x) = 1 + \frac{4}{1+x} + (5+x) = (6+x) + \frac{4}{(1+x)}$

$\Rightarrow$ $f'(x) = 1 - \frac{4}{(1+x)^2} = 0;$

$x^2 + 2x - 3 = 0$

$\Rightarrow$ $x = -3, 1$

Now $f''(x) = \frac{8}{(1+x)^3},$

$f''(-3) = -ve,$

$f''(1) = +ve$

Hence minimum value at $x = 1$

$f(1) = \frac{(5+1)(2+1)}{(1+1)} = \frac{6\times3}{2} = 9.$

22. (c) $(3+5\omega+3\omega^2)^2 + (3+3\omega+5\omega^2)^2$

$= (3+3\omega+3\omega^2+2\omega)^2 + (3+3\omega+3\omega^2+2\omega^2)^2$

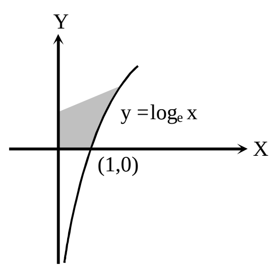
$(1+\omega+\omega^2 = 0, \omega^3 = 1)$

$= (2\omega)^2 + (2\omega^2)^2 = 4\omega^2 + 4\omega^4 = 4(-1) = -4.$

23. (d) Area

$A = \int_0^{\infty} \log x\, dx$

$= (x\log x - x)_0^{\infty} = \infty$



24. (d) $\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & -3 \end{vmatrix}$

$= 2(9-8) - 1(-3-2) - 1(4+3) = 7 - 7 = 0$

Hence, number of solutions is zero.

25. (c) n^{th} term of 1^{st} series $= 3 + (n-1)7 = 7n - 4$

n^{th} term of 2^{nd} series $= 63 + (n-1) = 2n + 61$

$\therefore$ we have, $7n - 4 = 2n + 61$

$\Rightarrow$ $n = 13$

26. (d) $\frac{192}{3}\int_{1/2}^{x} t^3 dt \le f(x) \le \frac{192}{2}\int_{1/2}^{x} t^3 dt$

$16x^4 - 1 \le f(x) \le 24x^4 - \frac{3}{2}$

$\int_{1/2}^{1}(16x^4 - 1)dx \le \int_{1/2}^{1} f(x)dx \le \int_{1/2}^{1}\left(24x^2 - \frac{3}{2}\right)dx$

$1 < \frac{26}{10} \le \int_{1/2}^{1} f(x)dx \le \frac{39}{10} < 12$

27. (a, d) Here, $0 < (x_1 - x_2)^2 < 1$

$\Rightarrow$ $0 < (x_1 + x_2)^2 - 4x_1x_2 < 1 \Rightarrow 0 < \frac{1}{\alpha^2} - 4 < 1$

$\Rightarrow$ $\alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

28. (b, c, d) $\frac{\pi}{2} < \alpha < \pi, \pi < \beta < \frac{3\pi}{2}$

$\Rightarrow$ $\frac{3\pi}{2} < \alpha + \beta < \frac{5\pi}{2} \Rightarrow \sin\beta < 0; \cos\alpha < 0$

$\Rightarrow$ $\cos(\alpha+\beta) > 0.$

29. (a, b) For the given line, point of contact for

$E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\left(\frac{a^2}{3}, \frac{b^2}{3}\right)$

and for $E_2 : \frac{x^2}{B^2} + \frac{y^2}{A^2} = 1$ is $\left(\frac{B^2}{3}, \frac{A^2}{3}\right)$

Point of contact $x + y = 3$ of and circle is (1, 2)

Also, general point on $x + y = 3$ can be taken as $\left(1 \mp \frac{r}{\sqrt{2}}, 2 \pm \frac{r}{\sqrt{2}}\right)$ where, $r = \frac{2\sqrt{2}}{3}$

So, required points are $\left(\frac{1}{3}, \frac{8}{3}\right)$ and $\left(\frac{5}{3}, \frac{4}{3}\right)$

Comparing with points of contact of ellipse,

$a^2 = 5, B^2 = 8$ $b^2 = 4, A^2 = 1$

$\therefore$ $e_1e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$ and $e_1^2 + e_2^2 = \frac{43}{40}$

30. (a, b, d) tangent at P, $xx_1 - yy_1 = 1$ intersects x axis at $M\left(\frac{1}{x_1}, 0\right)$

Slope of normal $= -\frac{y_1}{x_1} = \frac{y_1 - 0}{x_1 - x_2}$

$\Rightarrow \quad x_2 = 2x_1 \Rightarrow N \equiv (2x_1, 0)$

For centroid $\ell = \dfrac{3x_1 + \dfrac{1}{x_1}}{3}, m = \dfrac{y_1}{3}$

$\dfrac{d\ell}{dx_1} = 1 - \dfrac{1}{3x_1^2}$

$\dfrac{dm}{dy_1} = \dfrac{1}{3}, \dfrac{dm}{dx_1} = \dfrac{1}{3}\dfrac{dy_1}{dx_1} = \dfrac{x_1}{3\sqrt{x_1^2 - 1}}$

31. (a, c) Let $\int_0^{\pi} e^1(\sin^6 at + \cos^4 at)\ dt = A$

$I = \int_{\pi}^{2\pi} e^1(\sin^6 + at + \cos^4 at)\ dt$

Put $t = \pi + x \qquad dt = dx$

for $a = 2$ as well as $a = 4$

Similarly

$\int_{2\pi}^{3\pi} e^1(\sin^6 at + \cos^4 at)\ dt = e^{2\pi}A$

So, $L = \dfrac{A + e^{\pi}A + e^{2\pi}A + e^{3\pi}A}{A} = \dfrac{e^{4\pi} - 1}{e^{\pi} - 1}$

For both $a = 2, 4$

32. (d) $P(x) = ax^2 + b$ with a, b of same sign.

$P(P(x)) = a(x^2 + b)^2 + b$

If $x \in R$ or $ix \in R$

$\Rightarrow \quad x^2 \in R$

$\Rightarrow \quad P(x) \in R \quad \Rightarrow \quad P(P(x)) \neq 0$

Hence real or purely imaginary number cannot satisfy $P(P(x)) = 0$.

33. (c) $x^2 - 6x - 2 = 0$

$\Rightarrow \quad x^{10} - 6x^9 - 2x^8 = 0$

'α', β roots $\Rightarrow \quad a^{10} - 6\alpha^8 - 2\alpha^8 = 0 \qquad \ldots$ (i)

and $\beta^{10} - 6\beta^8 - 2\beta^8 = 0 \qquad \ldots$ (ii)

Equation (i) and (ii) we get,

$\alpha^{10} - \beta^{10} - 6(\alpha^9 - \beta^9) - 2(\alpha^8 - \beta^8) = 0$

$\Rightarrow \quad a_{10} - 6a_9 - 2a_8 = 0$

$\Rightarrow \quad \dfrac{a_{10} - 2a_8}{2a_9} = 3$

34. (a, d) Here, $0 < (x_1 - x_2)^2 < 1$

$\Rightarrow \quad 0 < (x_1 + x_2)^2 - 4x_1x_2 < 1$

$\Rightarrow \quad 0 < \dfrac{1}{\alpha^2} - 4 < 1$

$\Rightarrow \quad \alpha \in \left(-\dfrac{1}{2}, -\dfrac{1}{\sqrt{5}}\right) \cup \left(\dfrac{1}{\sqrt{5}}, \dfrac{1}{2}\right)$

35. (a) $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$

$x^2 - 5x + 5 = 1 \qquad x^2 + 4x - 60 = 0 \qquad x^2 - 5x + 5 = -1$

$x^2 - 5x + 4 = 0 \qquad x = -10, x = 6 \qquad x^2 - 5x + 6 = 0$

$x = 1, x = 4$

at x = 2, $x^2 + 4x - 60 = -48$ (even)

$\therefore \quad x = 2$ is valid

at $x = 3$, $x^2 + 4x - 60 = -39$ (odd)

$x = 3$ is invalid $\qquad x = 1, 2, 4, 6, -10$

36. (c) $4ax^2 + \dfrac{1}{x} \geq 1 \qquad \Rightarrow \quad y = 4ax^2 + \dfrac{1}{x}$

$\Rightarrow \quad y = \dfrac{dy}{dx} 8\alpha x - \dfrac{1}{x^2} = 0 \qquad \Rightarrow \quad x = \left(\dfrac{1}{8\alpha}\right)^{1/3}$

$\Rightarrow \quad f(x) = \dfrac{4\alpha x^3 + 1}{x} = \dfrac{1/2 + 1}{1/(8\alpha)^{1/3}} \qquad \Rightarrow \quad \dfrac{3}{2} \times (8\alpha)^{1/3} \geq 1$

$\Rightarrow \quad \alpha^{1/3} \geq \dfrac{1}{3} \Rightarrow \alpha \geq \dfrac{1}{27}$

37. (c) $x^2 - 2x\sec\theta + 1 = 0$

$\Rightarrow \quad x = \dfrac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2}$

$\Rightarrow \quad x = \sec\theta + \tan\theta, \sec\theta - \tan\theta \quad \Rightarrow \alpha_1 = \sec\theta - \tan\theta$

now $x^2 + 2x\tan\theta - 1 = 0$

$\Rightarrow \quad x = \dfrac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$\Rightarrow \quad x = -\tan\theta \pm \sec\theta$

$\Rightarrow \quad \alpha_2 = (\sec\theta - \tan\theta)$

$\Rightarrow \quad \beta_2 = -(\sec\theta + \tan\theta)$

$\therefore \quad \alpha_1 + \beta_2 = -2\tan\theta$

Alternate: (i) $x^2 - 2x\cos\theta + 1 = 0$

$x = \dfrac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2} = \sec\theta \pm \tan\theta$

$\Rightarrow \quad \alpha_1 = \sec\theta - \tan\theta$

$\Rightarrow \quad \beta_1 = \sec\theta + \tan\theta$

(ii) $x^2 + 2x\tan\theta - 1 = 0$

$\Rightarrow \quad x = \dfrac{-\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$\Rightarrow \quad x = -\tan\theta \pm \sec\theta$

$\Rightarrow \quad \alpha_2 = -\tan\theta + \sin\theta \qquad \beta_2 = -\tan\theta - \sec\theta$

$\Rightarrow \quad \alpha_1 + \beta_2 = -2\tan\theta$

38. (b, c, d) $ax + 2y = \lambda \quad 3x - 2y = \mu$

(a) $a=-3$ gives $\lambda=-\mu$ or $\lambda+\mu=0$ not for all λ, μ

(b) $a\neq -3$

$\Rightarrow \quad \Delta\neq 0$ where $\Delta=\begin{vmatrix} a & 2 \\ 3 & -2\end{vmatrix}=-2a-6$

$\therefore$ (b) is correct

(c) correct

(d) if $\lambda+\mu\neq 0$

$\Rightarrow \quad -3x+2y=\lambda$ ...(i)

& $\quad 3x-2y=\mu$ ...(ii)

Inconsistent $\Rightarrow$ (d) correct

39. (b) We have $\sum_{r=1}^{n}(x+r-1)(x+r)=10n$

$\Rightarrow \quad \sum_{r=1}^{n}(x^2+(2r-1)x+(r^2-r))=10n$

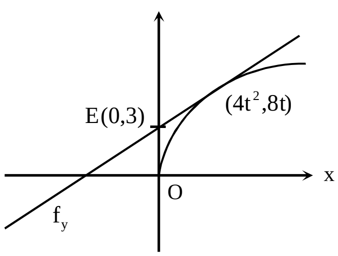
$\therefore$ On solving, we get $x^2+nx+\left(\frac{n^2-31}{3}\right)=0$

$\therefore \quad (2\alpha+1)=-n \Rightarrow \alpha=\frac{-(n+1)}{2}$ ...(i)

And $\alpha(\alpha+1)=\frac{n^2-31}{3}$ ...(ii)

Using equation (i) and (ii) $n^2=121$ or $n=11$

40. (a) $A\rightarrow 4; B\rightarrow 1; C\rightarrow 2; D\rightarrow 3$



Here, $y^2=16x, 0\leq y\leq 6$

Tangent at F, $yt=x+at^2$

at $\quad x=0, y=at=4t$

Also, $(4t^2, 8t)$ satisfy

$y=mx+c$

$\Rightarrow \quad 8t=4mt^2+3 \quad \Rightarrow \quad 4mt^2-8t+3=0$

$\therefore$ Area of $\Delta=\frac{1}{2}\begin{vmatrix} 0 & 3 & 1 \\ 0 & 4t & 1 \\ 4t^2 & 8t & 1\end{vmatrix}$

$=\frac{1}{2}\cdot 4t^2(3-4t)$

$A=2[3t^2-4t^3]$

$\therefore \quad \frac{dA}{dt}=2[6t-12t^2]$

$=-12t(2t-1)$

$\therefore$ Maximum at $t=\frac{1}{2}$ and

$4mt^2-8t+3=0$

$\Rightarrow \quad m-4+3=0 \Rightarrow$ (A) $m=1$

$G(0,4t)\Rightarrow G(0,2)$

(B) Area $=2\left(\frac{3}{4}-\frac{1}{2}\right)=\frac{1}{2}$

$(x_0, y_0)=(4t^2, 8t)=(1,4)$

(C) $y_0=4$

(D) $y_1=2$

41. (b) $A\rightarrow 3,5; B\rightarrow 3,5; C\rightarrow 1,2,4$

(A) $\because \quad y=\sin^{-1}(3x-4x^3)$

$$=\begin{cases} -\pi-3\sin^{-1}x, & -1\leq x\leq -\frac{1}{2} \\ 3\sin^{-1}x, & -\frac{1}{2}\leq x\leq \frac{1}{2} \\ \pi-3\sin^{-1}x, & \frac{1}{2}\leq x\leq 1 \end{cases}$$

$$\therefore \quad \frac{dy}{dx}.=\begin{cases} -\frac{3}{\sqrt{1-x^2}}, & -1\leq x\leq -\frac{1}{2} \\ \frac{3}{\sqrt{1-x^2}}, & -\frac{1}{2}\leq x\leq \frac{1}{2} \text{ (R, T)} \\ -\frac{3}{\sqrt{(1-x^2)}}, & \frac{1}{2}\leq x\leq 1 \end{cases}$$

(B) $\because \quad y=\cos^{-1}(4x^3-3x)$

$$=\begin{cases} 3\cos^{-1}x-2\pi, & -1\leq x\leq -\frac{1}{2} \\ 2\pi-3\cos^{-1}x, & -\frac{1}{2}\leq x\leq \frac{1}{2} \\ 3\cos^{-1}x, & \frac{1}{2}\leq x\leq 1 \end{cases}$$

$$\therefore \quad \frac{dy}{dx}=\begin{cases} \frac{-3}{\sqrt{1-x^2}}, & -1\leq x\leq -\frac{1}{2} \\ \frac{3}{\sqrt{1-x^2}}, & -\frac{1}{2}\leq x\leq \frac{1}{2} \text{ (R, T)} \\ \frac{-3}{\sqrt{(1-x^2)}}, & \frac{1}{2}\leq x\leq 1 \end{cases}$$

(C) $\because \quad y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$$= \begin{cases} 3\tan^{-1} x & , \quad -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \pi + 3\tan^{-1} x & , \quad x < -\frac{1}{\sqrt{3}} \\ -\pi + 3\tan^{-1} x & , \quad x > \frac{1}{\sqrt{3}} \end{cases}$$

$$\therefore \quad \frac{dy}{dx} = \begin{cases} \frac{3}{1+x^2} & , \quad -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}} \\ \frac{3}{1+x^2} & , \quad x < -\frac{1}{\sqrt{3}} \\ \frac{3}{1+x^2} & , \quad x > \frac{1}{\sqrt{3}} \end{cases}$$

42. (b) $A\to4; B\to5; C\to3; D\to3$

(A) $z = \frac{2i(x+iy)}{1-(x+iy)^2} = \frac{2i(x+iy)}{1-(x^2-y^2+2ixy)}$

Using $1-x^2 = y^2$

$Z = \frac{2ix-2y}{2y^2-2ixy} = -\frac{1}{y}.$

$\because \quad -1 \le y \le 1 \Rightarrow -\frac{1}{y} \le -1$ or $-\frac{1}{y} \ge 1.$

(B) For domain $-1 \le \frac{8.3^{x-2}}{1-3^{2(x-1)}} \le 1 \Rightarrow -1 \le \frac{3^x - 3^{x-2}}{1-3^{2x-2}} \le 1.$

Case-I: $\frac{3^x-3^{x-2}}{1-3^{2x-2}} - 1 \le 0$

$\Rightarrow \frac{(3^x-1)(3^{x-2}-1)}{(3^{2x-2}-1)} \ge 0 \Rightarrow x \in (-\infty, 0] \cup (1, \infty).$

Case-II: $\frac{3^x-3^{x-2}}{1-3^{2x}-2} + 1 \ge 0$

$\Rightarrow \frac{(3^{x-2}-1)(3^x+1)}{(3^x.3^{x-2}-1)} \ge 0 \Rightarrow x \in (-\infty, 1) \cup [2, \infty).$

So, $x \in (-\infty, 0] \cup [2, \infty).$

(C) $R_1 \to R_1 + R_3$

$$f(\theta) = \begin{vmatrix} 0 & 0 & 2 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix} = 2(\tan^2\theta + 1) = 2\sec^2\theta.$$

(D) $f'(x) = \frac{3}{2}(x)^{1/2}(3x-10) + (x)^{3/2} \times 3 = \frac{15}{2}(x)^{1/2}(x-2)$

Increasing, when $x \ge 2.$

43. (a) $A\to1; B\to2,3; C\to2,3,4,5; D\to3$

(A) $f'(x) > 0, \forall x \in (0, \pi/2)$

$f(0) < 0$ and $f(\pi/2) > 0$

So, one solution.

(B) Let (a, b, c) is direction ratio of the intersected line, then

$ak + 4b + c = 0$

$4a + kb + 2c = 0$

$\frac{a}{8-k} = \frac{b}{4-2k} = \frac{c}{k^2-16}$

We must have

$2(8-k) + 2(4-2k) + (k^2-16) = 0 \Rightarrow k = 2,4.$

(C) Let $f(x) = |x+2| + |x+1| + |x-1| + |x-2|$

$\Rightarrow$ k can take value 2, 3, 4, 5.

(D) $\int \frac{dy}{y+1} = \int dx$

$\Rightarrow f(x) = 2e^x - 1 \Rightarrow f(\ln 2) = 3$

44. (a, c, d) $|\vec{b} + \vec{c}| = |\vec{a}|$

$\Rightarrow |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c} = |\vec{a}|^2 \Rightarrow 48 + |\vec{c}|^2 + 48 = 144$

$\Rightarrow |\vec{c}| = 4\sqrt{3} \therefore \frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$

Also, $|\vec{a} + \vec{b}| = |\vec{c}|$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{c}|^2$

$\Rightarrow \vec{a}\cdot\vec{b} = -72 \quad \vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{a}\times\vec{b} = \vec{c}\times\vec{a} \Rightarrow |\vec{a}\times\vec{b} + \vec{c}\times\vec{a}| = 2|\vec{a}\times\vec{b}| = 48\sqrt{3}$

45. (c, d) $(Y^3Z^4 - Z^4Y^3)^T$

$= (Z^T)^4(Y^T)^3 - (Y^T)^3(Z^T)^4$

$= -Z^4Y^3 + Y^3Z^4$

$\Rightarrow$ symmetric $X^{44} + Y^{44}$ is symmetric

$X^4Z^3 - Z^3X^4$ skew symmetric

$X^{23} + Y^{23}$ skew symmetric.

46. (b, c) We get $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix}$

$= -648\alpha \; (R_3 \to R_3 - R_2; R_2 \to R_2 - R_1)$

$\begin{vmatrix} \alpha^2-2 & 4\alpha^2-2 & 9\alpha^2-2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 2 & 2 & 2 \end{vmatrix}$

$= -648\alpha \; (R_1 \to R_1 - R_2; R_3 \to R_3 - R_2)$

$$\Rightarrow \begin{vmatrix} -2\alpha^2 & -5\alpha^2 & -9\alpha^2-3 \\ -2\alpha & -2\alpha & 3+6\alpha \\ 0 & 0 & 2 \end{vmatrix} = -648\alpha$$

47. (b, d) Let the required plane be $x+z+\lambda y-1=0$

$\Rightarrow \quad \frac{|\lambda-1|}{\sqrt{\lambda^2+2}}=1 \Rightarrow \lambda=-\frac{1}{2}$

$\Rightarrow \quad P_3 \equiv 2x-y+2z-2=0$

Distance of P_3 from (α, β, γ) is 2

$\frac{|2\alpha-\beta+2\gamma-2|}{\sqrt{4\times1+4}}=2$

$\Rightarrow \quad 2\alpha-\beta+2\lambda+4=0$ and $2\alpha-\beta+2\lambda-8=0$

48. (a, b) Line L will be parallel to the line of intersection of P_1 and P_2

Let a, b and c be the direction ratios of line L

$\Rightarrow \quad a+2b-c=0$ and $2a-b+c=0 \Rightarrow a:b:c::1:-3:-5$

Equation of line L is $\frac{x-0}{1}=\frac{y-0}{-3}=\frac{z-0}{-5}$

Again foot of perpendicular from origin to plane P_1 is $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$

$\therefore$ Equation of project of line L on plane P_1 is

$\frac{x+\frac{1}{6}}{1}=\frac{y+\frac{2}{6}}{-3}=\frac{z-\frac{1}{6}}{-5}=k$

Clearly points $\left(0,-\frac{5}{6},-\frac{2}{3}\right)$ and $\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$ satisfy the line of projection i.e. M

49. (a, d) $P(at^2, 2at) \Rightarrow Q\left(\frac{16a}{t^2},-\frac{8a}{t}\right)$

$\Delta OPQ=\frac{1}{2}OP \cdot OQ$

$\Rightarrow \quad \frac{1}{2}\left|at\sqrt{t^2+4}\cdot\frac{a(4)}{t}\sqrt{\frac{16}{t^2}+4}\right|=3\sqrt{2}$

$t^2-3\sqrt{2}t+4=0 \Rightarrow t=\sqrt{2}, 2\sqrt{2}$

Hence, $P(at^2, 2at)\equiv P\left(\frac{t^2}{2}, t\right)$

$t=\sqrt{2} \Rightarrow P(1,\sqrt{2})$

$t=2\sqrt{2} \Rightarrow P(4,2\sqrt{2})$

50. (d) $x^2-x+1=0$

Roots are $-\omega,-\omega^2$

Let $\alpha=-\omega, \beta=-\omega^2$

$\alpha^{101}+\beta^{107}=(-\omega)^{101}+(-\omega^2)^{107}$

$=-(\omega^{101}+\omega^{214})$

$=-(\omega^2+\omega)=1$

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