# RELATIONS (XII, R. S. AGGARWAL)

## EXERCISE 1A (Pg.No.: 16)

1. Find the domain and range of the relation $R=\{(-1,1),(1,1),(2,4),(-2,4)\}$

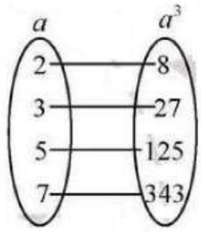
**Sol.** $dom(R)=\{-1,1,-2,2\}$, range $(R)=\{1,4\}$

**2.** Let R = {(a, $a^3$): a is a Prime Number less than 5}
Find the range of R.

**Sol.** Let A = { a : a is a prime number less then 5}
$\Rightarrow$ A = {2, 3}
Now, R = { (2, 8), (3, 27)}
By the definition of range R. Range(R) = {8, 27} Ans.

3. Let $R=\{(a,a^3): a \text{ is a prime number less than } 10\}$.
Find (i) $R$ (ii) dom$(R)$ and (iii) range $(R)$

**Sol.** $a$ is a prime number less than 10
$a=2,3,5,7$
$R=\{(2,8),(3,27),(5,125),(7,343)\}$
dom$(R)=\{2,3,5,7\}$, range $(R)=\{8,27,125,343\}$

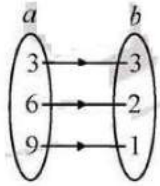



4. Let R = {x, y) : x + 2y = 8} be a
Relation on N.
Write range of R.

Sol $\because$ x + 2y = 8
$\Rightarrow$ x = 8 – 2y
Putting y = 1, we have x = 6 Putting y = 2, we have x = 4 Putting y = 3, we have x = 2
Here, R = { (2, 3), (4, 2), (6, 1) } Range (R) = (3, 2, 6)

5. List the elements of each of the following relations. Find the domain and range in each case.
$R_2=\{(a,b): a\in N, b\in N \text{ and } a+3b=12\}$

**Sol.** $R_2\{(a,b): a\in N, b\in N \text{ and } a+3b=12\}$
$R_2=\{(3,3),(6,2),(9,1)\}$,
dom$(R_2)=\{3,6,9\}$,
range $(R_2)=\{3,2,1\}$




6. Let $R=\{(a,b): b=|a-1|, a\in Z \text{ and } |a|<3\}$
Find the domain and range of R

Sol. Let, A = {a : a $\varepsilon$ z and |a| < 3}
$\Rightarrow$ A = {-2, -1, 0, 1, 2}
$\because$ R = {(a, b) : b = | a – 1 |, a $\varepsilon$ z and | a |< 3}
$\Rightarrow$ R = {(-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1)}

Clearly domain (R) = { -2, -1, 0, 1, 2}
Range(R) = {0, 1, 2, 3}

7. Let $R=\left\{\left(a,\frac{1}{a}\right): a\in N \text{ and } 1<a<5\right\}$

Find the domain and range of R

Sol. Let A = {a: a∈N, & 1 < a < 5}

⇒ A = {2, 3, 4}

$\therefore$ R = $\left\{\left(2,\frac{1}{2}\right),\left(3,\frac{1}{3}\right),\left(4,\frac{1}{4}\right)\right\}$

Domain = {2, 3, 4}    Range = $\left\{\frac{1}{2},\frac{1}{3},\frac{1}{4}\right\}$

8. Let $R=\{(a,b): a,b\in N \text{ and } b=a+5, a<4\}$

Find the domain and range of R

Sol. Let, A = {a : a ∈ N & a < 4}

⇒ A = {1, 2, 3}    ∴ R = {(1, 6), (2, 7), (3, 8)}

Clearly, Domain = {1, 2, 3}and Range = {6, 7, 8}

9. Let S be the set of all sets and let $R=\{(A,B): A\subset B\}$, i.e. A is proper subset of B. show that R is

(i) transitive    (ii) not reflexive    (iii) not symmetric

Sol. (i) transitivity: -

Let, A, B and C ∈ S, such that (A, B) & (B, C) ∈ R.

Let, A, B and C ∈ S, such that (A, B) & (B, C) ∈ R.

∵ (A, B) ∈ R ⇒ A ⊂ B ……(i)    (B, C) ∈ R ⇒ B ⊂ C ……(ii)

From (i) and (ii), we have A ⊂ C ⇒ (A, C) ∈ R

Thus, R is a transitive  Relation S.

(ii) Non – reflexive ; →

∵ A ⊄ A    ⇒ (A, A) ∈ R

thus, R is non reflexive.

(iii) Non. Symmetric :→

Let, A ⊂ B

∴ (A, B) ∈ R

But, B ⊄ A R

∴ (B, A) ∉ R ∵ (A, B) ∈ R & (B, A) ∉ R    ∴ R is non – symmetric

10. Let $A$ be the set of all points in a plane and let $O$ be the origin. Show that the relation $R$, defined by $R=\{(P,Q): OP=OQ\}$ is an equivalence relation.

**Sol.** Let $O$ denote the origin in the given plane. Then, $R=\{(P,Q): OP=OQ\}$.

We observe the following properties of relation $R$:

**Reflexivity :** For any point $P$ inset $A$, we have $OP=OP$

$\Rightarrow (P,P)\in R$. Thus, $(P,P)\in R$ for all $P\in A$. So, $R$ is reflexive.

**Symmetric :** Let $P$ and $Q$ be two points inset $A$ such that $(P,Q)\in R$

$\Rightarrow OP=OQ \Rightarrow OQ=OP \Rightarrow (Q,P)\in R$.

Thus, $(P,Q)\in R \Rightarrow (Q,P)\in R$ for $P,Q\in A$. So, $R$ is symmetric.

**Transitivity :** Let $P, Q$ and $S$ be three points in set $A$ such that $(P, Q) \in R$ and $(Q, S) \in R$

$\Rightarrow OP = OQ$ and $OQ = OS \quad \Rightarrow OP = OS \quad \Rightarrow (P, S) \in R$

So, $R$ is transitive. Hence, $R$ is an equivalence relation.

Let $P$ be a fixed point in set $A$ and $Q$ be a point in set $A$ such that $(P, Q) \in R$. Then, $(P, Q) \in R$.

$\Rightarrow OP = OQ \quad \Rightarrow Q$ moves in the plane in such a way that its distance from the origin.

$O(0, 0)$ is always same and is equal to $OP$.

$\Rightarrow$ Locus of $Q$ is circle with centre at the origin and radius $OP$.

Hence, the set of all points related to $P$ in the circle passing through $P$ with origin $O$ as centre.

11. On the set S of all real numbers, define a relation $R = \{(a, b) : a \le b\}$

Show that R is (i) reflexive (ii) transitive (iii) not symmetric

Sol. (i) Reflexivity : $\rightarrow$

Let, a be an arbitrary element on S.

$\because$ a $\le$ a $\Rightarrow$ (a, a) $\in$ R thus, R is reflective.

(ii) Transitivity : $\rightarrow$

Let, a, b & c $\in$ s such that (a, b) and (b, c) $\in$ s

$\because$ (a, b) $\in$ R $\Rightarrow$ a $\le$ b .......(i) and (b, c) $\in$ R $\Rightarrow$ b $\le$ c ...(ii)

from (i) and (ii) we have a $\le$ c $\Rightarrow$ (a, c) $\in$ R

Thus, R is transitive.

(iii) Non symmetry : $\rightarrow$

$\because$ $5 \le 6 \Rightarrow (5, 6) \in R$

*But*, $6 \nleq 5 \Rightarrow (6, 5) \in R$

thus, R is non symmetric

12. Let $A = \{1, 2, 3, 4, 5, 6\}$ and let $R = \{(a, b) : a, b \in A \text{ and } b = a + 1\}$

Show that R is (i) not reflexive (ii) not symmetric and (iii) not transitive

Sol. In roster form,

R = {(1, 2), (2, 3), (3, 4), (4, 5) (5, 6)}

Non – reflective: $\rightarrow$

$\because$ (1, 1) $\in$ R, but 1 $\in$ A thus, R is non-reflective.

Non – symmetric : $\rightarrow$ $\quad \because$ (1, 2) $\in$ R but, (2, 1) $\notin$ R

thus, R is non-symmetric.

Non transitive: $\rightarrow$ $\quad \because$ (1, 2) $\in$ R & (2, 3) $\in$ R

But, (1, 3) $\notin$ R $\quad$ thus, R is non- transitive

## EXERCISE 1B (Pg. No.: 18)

**1.** Define a relation on a set. What do you mean by the domain and range of a relation?

**Sol.** Relation in a set :- A relation $R$ in a set $A$ is a subset of $A \times A$.

Thus, $R$ is a relation is a set $A \Leftrightarrow R \subseteq A \times A$, if $(a, b) \in R$, then we say that $a$ is related to $b$ and write, $aRb$. If $(a, b) \notin R$, then we say that $a$ is not related to $b$ and write $a \not R b$.

Domain and range of a relation. Let $R$ be a relation in a set $A$. Then, the set of all first co-ordinate of element of $R$ is called the domain of $R$, written as $\text{dom}(R)$ and the set of all second co-ordinate of $R$ is called the range of $R$, write as range $(R)$.

2. Let A be the set of all triangles in a plane show that the relation $R = \{(\Delta_1, \Delta_2) : \Delta_1 \sim \Delta_2\}$ is an equivalence relation on A

Sol. Reflectivity:$\rightarrow$

Let, $\Delta$ be an arbitrary element on A.

$\because \Delta \sim \Delta \Rightarrow (\Delta, \Delta) \in R \ \forall \ \Delta \in R$ thus, R is reflective

Symmetricity :$\rightarrow$

Let, $\Delta_1$ and $\Delta_2 \in A$, such that $(\Delta_1, \Delta_2) \in R$

$\because (\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \sim \Delta_2$

$\Rightarrow \Delta_2 \sim \Delta_1 \Rightarrow (\Delta_2, \Delta_1) \in R$

thus, R is symmetric relation.

Transitivity : $\rightarrow$

Let, $\Delta_1$, $\Delta_2$ and $\Delta_3 \in$ a such that, $(\Delta_1, \Delta_2$ & $(\Delta_2, \Delta_3) \in R$

$\because (\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \sim \Delta_2$ .....(i)

& $(\Delta_2, \Delta_3) \in R \Rightarrow \Delta_2 \sim \Delta_3$ .....(ii)

From (i) and (ii) we have

$\Delta_1 \sim \Delta_3 \Rightarrow (\Delta_1, \Delta_3) \in R$

thus, R is transitive.

3. Let $R = \{(a, b) : a, b \in Z$ and $(a+b)$ is even$\}$. Show that $R$ is an equivalence relation on $Z$.

**Sol.** $R = \{(a, b) : a, b \in Z \text{ and } a+b \text{ is even}\}$

**Reflexive :** Let $a \in Z$ then $a + a = 2a$, which is even. $\therefore (a, a) \in R$, hence it is reflexive.

**Symmetric :** Let $(a, b) \in Z$ then $a + b = \text{even} \Rightarrow b + a = \text{even}$. $\therefore (b, a)$ also belongs to $R$.

$(a, b) \in R \Rightarrow (b, a) \in R$. Here $R$ is symmetric also.

**Transitive :** Let $(a, b)$ and $(b, c) \in R$ then $a + b = \text{even} = 2k$ and $b + c = \text{even} = 2r$

Adding then, $a + 2b + c = 2k + 2r \Rightarrow a + c = 2(k + r - b) \Rightarrow a + c = \text{even} \quad \therefore (a, c) \in R$

Hence, it is transitive also.

$\therefore$ Since, $R$ is reflexive, symmetric and transitive. Hence, $R$ is an equivalence.

4. Let $R = \{(a, b) : a, b \in Z$ and $(a-b)$ is divisible by $5\}$. Show that $R$ is an equivalence relation on $Z$.

**Sol.** $(a, b) \in R \Leftrightarrow a - b$ is divisible by 5.

**(i) Reflexive :** All $(a, a) \in R$ as $a - a = 0$ which is divisible by 5. Hence, $R$ is reflexive.

**(ii) Symmetric :** If $(a, b) \in R$, then $a - b$ is divisible by 5.

$\therefore\ a-b=5k$ and $(b-a)=-5k \quad \therefore\ (b-a)$ is also divisible by 5.

$\therefore\ (b,\ a)\in R \quad \therefore\ R$ is symmetric also.

**(iii) Transitive :** If $(a,\ b)$ and $(b,c)\in R$ then we must have according to definition of $R$

$a-b$ is divisible by 5. $\therefore\ a-b=5m$ and $b-c$ is divisible by 5

$b-c=5n$

Adding then we get, $a-c=5(m+n)$

$\therefore\ (a-c)$ is also divisible by 5. Hence, $(a,\ c)\in R$, hence the given ration is transitive also.

$\therefore\ R$ is reflexive, symmetric and transitive. Hence $R$ is an equivalence relation.

5. Show that the relation $R$ defined on the set $A=\{1,\ 2,\ 3,\ 4,\ 5\}$, given by $R=\{(a,\ b):|a-b| \text{ is even}\}$ is an equivalence relation.

**Sol.** We have, $R=\{(a,\ b):|a-b| \text{ is even}\}$, where $a,\ b\in A=\{1,\ 2,\ 3,\ 4\}$

We observe the following proposition of relation $R$:

**Reflexivity :** For any $a\in A$, we have $|a-a|=0$, which is even.

$\therefore\ (a,\ a)\in R$ for all $a\in A$. So, $R$ is reflexive.

**Symmetry :** Let $(a,\ b)\in R \Rightarrow |a-b|$ is even $\Rightarrow |b-a|$ is even $\Rightarrow (b,\ a)\in R$

Thus, $(a,\ b)\in R \Rightarrow (b,\ a)\in R$. So, $R$ is symmetric.

**Transitivity :** Let $(a,\ b)\in R$ and $(b,\ c)\in R$. Then, $(a,\ b)\in R$ and $(b,\ c)\in R$

$\Rightarrow |a-b|$ is even and $|b-c|$ is even

$\Rightarrow$ ($a$ and $b$ both are even or both are odd) and ($b$ and $c$ both are even or both are odd).

Now two cases arise :

**Case I :** When $b$ is even in this case, $(a,\ b)\in R$ and $(b,\ c)\in R$

$\Rightarrow |a-b|$ is even and $|b-c|$ is even $\Rightarrow a$ is even and $c$ is even [$\because b$ is even]

$\Rightarrow |a-c|$ is even $\Rightarrow (a,\ c)\in R$

**Case II :** When $b$ is odd in this case, $(a,\ b)\in R$ and $(b,\ c)\in R$

$\Rightarrow |a-b|$ is even and $|b-c|$ is even $\Rightarrow a$ is odd and $c$ is odd [$\because\ b$ is odd]

$\Rightarrow |a-c|$ is even $\Rightarrow (a,\ c)\in R$. Thus, $(a,\ b)\in R$ and $(b,\ c)\in R \Rightarrow (a,\ c)\in R$

So, $R$ is transitive. Hence, $R$ is an equivalence relation.

6. Show that the relation $R$ on $N\times N$, defined by $(a,\ b)R(c,\ d)\Leftrightarrow a+d=b+c$ is an equivalence relation.

**Sol.** We observe the following proposition of relation $R$:

**Reflexivity :** Let $(a,\ b)$ be an arbitrary element of $N\times N$

Then, $(a,\ b)\in N\times N \Rightarrow (a,\ b)\in N \Rightarrow a+b=b+a$

$\Rightarrow (a,\ b)R(a,\ b)$. Thus, $(a,\ b)R(a,\ b)$ for all $(a,\ b)\in N\times N$. So, $R$ is reflexive on $N\times N$.

**Symmetry :** Let $(a,\ b),\ (c,\ d)\in N\times N$ be such that $(a,\ b)R(c,\ d)$. Then, $(a,\ b)R(c,\ d)$

$\Rightarrow a+d=b+c \Rightarrow c+b=d+a$ [By commutativity of order on $N$ ]

$\Rightarrow (c,\ d)R(a,\ b)$, Thus $(a,\ b)R(c,\ d) \Rightarrow (c,\ d)R(a,\ b)$ for all $(a,\ b),\ (c,\ d)\in N\times N$

So, $R$ is symmetric on $N \times N$.

**Transitivity :** Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b)R(c, d)$ and $(c, d)R(e, f)$

Then, $(a, b)R(c, d) \Rightarrow a+d=b+c$, $(c, d)R(e, f) \Rightarrow c+f=d+e$

$\Rightarrow (a+d)+(c+f)=(b+c)+(d+e) \Rightarrow a+f=b+e \Rightarrow (a, b)R(e, f)$

Thus, $(a, b)R(c, d)$ and $(c, d)R(e, f) \Rightarrow (a, b)R(e, f)$ for all $(a, b), (c, d), (e, f) \in N \times N$

So, $R$ is transitive on $N \times N$.

Hence, $R$ being reflexive, symmetric and transitive is an equivalence relation on $N \times N$.

7. Let $S$ be the set of all real numbers and let $R = \{(a, b) : a, b \in S \text{ and } a = \pm b\}$

Show that $R$ is an equivalence relation on $S$.

**Sol.** As $a = \pm b$ and $a^2 = b^2$ have same meaning.

$\therefore$ Given relation $R$ becomes $R = \{(a, b) : a, b \in S \text{ and } a^2 = b^2\}$

Now, $(a, a) \in R \quad \because a^2 = a^2$ is true. $\therefore R$ is reflexive and if $(a, b) \in R$

$\Rightarrow a^2 = b^2 \Rightarrow b^2 = a^2 \Rightarrow (b, a) \in R \quad \therefore R$ is symmetric

Also, if $(a, b) \in R$ and $(b, c) \in R \Rightarrow a^2 = b^2$ and $b^2 = c^2 \Rightarrow a^2 = c^2 \Rightarrow (a, c) \in R$

$\therefore R$ is transitive.

8. Let $S$ be the set of all points in a plane and let $R$ be a relation in $S$ defined by $R = \{(A, B) : d(A, B) < 2 \text{ units}\}$, where $d(A, B)$ is the distance between the points $A$ and $B$. Show that $R$ is reflexive and symmetric but not transitive.

**Sol.** **(i) Reflexive :** $d(A, A) < 2 \Rightarrow (A, A) \in R$

**(ii) Symmetric :** $(A, B) \in R \Rightarrow d(A, B) < 2 \Rightarrow d(B, A) < 2 \quad [\because d(B, A) = d(A, B)]$

$\Rightarrow (B, A) \in R$

**(iii) Transitive :** Consider points $A(0, 0)$, $B(1.5, 0)$, $C(3, 0)$

Then, $d(A, B) = 1.5$, $d(B, C) = 1.5$ and $d(A, C) = 3$.

$\therefore d(A, C) < 2$ is not true.

Hence, $R$ is reflexive and symmetric but not transitive.

9. Let $S$ be the set of all real numbers. Show that the relation $R = \{(a, b) : a^2 + b^2 = 1\}$ is symmetric but neither reflexive nor transitive.

**Sol.** $S = \{(a, b) : a^2 + b^2 = 1\}$

**Reflexive :** The given relation $S$ is on the set of real numbers.

Let $a \in R$ then $a^2 + a^2 \neq 1$ for $a = 2, 3...$ Hence, $S$ is not reflexive.

**Symmetric :** Let $(a, b) \in R$ then we must have $a^2 + b^2 = 1$

$\therefore (b, a) \in R$, Hence $(a, b) \in R \Rightarrow (b, a) \in R$. $\therefore S$ is symmetric.

**Transitive :** Let $(a, b) \in R$ and $(b, c) \in R$.

$\because (\cos 30°, \sin 30°) \in R$ and $(\sin 30°, \cos 30°) \in R$, but $(\cos 30°, \cos 30°) \notin R$

Hence, $S$ is not transitive. $\therefore S$ is symmetric but neither reflexive nor transitive

10. Let $R=\{(a, b): a=b^2\}$ for all $a, b \in N$. Show that $R$ satisfies none of reflexivity, symmetric and transitivity.

**Sol.** We have, $R=\{(a, b): a=b^2\}$ where $a, b \in N$

**Reflexivity :** We observe that $2 \neq (2)^2 \Rightarrow 2$ is not related to 2, i.e., $(2, 2) \notin R$

So, $R$ is not reflexive

**Symmetry :** We observed that $4=(2)^2$ $(4, 2) \in R$ but $(2, 4) \notin R$ $[\because 4^2 \neq 2]$

So $R$ is not symmetric.

**Transitive :** Clearly, $(16, 4) \in R$ and $(4, 2) \in R$ but $(16, 2) \notin R$. Hence not transitive.

11. Show that the relation $R=\{(a, b): a>b\}$ on $N$ is transitive but neither reflexive nor symmetric.

**Sol.** We have, $R=\{(a, b): a>b\}$, where $a, b \in R$.

**Reflexivity :** For any $a \in R$, we have, $(a>b)$

$\Rightarrow (a, b) \notin R$ for all $a \notin R$ $\Rightarrow$ so is not reflexive.

**Symmetric :** We observe that $(3, 4) \in R$ but $(4, 3) \notin R$. So, $R$ is not symmetric.

**Transitive :** Let $(a, b) \in R$ and $(b, c) \in R$. Then, $(a, b) \in R$.

$\Rightarrow a>b$ and $b>c$ $\Rightarrow a>c$ $\Rightarrow (a, c) \in R$

So, $R$ is transitive. Hence, $R$ is transitive but neither reflexive nor symmetric.

12. Let $A=\{1, 2, 3\}$ and $R=\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$. Show that $R$ is reflexive but neither symmetric nor transitive.

**Sol.** Since $1, 2, 3 \in A$ and $(1, 1), (2, 2), (3, 3) \in R$ is for each $a \in A$, $(a, a) \in R$.

So, $R$ is reflexive. We observe that $(1, 2) \in R$ but $(2, 1) \notin R$. So, $R$ is not symmetric.

Also, $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$. So, $R$ is not transitive.

13. Let $A=\{1, 2, 3, 4\}$ and $R=\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (3, 2)\}$. Show that $R$ is reflexive and transitive but not symmetric.

**Sol.** $R=\{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3), (3, 2)\}$

As $(1, 1), (2, 2), (3, 3), (4, 4) \in R$ is reflexive and we observe that $(1, 2) \in R$ but $(2, 1) \in R$.

So, $R$ is not symmetric. Also, $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \in R$. So, $R$ is transitive.