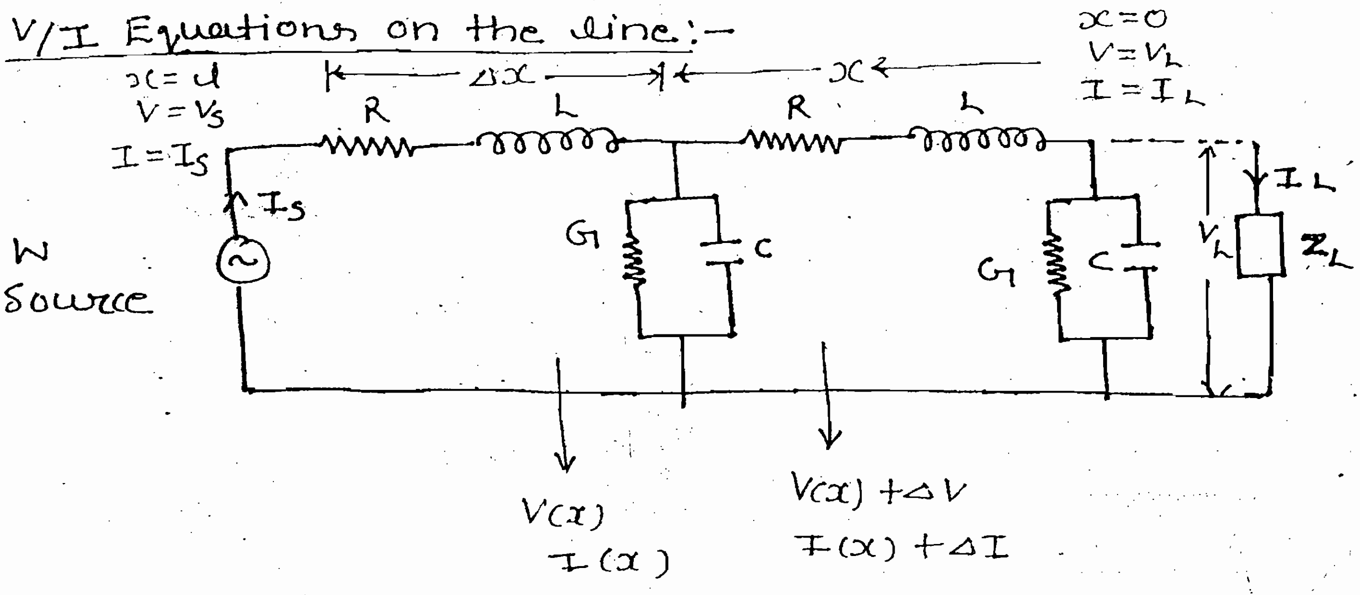
V/I Equations on the line:-



Z_{series} of Δx length $= (R + j\omega L) \cdot \Delta x$

Y_{shunt} of Δx length $= (G + j\omega C) \cdot \Delta x$

$$\Delta V = I\,(R + j\omega L) \cdot \Delta x$$

$$\frac{\Delta V}{\Delta x} = \frac{dV}{dx} = (R + j\omega L)\, I \quad \text{— (I)}$$

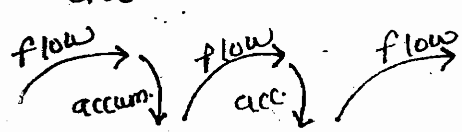
$$\frac{\Delta I}{\Delta x} = \frac{dI}{dx} = (G + j\omega C)\, V \quad \text{— (II)}$$

Take I from (I) & put in (II)

$$\frac{d}{dx}\left[\left(\frac{1}{R + j\omega L}\right)\frac{dV}{dx}\right] = (G + j\omega C)\, V$$

$$\frac{d^2V}{dx^2} = (R + j\omega L)(G + j\omega C) \quad \text{— (III)}$$

$$\frac{d^2I}{dx^2} = (R + j\omega L)(G + j\omega C) \quad \text{— (IV)}$$



$\longrightarrow$ Meaning of Harmonic

Note:-

If the source is time harmonic at one end the effect is space harmonic all along the length of the line.

Let $\sqrt{(R+j\omega L)(G+j\omega C)} = \gamma$ (per m)
Unit

Eq-(III) & (IV) can be written as

$$\frac{d^2V}{dx^2} - \gamma^2 V = 0$$

$$\frac{d^2I}{dx^2} - \gamma^2 I = 0$$

The V/I solution on the line is

$$V(x) = c_1 e^{-\gamma x} + c_2 e^{\gamma x} \quad \text{—(V)}$$

$$I(x) = c_3 e^{-\gamma x} + c_4 e^{\gamma x} \quad \text{—(VI)}$$

Applying initial conditions, $(x=0, V=V_L, I=I_L)$

$$V_L = c_1 + c_2 \quad \text{—(VII)}$$

$$I_L = c_3 + c_4 \quad \text{—(VIII)}$$

Using eq-(I) in eq-(V)

$$\frac{dv}{dx} = -\gamma c_1 e^{-\gamma x} + \gamma c_2 e^{\gamma x} = (R+j\omega L)I$$

Put $x = 0$

$$\underline{c_2 - c_1} = \frac{(R+j\omega L)I_L}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

$$= \sqrt{\frac{R+j\omega L}{G+j\omega C}}\, I_L = I_L Z_0$$

where $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$I_L Z_0 = c_2 - c_1 \quad \text{—(IX)}$$

Similarly $\frac{V_L}{Z_0} = c_4 - c_3$ —(X)

$$\boxed{c_1 = \frac{V_L - I_L Z_0}{2} \qquad c_2 = \frac{V_L + I_L Z_0}{2}}$$

$$\boxed{C_3 = \frac{I_L - V_L/Z_0}{2}, \quad C_4 = \frac{I_L + V_L/Z_0}{2}}$$

$$V(x) = \frac{V_L}{2}\left[\left(1 - \frac{Z_0}{Z_L}\right)e^{-\gamma x} + \left(1 + \frac{Z_0}{Z_L}\right)e^{\gamma x}\right]$$

$$V(x) = \frac{V_L(Z_L + Z_0)}{2Z_L}\left[e^{\gamma x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-\gamma x}\right]$$

$$V(x) = V_0\left(e^{\gamma x} + \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)e^{-\gamma x}\right)$$

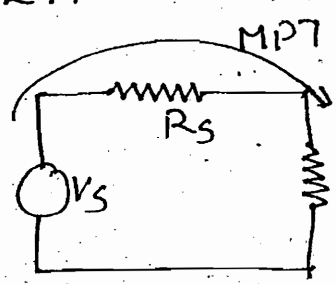
$$I(x) = \frac{I_L(Z_L + Z_0)}{2Z_0}\left[e^{\gamma x} + \left(\frac{Z_0 - Z_L}{Z_0 + Z_L}\right)e^{-\gamma x}\right]$$

$e^{\gamma x} \longrightarrow -x$, source - load forward wave

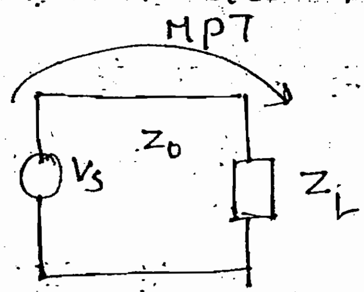
$e^{-\gamma x} \longrightarrow +x$, load - source, reflected wave

→ Every transmission line has two waveforms i.e. the forward and reflected and the reflected wave does not exist in one single condition i.e. $Z_L = Z_0$. This is called as matched condition and this is a condition where complete source power is absorbed by the load.

→ This is similar to max. power transfer in N/w or EM wave reflections b/w materials.



$R_S = R_L$

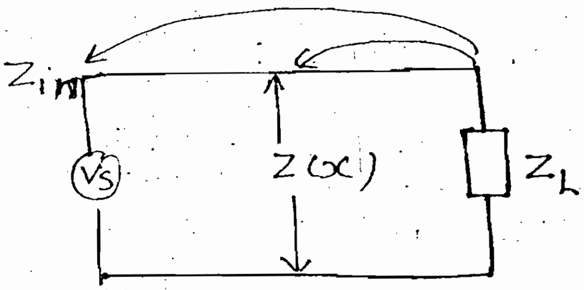


$Z_L = Z_0$

Note:-

- Z_0 is a unique impedance of the line with which if the line is terminated the reflections on the line and hence called as characteristic impedance

Impedance on line (z) and i/p impedance (Z_{in}):-



Note:-

Due to the loaded one end and primary constan n the line, impedance is different on different point of line such that

$$Z(x) = \frac{V(x)}{I(x)} \qquad Z_{in} = Z(l)$$

$$V(x) = \frac{V_L}{Z_L}\left(\frac{(Z_L+Z_0)}{2} e^{\gamma x} + \frac{(Z_L-Z_0)}{2} e^{-\gamma x} \right)$$

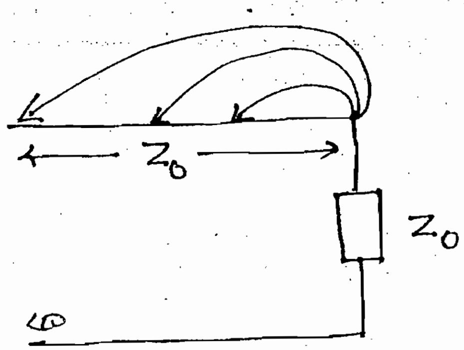
$$V(x) = V_L \cosh(\gamma x) + I_L Z_0 \sinh(\gamma x)$$

Similarly

$$I(x) = I_L \cosh(\gamma x) + \frac{V_L}{Z_0} \sinh(\gamma x)$$

$$Z(x) = Z_0 \left[\frac{Z_L \cosh(\gamma x) + Z_0 \sinh(\gamma x)}{Z_0 \cosh(\gamma x) + Z_L \sinh(\gamma x)} \right]$$

∴ If $Z_L = Z_0$ then $Z(x) = Z_0 = Z_L = Z_{in}$



Matched line

→ For a matched line the impedance anywhere on the line is also same Z_0

Note:–

Z_0 is that unique impedance with which if the line is terminated impedance on the line is also the same

Short circuit and open circuit lines:–

→ If $Z_L = 0$ ⇒ SC line.

$$Z_{in} = Z_{SC} = Z_0 \tanh(\gamma x)$$

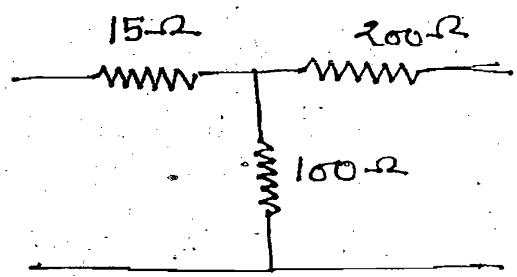
→ If $Z_L = \infty$ ⇒ O.C line

$$Z_{in} = Z_{OC} = Z_0 \coth(\gamma x)$$

$$\Rightarrow \boxed{Z_0 = \sqrt{Z_{SC} \cdot Z_{OC}}}$$

It is geometric mean of S.C and O.C i/p impedances.

eg:–

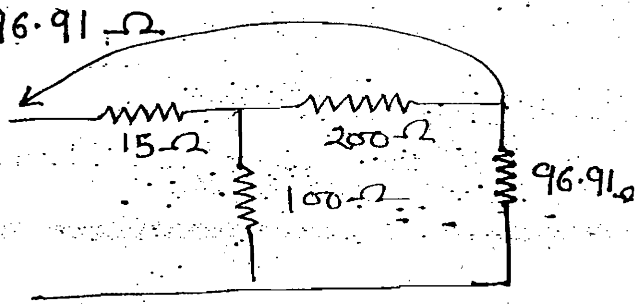


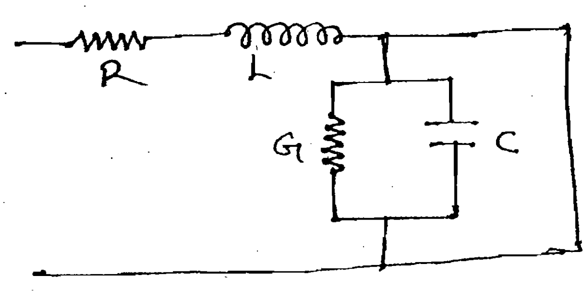
$$Z_{OC} = 115\Omega$$

$$Z_{SC} = 15 + \frac{100 \cdot 200}{100 + 200}$$

$$= 81.67\Omega$$

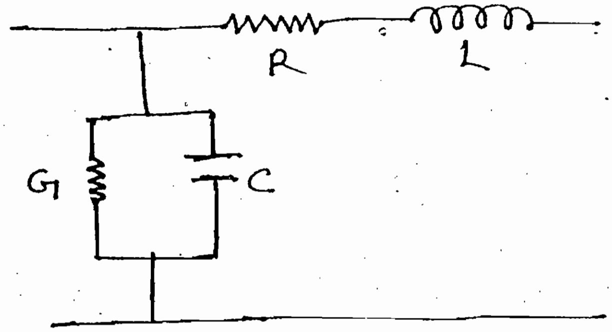
$$Z_0 = \sqrt{81.67 \times 115} = 96.91\Omega$$

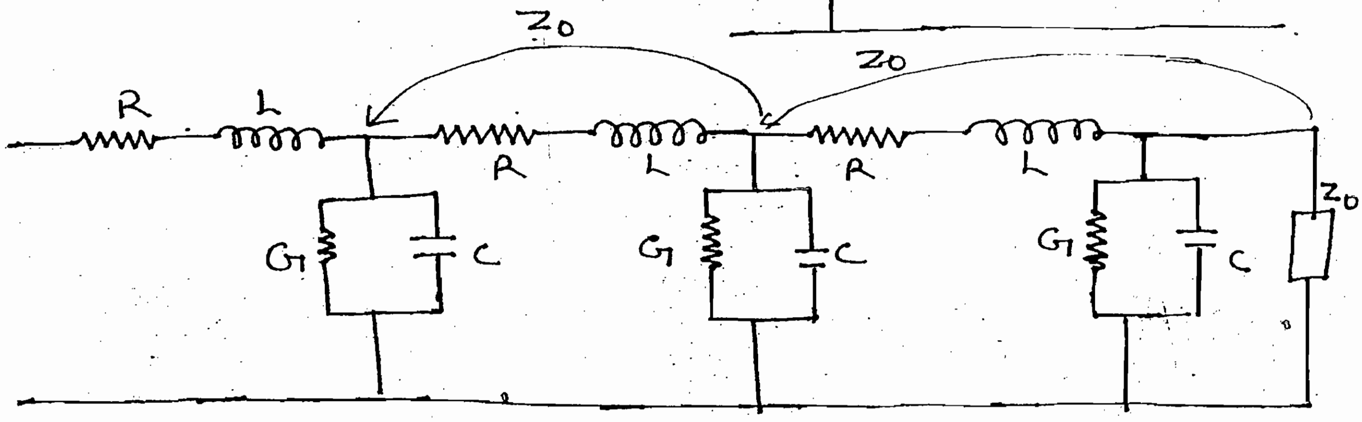




$$Z_{SC} = R + j\omega L$$

$$Z_{OC} = \frac{1}{G + j\omega C}$$





<u>Note:-</u>

- Any N/w either discrete or continous when terminated with its characteristics impedance the same impedance appears on other end
- For a transmission line whose every RLGC branch having Z_0 characteristics impedance when terminated with Z_0 at one end the loading effect appears to be the same anywhere on the line.
- Lossless line and distortionless line,

$$\gamma = \sqrt{R}$$

<u>Lossless line and distortionless line:-</u>

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \text{propogation constant}$$

$$= \alpha + j\beta$$

If $\alpha = 0$ everywhere on the line, the line is said to be lossless

(I) $R = G = 0$

(II) $\left.\begin{array}{l} \omega L >> R \\ \omega C >> G \end{array}\right\}$ High frequency lines

$$\gamma = j\omega\sqrt{LC} \quad \text{(purely imaginary)}$$
$$= j\beta$$
$$\beta = \omega\sqrt{LC}$$
$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_R}} = \frac{3\times 10^8}{\sqrt{\epsilon_R}}$$
$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} = \text{real}$$
$$= \sqrt{\frac{\mu_0 \ln(b/a)}{2\pi} \frac{\ln(b/a)}{2\pi\epsilon}}$$
$$= \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_R}} \cdot \frac{\ln(b/a)}{2\pi}$$
$$\boxed{Z_0 = \frac{60 \ln(b/a)}{\sqrt{\epsilon_R}}} \longrightarrow \text{Co-Axial cable}$$

→ Z_0 is a design aspect of the line and depends on the cable dimensions but not on R, L, G, C or operational frequency of the line.

$$\boxed{Z_0 = \frac{120 \ln(b/a)}{\sqrt{\epsilon_R}}} \longrightarrow \text{Parallel wire.}$$

Distortionless line:-

If the phase is linear the wave is said to distortionless i.e. $\beta \propto \omega$ → For distortionless line

→ Equal Rise time = Fall time is characteristic of distortionless line

Series arm time constant = Shunt arm time constant

$$\frac{L}{R} = \frac{C}{G} \Rightarrow \boxed{LG = RC}$$

$$\gamma = \sqrt{R\left(1+\frac{j\omega L}{R}\right) G\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG}\left(1+\frac{j\omega L}{R}\right) = \text{complex}$$

$$Z_0 = \sqrt{\frac{R\,(1+j\omega L/R)}{G\,(1+j\omega C/G)}} = \sqrt{\frac{R}{G}} = \text{Real}$$

Note:-

Every lossless line $\Rightarrow$ Distortionless $\not\Rightarrow$ lossless

$R = G = 0 \longrightarrow LG = RC$

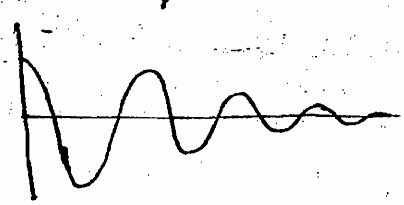
$\beta = \omega\sqrt{LC} \longrightarrow \beta \propto \omega$

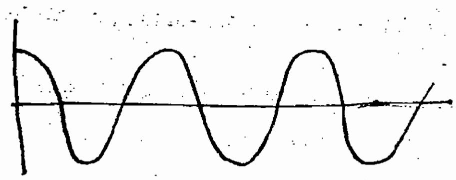
Workbook:-

1) $$Z_{in} = Z_0\left[\frac{Z_L \cosh(j\beta l) + Z_0 \sinh(j\beta l)}{Z_0 \cosh(j\beta l) + Z_L \sinh(j\beta l)}\right]$$

$$\cosh\theta = \frac{e^{\theta}+e^{-\theta}}{2} \qquad \cosh(j\theta) = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow = \cos\theta$





$\sinh(j\theta) = j\sin\theta$

$$Z_{in} = Z_0 \left[\frac{Z_L \cos\beta l + j Z_0 \sin\beta l}{Z_0 \cos\beta l + j Z_L \sin\beta l} \right]$$

(I) $l = \frac{\lambda}{8}$ $\qquad \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \frac{\pi}{4}$

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0}{Z_0 + j Z_L} \right]$$

(II) $l = \lambda/4$ $\qquad \beta l = \frac{\pi}{2}$ $\qquad Z_{in} = \frac{Z_0^2}{Z_L}$

(III) $l = \frac{\lambda}{2}$ $\qquad \beta l = \pi$ $\qquad Z_{in} = Z_L$

$Z_L = j50$ $\qquad Z_0 = 50\,\Omega$

$$Z_{in} = 50 \left[\frac{j50 + j50}{50 + j50} \right] = \infty \rightarrow O/C$$

$$Z_{in\,(\lambda/4)} = \frac{50 \times 50}{j\,50} = -j\,50\,\Omega$$

$$Z_{in\,(\lambda/2)} = Z_L = j\,50\,\Omega$$

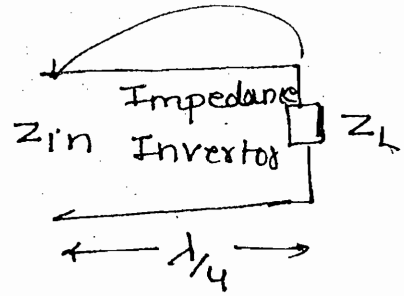
Note 1:-

→ βl is a critical aspect of the line that decides the nature and performance of the line and it is called as electrical paths line

→ length/λ relationship is a critical aspect.

Note 2:-

$l = \lambda/4$ $\qquad Z_{in} = \frac{Z_0^2}{Z_L}$



It's one end impedance has opposite nature to other end impedance hence called as impedance invertor

Note 3:—

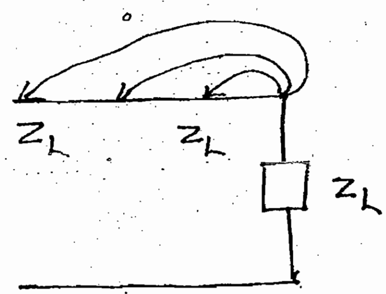
$l = \lambda/2 \qquad Z_{in} = Z_L$

It is called as impedance reflector

Note 4:—

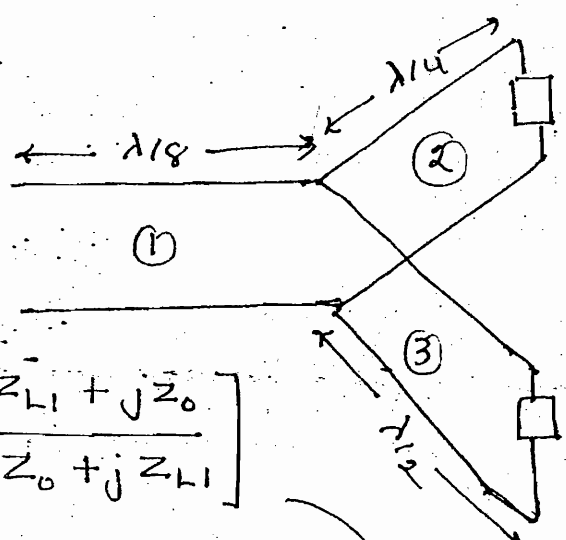
If $l \to 0$ or $\lambda \to \infty$ where $\beta l \to 0$

$Z(x) = Z_L$



For electrically short line the wire is a simple connecting wire or a ~~short~~ ckt.

5.



$$Z_{in2}\,(\lambda/4) = \frac{R_0^2}{R_0/2} = 2R_0$$

$$Z_{in3}\,(\lambda/2) = 2R_0$$

$$Z_{in1}\,(\lambda/8) = Z_0\left[\frac{Z_{L1} + jZ_0}{Z_0 + jZ_{L1}}\right]$$

$$Z_{L1} = Z_{in2} \,||\, Z_{in3}$$

$$= R_0\left[\frac{R_0 + jR_0}{R_0 + jR_0}\right]$$

$$= R_0$$

When ... interchanged

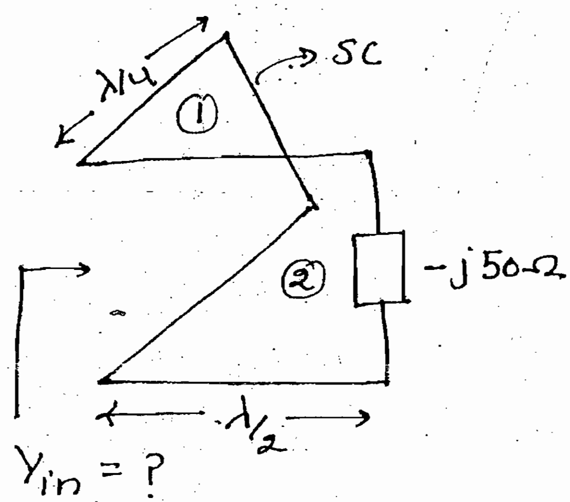
$$Z_{in2} \; (\lambda/4) = \frac{R_o^2}{2R_o} = \frac{R_o}{2}$$

$$Z_{in3} \; (\lambda/2) = \frac{R_o}{2}$$

$$Z_{in1} \; (\lambda/8) = Z_o\left[\frac{Z_{L1} + jZ_o}{Z_o + jZ_{L1}}\right] = R_o\left[\frac{R_o/4 + jR_o}{R_o + j\frac{R_o}{4}}\right]$$

$$= R_o\left[\frac{1+j4}{4+j}\right]$$

$$Y_{in} = Y_{in1} + Y_{in2}$$



$$Y_{in1} \; (\lambda/4) = \frac{Z_L}{Z_o^2} = 0$$

$$Y_{in2} \; (\lambda/2) = \frac{1}{Z_L} = \frac{1}{-j50}$$

$$= j0.02$$

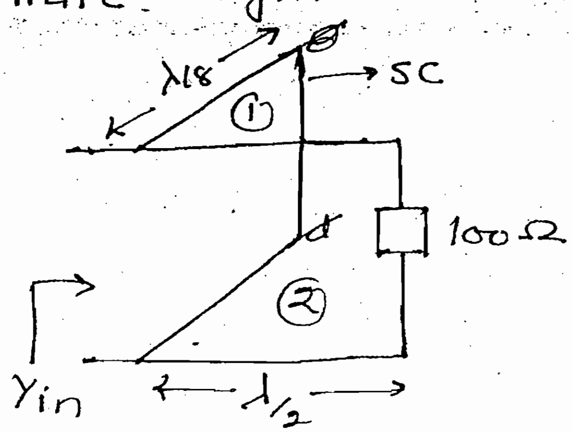
$$Z_{in} = (Z_{in1} \,||\, Z_{in2})$$

$$= (\infty \,||\, -j50) = -j50$$

$$Y_{in} = j0.02 \mho$$

Stub ⇒ SC line of finite length

$$Y_{in} = Y_{in1} + Y_{in2}$$



$$Z_{in1} \; (\lambda/8) = Z_o\left[\frac{0 + jZ_o}{Z_o + j0}\right]$$

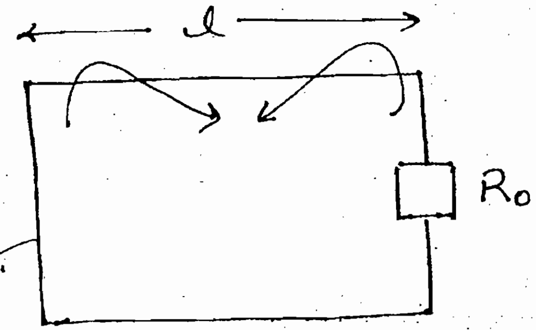
$$= jZ_o = j50\,\Omega$$

$$Y_{in1} = \frac{1}{j50} = -j0.02$$

$$Y_{in2} = \frac{1}{Z_L} = \frac{1}{100} = 0.01$$

$$Y_{in} = Y_{in1} + Y_{in2} = 0.01 - j0.02 \, ℧$$

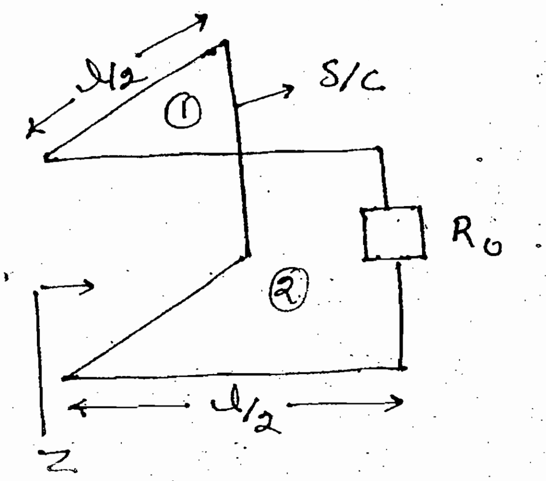
8.



$$l = \lambda/4$$

$$Z_{in1} = R_0 \left[\frac{0 + jR_0}{R_0 + j0}\right]_{SC}$$

$$= jR_0$$



$$Z_{in2} = R_0$$

$$Z = \frac{jR_0 \cdot R_0}{jR_0 + R_0}$$

$$= \frac{jR_0}{j+1}$$

$l = \lambda$ $\quad Z = R_0$

$l = \lambda/8$ $\quad Z = SC$

9. $V_p = 2 \times 10^8$ $\quad f = 10\,MHz$ $\quad \lambda = 20\,m$, $l = 10\,m$ $\quad l = \lambda/2$

$$Z_{in} = Z_L = 30 - j40\,\Omega$$

10. $f' = 5\,MHz$ $\quad \lambda' = 40\,m$ $\quad l = \frac{\lambda'}{4}$

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{50 \times 50}{30 - j40} = 30 + j40$$

11. $$Z_{in} = j60 = Z_{sc} = Z_0 \tanh(j\beta l)$$

$$= jZ_0 \tan \beta l$$

$$\beta l = \frac{\pi}{4}$$

$$= jZ_0$$

$$Z_0 = 60\,\Omega$$

2. $f' = 2f$

$12\,MHz \longrightarrow 24\,MHz$

$\lambda' = \lambda/2$

$\beta' l = 2\beta l = \pi/2$

$\therefore Z_{in} = Z_{sc} = jZ_0 \tan \pi/2 = \infty$

3. $f = 12\,MHz$

$Z_{in} = j60 = j\omega L \Rightarrow L = \dfrac{60}{2\pi \times 12 \times 10^6}$

$= \dfrac{2.5}{\pi}\,\mu H$

<u>Summary</u>:—

$Z_{sc} = jZ_0 \tan \beta l$

$\left.\begin{array}{l} 0 < \beta l < \frac{\pi}{2} \\ 0 < l < \lambda/4 \end{array}\right\}$ Z_{in} is inductive

$\left.\begin{array}{l} \frac{\pi}{2} < \beta l < \pi \\ \lambda/4 < l < \lambda/2 \end{array}\right\}$ Z_{in} is capacitive

$Z_{oc} = -jZ_0/\tan \beta l$

Z_{in} inductive $\longrightarrow$ Z_{in} is capacitive

Z_{in} is capacitive $\longrightarrow$ Z_{in} is inductive.

→ one end of a line is S.C or O.C then the other end is purely reactive.

14. (a) $Z_{in} = -jZ_0$ For S/C line $\lambda/8$ length X

$\quad\quad \longrightarrow jZ_0$

(b) $Z_{in} = \pm j\infty$ for SC line $\lambda/4$ length

$$Z_{SC} = jZ_0 \tan\beta l$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

X (c) $Z_{in} = -jZ_0 \longrightarrow$ O/C line $\rightarrow \lambda/2$ length

$\downarrow$

$$Z_{in} = Z_L$$

(d) $Z_{in} = Z_0$

15. $$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{LC}{C^2}} = \frac{\sqrt{\mu_0 \epsilon_0 \epsilon_R}}{c} = \frac{\sqrt{\epsilon_R}}{V_p \cdot c}$$

16. (a) α = even , β = odd X

(b) α = constant $\beta = k\omega$

(c) $\frac{L}{C}$ = constant $= \frac{R}{G}$ X

(d) Matched line ($Z_L = Z_0$) X

17. $\beta l = \omega\sqrt{LC}\, l = 108°$

Reflection Coefficient :-

$$V(x) = \underbrace{V_0 e^{j\beta x}}_{V_f} + \underbrace{V_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta x}}_{V_r}$$

$\frac{V_r}{V_f}$ = Reflection coefficient for the voltages anywhere on the line $= \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j2\beta x}$

$= \Gamma(x)$

$$\boxed{\Gamma(x) = \frac{Z(x) - Z_0}{Z(x) + Z_0}}$$

At $x = 0$ $\quad \Gamma_{at\ load} = \frac{Z_L - Z_0}{Z_L + Z_0}$ = complex

$= |\Gamma| \angle\theta = \left|\frac{V_r}{V_f}\right| \angle\theta$

$|\Gamma| \rightarrow$ It is the ratio of reflected voltage to the forward voltage amplitude

$\angle\theta \rightarrow$ It is the time delay or phase difference b/w the reflected and forward voltage.

Γ is the measure of mismatch b/w the expected impedance and the actual impedance Z_L at the load.

$\Gamma(x)$ is the measure of mismatch b/w the expected impedance Z_0 and the actual impedance $Z(x)$ anywhere on the line.

Note. –

(i) $\Gamma_I = \dfrac{Z_0 - Z_L}{Z_0 + Z_L} = -\Gamma_V$

If V_f and I_f they are inphase

$$V_0 e^{j\beta x} + V_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta x}$$

$$I_0 e^{j\beta x} - I_0 \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) e^{-j\beta x}$$

then V_r and I_r are out of phase by 180°

(ii) $0 \leq |\Gamma| \leq 1$

Matched line

Complete Reflection (Mismatch)