ELECTROSTATIC POTENTIAL AND CAPACITANCE

CHAPTER – 2

ELECTROSTATIC POTENTIAL AND CAPACITANCE

Electric Field is Conservative

In an electric field work done by the electric field in moving a unit positive charge fromone point to the other, depends only on the position of those two points and does notdepend on the path joining them.

Electrostatic Potential

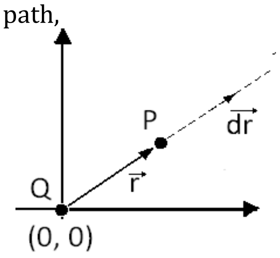
Electrostatic potential is defined as "Work required to be done against the force by electric field in bringing a unit positive charge from infinite distance to the given point inthe electric field us called the electrostatic potential (V) at that point". According to above definition the electric potential at point P is given by the formula

$V_p = -\int_{\infty}^{p} \vec{E}\,.\,\overrightarrow{dr}$

Electric potential is scalar quantity. SI units (J/C) called as volt (V).

Potential at a point Due to a Point Charge

Consider a point charge positive Q at the origin. To deter let P be the point at a distance 'r' from origin of coordinate axis. Since work done in electric field is independent of path,



We will consider radial path as shown in figure.

According to definition of electric potential we can use the equation $V_p = -\int_{\infty}^{p} \vec{E}\,.\,\overrightarrow{dr}$ And electric field E is $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec{r}$, given by.

$V_p = -\int_{\infty}^{p} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^3}\vec{r}\,.\,\overrightarrow{dr}$

$V_p = -\int_{\infty}^{p} \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,.\,dr$

$V_p = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]$

$V_p = \frac{Q}{4\pi\varepsilon_0 r}$

Electric Potential Due to Group of Point Charge

The potential at any point due to group of point charges is the algebraic sum of the potentials contributed at the same point by all the individual point charges

$V = V_1 + V_2 + V_3 + \ldots$

Electric Potential Difference

Electric potential difference is defined as "Work required to be done to take a unit positivecharge from one point (say P) to another point (say Q) against the electric field According to formula for potential at point P

$$V_P = -\int_{\infty}^{p} \vec{E} \cdot \overrightarrow{dr}$$

Thus, potential at point Q is given by

$$V_Q = -\int_{\infty}^{Q} \vec{E} \cdot \overrightarrow{dr}$$

From above formula potential difference between points Q and P is given by

$$V_Q - V_P = \left(-\int_{\infty}^{Q} \vec{E}\cdot\overrightarrow{dr}\right) - \left(-\int_{\infty}^{P} \vec{E}\cdot\overrightarrow{dr}\right)$$

$$V_Q - V_P = \int_{Q}^{\infty} \vec{E}\cdot\overrightarrow{dr} + \int_{\infty}^{P} \vec{E}\cdot\overrightarrow{dr}$$

$$V_Q - V_P = \int_{Q}^{P} \vec{E}\cdot\overrightarrow{dr}$$

$$V_Q - V_P = -\int_{P}^{Q} \vec{E}\cdot\overrightarrow{dr}$$

Or work done in moving charge from point P to point Q

SI unit of potential is V and dimensional formula $[M^1 L^2 T^{-3} A^{-1}]$

Electrostatic Potential Energy

The electric potential energy is defined as "The work required to be done against theelectric field in bringing a given charge *(q)*, from infinite distance to the given point in the electric field motion without acceleration is called the electric potential energy of that charge at that point."

From definition of electric potential energy and the electric potential we can write electricpotential energy of charge q at point P, as

$U_p = -\int_{\infty}^{p} q\vec{E}\cdot\overrightarrow{dr} = q\int_{\infty}^{p}\vec{E}\cdot\overrightarrow{dr} = qV_p$

The absolute value of the electric potential energy is not at all important, only the difference in its value is important. Here, in moving a charge q, from point P to Q, withoutacceleration, the work required to be done by the external force, shows the difference in the electric potential energies $(U_Q - U_P)$ of this charge q, at those two points.

$$U_Q - U_P = -q\int_{P}^{Q}\vec{E}\cdot\overrightarrow{dr}$$

Electric potential energy is of the entire system of the sources producing the field and the

Charge, for some configuration, and when the configuration changes the electric potential energy of the system also changes.

Potential Energy of a System of Two Point Charges

The potential energy possessed by a system of two - point charges q_1 And q_2 Separated by a distance ' r ' is the work done required to bring them to these arrangements from infinity. This electrostatic potential energy is given by

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

Electric Potential Energy of a System of Point Charges

The electric potential energy of such a system is the work done in assembling this system starting from infinite separation between any two-point charges.

For a system of point charges $q_1, q_2, q_3 \ldots . q_n$ The potential energy is

$$U = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}(i \neq j)$$

It simply means that we have to consider all the pairs that are possible

Important points related electrostatic potential energy

(i) Work done required by an external agency to move a charge q from A to B in an electric field with constant speed

(ii) When a charge q is let free in an electric field, it loses potential energy and gains kinetic energy, if it goes from A to B, then loss in potential energy = gain in kinetic energy

$$q(V_B - V_A) = \frac{1}{2}mV_B^2 - \frac{1}{2}mV_A^2$$

Q. Find work done by some external force in moving a charge $q = 2\mu C$ from infinity to a point where electric potential is 10^4 V

Sol. Work $W = Vq = (10^4\ V)(2\times10^{-6} C) = 2\times10^{-2}$ J

Q. The electric field at distance r perpendicularly from the length of an infinitely long wire is $E_{(r)} = \frac{\lambda}{2\pi\varepsilon_0 r}$, where λ is the linear charge density of the wire. Find the potential at a point having distance b from the wire with respect to a point having distance a from the wire $(a > b)$

Sol. Let V_a be reference point thus $V_a = 0$

$V_b - V_a = -\int_a^b \vec{E}\cdot\overrightarrow{dr}$

$V_b - V_a = -\int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}dr \because (\vec{E} \parallel \overrightarrow{dr})$

$V_b - V_a = -\frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r}$

$V_b - V_a = -\frac{\lambda}{2\pi\varepsilon_0}[\ln r]_a^b$

$V_b - V_a = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right)$

$\because V_a = 0$ reference

$V_{ba} = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{a}{b}\right)$

Q. An electric field is represented by $\vec{E} = Ax\hat{\imath}$, where $A = 10\ \text{V/m}^2$. Find the potential of the origin with respect to the point (10,20)m

Sol. $\vec{E} = Ax\hat{\imath} = 10x\hat{\imath}$

$V_{(0,0)} - V_{(10,0))} = -\int_{(10,20)}^{(0,0)} \vec{E}\cdot\overrightarrow{dr}$

$V_{(0,0)} - V_{(10,0))} = -\int_{(10,20)}^{(0,0)} (10x\hat{\imath})\cdot(dx\hat{\imath} + dy\hat{\jmath})$

$V_{(0,0)} - V_{(10,2))} = -\int_{10}^{0} (10xdx)$

$V_{(0,0)} - V_{(10,00)} = -10\left[\frac{x^2}{2}\right]_{10}$

$V_{(0,0)} - V_{(10,0))} = [0 - (-500)]$

$V_{(0,0)} - V_{(10,0)} = 500\ \text{V}$

Since V (10,20) is to be taken zero V (0, 0) = 500 volts

Electric Potential Due to Continuous Charge Distribution

The electric potential due to continuous charge distribution is the sum of potential of all the infinitesimal charge elements in which the distribution may be divided

$$\text{V} = \int \text{Dv} = \int \frac{dq}{4\pi\varepsilon_0 r}$$

Electric Potential Due to a Charged Ring

A charge Q is uniformly distributed over the circumference of a ring. Let us calculate the electric potential at an axial point at a distance r from the centre of the ring. The electric potential at P due to the charge element dq of the ring is given by

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{Z} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{R^2 + r^2}}$$

Hence electric potential at P due to the uniformly charged ring is given by

$$V = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{R^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{R^2+r^2}}\int dq$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+r^2}}$$

Electric Potential Due to a Charged Disc at a Point on The Axis

A non-conducting disc of radius ' R ' has a uniform surface charge density $\sigma\text{C/M}^2$ To calculate the potential at a point on the axis of the disc at a distance from its centre. Consider a circular element of disc of radius X′ And thickness dx. All points on this ring are at the same distance $Z = \sqrt{x^2 + r^2}$, from the point P. The charge on the ring is dq = σA dq = $\sigma(2\pi x dx)$ and so the potential due to the ring is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{Z} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi x dx)}{\sqrt{x^2+r^2}}$$

Since potential is scalar, there are no components. The potential due to the whole disc is given by

$$V = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{x}{\sqrt{x^2+r^2}}dx = \frac{\sigma}{2\varepsilon_0}\left[(x^2+r^2)^{1/2}\right]_0^R$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[(R^2+r^2)^{1/2} - r\right]$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[r\left(\frac{R^2}{r^2}+1\right)^{1/2} - r\right]$$

For large distance R/r ≪ 1 thus

$$r\left(\frac{R^2}{r^2}+1\right)^{1/2} \approx r\left(1 + \frac{R^2}{2r^2}\right)$$

Substituting above value in equation for potential

$$V = \frac{\sigma}{2\varepsilon_0}\left[r\left(1+\frac{R^2}{2r^2}\right) - r\right]$$

$$V = \frac{\sigma}{2\varepsilon_0}\frac{R^2 r}{2r^2}$$

$$V = \frac{\sigma}{2\varepsilon_0}\frac{R^2}{2r}\left(\frac{\pi}{\pi}\right)$$

Since $Q = \sigma\pi R^2$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Thus, at large distance, the potential due to disc is the same as that of point charge

Electric Potential Due to A Shell

A shell of radius R has a charge Q uniformly distributed over its surface.

(a) At an external point

At point outside a uniform spherical distribution, the electric field is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

Since $\vec{E}$ Is radial, $\vec{E}\cdot\overrightarrow{dr} = \text{E}dr$

Since $V(\infty) = 0$, we have

$$V(r) = -\int_0^r \vec{E}\cdot\overrightarrow{dr} = -\int_0^r \frac{Q}{4\pi\varepsilon_0 r^2}dr = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_\infty$$

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}\ (r > R)$$

Thu potential due to uniformly charged shell is the same as that due to a point charge Q at the Centre of the shell.

(b) At an internal point

At point inside the shell, $E = 0$. So, work done in bringing a unit positive charge from a point on the surface to any point inside the shell is zero. Thus, the potential has a fixed value at all points within the spherical shell and is equal to the potential at the surface.

Above results hold for a conducting sphere also whose charge lies entirely on the outer surface.

Electric Potential Due to A Non-Conducting Charged Sphere

A charge Q is uniformly distributed through a non-conducting volume of radius R.

(a) Electric potential at external point is given by equation. ' r ' is the distance of point from the center of the sphere

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

(b) Electric potential at an internal point is given by equation

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2R^3}[3R^2 - r^2]$$

Here R is the radius of the sphere and r is the distance of point from the centre.

Relation Between the Electric Field and Electric Potential

We know that electric potential from electric field is given by
$V_P = -\int_{\infty}^{P} \vec{E} \cdot \overrightarrow{dr}$
And potential difference between two points is given by
$V_Q - V_P = -\int_{P}^{Q} \vec{E} \cdot \overrightarrow{dr}$
If points P and Q are very close to each other, then for such a small displacement $\overrightarrow{dr}$, integration is not required and only term $\vec{E} \cdot \overrightarrow{dr}$ can be kept thus $dV = -\vec{E} \cdot \overrightarrow{dr}$

i) If $\overrightarrow{dr}$, is the direction of electric field $\vec{E}, \vec{E} \cdot \overrightarrow{dr} = Edr\cos\theta = Edr$
$dV = -Edr$
$E = -\frac{dV}{dr}$

This equation gives the magnitude of electric field in the direction of displacement $\overrightarrow{dr}$.
Here $\frac{dV}{dr}$ = potential difference per unit distance. It is called the potential gradient. Its unit is Vm^{-1}, which is equivalent to N/C.

(ii) If $\overrightarrow{dr}$, is not in the direction of $\vec{E}$, but in some other direction, the $-\frac{dV}{dr}$ would give us the component of electric field in the direction of that displacement
If electric field is in X direction and displacement is in any direction (in three dimensions) then
$\vec{E} = E_x\hat{\imath}$ and $\overrightarrow{dr} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$
$\therefore dV = -(E_x\hat{\imath}) \cdot (dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}) = -E_x dx$
$\therefore E_x = -\frac{dV}{dx}$

Similarly, if the electric field is Y and only in Z direction respectively, we would get $E_y = -\frac{dV}{dy}$ and $E_z = -\frac{dV}{dz}$
Now if the electric field also have three (x, y, z) components, then
$E_x = -\frac{\partial V}{\partial x}.\ E_y = -\frac{\partial V}{\partial y}.\ E_z = -\frac{\partial V}{\partial z}.$
And $\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial v}{\partial z}\hat{k}\right).$
Here $\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}$ shows the partial differentiation of $V(x, y, z)$ with respect to x, y, z respectively. Moreover, the potential differentiation of $V(x, y, z)$ with respect to x means the differentiation of V with respect to x only, by taking y and z in the formula of V as constant

Q. The electric potential in a region is represented as $V = 2x + 3y - z$. Obtain expression for the electric field strength

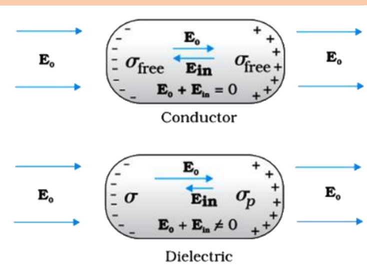
Sol. We know
$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial v}{\partial z}\hat{k}\right)$
Here
$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}[2x + 3y - z] = 2$
$\frac{\partial V}{\partial y} = \frac{\partial}{\partial x}[2x + 3y - z] = 3$
$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}[2x + 3y - z] = -1$
$\vec{E} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$

Q. The electrical potential due to a point charge is given by $V = \frac{1}{4\pi s_0}\frac{Q}{r}$.
Determine-
a) the radial component of the electric field
b) the x-component of the electric field

Sol.
a) The radial component of electric field
$E_r = -\frac{dV}{dr} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}.$
(b) In terms of rectangular components, the radial distance $r = (x^2 + y^2 + z^2)^{1/2}$; therefore, the potential function
$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x^2+y^2+z^2)^{\frac{1}{2}}}$
To find the x-component of the electric field, we treat y and z constants. Thus
$E_x = -\frac{\partial V}{\partial x}$ $\quad E_x = \frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+v^2+z^2)^{\frac{3}{2}}}$ $\quad E_x = \frac{1}{4\pi\varepsilon_0}\frac{r^3}{r^3}$

Dielectrics and Polarization

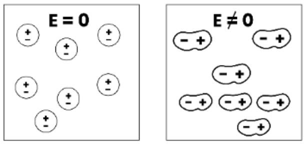
Dielectrics, in general, can be described as materials that are very poor conductors of electric current. They are basically insulators and contain no free electron. Dielectrics can be easily polarized when an electric field is applied to it. Thus, their behavior in an electric field is entirely different from that of conductors as would be clear from the following discussion.



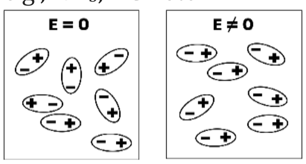
the difference in behavior of a conductor and a dielectric in an external electric field.

Types of dielectrics: Dielectrics are two types:

Non-Polar Dielectrics: When the Centre of positive charge coincides with the center of negative charge in a molecule, e.g., Nitrogen, Oxygen, CO_2 etc.



Polar Dielectrics: When the center of positive and negative charges does not coincide because of the asymmetric shape of the molecules, e.g., NH_3, HCL etc.



The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarized by the external field. Substances for which this assumption is true are called linear isotropic dielectrics. The induced dipole moments of different molecules add up giving a net dipole moment of the dielectric in the presence of the external field. Thus, in either case, whether polar or non-polar, a dielectric develops a net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarization and is denoted by P. For linear isotropic dielectrics

$$P = \chi_e E$$

where is a constant characteristic of the dielectric and is known as the electric susceptibility of the dielectric medium. It is possible to relate to the molecular properties of the substance.

Q. *Electric field inside the capacitor is 50 V/m and the dielectric constant = 4.5. What is polarization?*

Sol. Given: Dielectric Constant, $\varepsilon_r = 4.5$

Electric Field, E = 50 V/m

Susceptibility, $\chi_e = \varepsilon_r - 1 = 4.5 - 1 = 3.5$

Polarization, $P = \chi_e E = 3.5 \times 50 = 175\ C/m^2$

Hence, the polarization is $175\ C/m^2$.

Q. The relative dielectric constant of polystyrene is 3.5. What is the polarization produced when a 1.5 mm thick sheet of polystyrene is subjected to 240 V?

Sol. Given- Dielectric constant, $\varepsilon_r = 3.5$

$\varepsilon_0 = 8.85 \times 10^{-12}$ C / V m

Thickness, d = 1.5 mm = 1.5×10^{-3} m

Voltage, V = 240 V

Electric Field, $E = V/d = 240 / (1.5 \times 10^{-3})$ V/m $= 1.6 \times 10^5$ V/m

Polarization, $P = \varepsilon_0 (\varepsilon_r - 1) E$

$= 8.85 \times 10^{-12} \times (3.5 - 1) \times 1.5 \times 10^5$ C/m2

Capacitor

As we know that capacitance of a conductor depends on the presence of other conductors nearby. This fact is used to make a capacitor.

Capacitance $\boxed{C = \dfrac{q}{\Delta V}}$

Energy stored in capacitor

If C = capacitance of condenser,

Q = charge on condenser,

V = potential it is raised to them

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

This energy is stored as the electric field between plates. Then

energy density of the field is $\frac{1}{2}\varepsilon_0 E^2$

Parallel plate capacitor

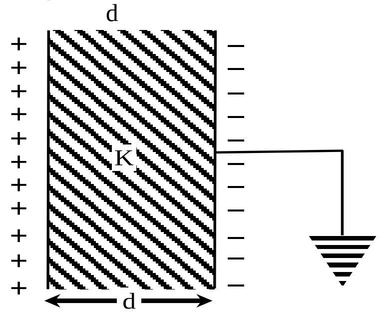
$$C = \frac{\epsilon_0 KA}{d}$$

A- area of each plate,

d - distance between plates,

K-dielectric constant of the medium between the plates.

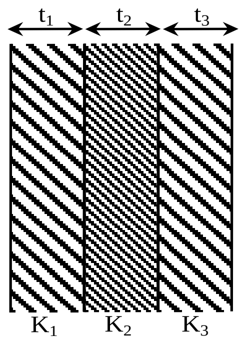
$$C = \frac{\epsilon_0 A}{d}$$



(ii) When a number of mediums are placed between the plates-

$$C = \frac{\epsilon_0 A}{\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3}\right)}$$

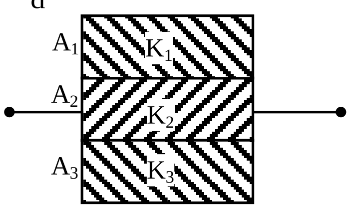
where $t_1 + t_2 + t_3 = d$



(iii) In this combination equivalent capacitance

$C = \frac{K_1 \in_0 A_1}{d} + \frac{K_2 \in_0 A_2}{d} + \frac{K_3 \in_0 A_3}{d}$

$C = \frac{\in_0}{d} (K_1A_1 + K_2A_2 + K_3A_3)$



(iv) If a metal sheet of thickness 't' is kept between the two plates, then $C = \frac{\in_0 A}{d - t}$

(v) If a thin metal foil is placed between the plates, then the capacity remains unaffected.

(vi) $\frac{C_{medium}}{C_{air}} = K$

Note

A parallel plate capacitor is connected to a battery. What will happen when

Case-I Distance between the plates is reduced-

(i) Same potential difference
(ii) Increased capacitance
(iii) Charge will increase
(iv) Electric field will increase
(v) Energy stored will increase.

Case-II Dielectric is kept between the plates-

(i) Same potential difference
(ii) Capacitance will increase
(iii) Charge will increase
(iv) No change in electric field
(v) Energy stored will increase, this increase is a result of work done to place the dielectric between the plates.

Note

A parallel plate capacitor is charged and then battery is removed, when

Case-I Distance between the plates is decreased-

(a) Charge will remain the same
(b) Capacitance will increase
(c) Potential difference across plates will decrease
(d) No change in electric field.
(e) Energy stored will decrease. This decrease in energy is due to work done in bringing the plates closer.

Case -II A dielectric is placed between the plates-

(a) No change in charge
(b) Increase in capacitance
(c) Potential difference will decrease
(d) Electric field between the plates will decrease.
(e) Energy stored will be reduced.

Note

(i) If nothing is mentioned then assume battery is disconnected and Q is constant.
(ii) A parallel plate capacitor is connected to battery (V-constant) and a slab of dielectric constant K is inserted between the plates then total energy given by battery is divided into two parts.
(a) Half is used to insert the slab
(b) Half is stored in form of E.P.E.

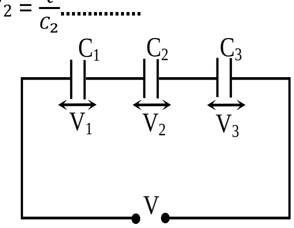
Combination of Capacitor

There are two possible combinations-
(1) Series (2) Parallel

Series combination

(1) Charge on each condenser is same i.e.
$Q = C_1V_1 = C_2V_2 = C_3V_3 = ...$

(2) Potential difference across each condenser is inversely proportional to it's capacity
i.e. V µ 1/C
So, $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$..............



(3) Total potential difference (V) in the circuit is sum of the potential differences across each capacitor i.e.

$V = V_1 + V_2 + V_2 + V_3$ ....

or $V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + ...$

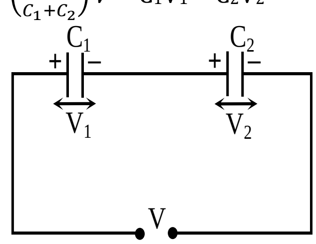
$\Rightarrow \frac{V}{Q} = \frac{1}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + ...$

Series combination of two capacitors

(1) $C = \frac{C_1C_2}{C_1+C_2}$

(2) $V_1 = \frac{C_2}{C_1+C_2}V$ & $V_2 = \frac{C_1}{C_1+C_2}V$

(3) $Q = \left(\frac{C_1C_2}{C_1+C_2}\right)V = C_1V_1 = C_2V_2$

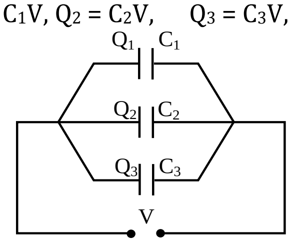


Parallel Combination

(i) Potential difference across each capacitor is same and is equal to the potential difference applied across the circuit.

(ii) Charge on each capacitor is proportional to its capacitance i.e.

$Q_1 = C_1V$, $Q_2 = C_2V$, $Q_3 = C_3V$, ... $Q \propto C$



(iii) Total charge in circuit

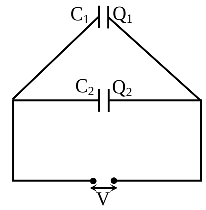
$Q = Q_1 + Q_2 + Q_3 + ...$

(iv) If C is the total capacitance of the circuit, then

$C = C_1 + C_2 + C_3 + ...$

For parallel combination of two capacitances-

(1) $C = C_1 + C_2$



(2) $Q_1 = \frac{C_1}{C_1+C_2} Q = C_1V$

$$Q_2 = \frac{C_2}{C_1+C_2} Q = C_2V$$

(3) The total energy stored in parallel combination of two capacitors is

$U = U_1 + U_2 = \frac{1}{2} C_1V^2 + \frac{1}{2} C_2V^2$

$= \frac{1}{2} V^2(C_1 + C_2)$

Q. Two capacitors of capacitance $C_1 = 6\mu F$ and $C_2 = 3\mu F$ are connected in series across a cell of emf 18 V. Calculate

(i) the equivalent capacitance

(ii) the potential difference across each capacitor

(iii) the charge on each capacitor

Sol. (i) $C = \frac{C_1C_2}{C_1+C_2} = \frac{6\times3}{6+3} = 2\mu F$

(ii) $V_1 = \left(\frac{C_2}{C_1+C_2}\right)V = \left(\frac{6}{6+3}\right) \times 18 = 12$ V

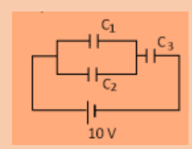
(iii) In series combination charge on each capacitor is same $Q = CV = 2 \times 10^{-6} \times 18 = 36\mu C$

Q. In the circuit shown the capacitors are $C_1 = 15\mu F, C_2 = 10\mu F$ and $C_3 = 25\mu F$. Find

(i) the equivalent capacitance of the circuit

(ii) the charge on each capacitor

(iii) the potential difference across each capacitor



Sol. (i) C_1 and C_2 are parallel thus $C_{12} = 15 + 10 = 25\mu F$

This C_{12} is in series with C_3 thus

$C = 12.5\mu F$ $\frac{1}{C} = \frac{1}{25} + \frac{1}{25} = \frac{2}{25}$

(ii) Q is the total charge supplied by the cell $= CV = (12.5 \times 10)C$

Charge on $C_1 = Q_1 = \left(\frac{C_1}{C_1+C_2}\right) Q = \left(\frac{15}{15+10}\right) \times 125 = 75\mu C$

Charge on $C_2 = Q_2 = \left(\frac{C_2}{C_1+C_2}\right) Q = \left(\frac{10}{15+10}\right) \times 125 = 50\mu C$

Charge on $C_3 = Q = 125\mu C$

(iii) p.d across $C_1 = V_1 = Q_1/C_1 = 75/15 = 5$ V

p.d across $C_2 = V_2 = V_1 = 5$ V

p.d across $C_3 = V_3 = Q_3/C_3 = 125/25 = 5$ V

Energy Stored in Charged Capacitor

In order to establish a charge on the capacitor, work has to be done on the charge. This work is stored in the form of the potential energy of the charge. Such a potential energy is called the energy of capacitor.

Suppose the charge on a parallel plate capacitor is Q. In this condition each plate of the capacitor is said to be lying in the electric field of the other plate.

The magnitude of the uniform electric field produced by one plate of capacitor is $= \frac{\sigma}{2\varepsilon_0}$ Where $\sigma i \frac{Q}{A}$ and A is area of plate

Hence taking arbitrarily the potential on this plate as zero, that of the other plate at distance d from it will be $= \frac{\sigma}{2\varepsilon_0} d$

The potential energy of the second plate will be = (potential) (charge Q on it)

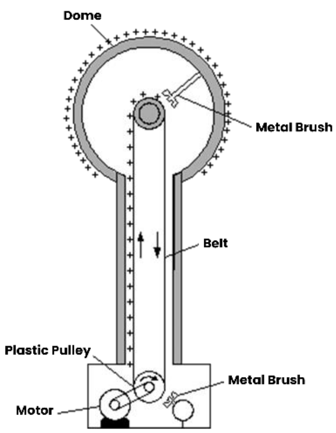
Potential energy stored in capacitor $= \frac{\sigma}{2\varepsilon_0} dQ$

$U_E = \frac{\sigma dQ}{2\varepsilon_0} = \frac{Q}{A}\frac{dQ}{2\varepsilon_0} = \frac{Q^2}{2(\varepsilon_0 A/d)} = \frac{1}{2}\frac{Q^2}{C}$

OR

$U_E = \frac{1}{2}CV^2 = \frac{1}{2}VQ$

Van De Graff Generator



Principle:

Suppose there is a positive charge Q, on an insulated conducting spherical shell of radius R, as shown in figure. At the centre of this shell, there is a conducting sphere of radius r $(r < R)$, having a charge q.

Here electric potential on the shell of radius R is,

$$V_R = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

And the electric potential on the spherical shell of radius r is,

$$V_r = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

It is clear from these two equations that the potential on the smaller sphere is more and the potential difference between them is

$$V_r - V_R = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right) - \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

$$V_r - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Hence if the smaller sphere is brought in electrical contact with bigger sphere, then the charge goes from smaller to bigger sphere. Thus, charge can be accumulated to a very large amount on the bigger sphere and there by its potential can be largely increased

SUMMARY

- **Electrostatic Potential at a Point:**
  It is the work done by per unit charge by an external agency, in bringing a charge from infinity to that point.
- **Electrostatic Potential due to a Charge at a Point:**
  $$V(r) = \frac{2}{4\pi\varepsilon_0}\frac{Q}{r}$$
- **The electrostatic potential at a point with position vector r due to a point dipole of dipole moment p place at the origin is**
  $$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\vec{r}}{r^2}$$
  The result is true also for a dipole (with charges –q and q separated by 2a) for r >> a.
- **For a charge configuration q_1, q_2, …… q_n with position vectors r_1, r_2, ….r_n, the potential at a point P is given by the superposition principle,**
  $$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1p}} + \frac{q_2}{r_{2p}} + \cdots + \frac{q_n}{r_{np}}\right)$$
  Where r_{1p} is the distance between q_1, and P, as and so on.
- **Electrostatics Potential Energy Stored in a System of Charges:**
  It is the work done (by an external agency) in assembling the charges at their locations.
- **Electrostatic Potential Energy of Two Charges q_1, q_2, at r_1, r_2:**
  $$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$
  Where r_{12} is distance between q_1 and q_2
- **Potential Energy of a Charge q in an External Potential:**
  V(r)= qv(r)
- **Potential Energy of a Dipole of Dipole Moment p in a Uniform Electric Field:**
  E = –p.E.
- **Equipotential Surface:**
  An equipotential surface is a surface over which potential has a constant value.
  (A) For a point charge, concentric spheres centered at a location of the charge are equipotential surfaces.
  (B) The electric field E at a point is perpendicular to the equipotential surface through the point.
  (C) E is in the direction of the steepest decrease of potential.
- **Capacitance C of a System of Two Conductors Separated by an Insulator:**
  It is defined as,
  $$C = \frac{Q}{V}$$
  Where Q and – Q are the charges on the two conductors V is the potential difference between them.
- Capacitance is determined purely geometrically, by the shapes, sizes, and relative positions of the two conductors.
- **Capacitance C of a parallel plate capacitor (with vacuum between the plates):**
  $$C = \varepsilon_0 \frac{A}{d}$$
  Where A is the area of each plate and d the separation between them.
- **For capacitors in the series combination:**
  The total capacitance C is
  $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots.$$
- **For capacitors in the parallel combination:**
  The total capacitance C is
  $$= {}_1 + C_2 + C_3 + \ldots\ldots$$
  Where C_1, C_2, C_3 C ... C are individual capacitances
- **The energy U stored in a capacitor of capacitance C, with charge Q and voltage V:**
  $$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$
- **The electric energy density (energy per unit volume) in a region with electric field:**
  $$(1/2)\varepsilon_0 E^2$$
- **The potential difference between the conductor (radius r_0) inside & outside spherical shell (radius R):**
  $$\phi(r_0) - \phi(R) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_0} - \frac{1}{R}\right)$$
  Which is always positive.
- **When the medium between the plates of a capacitor filled with an insulating substance: Changes observed are as follows:**
  i. Polarization of the medium gives rise to a field in the opposite direction.
  ii. The net electric field inside the insulating medium is reduced.
  iii. Potential difference between the plates is thus reduced.
  iv. Capacitance C increases from its value when there is no medium (vacuum).
- **Electrostatic Shielding:**
  A conductor has a cavity with no charge inside the cavity, then no matter what happens outside the conductor. Even if there are intense electric fields outside the conductor, the cavity inside has, shielding whatever is inside the cavity from whatever is outside the cavity. This is called electrostatic shielding.

MIND MAP

Electrostatic Potential & Capacitance

Electric Potential

- Work done per unit test charge by an external agent in moving the test charge from reference point to the desired point S.I. unit J/C

  V_0 = Work done/charge

In 1774, Alessandro Volta wrote treatise " on the forces of attraction of electric fire"

Electric potential due to a point charge

$V_P = Q/4\pi\varepsilon_0 r$

Potential due to a system of charges

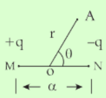
$$V = \frac{1}{4\pi\varepsilon_0}\sum \frac{Q_i}{r_i}$$

Potential due to a dipole

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

Where $p = qd$

$\theta = \angle AON$

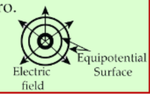


At $\theta = 0$, $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$ [axial position]

At, $\theta = 90°$, $V = 0$ [equitorial position]

Equipotential Surface

- Potential is same at all the points of the surface
- Component of electric field parallel to an equipotential surface is zero.



Electric Potential Energy

- It is negative of work done by the electric force as the configuration of the system changes.
- $Ur_2 - Ur_1 = -W = \frac{q_1 \cdot q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$
- If the separation between charges is 'r' then $U_{(r)} = \frac{q_1\, q_2}{4\pi\varepsilon_0 r}$
- Potential Difference,

  $V_B - V_A = \frac{U_B - U_A}{q}$

  $U_B - U_A$ = change in Potential Energy

  q = test charge

Potential Energy of a dipole

$dU = pE\sin\theta d\theta$

If we choose P.E. of dipole to be zero when

$\theta = 90°$ then

$$U_\theta - U_{90°} = \int_{90°}^{\theta} pE\sin\theta\, d\theta$$

$U_\theta = -pE\cos\theta = -\vec{p}\times\vec{E}$

If it is rotated through angle θ against the torque

Conductors and Insulators

Conductors	Insulators
Such a material Which when placed in an electric field, the free electrons move in a direction opposite to the field.	Such a material in which electrons are tightly bound, & when exposed in an electric field, Electrons does not move *i.e.* having no free electrons

- Electric field inside a conductor is zero
- Electric field is always perpendicular to the charged surface
- In static state, there will be no additional charge in a conductor

Dielectric

- $F = \frac{q_1 q_2}{4\pi K\varepsilon_0 r^2}$, K = dielectric constant of medium
- A dielectric is an electrical insulator that can be polarized by an applied electric field.

Capacitance, $C = \frac{Q}{V}$

Capacitance of a parallel plate capacitor

$C = K\varepsilon_0 A/d$, K = dielectric constant

Capacitance when material slab inserted between them

$C = K\varepsilon_0 A/[Kd - x(K-1)]$

where x = thickness of the slab inserted

Capacitance of a spherical capacitor

$$C = 4\pi\varepsilon_0 / \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For isolated sphere

$C = 4\pi\varepsilon_0 R$

Energy stored in a capacitor

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

Series grouping of capacitors

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + - - - - + \frac{1}{C_n}$$

For two, $C = \frac{C_1 C_2}{C_1 + C_2}$

Parallel grouping of capacitors

$C = C_1 + C_2 + C_3 + - - - - + C_n$

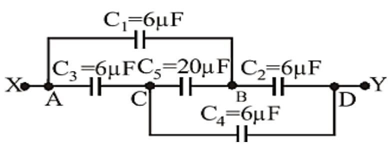
for two, $C = C_1 + C_2$

PRACTICE EXERCISE

MCQ

Q1. A parallel plate condenser is immersed in an oil of dielectric constant 2. The field between the plates is
(a) increased, proportional to 2
(b) decreased, proportional to $\frac{1}{2}$
(c) increased, proportional to -2
(d) decreased, proportional to $-\frac{1}{2}$

Q2. What is the effective capacitance between points *X* and *Y*?



(a) 24 μ*f* (b) 18 μ*f*
(c) 12 μ*f* (d) 6 μ*f*

Q3. Two identical particles each of mass m and having charges -q and +q are revolving in a circle of radius r under the influence of electric attraction. Kinetic energy of each particle is $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$
(a) $kq^2/4r$ (b) $kq^2/2r$
(c) $kq^2/8r$ (d) kq^2/r

Q4. A parallel plate condenser with a dielectric of dielectric constant *K* between the plates has a capacity *C* and is charged to a potential *V* volt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is
(a) zero (b) $\frac{1}{2}(K-1)CV^2$
(c) $\frac{CV^2(K-1)}{K}$ (d) $(K-1)\,CV^2$

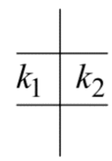
Q5. If a slab of insulating material 4×10^{-5} m thick is introduced between the plates of a parallel plate capacitor, the distance between the plates has to be increased by 3.5×10^{-5} m to restore the capacity to original value. Then the dielectric constant of the material of slab is
(a) 8 (b) 6
(c) 12 (d) 10

Q6. A unit charge moves on an equipotential surface from a point A to point B, then
(a) $V_A - V_B = +$ ve (b) $V_A - V_B = 0$
(c) $V_A - V_B = -$ ve (d) it is stationary

Q7. Identify the false statement.
(a) Inside a charged or neutral conductor, electrostatic field is zero
(b) The electrostatic field at the surface of the charged conductor must be tangential to the surface at any point
(c) There is no net charge at any point inside the conductor
(d) Electrostatic potential is constant throughout the volume of the conductor

Q8. The 1000 small droplet of water each of radius r and charge Q, make a big drop of spherical shape. The potential of big drop is how many times the potential of one small droplet?
(a) 1 (b) 10
(c) 100 (d) 1000

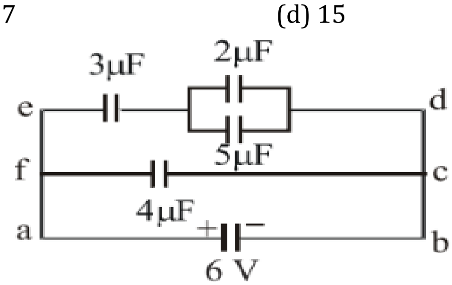
Q9. A parallel plate condenser is filled with two dielectrics as shown. Area of each plate is A m^2 and the separation is *t m*. The dielectric constants are k_1 and k_2 respectively. Its capacitance in farad will be



(a) $\frac{\varepsilon_o A}{t}(k_1+k_2)$ (b) $\frac{\varepsilon_o A}{t}.\frac{k_1+k_2}{2}$
(c) $\frac{2\varepsilon_o A}{t}(k_1+k_2)$ (d) $\frac{\varepsilon_o A}{t}.\frac{k_1-k_2}{2}$

Q10. A one microfarad capacitor of a TV is subjected to 4000 V potential difference. The energy stored in capacitor is
(a) 8J (b) 16J
(c) 4×10^{-3} J (d) 2×10^{-3}J

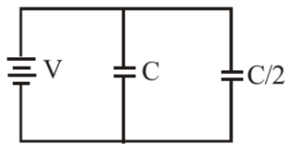
Q11. In the circuit given below, the charge in μc, on the capacitor having 5 μf is
(a) 4.5 (b) 9
(c) 7 (d) 15



Q12. An alpha particle is accelerated through a potential difference of 10^6 volt. Its kinetic energy will be
(a) 1 mev (b) 2 mev
(c) 4 mev (d) 8 mev

Q13. A parallel plate capacitor having a separation between the plates d, plate area A and material third of the material is replaced by another material with dielectric constant 2K, so that effectively there are two capacitors one with area 1/3A, dielectric constant 2K and another with area 2/3A and dielectric constant K. If the capacitance of this new capacitor is C then $\frac{C}{C_0}$ is
(a) 1 (b) 4/3
(c) 2/3 (d) 1/3

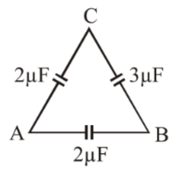
Q14. Two condensers, one of capacity C and other of capacity C/2 are connected to a V-volt battery, as shown. The work done in charging fully both condensers is



(a) $\frac{1}{4}CV^2$ (b) $\frac{3}{4}CV^2$

(c) $\frac{1}{2}CV^2$ (d) $2CV^2$

Q15. Three capacitors are connected in the arms of a triangle ABC as shown in figure 5 V is applied between A and B. The voltage between B and C is



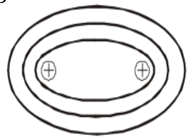
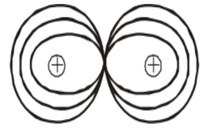
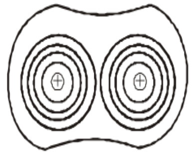
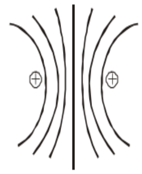
(a) 2V (b) 1V
(c) 3V (d) 1.5V

Q16. Two parallel metal plates having charges + Q and – Q face each other at a certain distance between them. If the plates are now dipped in kerosene oil tank, the electric field between the plates will
(a) remain same (b) become zero
(c) increases (d) decrease

Q17. Choose the wrong statement about equipotential surfaces.
(a) It is surface over which the potential is Constant
(b) The electric field is parallel to the equipotential surface
(c) The electric field is perpendicular to the equipotential surface
(d) The electric field is in the direction of steepest decrease of potential

Q18. Equipotential at great distance from a collection of charges whose total sum is not zero area approximately
(a) spheres (b) planes
(c) paraboloids (d) ellipsoids

Q19. Which of the following figure shows the correct equipotential surfaces of a system of two positive charges?

(a) 
(b) 
(c) 
(d) 

Q20. Two identical metal plates are given positive charges Q_1 and Q_2 ($<Q_1$) respectively. If they are now brought close together form a parallel plate capacitor with capacitance C, the potential difference between them is

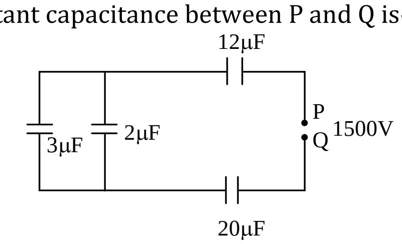
(a) $\frac{Q_1+Q_2}{2C}$ (b) $\frac{Q_1-Q_2}{C}$

(c) $\frac{Q_1-Q_2}{C}$ (d) $\frac{Q_1-Q_2}{2C}$

Q21. The capacitor of capacitance 4mf and 6mf are connected in series. A potential difference of 500 volts is applied to the outer plates of the two-capacitor system. Then the charge on each capacitor is numerically-
(a) 6000 C (b) 1200C
(c) 1200 mc (d) 6000mc

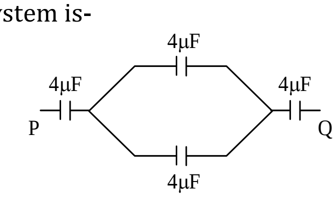
Q22. Three capacitors each of capacitance 1mf are connected in parallel. To this combination, a fourth capacitor of capacitance 1mf is connected in series. The resultant capacitance of the system is-
(a) 4mf (b) 2mf
(c) 4/3 mf (d) 3/4 mf

Q23. In the circuit diagram shown in the adjoin figure, the resultant capacitance between P and Q is-



(a) 47 mf (b) 3mf
(c) 60 mf (d) 10 mf

Q24. Four condensers each capacity 4mf are connected as shown in figure $V_P - V_Q$ = 15 volts. The energy stored in the system is-



(a) 2400 ergs (b) 1800ergs
(c) 3600 ergs (d) 5400 ergs

Q25. Two capacitors each of 0.5 mf capacitance are connected in parallel and are then charged by 200 volts. D.C. supply. The total energy of their charges (in joules) is-
(a) 0.01 (b) 0.02 (c) 0.04 (d) 0.06

ASSERTION AND REASONING

Directions: These questions consist of two statements, each printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.
(a) If both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) If both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) If the Assertion is correct but Reason is incorrect.
(d) If both the Assertion and Reason are incorrect.

Q1. Assertion: If the distance between parallel plates of a capacitor is halved and dielectric constant is three times, then the capacitance becomes 6 times.
Reason: Capacity of the capacitor does not depend upon the nature of the material.

Q2. Assertion: A parallel plate capacitor is connected across battery through a key. A dielectric slab of dielectric constant K is introduced between the plates. The energy which is stored becomes K times.
Reason: The surface density of charge on the plate remains constant or unchanged.

Q3. Assertion: According to classical theory the proposed path of an electron in Rutherford atom model will be parabolic.
Reason: According to electromagnetic theory an accelerated particle continuously emits radiation.

Q4. Assertion: Bohr had to postulate that the electrons in stationary orbits around the nucleus do not radiate.
Reason: According to classical physics all moving electrons radiate.

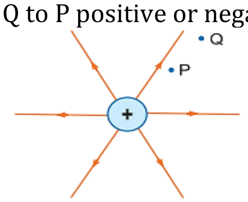
Q5. Assertion: Electrons in the atom are held due to coulomb forces.
Reason: The atom is stable only because the centripetal force due to Coulomb's law is balanced by the centrifugal force.

VERY SHORT ANSWER QUESTIONS

Q1. Name the physical quantity whose SI unit is JC^{-1}. Is it a scalar or a vector quantity?

Q2. Why is the electrostatic potential inside a charged conducting shell constant throughout the volume of the conductor?

Q3. Figure shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? Give reason.

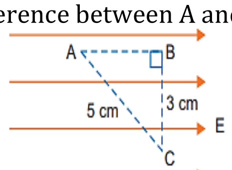


Q4. Depict the equipotential surfaces for a system of two identical positive point charges placed a distance 'd' apart.

Q5. Why do the equipotential surfaces due to a uniform electric field not intersect each other?

SHORT ANSWER QUESTIONS

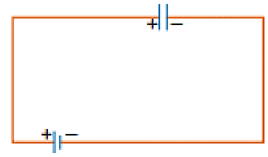
Q1. Three points A, B and C lie in a uniform electric field (E) of 5 × 103 NC–1 as shown in the figure. Find the potential difference between A and C.



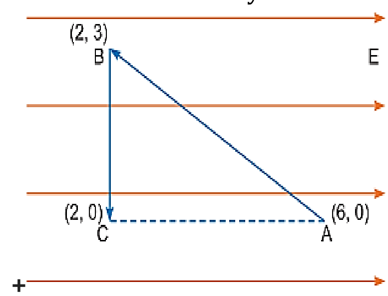
Q2. Plot a graph comparing the variation of potential 'V' and electric field 'E' due to a point charge 'Q' as a function of distance 'R' from the point charge.

Q3. What is electrostatic shielding? How is this property used in actual practice? Is the potential in the cavity of a charged conductor zero?

Q4. Why does current in a steady state not flow in a capacitor connected across a battery? However momentary current does flow during charging or discharging of the capacitor. Explain.



Q5. A test charge 'q' is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure.
(i) Calculate the potential difference between A and C.
(ii) At which point (of the two) is the electric potential more and why?



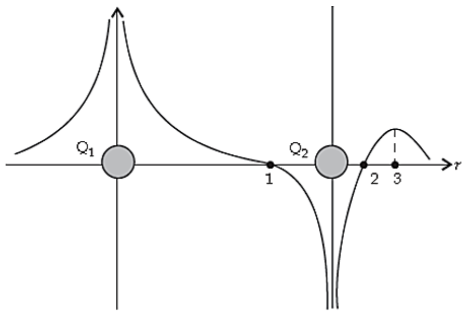
LONG ANSWER QUESTIONS

Q1. Derive an expression for the electric potential at a point due to an electric dipole. Mention the contrasting features of electric potential of a dipole at a point as compared to that due to a single charge.

Q2. (a) Explain briefly, using a proper diagram, the difference in behavior of a conductor and a dielectric in the presence of external electric field.
(b) Define the term polarization of a dielectric and write the expression for a linear isotropic dielectric in terms of electric field.

CASE STUDY QUESTIONS

Q1. Read the para given below and answer the questions that follow:
Potential of Two Point Charges. The potential at any observation point P of a static electric field is defined as work done by the external agent (or negative of work done by electrostatic field) in slowly bringing a unit positive point charge form infinity to the observation point. The figure given below shows the potential variation along the line of charges. Two point charges Q, and Q, lie along a line at a distance from each other.



(i) At which the point 1,2 and 3 is the electric field is zero?
(a) 1 (b) 2
(c)3 (d)both (a) and (b)

(ii) The signs of charges Q_1 and Q_2 respectively are
(a) Positive and negative
(b) Negative and positive
(c) Positive and positive
(d) Negative and negative

(iii) Which of the charges Q_1 and Q_2 is greater in magnitude?
(a) Q_1
(b) Q_2
(c) Same
(d) Cannot determined

(iv) which of the following statement is not true?
(a) Electrostatic force is a conservative force
(b) Potential energy of charge q at a point is the work done per unit charge in bringing a charge form any point to infinity.
(c) When two like charges lie infinite distance apart, their potential energy is zero.
(d) Both (a) and (c)

NUMERICAL TYPE QUESTIONS

Q1. A bullet of mass 2 g is having a charge of 2 μc. Through what potential difference must it be accelerated, starting from rest, to acquire a speed of 10 m/s?

Q2. The electric potential at a point in free space due to charge Q coulomb is $Q \times 10^{11}$ Volts. Determine the he electric field at that point.

Q3. In a certain region of space with volume 0.2 M^3, the electric potential is found to be 5 V throughout. Then determine the magnitude of the electric field in this region.

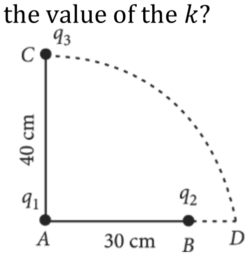
Q4. Two metal spheres, one of radius R and the other of radius $2R$ respectively have the same surface charge density σ. They are brought in contact and separated. What will be the new surface charge densities on them?

Q5. A hollow metallic sphere of radius 10 cm is charged such that potential of its surface is 80 V. Then what will be the potential at the center of the sphere.

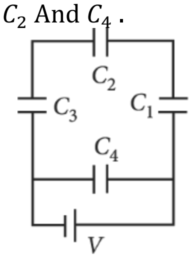
Q6. Two metallic spheres of radii 1 cm and 2 cm are given charges 10^{-2} C and 5×10^{-2} C respectively. If they are connected by a conducting wire, then find the final charge on the smaller sphere.

Q7. An electric dipole of moment p is placed in the position of stable equilibrium in uniform electric field of intensity E. This is rotated through an angle θ from the initial position. Then fine the potential energy of the electric dipole in the final position.

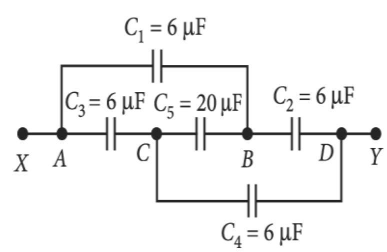
Q8. Two charges q_1 And q_2 Are placed 30 cm apart, as shown in the figure. A third charge q_3 Is moved along the arc of a circle of radius 40 cm from C to D. The change in the potential energy of the system is $\frac{q_3}{4\pi\varepsilon_0}k$, what will be the value of the k?



Q9. A network of four capacitors of capacity equal to $C_1 = C, C_2 = 2C, C_3 = 3C$ and $C_4 = 4C$ are connected to a battery as shown in the figure. Determine the ratio of the charges on C_2 And C_4 .



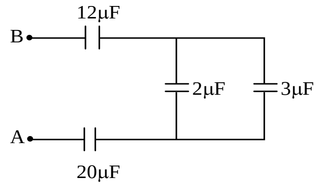
Q10. What is the effective capacitance between points X and Y?



HOMEWORK EXERCISE

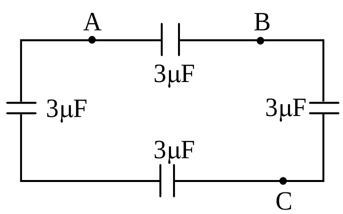
MCQ

Q1. Four capacitors are connected as shown in the adjoining figure. The potential difference between A and B is 1500 volts. The energy stored in 2mf capacitance will be-



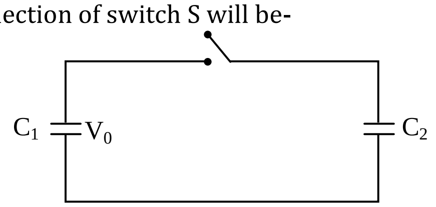
(a) 5.06J (b) 0.506J
(c) 50.6J (d)none of the

Q2. Four capacitors of each capacity 3mf are connected as shown in the adjoining figure. The ratio of equivalent capacitance between A and B and between A and C will be-



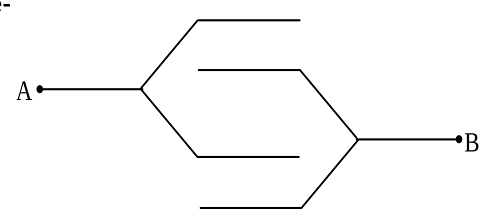
(a) 4 : 3 (b) 3 : 4
(c) 2 : 3 (d) 3 : 2

Q3. A capacitor of capacity C_1 is charged to the potential of V_0. On disconnecting with the battery, it is connected with a capacitor of capacity C_2 as shown in the adjoining figure. The ratio of energies before after the connection of switch S will be-



(a) $(C_1 + C_2)/C_1$ (b) $C_1 / (C_1 + C_2)$
(c) C_1C_2 (d) C_1/C_2

Q4. Four plates of the same area A of cross-section are joined as shown in the figure. The distance between each plate is d. The equivalent capacity across AB will be-



(a) $\frac{2\varepsilon_0 A}{d}$ (b) $\frac{3\varepsilon_0 A}{d}$
(c) $\frac{3\varepsilon_0 A}{2d}$ (d) $\frac{\varepsilon_0 A}{d}$

Q4. A capacitor 4 mf charged to 50V is connected to another capacitor of 2mf charged to 100V with plates of like charges connected together. The total energy before and after connection in multiples of (10^{-2} J) is-
(a) 1.5 and 1.33 (b) 1.33 and 1.5
(c) 3.0 and 2.67 (d) 2.67 and 3.0

Q5. Two capacitors of 3pf and 6pf are connected in series and a potential difference of 5000V is applied across the combination. They are then disconnected and reconnected in parallel. The potential between the plates is-
(a) 2250V (b) 1111V
(c) 2.25×10^6V (d) 1.1×10^6 V

Q6. Capacitors are used in electrical circuits where appliances need rapid:
(a) Current (b) Voltage
(c) Watt (d) Resistance

Q7. Which of the following is called electrical energy tank?
(a) Resistor (b) Inductance
(c) Capacitor (d) Motor

Q8. The net charge on a capacitor is
(a) 2q (b) q/2
(c) 0 (d) infinity

Q9. If the maximum circumference of a sphere is 2 m, then its capacitance in water would be
(Dielectric constant of water = 81)
(a) 27.65 pf (b) 2385 pf
(c) 236.5 pf (d) 2865 pf

Q10. Consider a uniform electric field in the z-direction. The potential is a constant
(a) for any x for a given z
(b) for any y for a given z
(c) on the x-y plane for a given z
(d) all of these

ASSERTION AND REASONING

Directions: These questions consist of two statements, each printed as Assertion and Reason. While answering these questions, you are required to choose any one of the following four responses.
(a) If both Assertion and Reason are correct and the Reason is a correct explanation of the Assertion.
(b) If both Assertion and Reason are correct but Reason is not a correct explanation of the Assertion.
(c) If the Assertion is correct but Reason is incorrect.
(d) If both the Assertion and Reason are incorrect.

Q1. Assertion- The capacity of a given conductor remains same even if charge is varied on it.
Reason- Capacitance depends upon nearly medium as well as size and shape of conductor.

Q2. Assertion - The whole charge of a conductor cannot be transferred to another isolated conductor.
Reason- The total transfer of charge from one to another is not possible.

VERY SHORT ANSWER QUESTIONS

Q1. "For any charge configuration, equipotential surface through a point is normal to the electric field." Justify.

Q2. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 V. What is the potential at the center of the sphere?

Q3. Do free electrons travel to region of higher potential or lower potential?

SHORT ANSWER QUESTIONS

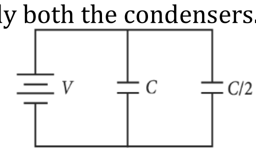
Q1. The two graphs are drawn below, show the variations of electrostatic potential (V) with 1 /r (r being the distance of field point from the point charge) for two-point charges q_1 and q_2.
(i) What are the signs of the two charges?
(ii) Which of the two charges has the larger magnitude and why?

Q2. Draw 3 equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along Z-direction. How are these surfaces different from that of a constant electric field along Z-direction?

NUMERICAL TYPE QUESTIONS

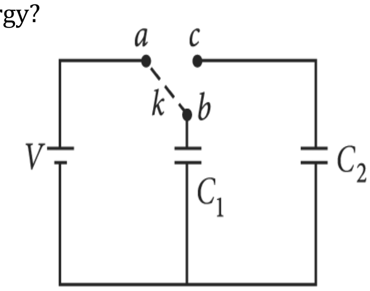
Q1. A capacitor is charged with a battery and energy stored is U. After disconnecting battery another capacitor of same capacity is connected in parallel to the first capacitor. Then find the energy stored in each capacitor.

Q2. Two condensers, one of capacity C and other of capacity $C/2$ are connected to a V-volt battery, as shown in the figure. Then determine the work done in charging fully both the condensers.



Q3. A series combination of n_1 Capacitors, each of value C_1, is charged by a source of potential difference 4V. When another parallel combination of n_2 Capacitors, each of volume C_2, is charged by a source of potential difference V, it has the same (total) energy stored in it, as the first combination has. Determine the value of C_2, in terms of C_1.

Q4. Two identical capacitors C_1 And C_2 Of equal capacitance are connected as shown in the circuit. Terminals a and b of the key k are connected to charge capacitor C_1 Using battery of emf V volt. Now disconnecting a and b, the terminals b and c are connected. Due to this, what will be the percentage loss of energy?



Q5. The capacitance of a parallel plate capacitor with air as medium is 6 μf. With the introduction of a dielectric medium, the capacitance becomes 30 μf. Find the permittivity of the medium. ($\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ M^{-2}}$)

Q6. There is an electric field E in x-direction. If the work done on moving a charge of 0.2 C through a distance of 2 m along a line making an angle 60° with x-axis is 4 J, then what is the value of E ?

Q7. An electric dipole of moment $\vec{p}$ Is lying along a uniform electric field $\vec{E}$. Find the work done in rotating the dipole by 90°.

Q8. If potential (in volts) in a region is expressed as $V(x, y, z) = 6xy - y + 2yz$, then what will be the electric field (in N/C) at point (1, 1, 0)?

Q9. In a region, the potential is represented by $V(x, y, z) = 6x - 8xy - 8y + 6yz$, where V is in volts and x, y, z are in metres. Then find the electric force experienced by a charge of 2 coulomb situated at point (1, 1, 1).

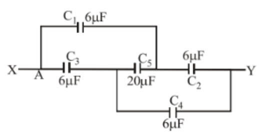
Q10. A parallel plate condenser with oil between the plates (dielectric constant of oil $K = 2$) has a capacitance C. If the oil is removed, then find the capacitance of the capacitor.

PRACTICE EXERCISE SOLUTIONS

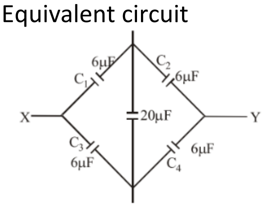
MCQ

S1. (b) In oil, C becomes twice, V becomes half. Therefore, E = V/d becomes half.

S2. (d)

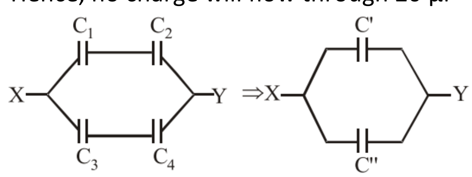


Equivalent circuit

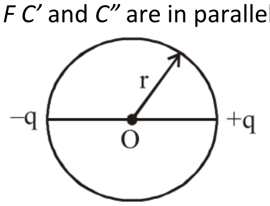


Here, $\frac{C_1}{C_3} = \frac{C_2}{C_4}$

Hence, no charge will flow through 20 μf



C_1 and *C_2* are in series, also *C_3* and *C_4* are in series. Hence, *C′* = 3 μ*f*, *C″* = 3 μ *F C′* and *C″* are in parallel.



Hence net capacitance = *C′* + *C″* = 3 + 3 = 6 μ*f*

S3. (c) $\frac{mv^2}{r} = \frac{kq^2}{(2r)^2}; \text{mv}^2 = \frac{\text{kq}^2}{4r}$

Kinetic energy of each particle

$= \frac{1}{2}\text{mv}^2 = \frac{\text{kq}^2}{8r}$

S4. (a) The potential energy of a charged capacitor is given by $U = \frac{Q^2}{2C}$.

If a dielectric slab is inserted between the plates, the energy is given by

$\frac{Q^2}{2KC}$, where *K* is the dielectric constant.

Again, when the dielectric slab is removed slowly its energy increases to initial potential energy. Thus, work done is zero.

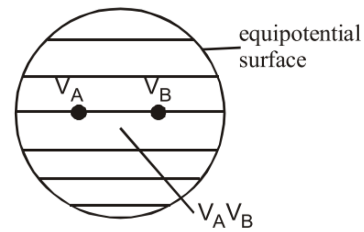
S5. (a) As x = $t\left(1 - \frac{1}{K}\right)$, where x is the addition distance of plate, to restore the capacity of original value.

∴ $3.5 \times 10^{-5} = 4 \times 10^{-5}\left(1 - \frac{1}{K}\right)$.

Solving, we get, K = 8.

S6. (b) At. Equipotential surface, the potential is same at any point i.e., $V_A = V_B$ as shown in figure. Hence no work is required to move unit change from one point to another i.e.,

$V_A\text{-}V_B = \frac{W}{\text{unit charge}} = 0 \Rightarrow W = 0$



S7. (b)
(i) Electrostatic field is zero inside a charged conductor or neutral conductor.
(ii) Electrostatic field at the surface of a charged conductor must be normal to the surface at every point.
(ii) There is no net charge at any point inside the conductor and any excess charge must reside at the surface.
(iv) Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.
(v) Electric field at the surface of a charged conductor is

$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$

S8. (c) Volume of big drop = 1000 × volume of each small drop

$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \Rightarrow R = 10r$

$\because \text{V}=\frac{\text{kq}}{r}\text{andV'}=\frac{\text{kq}}{R} \times 1000$

Total charge on one small droplet is q and on the big drop is 1000q.

$\Rightarrow \frac{\text{V'}}{V} = \frac{1000r}{R} = \frac{1000}{10} = 100$

$\because \text{V'} = 100V$

S9. (b) The two capacitors are in parallel so

$C = \frac{\varepsilon_0 A}{t \times 2}(k_1 + k_2)$

S10. (a) $E = \frac{1}{2}\text{CV}^2 = \frac{1}{2} \times 1 \times 10^{-6} \times (4000)^2 = 8\text{J}.$

S11. (b) Potential difference across the branch de is 6 V. Net capacitance of de branch is 2.1 μf

So, q = CV

⇒ q = 2.1 × 6 μc

⇒ q = 12.6 μc

Potential across 3 μf capacitance is

$V = \frac{12.6}{3} = 4.2\text{volt}$

Potential across 2 and 5 combinations in parallel is 6-4.2 = 1.8V

So, q′ = (1.8)(5) = 9 μc

S12. (b) Charge on α particle, q = 2 e.

K.E. = work done = q × V = 2e × 10^6V =2 mev.

S13. (b) $C_0 = \frac{k\epsilon_0 A}{d}$

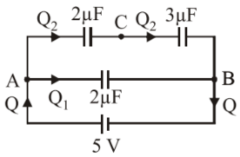
$C = \frac{k\epsilon_0 2}{3d} + \frac{2k\epsilon_0 A}{3d} = \frac{4}{3}\frac{k\epsilon_0 A}{d}$

$\because \frac{C}{C_0} = \frac{\frac{3d}{k\epsilon_0 A}}{d} = \frac{4}{3}$

S14. (b) Work done = Change in energy

$= \frac{1}{2}\left(C + \frac{C}{2}\right)V^2 = \frac{1}{2}\left(\frac{3C}{2}\right)V^2 = \frac{3CV^2}{4}$

S15. (a) The equivalent circuit diagram as shown in the figure.



The equivalent capacitance between A and B is

$C_{eq} = \frac{2\mu F \times 3\mu F}{2\mu F + 3\mu F} + 2\mu F = \frac{16}{5}\mu F$

Total charge of the given circuit is

$Q = \frac{16}{5}\mu F \times 5V = 16\mu C$

$Q_1 = (2\mu F) \times 5V = 10\mu C$

$\therefore Q_2 = Q - Q_1 = 16\mu C - 10\mu C = 6\mu C$

☐ Voltage between B and C is

$V_{BC} = \frac{Q_2}{3\mu F} = \frac{6\mu C}{3\mu F} = 2V$

S16. (d) Electric field

$E = \frac{\sigma}{\varepsilon} = \frac{Q}{A\varepsilon}$

$\mathcal{E}$ of kerosine oil is more than that of air. As $\mathcal{E}$ increases, E decreases.

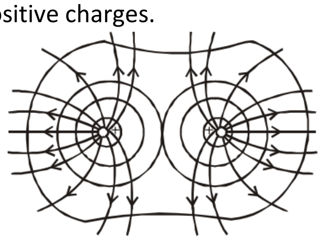
S17. (b) Electric lines of force are always perpendicular to an equipotential surface.

S18. (a) Here we have to find out the shape of equipotential surface, these surfaces are perpendicular to the field lines, so there must be electric field which cannot be without charge. So, the collection of charges, whose total sum is not zero, with regard to great distance can be considered as a point charge. The equipotential due to point charge are spherical in shape as electric potential due to point charge q is given by

$V = K_e \frac{q}{r}$

This suggest that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points which are at same potential, form spherical surface. The lines of field from point charge are radial. So, the equipotential surface perpendicular to field lines from a sphere.

S19. (c) Equipotential surfaces are normal to the electric field lines. The following figure shows the equipotential surfaces along with electric field lines for a system of two positive charges.



S20. (c) Let plate A plate B be carrying charges Q_1 and Q_2 respectively. When they are brought closer, they induce equal and opposite charges on each other i.e. $-Q_2$ on plate A and $-Q_1$ on plate B.

Therefore, net charge on plate A = $Q_1 - Q_2$ and

Net charge on plate B = $-(Q_1 - Q_2)$,

So, the charge on the capacitor = $Q_1 - Q_2$.

∴ Potential different between the plates

$V = \frac{Q_1 - Q_2}{C}$

S21. (c) $C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = 2.4\ \mu f$

Charge flown through the circuit

= $2.4 \times 500 \times 10^{-6}$C = 1200

S22. (d) Resultant of parallel combination

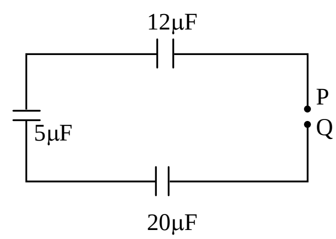
= 3 μf

Total capacitance of combination is

$C_{eq} = \frac{3\mu F \times 1\mu F}{(3+1)\mu F}$

$C_{eq} = \frac{3}{4}\ \mu f$

S23. (b) The given circuit can be drawn as



Where C = (3 + 2) μf = 5 μf

$\frac{1}{C_{PQ}} = \frac{1}{5} + \frac{1}{20} + \frac{1}{12} = \frac{20}{60} = \frac{1}{3}$

C_{PQ} = 3 μf

S24. (b) Total capacitance of given system]

$\frac{1}{C_{eq}} = \frac{5}{8}$ Þss $C_{eq} = \frac{8}{5}\ \mu f$

Energy stored = $\frac{1}{2} c_{eq} v^2$

= $\frac{1}{2} \times \frac{8}{5} \times 10^{-6} \times 225$

= 180×10^{-6} joule

= $180 \times 10^{-6} \times 10^{7}$ ergs

= 1800 ergs

S25. (b) C_{eq} = 1 μf, E = $\frac{1}{2} c_{eq} v^2$

Þ E = $\frac{1}{2} \times 1 \times 10^{-6} \times 200 \times 200$

= 2×10^{-2} = 0.02J

ASSERTION AND REASONING

S1. (b) Explanation- C = [($A\epsilon_0$k)/d]

Thus, C is not function of nature of material.

If d → (d/2) & k → 3k then C' = [($A\epsilon_0$ × 3k)/(d/2)] = [($6A\epsilon_0$)/d] = 6C

I.e., Capacitance becomes 6 times. Hence both statements are true but reason is not explanation of assertion.

S2. (c) If dielectric slab of dielectric constant k is filled in between the plates of a capacitor while charging it, the potential difference between the plates does not change. But capacity becomes k times

∴ $V^1 = V$, $C^1 = kc$

Energy stored in capacitor = V^1 = (1/2) $C^1V\ ^{(1)2}$ = (1/2) $(kc)V^2$ = kv

Thus, energy stored becomes k times.

Surface charge density = σ = (q^1/A) = $[(C^1V^1)/A]$ = [(kcv)/A] = (kq/A) = kσ.

∴ Assertion is true & reason is false

S3. (d) According to classical electromagnetic theory, an accelerated charged particle continuously emits radiation. As electrons revolving in circular paths are constantly experiencing centripetal acceleration, hence they will be losing their energy continuously and the orbital radius will go on decreasing, form spiral and finally the electron will fall in the nucleus.

S4. (b) Bohr postulated that electrons in stationary orbits around the nucleus do not radiate. This is the one of Bohr's postulates, according to this the moving electrons radiates only when they go from one orbit to the next lower orbit.

S5. (c) According to postulates of Bohr's atom model the electron revolves around the nucleus in fixed orbit of definite radii. As long as the electron is in a certain orbit it does not radiate any energy.

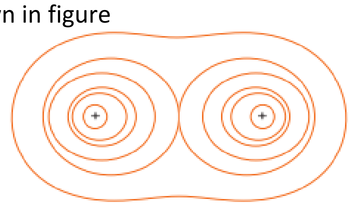
VERY SHORT ANSWER QUESTIONS

S1. Electric potential. It is a scalar quantity

S2. E = 0 inside the conductor & has no tangential component on the surface. No work is done in moving charge inside or on the surface of the conductor and potential is constant.

S3. The work done by the field is negative. This is because the charge is moved against the force exerted by the field.

S4. Equipotential surfaces due to two identical charges is shown in figure



S5. This is because at the point of intersection there will be two values of electric potential, which is not possible.

SHORT ANSWER QUESTIONS

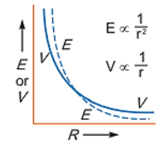
S1. The line joining B to C is perpendicular to electric field, so potential of

B = potential of C i.e., $V_B = V_C$

Distance AB =4 cm

Potential difference between A and C = E × (AB) = 5 × 10^3 × (4 × 10^{-2}) = 200 volt

S2. The graph of variation of potential and electric field due to a point charge Q with distance R from the point charge is shown in figure.



S3. Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence. The field inside a conductor is zero. This is known as electrostatic shielding. Sensitive instruments are shielded from outside electrical influences by enclosing them in a hollow conductor. During lightning it is safest to sit inside a car, rather than near a tree. The metallic body of a car becomes an electrostatic shielding from lightening. Potential inside the cavity is not zero. Potential is constant.

S4. (i) In the steady state no current flows through capacitor because, we have two sources (battery and fully charged capacitor) of equal potential connected in opposition.

(ii) During charging or discharging there is a momentary flow of current as the potentials of the two sources are not equal to each other

S5. (i) Since electric field is conservative in nature, the amount of work done will depend upon initial and final positions only.

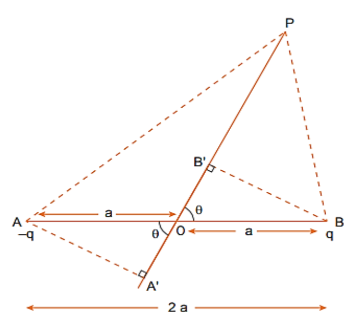
Work done, W= $\vec{F}.\vec{d}$=$q\vec{E}.\vec{d}$= $qEcos180°$= -4qe

Hence, V_A-V_C=W/q =-4E

(ii) $V_C > V_A$, because direction of electric field is in decreasing potential.

LONG ANSWER QUESTIONS

S1.



Potential at a point due to a dipole. Suppose, the negative charge −q is placed at a point A and the positive charge q is placed at a point B (fig.), the separation AB = 2a. The middle point of AB is O. The potential is to be evaluated at a point P where OP = r and ∠POB = θ. Also, let r >> a.

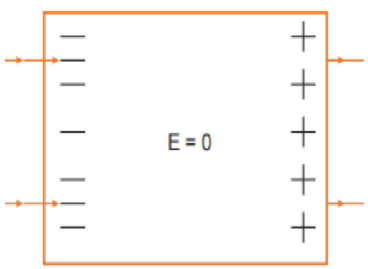
Let AA' be the perpendicular from A to PO and BB' be the perpendicular from B to PO. Since a is very small compared to r,

$AP = A'P = OP + OA'$
= Op + AO cosθ
Similarly,
$BP = B'P = OP - OB'$
= r- a cosθ
The potential at P due to the charge –q is
$V_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{AP} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r+acos\theta}$
The potential at P due to the charge q is
$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r-acos\theta}$
The net potential at P due to the dipole is
$V = V_1 + V_2$
$=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r-acos\theta} - \frac{q}{r+acos\theta}\right)$
$=\frac{1}{4\pi\varepsilon_0}\frac{q2acos\theta}{r^2-a^2 \text{Cos}^2\theta}$
$V = \frac{1}{4\pi\varepsilon_0}\frac{pcos\theta}{r^2}$

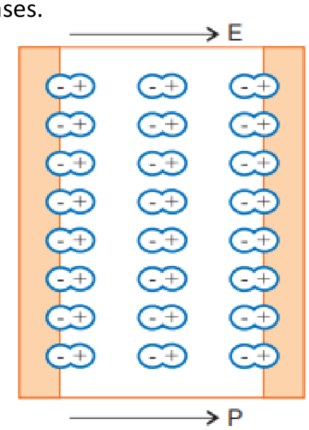
(i) When point P lies on the axis of dipole, then θ = 0°
$cos\theta = cos0° = 1$
$V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$

(ii) When point P lies on the equatorial plane of the dipole, then
$cos\theta = cos90° = 0$
V=0
It may be noted that the electric potential at any point on the equatorial line of a dipole is zero.

S2.



(a) For conductor: Due to induction the free electrons collect on the left face of slab creating equal positive charge on the right face. Internal electric field is equal and opposite to external field; hence net electric field (inside the conductor) is zero.
For dielectric: Due to alignment of atomic dipoles along E, the net electric field within the dielectric decreases.



(b) The net dipole moment developed per unit volume in the presence of external electric field is called polarization vector $\vec{P}$
$$\vec{P} = \chi_e \vec{E}$$

CASE STUDY QUESTIONS

S1. (i). (c), (ii). (a), (iii). (b), (iv). (b)

NUMERICAL TYPE QUESTIONS

S1. Using $\frac{1}{2}mv^2 = qV$
$V = \frac{1}{2} \times \frac{2\times10^{-3}\times10\times10}{2\times10^{-6}} = 50$ kv

S2. $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r} = Q.10^{11}$ Volts
∴ $\frac{1}{r} = 4\pi\varepsilon_0 \times 10^{11}$
$E = \frac{\text{Potential}}{r} = Q\cdot10^{11} \times 4\pi\varepsilon_0\cdot10^{11}$
⇒ $E = 4\pi\varepsilon_0\cdot Q\cdot10^{22}$ Volt/m

S3. Electric field in a region, $E = -\frac{dV}{dr}$
But here electric potential is constant. Therefore, electric field will be zero.

S2. **(b)** Using $\frac{1}{2}mv^2 = qV$
$V = \frac{1}{2} \times \frac{2\times10^{-3}\times10\times10}{2\times10^{-6}} = 50$ kv

S4. Before contact, $Q_1 = \sigma\cdot4\pi R^2$ And $Q_2 = \sigma\cdot 4\pi(2R)^2$
As, surface charge density, $\sigma = \frac{\text{Net charge }(Q)}{\text{Surface area }(A)}$
Now, after contact, $Q_1' + Q_2' = Q_1 + Q_2 = 5Q_1 = 5(\sigma\cdot4\pi R^2)$ ...(i)
They will be at equal potentials, so,
$\frac{Q_1'}{R} = \frac{Q_2'}{2R} \Rightarrow Q_2' = 2Q_1'$
∴ $3Q_1' = 5(\sigma\cdot4\pi R^2)$ (From equation (i))
And $Q_2' = \frac{10}{3}(\sigma\cdot4\pi R^2)$
∴ $\sigma_1 = \frac{5}{3}\sigma$ and $\sigma_2 = \frac{5}{6}\sigma$

S5. Potential inside the sphere is the same as that on the surface *i.e.*, 80 V.

S6. Radii of spheres $R_1 = 1$ cm $= 1\times10^{-2}$ Cm;
$R_2 = 2$ cm $= 2\times10^{-2}$ M and charges on sphere;
$Q_1 = 10^{-2}$ C and $Q_2 = 5\times10^{-2}$ C
Common potential $(V) = \frac{\text{Total charge}}{\text{Total capacity}} = \frac{Q_1+Q_2}{C_1+C_2}$
$= \frac{(1\times10^{-2})+(5\times10^{-2})}{4\pi E_0 10^{-2}+4\pi E_0(2\times10^{-2})} = \frac{6\times10^{-2}}{4\pi E_0(3\times10^{-2})}$
Therefore, final charge on smaller sphere = C_1V
$= 4\pi E_0 \times 10^{-2} \times \frac{6\times10^{-2}}{4\pi E_0\times3\times10^{-2}} = 2\times10^{-2}$ C

S7. To orient the dipole at any angle θ from its initial position, work has to be done on the dipole from θ = 0° to θ.
∴Potential energy = $pE(1 - \text{Cos}\,\theta)$

S8. The potential energy when q_3 Is at point C

$U_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_3}{0.40} + \frac{q_2q_3}{\sqrt{(0.40)^2+(0.30)^2}}\right]$

The potential energy when q_3 Is at point D

$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_3}{0.40} + \frac{q_2q_3}{0.10}\right]$

Thus, change in potential energy is $\Delta U = U_2 - U_1$

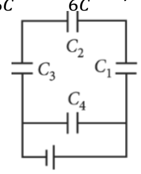
$\Rightarrow \quad \frac{q_3}{4\pi\varepsilon_0}k = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_3}{0.40} + \frac{q_2q_3}{0.10} - \frac{q_1q_3}{0.40} - \frac{q_2q_3}{0.50}\right]$

$\Rightarrow \quad k = \frac{5q_2-q_2}{0.50} = \frac{4q_2}{0.50} = 8q_2$

S9. C_1, C_2 And C_3 Are in series

$\frac{1}{C'} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C}$

Or, $\frac{1}{C'} = \frac{6+3+2}{6C} = \frac{11}{6C}$ Or, $C' = \frac{6C}{11}$



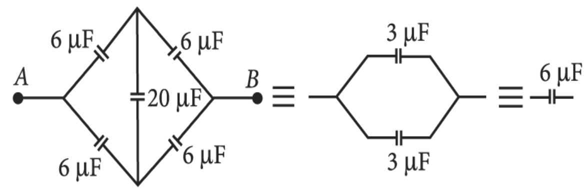
As the capacitors C_1, C_2 And C_3 Are in series so the charge on each capacitor is

$Q' = \frac{6}{11}CV$

Also charge on capacitor C_4 Is $Q = 4CV$

$\therefore \quad$ Ratio $= \frac{Q'}{Q} = \frac{6CV}{11\times 4CV} = \frac{3}{22}$

S10. The given circuit can be simplified as



HOMEWORK EXERCISE SOLUTIONS

MULTIPLE CHOICE QUESTIONS

1. **(d)** $C = 2 + 3 = 5\,\mu f$

$$V_{AB} = \frac{q}{12} + \frac{q}{20} + \frac{q}{5} = \frac{q}{3}$$

$$1500 = \frac{q}{3} \quad \Rightarrow q = 4500 \text{ mc}$$

Hence potential difference across

$2\mu f$ capacitor $= \frac{4500}{5} = 900$ V

$$\text{Energy stored} = \frac{1}{2} \times 2 \times 10^{-6}(900)^2 = 0.81 \text{ J}$$

S2. **(a)** $\text{Energy} = \frac{q^2}{2C}$

$$\frac{E_{before}}{E_{after}} = \frac{q^2/2C_1}{q^2/[2(C_1+C_2)]}$$

(Q q remains same)

$$\frac{E_{before}}{E_{after}} = \frac{C_1+C_2}{C_1}$$

S3. **(b)** The arrangement shown in the figure is equivalent to three capacitors in parallel hence resultant capacitance $= \frac{3\varepsilon_0 A}{d}$

S4. **(a)** The total energy before connection

$$= \frac{1}{2} \times 4 \times 10^{-6} \times (50)^2 + \frac{1}{2} \times 2 \times 10^{-6} \times (100)^2$$

$= 1.5 \times 10^{-2}$ J

When connected in parallel

$= 4 \times 50 + 2 \times 100 = 6 \times V \Rightarrow V = \frac{200}{3}$

Total energy after connection

$$= \frac{1}{2} \times 400 \times 10^{-6} \times \frac{200}{3} = 1.33 \times 10^{-2} \text{ J}$$

S5. **(b)** $\frac{1}{C} = \frac{1}{3} + \frac{1}{6} \quad \Rightarrow C = 2\text{pf}$

Total charge $= 2 \times 10^{-12} \times 5000 = 10^{-8}$ C

The new potential when the capacitors are connected in parallel is

$$V = \frac{10^{-8}}{(3+6)\times 10^{-12}} = 1111\text{V}$$

S6. **(a)** In this, we find out why capacitors are used in electrical appliances and why these appliances need a capacitor. A capacitor is a device that stores electrical energy in the form of an electrical field. But a capacitor does not have a new electron that means it does not produce electrical energy on its own. So the capacitors are used in electrical circuits where appliances need current.

S7. **(c)** Capacitor stores electrical energy whereas inductor stores magnetic energy. Hence, capacitor is called the electrical energy tank.

S8. **(c)** Every condenser is made with two plates. The charge on one plate is +Q and other is −Q. Thus, total charge of condenser is Q t =+Q−Q=0

S9. **(d)** $\varepsilon_0 = 8.85\times10^{-12}$

$E_r = 81$

Perimeter, $L = 2\pi r = 2$m

$R = 1/\pi$

The capacitance in water,

$C = 4\pi\varepsilon_0 \varepsilon_r r$

$C = 4\pi \times 8.85\times10^{-12} \times 81 \times 1/\pi$

C =2865pf

10. **(d)** The equipotential surface is the area that is the locus of all vertices with the same potential. These same equipotential points are the spots inside an electric field that have the same electric potential. An equipotential line is formed when these locations are joined by a line or a curve. Whether such points are found on an area, the surface is referred to be an equipotential surface. According to the information provided, the electric field in the z-direction appears uniform, and the equipotential surfaces are in the XY plane. As a result, the potential for a given z is continuous or unchanged in the x-y plane, for every x and y throughout this plane.

ASSERTION AND REASONING

S1. **(a)** Capacitance is basically a geometrical quantity.

S2. **(d)** The whole charge of a conductor can be transferred to another isolated conductor, if it is placed inside the hollow insulated conductor and connected with it

VERY SHORT ANSWER QUESTIONS

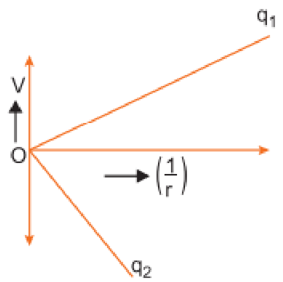
S1. The work done in moving a charge from one point to another on an equipotential surface is zero. If electric field is not normal to the equipotential surface, it would have non-zero component along the surface. In that case work would be done in moving a charge on an equipotential surface.

S2. Potential at centre of sphere = 10 V. Potential at all points inside the hollow metal sphere (or any surface) is always equal to the potential at its surface.

S3. Free electrons would travel to regions of higher potentials as they are negatively charged.

Short Answer Questions

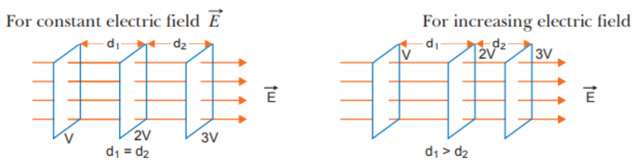
S1. (i) The potential due to positive charge is positive and due to negative charge, it is negative, so, q_1 is positive and q_2 is negative.



(ii) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

The graph between V and 1 /r is a straight line passing through the origin with slope $\frac{q}{4\pi\varepsilon_0}$. As the magnitude of slope of the line due to charge q_2 is greater than that due to q_1, q_2 has larger magnitude.

S2.



Difference: For constant electric field, the equipotential surfaces are equidistant for same potential difference between these surfaces; while for increasing electric field, the separation between these surfaces decreases, in the direction of increasing field, for the same potential difference between them.

NUMERICAL TYPE QUESTIONS

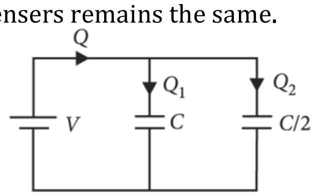
S1. Let q be the charge on each capacitor.

$\therefore$ Energy stored, $U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$

Now, when battery is disconnected and another capacitor of same capacity is connected in parallel to the first capacitor, then voltage across each capacitor, $V = \frac{q}{2C}$

$\therefore$ Energy stored $= \frac{1}{2}C\left(\frac{q}{2C}\right)^2 = \frac{1}{4}\cdot\frac{1}{2}\frac{q^2}{C} = \frac{1}{4}U$

S2. As the capacitors are connected in parallel, therefore potential difference across both the condensers remains the same.



$\therefore \quad Q_1 = CV;$

$Q_2 = \frac{C}{2}V$

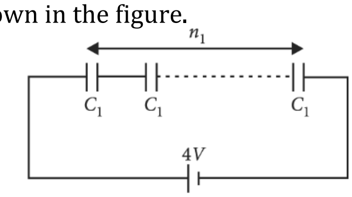
Also, $\quad Q = Q_1 + Q_2$

$= CV + \frac{C}{2}V = \frac{3}{2}CV$

Work done in charging fully both the condensers is given

By $W = \frac{1}{2}QV = \frac{1}{2}\times\left(\frac{3}{2}CV\right)V = \frac{3}{4}CV^2.$

S3. A series combination of n_1 Capacitors each of capacitance C_1 Are connected to $4V$ source as shown in the figure.



Total capacitance of the series combination of the capacitor is

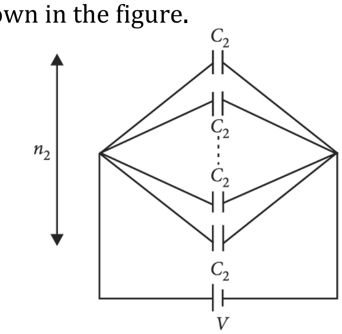
$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_1} +$................upto n_1 Terms = $\frac{n_1}{C_1}$

Or $\quad C_s = \frac{C_1}{n_1}$ ...(i)

Total energy stored in a series combination of the capacitors is

$U_s = \frac{1}{2}C_s(4V)^2 = \frac{1}{2}\left(\frac{C_1}{n_1}\right)(4V)^2$ (Using (i)) ...(ii)

A parallel combination of n_2 Capacitors each of capacitance C_2 Are connected to V source as shown in the figure.



Total capacitance of the parallel combination of capacitors is $C_p = C_2 + C_2 +$ .... + upto n_2 Terms = n_2C_2

Or $\quad C_p = n_2C_2$ ...(iii)

Total energy stored in a parallel combination of capacitors is

$U_p = \frac{1}{2}C_pV^2 = \frac{1}{2}(n_2C_2)(V)^2$ (Using (iii)) ...(iv)

According to the given problem,

$$U_s = U_p$$

Substituting the values of U_s And U_p From equations (ii) and (iv), we get

$\frac{1}{2}\frac{C_1}{n_1}(4V)^2 = \frac{1}{2}(n_2C_2)(V)^2$

Or $\quad \frac{16C_1}{n_1} = n_2C_2$ Or $C_2 = \frac{16C_1}{n_1n_2}$

S4. As we know that, loss of electrostatic energy,

$E_{Loss} = \frac{1}{2}\frac{C_1C_2}{(C_1+C_2)}V^2 = \frac{1}{2}\times\frac{C^2}{2C}V^2$

$= \frac{1}{2}\left(\frac{1}{2}CV^2\right) = \frac{1}{2}E \qquad [\because C_1 = C_2 = C]$

$\therefore$ Percentage of loss of energy $= \frac{\frac{1}{2}E}{E}\times 100\%$

$= \frac{1}{2}\times 100\% = 50\%.$

S5. Given: Capacitance without dielectric, $C = 6\ \mu f$ and capacitance with dielectric, $C' = 30\ \mu f$.

$\therefore$ Dielectric constant, $K = \frac{C'}{C} = \frac{30}{6} = 5.$

Now, permittivity of the medium, $\varepsilon = K\text{E}_0$

$= 5 \times 8.85 \times 10^{-12} = 0.44 \times 10^{-10}\ C^2\ \text{N}^{-1}\ \text{M}^{-2}$

S6. Charge $(q) = 0.2$ C; Distance $(d) = 2$ m;
Angle $\theta = 60°$ and work done $(W) = 4$ J
Work done in moving the charge (W)
$= F.d \text{ Cos } \Theta = qED \text{ Cos } \Theta$

$E = \frac{W}{qd \text{ Cos } \Theta} = \frac{4}{0.2 \times 2 \times \text{Cos } 60°} = \frac{4}{0.4 \times 0.5}$

$= 20$ N/C

S7. Work done in deflecting a dipole through an angle θ is given by

$$W = \int_0^\theta pE \text{ Sin } \Theta d\theta = pE(1 - \text{Cos } \Theta)$$

Since $\theta = 90°$

$\therefore \quad W = pE(1 - \text{Cos } 90°)$ Or, $W = pE$

S8. The electric field $\vec{E}$ And potential V in a region are related as $\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right]$

Here, $V(x, y, z) = 6xy - y + 2yz$

$\therefore \quad \vec{E} = -\left[\frac{\partial}{\partial x}(6xy - y + 2yz)\hat{\imath} + \frac{\partial}{\partial y}(6xy - y + 2yz)\hat{\jmath} + \frac{\partial}{\partial z}(6xy - y + 2yz)\hat{k}\right]$

$= -[(6(1))\hat{\imath} + (6(1) - 1 + 2(0))\hat{\jmath} + (2(1)))\hat{k}] = -(6\hat{\imath} + 5\hat{\jmath} + 2\hat{k})$

S9. Here, $V(x, y, z) = 6x - 8xy - 8y + 6yz$
The x, y and z components of electric field are

$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(6x - 8xy - 8y + 6yz)$

$= -(6 - 8y) = -6 + 8y$

$E_y = -\frac{\partial Y}{\partial y} = -\frac{\partial}{\partial y}(6x - 8xy - 8y + 6yz)$

$= -(-8x - 8 + 6z) = 8x + 8 - 6z$

$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}(6x - 8xy - 8y + 6yz) = -6y$

$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k}$

$= (-6 + 8y)\hat{\imath} + (8x + 8 - 6z)\hat{\jmath} - 6y\hat{k}$

At point (1, 1, 1)

$\vec{E} = (-6 + 8)\hat{\imath} + (8 + 8 - 6)\hat{\jmath} - 6\hat{k} = 2\hat{\imath} + 10\hat{\jmath} - 6\hat{k}$

The magnitude of electric field $\vec{E}$ Is

$\vec{E} = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(2)^2 + (10)^2 + (-6)^2}$

$= \sqrt{140} = 2\sqrt{35}\ \text{N C}^{-1}$

Electric force experienced by the charge

$F = qE = 2\ \text{C} \times 2\sqrt{35}\ \text{N C}^{-1} = 4\sqrt{35}\ \text{N}$

S10. Capacitance of capacitor with oil between the plate, $C = \frac{K\varepsilon_0 A}{d}$

If oil is removed capacitance, $C' = \frac{\varepsilon_0 A}{d} = \frac{C}{K} = \frac{C}{2}$