THREE-DIMENSIONAL GEOMETRY

CHAPTER – 11

THREE-DIMENSIONAL GEOMETRY

DIRECTION COSINES AND DIRECTION RATIOS OF A LINE

Direction cosines of a line

If the directed line OP makes angles α, β, and γ with positive X-axis, Y-axis and Z-axis respectively, then cos α, cos β, and cos γ, are called direction cosines of a line.

They are denoted by l, m, and n. Therefore, l = cos α, m = cos β and n = cos γ. Also, sum of squares of direction cosines of a line is always 1, i.e. $l^2 + m^2 + n^2 = 1$ or $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

Note

Direction cosines of a directed line are unique.

Example

If a line makes angles 90°, 135°, 45° with the x, y and z-axes respectively, find its direction cosines.

Solution: Let the direction cosines of the line be l, m and n.

Here let α = 90°, β = 135° and γ = 45°

As we know,

l = cos α, m = cos β and n = cos γ

Thus, direction cosines are:

l = cos 90° = 0

m = cos 135°= cos (180° – 45°) = -cos 45° = -1/√2

n = cos 45° = 1/√2

Therefore, the direction cosines of the line are 0, -1/√2, 1/√2.

Direction Ratios of a Line

Number proportional to the direction cosines of a line, are called direction ratios of a line.

(i) If a, b and c are direction ratios of a line, then $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

(ii) If a, b and care direction ratios of a line, then its direction cosines are

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

(iii) Direction ratios of a line PQ passing through the points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) are $x_2 - x_1$, $y_2 - y_1$ and $z_2 - z_1$ and direction cosines are $\frac{x_2-x_1}{|\overrightarrow{PQ}|}, \frac{y_2-y_1}{|\overrightarrow{PQ}|}, \frac{z_2-z_1}{|\overrightarrow{PQ}|}$

Note

(i) Direction ratios of two parallel lines are proportional.
(ii) Direction ratios of a line are not unique.

Example

Show that the points (2, 3, 4), (−1, −2, 1), (5, 8, 7) are collinear.

Solution: If the direction ratios of two lines segments are proportional, then the lines are collinear.
Let the given points are: A (2, 3, 4), B (−1, −2, 1) and C (5, 8, 7)
Direction ratio of the line joining the points A (2, 3, 4) and B (−1, −2, 1), are:

((−1−2), (−2−3), (1−4)) = (−3, −5, −3)
Where, $a_1 = -3$, $b_1 = -5$, $c_1 = -3$
Direction ratio of the line joining the points B (−1, −2, 1) and C (5, 8, 7) are:
[(5− (−1)), (8− (−2)), (7−1)] = (6, 10, 6)
Where, $a_2 = 6$, $b_2 = 10$ and $c_2 = 6$
Since, it is clear that the direction ratios of AB and BC are of same proportions, hence,
$a_2/a_1 = 6/-3 = -2$
$b_2/b_1 = 10/-5 = -2$
$c_2/c_1 = 6/-3 = -2$
Therefore, the points A, B and C are

LINE

A line or straight line is curve such that all points on the line segment joining two points of it lies on it.
A line in space can be determined uniquely, if
(i) Its direction and the coordinated of a point on it are known.
(ii) It passes through two given points.

Straight line

A straight line is a curve, such that all the points on the line segment joining any two points of it lies on it.

Equation of a Line through a Given Point and parallel to a given vector $\vec{b}$ Vector form $\vec{r} = \vec{a} + \lambda\vec{b}$ where, $\vec{a}$= Position vector of a point through which the line is passing $\vec{b}$ = A vector parallel to a given line Cartesian form

$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

where, (x_1, y_1, z_1) is the point through which the line is passing through and a, b, c are the direction ratios of the line.

If l, m, and n are the direction cosines of the line, then the equation of the line is
$\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$

Remember point: Before we use the DR' s of a line, first we have to ensure that coefficients of x, y and z are unity with a positive sign.

Example

Find the vector equation of the line which passes through the point A (1,2,3) and is parallel to the vector $3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

Solution: It is given that, the line passes A (1,2,3). Therefore, its position vector through A is $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and also the vector parallel to the line is $\vec{b} = 3\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ The vector equation of the line which passes through the point A and parallel the vector $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\vec{b}$ where λ is some scalar.
Therefore, the required vector equation of the line is
$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(3\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$

EQUATION OF LINE PASSING THROUGH TWO GIVEN POINTS

VECTOR FORM

If $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$, $\lambda \in R$ where a and b are the position vectors of the points through which the line is passing.

CARTESIAN FORM

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$,where, (x_1, y_1, z_1) and (x_2, y_2, z_2) are the points through which the line is passing

Example

Find the vector equation of the line passing through two points A (3,4, −6) and B (5, −2,7).

Solution: Given: A (3,4,−6)and B(5,−2,7)

Let $\vec{a}$ and $\vec{b}$ be the position vectors of the two points A(x_1,y_1,z_1) and B(x_2,y_2,z_2) respectively that lying on the line.

Thus, $\vec{a} = 3\hat{\imath} + 4\hat{\jmath} - 6\hat{k}$ and $\vec{b} = 5\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$

As we know, the vector equation of the line passing through the two points is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ where λ is some scalar quantity

Therefore, $\vec{r} = 3\hat{\imath} + 4\hat{\jmath} - 6\hat{k} + \lambda[(5\hat{\imath} - 2\hat{\jmath} + 7\hat{k}) - (3\hat{\imath} + 4\hat{\jmath} - 6\hat{k})]$

$\Rightarrow \vec{r} = 3\hat{\imath} + 4\hat{\jmath} - 6\hat{k} + \lambda[(5\hat{\imath} - 2\hat{\jmath} + 7\hat{k}) - (3\hat{\imath} + 4\hat{\jmath} - 6\hat{k})]$

$\therefore \vec{r} = 3\hat{\imath} + 4\hat{\jmath} - 6\hat{k} + \lambda(2\hat{\imath} - 6\hat{\jmath} + 13\hat{k})$

Example

Find the Cartesian equation of the line passing through A (−2,1,0) and B (3,4, −1)

Solution: Given: A (−2,1,0) and B (3,4, −1)

As we know, the Cartesian equation of the line passing through two points A (x_1, y_1, z_1) & B (x_2, y_2, z_2) is $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

Therefore, the Cartesian equation of the line passing through A (−2,1,0) and B (3,4, − 1) is

$\Rightarrow \frac{x+2}{3+2} = \frac{y-1}{4-1} = \frac{z-0}{-1-0}$ $\Rightarrow \frac{x+2}{5} = \frac{y-1}{3} = \frac{z-0}{-1}$

ANGLE BETWEEN TWO LINES VECTOR FORM

Angle between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is

$$\cos\theta = \left|\frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}|\cdot|\vec{b_2}|}\right|$$

Cartesian form: If θ is the angle between the lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$, then $\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

or $\sin\theta = \frac{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-b_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2}}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}$

Also, angle (θ) between two lines with direction cosines, l_1, m_1, n_1 and l_2, m_2, n_2 is given by $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$

Or

$\sin\theta = \sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2}$

Example

Find the angle between the pair of lines by

$\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$

Solution: Here $\vec{b_1} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b_2} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

The angle θ between the two lines is given by

$\text{Cos}(\theta) = \left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|}\right| = \left|\frac{(\hat{\imath}+2\hat{\jmath}+2\hat{k})(3\hat{\imath}+2\hat{\jmath}+6\hat{k})}{\sqrt{1+4+4}.\sqrt{9+4+36}}\right| = \left|\frac{3+4+12}{3\times7}\right| = \frac{19}{21}$

Hence $\theta = \cos^{-1}\left(\frac{19}{21}\right)$

Condition of Perpendicularity: Two lines are said to be perpendicular, when in vector form $\vec{b_1}\cdot\vec{b_2}$ =0; in cartesian form $a_1a_2 + b_1b_2 + c_1c_2 = 0$ or $l_1l_2 + m_1m_2 + n_1n_2 = 0$ [direction cosine form]

Condition that Two Lines are Parallel: Two lines are parallel, when in vector form $\vec{b_1}\cdot\vec{b_2}=|\vec{b_1}||\vec{b_2}|$ in cartesian form $\frac{\vec{a_1}}{\vec{a_2}}=\frac{\vec{b_1}}{\vec{b_2}}=\frac{\vec{c_1}}{\vec{c_2}}$
or $\frac{\vec{l_1}}{\vec{l_2}}=\frac{\vec{m_1}}{\vec{m_2}}=\frac{\vec{n_1}}{\vec{n_2}}$
[direction cosine form]

SKEW-LINE

If two lines do not meet and not parallel , then they are known as skew line.

SHORTEST DISTANCE BETWEEN TWO LINES

Two non-parallel and non-intersecting straight lines, are called skew lines. For skew lines, the line of the shortest distance will be perpendicular to both the lines.

Vector form: If the lines are $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\lambda\vec{b_2}$. Then, shortest distance

$$d=\left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}\times\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

Where $\vec{a_1},\vec{a_2}$ are position vectors of point through which the line is passing and $\vec{b_1}$, $\vec{b_2}$are the vectors in the direction of a line.

Cartesian form: If the lines are $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$.

Then, shortest distance,

$$d=\left|\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_1b_2-a_2b_1)^2}}\right|$$

Example

Find the shortest distance between the lines l_1 and l_2 whose vector equation are $\vec{r}=\hat{\imath}+\hat{\jmath}+\lambda(2\hat{\imath}-\hat{\jmath}+\hat{k})$ and $\vec{r}=2\hat{\imath}+\hat{\jmath}-\hat{k}+\mu(3\hat{\imath}-5\hat{\jmath}+2\hat{k})$

Solution: $\vec{a_1}=\hat{\imath}+\hat{\jmath}$, $\vec{a_2}=2\hat{\imath}+\hat{\jmath}-\hat{k}$,$\vec{b_1}=2\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{b_2}=3\hat{\imath}-5\hat{\jmath}+2\hat{k}$
$\vec{a_2}-\vec{a_1}=\hat{\imath}-\hat{k}$, $\vec{b_1}\times\vec{b_2}=3\hat{\imath}-\hat{\jmath}-7\hat{k}$ and $|\vec{b_1}\times\vec{b_2}|=\sqrt{59}$

Hence , the shortest distance between the given lines is given by $d=\frac{10}{\sqrt{59}}$

Distance between two Parallel Lines: If two lines l_1 and l_2 are parallel, then they are coplanar. Let the lines be $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_1}+\mu\vec{b_1}$, then the distance between parallel lines is

$$\left|\frac{\vec{b}\times(\vec{a_2}-\vec{a}_1)}{|\vec{b}|}\right|$$

Note

If two lines are parallel, then they both have same DR's.

Distance between Two Points: The distance between two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) is given by PQ $=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Mid-point of a Line: The mid-point of a line joining points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) is given by $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$

Example

Find the midpoint of a line segment joining points $P_1=(4,-6,8)$ and $P_2=(4,3,-5)$

We'll use the midpoint formula for the midpoint M between points in three dimensions.

$m=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$

where $P_1=(x_1,y_1,z_1)$ and $P_2=(x_2,y_2,z_2)$. We'll plug in the points we've been given, using $P_1=(4,-6,8)$ and $P_2=(4,3,-5)$.

$m=(\frac{4+4}{2},\frac{-6+3}{2},\frac{8-5}{2})=(4,-1.5,1.5)$

PLANE

A plane is a surface such that a line segment joining any two points of it lies wholly on it. A straight line which is perpendicular to every line lying on a plane is called a normal to the plane.

EQUATION OF A PLANE

(i)Equation of plane in normal form

Vector form: The equation of plane in normal form is given by $\vec{r}\cdot\hat{n}=d$, where $\vec{n}$ is a vector which is normal to the plane.

Cartesian form: The equation of the plane is given by ax + by + cz = d, where a, b and c are the direction ratios of plane and d is the distance of the plane from origin.

Another equation of the plane is lx + my + nz = p, where l, m, and n are direction cosines of the perpendicular from origin and p is a distance of a plane from origin.

Note

If d is the distance from the origin and l, m and n are the direction cosines of the normal to the plane through the origin, then the foot of the perpendicular is (ld, md, nd).

Example

Find the vector equation of a plane which is at a distance of 8 units from the origin and which is normal to the vector $2\hat{\imath}+\hat{\jmath}+2\hat{k}$

Solution: Here, d = 8 and $\vec{n} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{\imath}+\hat{\jmath}+2\hat{k}}{3}$

Hence, the required equation of the plane is

$\vec{r}.\left(\frac{2}{3}\hat{\imath}+\frac{1}{3}\hat{\jmath}+\frac{2}{3}\hat{k}\right)= 8 \qquad \vec{r}.\ (2\hat{\imath}+\hat{\jmath}+2\hat{k}) = 24$

(ii) Equation of a Plane Perpendicular to a given Vector and Passing Through a given Point

Vector form: Let a plane passes through a point A with position vector $\vec{a}$ and perpendicular to the vector $\vec{n}$ then $(\vec{r} - \vec{a})\cdot \vec{n} = 0$ This is the vector equation of the plane.

Cartesian form: Equation of plane passing through point (x_1, y_1, z_1) is given by $a (x - x_1) + b (y - y_1) + c (z - z_1) = 0$ where, a, b and c are the direction ratios of normal to the plane.

Example

Find the vector equation of plane passing through a point (2, -1, 3), and having the direction ratios of its normal as (5, 2, 4).

Solution: The coordinates of the point (2, -1, 3) can be represented as a position vector $\vec{a} = 2\hat{\imath}-\hat{\jmath}+3\hat{k}$. And the normal having the direction ratios (5, 2, 4) can be represented as a vector $\vec{N} = 5\hat{\imath}+2\hat{\jmath}+4\hat{k}$.

The equation of a line passing through a point and having a normal is $(\vec{r}-\vec{a}).\vec{N}=0$

$\left(\vec{r}-(2\hat{\imath}-\hat{\jmath}+3\hat{k})\right).(5\hat{\imath}+2\hat{\jmath}+4\hat{k}) = 0$

Therefore, $\left(\vec{r}-(2\hat{\imath}-\hat{\jmath}+3\hat{k})\right).(5\hat{\imath}+2\hat{\jmath}+4\hat{k}) = 0$

is the required vector equation of plane.

(iii) Equation of Plane Passing through Three Non-collinear Points

Vector form: If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are the position vectors of three given points, then equation of a plane passing through three non-collinear points is $(\vec{r} - \vec{a})\cdot\{(\vec{b}-\vec{a}) \times (\vec{c} - \vec{a})\}=0$.

Cartesian form: If (x_1, y_1, z_1) (x_2, y_2, z_2) and (x_3, y_3, z_3) are three non-collinear points, then equation of the plane is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

If above points are collinear, then

$$\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0.$$

(iv) Equation of Plane in Intercept Form:

If a, b and c are x-intercept, y-intercept and z-intercept, respectively made by the plane on the coordinate axes, then equation of plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Example

Write the equation of the plane whose intercepts on the co- ordinates axes are -4, 2 and 3 respectively.

Solution: We know that the equation of a plane having a, b and c intercept on the coordinate axes is given by $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}= 1$

Here a = -4, b = 2, and c = 3. So, the equation of the required plane is

$\frac{x}{-4}+\frac{y}{2}+\frac{z}{3}= 1 \qquad \Rightarrow -3x + 6y + 4z =12$

(v) Equation of Plane Passing through the Line of Intersection of two given Planes

Vector form:

If equation of the planes are $\vec{r}\cdot\widehat{n_1} = d_1$ and $\vec{r}\cdot\widehat{n_2} = d_2$, then equation of any plane passing through the intersection of planes is $\vec{r}\cdot(\widehat{n_1}+\lambda\widehat{n_2})=d_1+\lambda d_2$ where, λ is a constant and calculated from given condition.

Cartesian form: If the equation of planes are $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$, then equation of any plane passing through the intersection of planes is $a_1x + b_1y + c_1z - d_1 + \lambda (a_2x + b_2y + c_2z - d_2) = 0$ where, λ is a constant and calculated from given condition.

COPLANARITY OF TWO LINES

Vector form: If two lines $\vec{r} =\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} +\mu\vec{b_2}$ are coplanar, then $(\vec{a_2}-\vec{a_1})\cdot(\vec{b_2}-\vec{b_1})=0$

Cartesian form: If two lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are

Coplanar, then $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$

Example

Find the equation of the plane that passes through the intersection of the planes $\vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k}) = 6$ and $\vec{r}.(2\hat{\imath}+3\hat{\jmath}+4\hat{k}) = -5$ and passing through the point (1, 1, 1).

Solution: You must note that the vector equations of the two planes are given so the equation of the required (third) plane will also be in vector form.

Given: $\pi_1 = \vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k}) = 6$ $\qquad$ $\pi_2 = \vec{r}.(2\hat{\imath}+3\hat{\jmath}+4\hat{k}) = -5$

Now, we already know that the equation of the required plane is $\pi_1+\lambda\pi_2=0$ i.e.

$[\vec{r}.(\hat{\imath}+\hat{\jmath}+\hat{k}) - 6+\lambda\,[\vec{r}.(2\hat{\imath}+3\hat{\jmath}+4\hat{k})+5\,]=0$

$\vec{r}.[\,(\hat{\imath}+\hat{\jmath}+\hat{k})+\lambda.(2\hat{\imath}+3\hat{\jmath}+4\hat{k})] - 6 + 5\lambda=0$

$(x\hat{\imath}+y\hat{\jmath}+z\hat{k}).[(\hat{\imath}+\hat{\jmath}+\hat{k})+\lambda.(2\hat{\imath}+3\hat{\jmath}+4\hat{k})]-6+5\lambda=0$

Now since the required plane passes through (1,1,1), the point must satisfy the equation of the plane. Putting x =1 , y = 1, z = 1 we have –

$(\hat{\imath}+\hat{\jmath}+\hat{k}).[\,(\hat{\imath}+\hat{\jmath}+\hat{k})+\lambda.(2\hat{\imath}+3\hat{\jmath}+4\hat{k})]-6+5\lambda=0$

$(\hat{\imath}+\hat{\jmath}+\hat{k})\,[(1+2\lambda)\hat{\imath}+(1+3\lambda)\hat{\jmath}+(1+4\lambda)\hat{k}]-6+5\lambda=0$

Taking the dot product, we have:

$1+2\lambda+1+3\lambda+1+4\lambda-6+5\lambda=0$ $\qquad$ $\lambda=\frac{3}{14}$

Replacing the value of λ in the equation of the required plane gives us –

$\vec{r}.\left((1+\frac{3}{4})\hat{\imath}+(1+\frac{9}{14})\hat{\jmath}+(1+\frac{6}{7})\hat{k}\right) = 6 - \frac{15}{14}$

$\vec{r}.\left(\frac{10}{7}\hat{\imath}+\frac{231}{14}\hat{\jmath}+\frac{13}{7}\hat{k}\right) = \frac{69}{14}$

$\vec{r}.(20\hat{\imath}+23\hat{\jmath}+26\hat{k}) = 69$

is the required equation of the plane in Vector form

ANGLE BETWEEN TWO PLANES

Vector form: If $\vec{n_1}$ and $\vec{n_2}$ are normal to the planes and θ be the angle between the planes $\vec{r}\cdot\hat{n_1}=d_1$ and $\vec{r}\cdot\hat{n_2}=d_2$, then θ is the angle between the normals to the planes drawn from some common points.

$$\cos\theta = \left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}\right|.$$

Note

The planes are perpendicular to each other, if $\vec{n_1}.\vec{n_2}=0$ and parallel, if $\vec{n_1}.\vec{n_2}=|\vec{n_1}||\vec{n_2}|$

Cartesian form: If the two planes are $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$, then

$$\cos\theta = \frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}.$$

Note

Planes are perpendicular to each other, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$ and planes are parallel, if

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Example

Calculate the angle between the two planes given by the equation 2x + 4y – 2z = 5 and 6x – 8y – 2z = 14.

Solution: As mentioned above, the angle between two planes is equal to the angle between their normal.

Normal vectors to the above planes are represented by:

$n_1 = 2\hat{\imath}+4\hat{\jmath}-2\hat{k}$ and $n_2 = 6\hat{\imath}-8\hat{\jmath}-2\hat{k}$

$\cos\theta = \left|\frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\right|$

$\cos\theta = \left|\frac{(2\hat{\imath}+4\hat{\jmath}-2\hat{k}).(6\hat{\imath}-8\hat{\jmath}-2\hat{k})}{\sqrt{4+16+4}.\sqrt{36+64+4}}\right| = \left(\frac{2\sqrt{39}}{39}\right)$

$\theta = \cos^{-1}\left(\frac{2\sqrt{39}}{39}\right)$

DISTANCE OF A POINT FROM A PLANE

Vector form: The distance of a point whose position vector is $\vec{a}$ from the plane $\vec{r}\cdot\hat{n}$ = dis $|d-\vec{a}\ \hat{n}|$

Note

(i) If the equation of the plane is in the form $\vec{r}\ \vec{n} = d$, where $\vec{n}$ is normal to the plane, then the perpendicular distance is $|\vec{a}\cdot\vec{n} - d|/|\vec{n}\,|$

(ii) The length of the perpendicular from origin O to the plane $\vec{r}\ \vec{n} = d/|\vec{n}|$ $\quad[\because \vec{a} = 0]$

Cartesian form: The distance of the point (x_1, y_1, z_1) from the plane Ax + By + Cz = D is

d = $\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$

Example

Find the distance of a point (2,5,-3) from the plane $\vec{r}.\left(6\hat{\imath}-3\hat{\jmath}+2\hat{k}\right)=4$

Solution: Here, $\vec{a}=2\hat{\imath}+5\hat{\jmath}-3\hat{k}, \vec{N}=6\hat{\imath}-3\hat{\jmath}+2\hat{k}$ and d = 4

Therefore, the distance of the point (2,5,-3) from the given plane is

$\frac{|(2\hat{\imath}+5\hat{\jmath}-3\hat{k}).(6\hat{\imath}-3\hat{\jmath}+2\hat{k})-4|}{|6\hat{\imath}-3\hat{\jmath}+2\hat{k}|}=\frac{|12-15-6-4|}{\sqrt{36+9+4}}=\frac{13}{7}$

ANGLE BETWEEN A LINE AND A PLANE

Vector form: If the equation of line is $\vec{r}=\vec{a}+\lambda\vec{b}$ and the equation of plane is $\vec{r}\cdot\hat{n}$= d, then the angle θ between the line and the normal to the plane is

$$\cos\theta=\left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right|$$

and so the angle Φ between the line and the plane is given by 90° – θ, i.e. sin (90° –θ) = cos θ

$$\sin\phi=\left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right|$$

Cartesian form: If a, b and c are the DR's of line and lx + my + nz + d = 0 be the equation of plane, then $\sin\theta=\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$

If a line is parallel to the plane, then al + bm + cn = 0 and if line is perpendicular to the plane, then $\frac{l}{a}=\frac{m}{b}=\frac{n}{c}$

Remember Points (i) If a line is parallel to the plane, then normal to the plane is perpendicular to the line. i.e. $a_1a_2+b_1b_2+c_1c_2=0$

(ii) If a line is perpendicular to the plane, then DR's of line are proportional to the normal of the plane. i.e. $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ where, a_1, b_1 and c_1 are the DR's of a line and a_2, b_2 and c_2 are the DR's of normal to the plane.

Example

Find the angle between the line

$\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane 10x+2y-11z = 3

Solution: Let Φ be the angle between the line and the normal to the plane. Converting the given equations into vector form, we have

$\vec{r}=\left(-\hat{\imath}+3\hat{k}\right)+\lambda\left(2\hat{\imath}+3\hat{\jmath}+6\hat{k}\right)$

And $\vec{r}.\left(10\hat{\imath}+2\hat{\jmath}-11\hat{k}\right)=3$

$\vec{b}=2\hat{\imath}+3\hat{\jmath}+6\hat{k}$ and $\vec{n}=10\hat{\imath}+2\hat{\jmath}-11\hat{k}$

$\sin\Phi=\left|\frac{(2\hat{\imath}+3\hat{\jmath}+6\hat{k}).(10\hat{\imath}+2\hat{\jmath}-11\hat{k})}{\sqrt{4+9+36}\sqrt{100+4+121}}\right|=\left|\frac{-40}{7\times15}\right|=\frac{8}{21}$

$\Phi=\sin^{-1}\left(\frac{8}{21}\right)$

QUESTIONS

MCQ

Q1. The projections of a vector on the three coordinate axis are " 6,-3,2" respectively. The direction cosines of the vector are :
(a) $\frac{6}{7}, \frac{-3}{7}, \frac{2}{7}$
(b) $\frac{-6}{7}, \frac{-3}{7}, \frac{2}{7}$
(c) $6, -3, 2$
(d) $\frac{6}{5}, \frac{-3}{5}, \frac{2}{5}$

Q2. **Statement-I:** The point A(3,1,6) is the mirror image of the point B(1,3,4) in the plane x – y + z = 5. **Statement-2:** The plane x – y + z = 5 bisects the line segment joining A(3,1,6) and B(1,3,4).
(a) Statement -1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement -1
(b) Statement -1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement -1
(c) Statement -1 is true, Statement-2 is false
(d) Statement -1 is false, Statement-2 is true

Q3. The points A (0,1,2) , B(2, -1, 3) and C (1,-3,1) are vertices of
(a) An isosceles right angled triangle
(b) Right angled triangle
(c) An equilateral triangle
(d) None of these

Q4. The distance of the point $(1, -5, 9)$ from the plane $x - y + z = 5$ measured along a straight line $x = y = z$ is :
(a) $3\sqrt{5}$ unit (b) $10\sqrt{3}$ unit
(c) $5\sqrt{3}$ unit $3\sqrt{10}$ unit

Q5. An equation of a plane parallel to the plane x - 2y + 2z-5 = 0 and at a unit distance from the origin is :
(a) x-2y+2z+5=0
(b) x-2y+2z-3=0
(c) x-2y+2z+1=0
(d) x-2y+2z-1=0

Q6. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to
(a) 0 (b) -1
(c) $\frac{2}{9}$ (d) $\frac{9}{2}$

Q7. Distance between two parallel planes 2x+y+2z=8 and 4x + 2y + 4z + 5 = 0 is :-
(a) $\frac{3}{2}$ (b) $\frac{5}{2}$
(c) $\frac{7}{2}$ (d) $\frac{9}{2}$

Q8. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have:
(a) Any value (b) Exactly one value
(c) Exactly two values (d) Exactly three values

Q9. A vector $\vec{n}$ is inclined to x-axis at 45°, to y-axis at 60° and at an acute angle to z-axis. If $\vec{n}$ is a normal to a plane passing through the point $(\sqrt{2}, -1, 1)$, then the equation of the plane is :
(a) $\sqrt{2}x$-y-z=2 (b) $\sqrt{2}x$+y+z=2
(c) $3\sqrt{2}x$ –4y-3z=7 (d) $4\sqrt{2}x$+7y+z=2

Q10. The acute angle between two lines such that the direction cosines l,m, n of each of them satisfy the equations ℓ +m+n=0 and $\ell^2 + m^2 - n^2 = 0$ is :-
(a) 30° (b) 45°
(c) 60° (d) 15°

Q11. Find the vector equation of the plane whose cartesian form of equation is 3x -4y +2z = 5.
(a) $\vec{r}.(3\hat{\imath} - 4\hat{\jmath} + 2\hat{k})$=5
(b) $\vec{r}.(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k})$=5
(c) $\vec{r}.(3\hat{\imath} - 4\hat{\jmath} - 2\hat{k}) = 5$
(d) $\vec{r}.(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}) = 5$

Q12. If the projections of a line segment on the y and z-axes in 3-dimensional space are 2,3 and 6 respectively, then the length of the line segment is :
(a) 7 (b) 9
(c) 12 (d) 6

Q13. If two lines L_1 and L_2 in space, are defined by
$L_1 = \{x = \sqrt{\lambda}y + (\sqrt{\lambda} - 1), z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda}\}$ and
$L_2 = \{x = \sqrt{\mu}y + 1 - \sqrt{\mu}), z = (1 - \sqrt{\mu})y + \sqrt{\mu}\}$ then L_1 is perpendicular to L_2, for all non-negative reals λ and μ, such that :
(a) $\lambda = \mu$ (b) $\lambda \neq \mu$
(c) $\sqrt{\lambda} + \sqrt{\mu} = 1$ (d) $\lambda + \mu = 0$

Q14. The equation of a plane through the line of intersection of the planes x+2y=3,y-2z+1=0, and perpendicular to the first plane is :
(a) 2x-y+7z=11
(b) $2x - y + 10z = 11$
(c) $2x - y - 9z = 10$
(d) $2x - y - 10z = 9$

Q15. The equation of the plane containing the line 2x-5y+z=3; x+y+4z=5, and parallel to the plane, x+3y+6z=1, is
(a) x+3y+6z=7
(b) 2x+6y+12z=-13
(c) 2x+6y+12z=13
(d) x+3y+6z=-7

Q16. The equation of the plane passing through the point $(1,2,-3)$ and perpendicular to the planes 3x+y-2z=5 and 2x-5y-z=7, is:

(a) 3x-10y-2z+11=0 (b) 6x-5y-2z-2=0

(c) 11x+y+17z+38 (d) 6x-5y+2z+10=0

Q17. The number of distinct real values of λ for which the lines $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{\lambda^2}$ and $\frac{x-5}{1}=\frac{y-4}{\lambda^2}=\frac{z+1}{2}$ are coplanar is

(a) 0, $\pm\sqrt{2}$ (b) 0,2,-2

(c) 1,-2,0 (d) 4,5,0

Q18. A ray of light is incident along a line which meets another line, 7x − y + 1 = 0, at the point (0, 1). The ray is then reflected from this point along the line, y + 2x = 1. Then the coefficient of x, y in the equation of the line of incidence of the ray of light is

(a) (41,38) (b) (-41, 38)

(c) (41,25) (d) (41, -25)

Q19. The angle between the lines $\frac{x-2}{2}=\frac{y-1}{7}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{y-4}{2}=\frac{z-5}{4}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\cos^{-1}\left(\frac{3}{8}\right)$

Q20. Find the vector equation of the plane through the point $3\hat{\imath}-\hat{\jmath}+2\hat{k}$ and parallel to the lines

$\vec{r}=-\hat{\jmath}+3\hat{k}+\lambda(2\hat{\imath}-5\hat{\jmath}-\hat{k})$

and $\vec{r}=\hat{\imath}-3\hat{\jmath}+\hat{k}+\mu(-5\hat{\imath}+4\hat{\jmath})$

(a) $\vec{r}\cdot(4\hat{\imath}+5\hat{\jmath}-17\hat{k})+27=0$

(b) $\vec{r}\cdot(4\hat{\imath}+5\hat{\jmath}-17\hat{k})=0$

(c) $\vec{r}\cdot(4\hat{\imath}+5\hat{\jmath}-17\hat{k})+17=0$

(d) None

Q21. Find the coordinates of the foot of perpendicular drawn from the origin to the plane " 2x-3y+4z-6=0.

(a) $\left(\frac{12}{29},-\frac{18}{29},\frac{-24}{29}\right)$ (b) $\left(\frac{-12}{29},-\frac{18}{29},\frac{24}{29}\right)$

(c) $\left(\frac{12}{29},-\frac{18}{29},\frac{24}{29}\right)$ (d) $\left(\frac{12}{29},\frac{18}{29},\frac{24}{29}\right)$

Q22. Two lines $L_1:\frac{x-5}{0}=\frac{y}{3-\alpha}=\frac{z}{-2}$ and $L_2:\frac{x-\alpha}{0}=\frac{y}{-1}=\frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

(a) 0 (b) 2

(c) 3 (d) 4

Q23. The distance of point $(2,1,-3)$ parallel to the vector $(2\hat{\imath}+3\hat{\jmath}-6\hat{k})$ from the plane $2x+y+z+8=0$ is :

(a) 50 (b) 70

(c) 90 (d) 45

Q24. The distance of the point (1, 0, 2) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane x – y + z = 16, is

(a) $2\sqrt{14}$ (b) 8

(c) $3\sqrt{21}$ (d) 13

Q25. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two perpendicular lines, then the direction cosine of the line which is perpendicular to both the lines, will be _______.

(a) $(m_1n_2-m_2n_1),(n_1l_2-l_1n_2),(l_1m_2-l_2m_1)$

(b) $(m_1n_2+m_2n_1),(n_1l_2+l_1n_2),(l_1m_2+l_2m_1)$

(c) $(m_1n_2-m_2n_1),(n_1l_2-l_1n_2),(l_1m_2+l_2m_1)$

(d) $(-m_1n_2-m_2n_1),(-n_1l_2-l_1n_2),(-l_1m_2-l_2m_1)$

Q26. Perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane " x + y + z = 3. The feet of perpendiculars lie on the line"

(a) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ (b) $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

(c) $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$ (d) $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

Q27. Find the angle between the line $\vec{r}=(\hat{\imath}+2\hat{\jmath}-\hat{k})+\lambda(\hat{\imath}-\hat{\jmath}+\hat{k})$ and the plane $\vec{r}.(2\hat{\imath}-\hat{\jmath}+\hat{k})=4$.

(a) $\sin^{-1}\frac{2\sqrt{2}}{3}$ (b) $\sin^{-1}\frac{1}{3}$

(c) $\sin^{-1}\frac{2}{3}$ (d) None

Q28. Which of the following is true

(a) Vector equation of a line that passes through two given points whose position vector are $\vec{a}$ and $\vec{b}$ is $\vec{r}=\vec{a}-\lambda\vec{b}$

(b) Vector equation of a line that passes through two given points whose position vector are $\vec{a}$ and $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$

(c) Vector equation of a line that passes through two given points whose position vector are $\vec{a}$ and $\vec{b}$ is $\vec{r}=-\vec{a}-\lambda\vec{b}$

(d) Vector equation of a line that passes through two given points whose position vector are $\vec{a}$ and $\vec{b}$ is $\vec{r}=-\vec{a}+\lambda\vec{b}$

Q29. Given, vectors $\vec{a}=2\hat{\imath}+3\hat{\jmath}-\hat{k}$ and $\vec{b}=\hat{\jmath}+\hat{k}$. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a}$ and $\vec{b}$.If $\vec{u}$ is perpendicular to$\vec{a}$ "and u·b=12, then " $\vec{u}$ is equal to :

(a) $2\hat{\imath}+8\hat{\jmath}+16\hat{k}$

(b) $4\hat{\imath}+4\hat{\jmath}+16\hat{k}$

(c) $-4\hat{\imath}+8\hat{\jmath}+8\hat{k}$

(d) $-4\hat{\imath}+8\hat{\jmath}+16\hat{k}$

Q30. Find the value of $\boldsymbol{\lambda}$ when the projection of $\vec{a}=(\lambda\hat{\imath}+\hat{\jmath}+4\hat{k})$ on $\vec{b}=(2\hat{\imath}+6\hat{\jmath}+3\hat{k})$ is 4units

(a) 3 (b) 2

(c) 5 (d) 7

SUBJECTIVE QUESTIONS

Q1. Find the equation of the line through the points (1, 2, 4) and (2, 4, 6) in vector form as well as in cartesian form.

Q2. Find the co-ordinates of those points on the line $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{6}$ which is at a distance of 10 units from point (1, − 2, 3).

Q3. Find the equation of the line drawn through point (3, 0, 1) to meet at right angles the line $\frac{x+1}{3}=\frac{y-2}{-2}=\frac{z+1}{-1}$

Q4. Show that the two lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect. Find also the point of intersection of these lines.

Q5. Find the length of the perpendicular from P (2, − 3, 1) to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z+2}{-1}$.

NUMERICAL TYPE QUESTIONS

Q1. The distance between the planes 2x − y + 2z = 4 and 6x − 3y + 6z = 9 ___________.

Q2. Find the distance of the point (1, 0, − 3) from the plane x − y − z = 9 measured parallel to the line $\frac{x-2}{2}=\frac{y+2}{3}=\frac{z-6}{-6}$ ______.

Q3. Find the angle between the lines x − 3y − 4 = 0, 4y − z + 5 = 0 and x + 3y − 11 = 0, 2y − z + 6 = 0 __________.

Q4. The length of the perpendicular from point (1, 2, 3) to the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$ is____.

Q5. The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is______.

TRUE AND FALSE

Q1. The plane through the point (1, 1, 1) which passes through the line of intersection of the planes x + y + z = 6 and 2x + 3y + 4z + 5 = 0 is 20x + 23y + 26z − 69 = 0

Q2. If the planes x − cy − bz = 0, cx − y + az = 0 and bx + ay − z = 0 pass through a straight line, then the value of $a^2 + b^2 + c^2 + 2abc$ is 0

Q3. The angle θ between the planes $\vec{r}\cdot\vec{n}_1 = d_1$ and $\vec{r}\cdot\vec{n}_2 = d_2$ is given by, $\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}$

Planes are perpendicular if $\vec{n}_1\cdot\vec{n}_2 = 0$ & planes are parallel if $\vec{n}_1 = \lambda\vec{n}_2$.

Q4. The distance between two parallel planes ax + by + cx + d = 0 and ax + by + cx + d’ = 0 is $\frac{|d-d'|}{\sqrt{a^2+b^2+c^2}}$

Q5. Work done against a constant force $\vec{F}$ over a displacement $\vec{s}$ is defined as $W=\vec{F}.\vec{s}$

ASSERTION AND REASONING

Direction (Q1- 5): Each of these questions contains two statements , one is assertion (A) and other is reason (R).Each of these questions also has four alternative choice, only one of which is the correct answer.

(a) Both A and R are individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false
(d) A is false but R is true

Q1. **Assertion(A) :** The image of the point P(1 , 3, 4) in the plane 2x -y + z +3 = 0 is (-3 , 5,2).
Reason(R): The image (x ,y, z) of a point (x_1, y_1, z_1) in a plane ax +by +cz +d = 0 is given by $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=-\frac{2(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$

Q2. **Assertion(A) :** Two sphere $x^2+y^2+z^2+2x+2y+2z+5=0$ and $x^2+y^2+z^2+3x+3y+7z+6=0$ touch each other , then the equation of common tangent is x + y + 5z +1 = 0
Reason (R): If two spheres S_1 and S_2 touch each other , then $S_1 - S_2 = 0$ is common tangent plane.

Q3. **Assertion (A) :** If $\langle l, m, n\rangle$ are direction cosines of a line , there can be a line ,whose direction cosines are $\langle\sqrt{\frac{l^2+m^2}{2}}, \sqrt{\frac{m^2+n^2}{2}}, \sqrt{\frac{n^2+l^2}{2}}\rangle$.
Reason (R): The sum of direction cosines of a line is unity.

Q4. **Assertion (A) :** Let us consider the straight line $L_1 = \frac{x+4}{1}=\frac{y-3}{2}=\frac{z+2}{3}$,then DC’s of L_1 is $\langle\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$
Reason (R): DR’ s of line L_1 is (1, 2 ,3)
∴ DR’s of $L_1 = \langle\frac{1}{\sqrt{1^2+2^2+3^2}}, \frac{2}{\sqrt{1^2+2^2+3^2}}, \frac{3}{\sqrt{1^2+2^2+3^2}}\rangle = \langle\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$

Q5. **Assertion (A) :** The plane x + y + z = 4 is rotated through 90° about its line of intersection with the plane x + y + 2z = 4. Its equation in the new position is −x − y + 2z − 28 = 0

Reason (R): Given planes are x + y + z = 4 ..... (a)

and x + y + 2z = 4 ..... (b)

Since the required plane passes through the line of intersection of planes (a) and (b)

$\therefore$ its equation may be taken as

x + y + 2z − 4 + k (x + y + z − 4) = 0

or (1 + k)x + (1 + k)y + (2 + k)z − 4 − 4k = 0 ..... (c)

Since planes (a) and (c) are mutually perpendicular,

$\therefore$ (1 + k) + (1 + k) + (2 + k) = 0

or, 4 + 3k = 0 or $k = -\dfrac{4}{3}$

Putting $k = -\dfrac{4}{3}$ in equation (c), we get −x − y + 2z − 28 = 0

This is the equation of the required plane.

HOMEWORK

MCQ

Q1. Find the d.c's of a line whose direction ratios are 2, 3, –6

(a) $\frac{2}{7}, \frac{2}{5}, \frac{2}{7}$ (b) $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

(c) $\frac{2}{7}, \frac{3}{4}, -\frac{2}{7}$ (d) $\frac{3}{7}, \frac{4}{7}, \frac{6}{7}$

Q2. The projections of a line segment on x, y and z axes are respectively 3, 4 and 5. Find the length and direction cosines of the line segment-

(a) $5\sqrt{3}$; $\frac{3}{5\sqrt{3}}, \frac{4}{5\sqrt{3}}, \frac{1}{\sqrt{3}}$ (b) $5\sqrt{2}$; $\frac{5}{5\sqrt{2}}, \frac{3}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$

(c) $5\sqrt{2}$; $\frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$ (d) $3\sqrt{2}$; $\frac{3}{3\sqrt{2}}, \frac{4}{3\sqrt{2}}, \frac{1}{\sqrt{2}}$

Q3. If the line through the points (4, 1, 2) and (5, λ, 0) is parallel to the line through the points (2, 1, 1) and (3, 3, –1), find λ.

(a) 3 (b) –3

(c) 2 (d) 4

Q4. If $\frac{x-1}{\ell} = \frac{y-2}{m} = \frac{z+1}{n}$ is the equation of the line through (1, 2, –1) & (–1, 0, 1), then (ℓ, m, n) is-

(a) (–1, 0, 1) (b) (1, 1, –1)

(c) (1, 2, –1) (d) (0, 1, 0)

Q5. The equation of a line passing through the point (–3, 2, –4) and equally inclined to the axes, are-

(a) x – 3 = y + 2 = z – 4 (b) x + 3 = y – 2 = z + 4

(c) $\frac{x+3}{1} = \frac{y-2}{2} = \frac{z+4}{3}$ (d) none of these

Q6. The co-ordinates of the foot of the perpendicular drawn from the point A (1, 0, 3) to the join of the point B (4, 7, 1) and C (3, 5, 3) are-

(a) $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ (b) (5, 7, 17)

(c) $\left(\frac{5}{3}, -\frac{7}{3}, \frac{17}{3}\right)$ (d) $\left(-\frac{5}{3}, \frac{7}{3}, -\frac{17}{3}\right)$

Q7. The equation of the plane through the three points (1, 1, 1), (1, –1, 1) and (–7, –3, –5), is-

(a) 3x – 4z + 1 = 0 (b) 3x – 4y + 1 = 0

(c) 3x + 4y + 1 = 0 (d) None of these

Q8. The equation of a plane which passes through (2, – 3, 1) and is normal to the line joining the points (3, 4, –1) and (2, – 1, 5) is given by-

(a) x + 5y – 6z + 19 = 0 (b) x – 5y + 6z – 19 = 0

(c) x + 5y + 6z + 19 = 0 (d) x – 5y – 6z – 19 = 0

Q9. Find the angle between the planes 2x – y + z = 6 and x + y + 2z = 3 is-

(a) π / 3 (b) π / 6

(c) π / 2 (d) 0

Q10. The equation of the plane passing through the line of intersection of the planes x + y + z = 1 and 2x + 3y – z + 4 = 0 and parallel to x-axis is-

(a) y – 3z – 6 = 0 (b) y – 3z + 6 = 0

(c) y – z – 1 = 0 (d) y – z + 1 = 0

SUBJECTIVE QUESTIONS

Q1. What is the angle between the lines whose direction cosines are

$-\frac{\sqrt{3}}{4}, \frac{1}{4}, -\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{4}, \frac{1}{4}, \frac{\sqrt{3}}{2}$

Q2. Find the angle between any two diagonals of a cube.

Q3. Find the shortest distance and the vector equation of the line of shortest distance between the lines given by $\vec{r} = 3\hat{\imath} + 8\hat{\jmath} + 3\hat{k} + \lambda \rightleftarrows (3\hat{\imath} - \hat{\jmath} + \hat{k})$ $\vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \mu\ (-3\hat{i} + 2\hat{j} + 4\hat{k})$

Q4. Prove that the shortest distance between any two opposite edges of a tetrahedron formed by the planes y +z = 0, x +z = 0, x +y = 0, x +y +z = $\sqrt{3}$ a is $\sqrt{2}$ a.

Q5. Find the equation of the plane upon which the length of normal from origin is 10 and direction ratios of this normal are 3, 2, 6.

NUMERICAL TYPE QUESTIONS

Q1. The value of k for which the planes 3x – 6y – 2z = 7 and 2x + y – kz = 5 are perpendicular to each other, is________.

Q2. The distance between the planes x + 2y + 3z + 7 = 0 and 2x + 4y + 6z + 7 = 0 is _________.

Q3. If the product of distances of the point (1, 1, 1) from the origin and the plane x – y + z + k = 0 be 5, then k = _______ .

Q4. Let L be the line of intersection of the planes 2x + 3y + z = 1 and x + 3y + 2z = 2. If L makes an angle α with the positive x-axis, then cos α equals ________.

Q5. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to _______.

TRUE AND FALSE

Q1. If a, b and c are direction ratios of a line, then $\frac{l}{a}=\frac{m}{b}=\frac{n}{c}$

Q2. The shortest distance between the lines l_1 and l_2 whose vector equation are $\vec{r}=\hat{\imath}+\hat{\jmath}+\lambda(2\hat{\imath}-\hat{\jmath}+\hat{k})$ and $\vec{r}=2\hat{\imath}+\hat{\jmath}-\hat{k}$ +μ $(3\hat{\imath}-5\hat{\jmath}+2\hat{k})$ is 7 units.

Q3. The distance of the point (x_1, y_1, z_1) from the plane Ax + By + Cz = D is

d = $\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$

Q4. The points (1, 2, 3) and (2, – 1, 4) lie on opposite sides of the plane x + 4y + z – 3 = 0.

Q5. The image of the point P (3, 5, 7) in the plane 2x + y + z = 0 is (– 9, – 1, 1).

ASSERTION AND REASONING

Direction(Q1- 5): Each of these questions contains two statements , one is assertion (A) and other is reason (R).Each of these questions also has four alternative choice, only one of which is the correct answer.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A.

(c) A is true but R is false

(d) A is false but R is true

Q1. **Assertion(A) :** The equation of the plane passing through (1, 2, 0) which contains the line $\frac{x+3}{3}=\frac{y-1}{4}=\frac{z-2}{-2}$ is 6x + 2y + 13z – 10 = 0.

Reason(R): Equation of any plane passing through (1, 2, 0) may be taken as a (x – 1) + b (y – 2) + c (z – 0) = 0 ..... (a) where a, b, c are the direction ratios of the normal to the plane

Q2. **Assertion(A) :** The equation of line x + y – z – 3 = 0 = 2x + 3y + z + 4 in symmetric from is

$$\frac{x-0}{4}=\frac{y+\frac{1}{4}}{-3}=\frac{z+\frac{13}{4}}{1}$$

Reason (R): The points (0, – 1, 0), (2, 1, – 1), (1, 1, 1), (3, 1, –3) are coplanar.

Q3. **Assertion (A) :** Bisector of acute/obtuse angle: First make both the constant terms positive. Then (i) $a_1a_2+b_1b_2+c_1c_2>0$ ⇒origin lies on obtuse angle

(ii) $a_1a_2+b_1b_2+c_1c_2<0$ ⇒ origin lies in acute angle

Reason (R): If θ is the angle between line $\frac{x-x_1}{\ell}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$ and the plane ax + by + cz + d = 0,

then sin θ = $\left[\frac{a\ell+bm+cn}{\sqrt{(a^2+b^2+c^2)}\sqrt{\ell^2+m^2+n^2}}\right]$.

Q4. **Assertion (A) :** The equation of a plane cutting intercept a, b, c on the axes is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

Reason (R): The equation of a plane passing through a point having position vector & normal to vector $\vec{n}$ is $(\vec{r}+\vec{a})\,\vec{n}.=0$ or $\vec{r}+\vec{n}=\vec{a}+\vec{n}$

Q5. **Assertion (A) :** The direction cosines of the normal to the plane x + 2y -3z+4 = 0 are $\langle\frac{-1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{-3}{\sqrt{14}}\rangle$

Reason (R): Shortest distance between the two parallel lines $\vec{r}=\vec{a}_1+\lambda\vec{b}$ & $\vec{r}=\vec{a}_2+\mu\vec{b}$ is

d = $\left|\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{|\vec{b}|}\right|$.

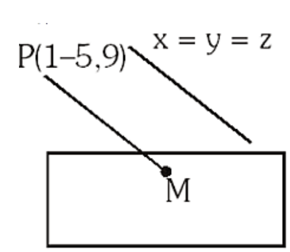
MCQ

S1. **(a)** Projection of a vector " $(\vec{r})$ on x-axis $= |\vec{r}|\ell = 6$
on y-axis $= |\vec{r}|m = -3$
on z-axis $= |\vec{r}|n = 2$
Apply $\ell^2 + m^2 + n^2 = 1$
$6 = 7\ell \Rightarrow \ell = \frac{6}{7}$ similarly $m = -\frac{3}{7}, n = \frac{2}{7}$

S2. **(b)** Mirror image of $B(1,3,4)$ in plane $x - y + z = 5$
$\frac{x-1}{1} = \frac{y-3}{-1} = \frac{z-4}{1} = \frac{-2(1-3+4-5)}{1-1+1} = 2$
$\Rightarrow x = 3, y = 1, z = 6$
∴ mirror image of $B(1,3,4)$ is $A(3,1,6)$
statement-1 is correct statement- 2 is true but it is not the correct explanation. because it is not necessary that to bisect the line joining $(1,3,4)$and
$(3,1,6)$ plane is always perpendicular to line AB

S3. **(a)** We observe that
AB = $\sqrt{(2-0)^2 + (-1-1)^2 + (3-2)^2} = 3$
BC = $\sqrt{(1-2)^2 + (-3+1)^2 + (1-3)^2} = 3$
CA = $\sqrt{(1-0)^2 + (-3-1)^2 + (1-2)^2} = 3\sqrt{2}$
Clearly, AB = BC and $AB^2 + BC^2 = AC^2$
Hence, triangle ABC is an isosceles right angled triangle

S4. (b)



eq^n. of a line || to x=y=z and passing through (1,-5,9) is $\frac{x-1}{1} = \frac{y+5}{1} = \frac{z-9}{1} = r$
Let is meets plane at $M(r+1, r-5, r+9)$
Put in equation of plane
$x - y + z = 5$
$r + 1 - r + 5 + r + 9 = 5$
$r = -10$
Hence M $(-9, -15, -1)$
Distance PM $= \sqrt{100 + 100 + 100} = 10\sqrt{3}$ unit

S5. **(b)** Let equation of plane parallel to x-2y+2z-5=0 be x-2y+2z=k…(i). Distance of plane from origin to (i) is 1
$\left|\frac{k}{\sqrt{9}}\right| = 1$
$k = \pm 3$. Equation of required plane is $x - 2y + 2z \pm 3 = 0$

S6. **(d)** $\begin{vmatrix} 3-1 & K+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & K+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0 \Rightarrow 2K - 9 = 0 \Rightarrow K = \frac{9}{2}$

S7. **(c)** $4x + 2y + 4z + 5 = 0$ and $4x + 2y + 4z - 16 = 0$
$\Rightarrow d = \left|\frac{d_2 - d_1}{\sqrt{A^2+B^2+C^2}}\right|$
$\Rightarrow d = \left|\frac{21}{\sqrt{36}}\right| = \frac{7}{2}$

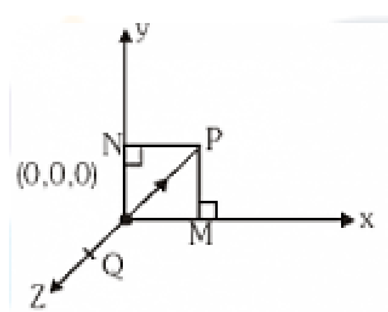
S8. **(c)** $(\bar{a} - \bar{b}) \cdot (\bar{c} \times \bar{d}) = 0$
$\Rightarrow \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & k \\ k & 2 & 1 \end{vmatrix} = 0$
$\Rightarrow k^2 + k = 0$
$\Rightarrow k(k+1) = 0$
$\Rightarrow k = 0$ or $k = -1$

S9. **(b)** Let α, β, γ be direction cosine of vector $\vec{n}, \alpha^2 + \beta^2 + \gamma^2 = 1 \Rightarrow \frac{1}{2} + \frac{1}{4} + \gamma^2 = 1$
$\gamma = \frac{1}{2}\left\{-\frac{1}{2} \text{ rejected}\right\} \vec{n} = \frac{\hat{\imath}}{\sqrt{2}} + \frac{\hat{\jmath}}{2} + \frac{\hat{k}}{2}$
Equation of plane
$\frac{1}{\sqrt{2}}(x - \sqrt{2}) + \frac{1}{2}(y + 1) + \frac{1}{2}(z - 1) = 0$
$\sqrt{2}x + y + z = 2$

S10. **(c)** $l + m + n = 0; \ell^2 + m^2 - n^2 = 0$
$\ell = -(m + n)$
$(m + n)^2 + m^2 - n^2 = 0$
$(m + n)[m + n + m - n] = 0$
$m + n = 0$ or $m = 0$
$m = -n$
Now n = 0 , $m = -l$
Then $l = \frac{1}{\sqrt{2}}, m = -\frac{1}{\sqrt{2}}, n = 0$
Then angle = cos θ = $\frac{1}{2}$
θ = $\frac{\pi}{3}$

S11. **(a)** The equation of the given plane is 3x -4y+2z =5
= $(x\hat{\imath} + y\hat{\jmath} + z\hat{k})(3\hat{\imath} - 4\hat{\jmath} + 2\hat{k}) = 5$
= $\vec{r}.(3\hat{\imath} - 4\hat{\jmath} + 2\hat{k}) = 5$

S12. **(a)**



∴ Projection of a line segment (op) on the x,y & z - axes are 2,3& 6 respectively.

$[OM = 2, ON = 3, OQ = 6]$
$\therefore P(2,3,6)$
$\therefore OP = \sqrt{2^2 + 3^2 + 6^2}$
$= \sqrt{49} = 7$

S13. (a) $L_1 = \{x = \sqrt{\lambda}y + (\sqrt{\lambda} - 1), z = (\sqrt{\lambda} - 1)y + \sqrt{\lambda}\}$
$\therefore L_1 : \frac{x-(\sqrt{\lambda}-1)}{\sqrt{\lambda}} = \frac{y-0}{1} = \frac{z-\sqrt{\lambda}}{(\sqrt{\lambda}-1)}$
$L_2 : \{x = \sqrt{\mu}y + (1 - \sqrt{\mu}), z = (1 - \sqrt{\mu})y + \sqrt{\mu}\}$
$\therefore L_2 : \frac{x-(1-\sqrt{\mu})}{\sqrt{\mu}} = \frac{y-0}{1} = \frac{z-\sqrt{\mu}}{(1-\sqrt{\mu})}$ $\therefore L_1 \perp^r L_2$
$\Rightarrow (\sqrt{\lambda})(\sqrt{\mu}) + 1 + (\sqrt{\lambda} - 1)(1 - \sqrt{\mu}) = 0$
$\Rightarrow \sqrt{\lambda}\sqrt{\mu} + 1 - \sqrt{\lambda}\sqrt{\mu} + \sqrt{\lambda} + \sqrt{\mu} - 1 = 0$
$\Rightarrow \sqrt{\lambda} + \sqrt{\mu} = 0 \Rightarrow \lambda = \mu = 0$

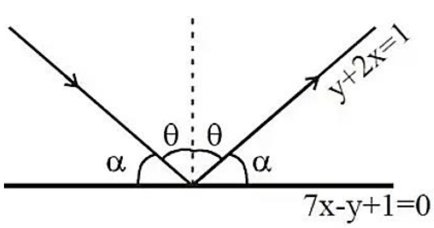
S14. (b) $(x + 2y - 3) + \lambda(y - 2z + 1) = 0$
$(1)(1) + (2 + \lambda)2 + (-2\lambda) \times 0 = 0$
$1 + 4 + 2\lambda = 0$
$\lambda = -\frac{5}{2}$
$(x + 2y - 3) - \frac{5}{2}(y - 2z + 1) = 0$
$2x - y + 10z - 11 = 0$

S15. (a) Let equation of plane parallel to $x + 3y + 6z = 1$ is $x + 3y + 6z = \lambda$ point on line of intersection is (4,1,0), it also lie on plane $x+3y+6z=\lambda$ so required plane is $x+3y+6z=7$

S16. (c) Normal vector of required plane is
$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix}$
$= -11\hat{i} - \hat{j} - 17\hat{k}$
Now, the plane passes through (1,2, -3),
$\therefore 11(x - 1) + (y - 2) + 17(z + 3) = 0$
$\Rightarrow 11x + y + 17z + 38 = 0$

S17. (a) Since the lines
$\frac{x-3}{1} = \frac{y-4}{2} = \frac{z+5}{\lambda^2}$ and $\frac{x-5}{1} = \frac{y-4}{\lambda^2} = \frac{z+1}{2}$
are coplanar Thus,
$\begin{vmatrix} 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \\ 5-3 & 4-4 & -1-(-5) \end{vmatrix} = 0$
$4\lambda^2 - 2(0) + \lambda^2(-2\lambda^2) = 0$
$2\lambda^2[2 - \lambda^2] = 0$
$\lambda = 0, \lambda = \pm\sqrt{2}$

S18. (b)



Let slope of incident ray be m.
$\therefore$ angle of incidence = angle of reflection
$\therefore \left|\frac{m-7}{1+7m}\right| = \left|\frac{-2-7}{1-14}\right| = \frac{9}{13}$
$\Rightarrow \frac{m-7}{1+7m} = \frac{9}{13}$ or $\frac{m-7}{1+7m} = -\frac{9}{13}$
$\Rightarrow 13m - 91 = 9 + 63m$ or $13m - 91 = -9 - 63m$
$\Rightarrow 50m = -100$ or $76m = 82$
$\Rightarrow m = -\frac{1}{2}$ or $m = \frac{41}{38}$
$\Rightarrow y - 1 = -\frac{1}{2}(x - 0)$ or $y - 1 = \frac{41}{38}(x - 0)$
i.e. $x+2y-2=0$ or $38y-38-41x=0$

S19. (c) Let α be the required angle,
$\cos \alpha = \frac{(2\hat{i}+7\hat{j}-3\hat{k})\cdot(-\hat{i}+2\hat{j}+4\hat{k})}{\sqrt{21}\times\sqrt{62}}$
$\cos \alpha = \frac{-2+14-12}{\sqrt{21}\times\sqrt{62}}$
$\cos \alpha = \frac{0}{\sqrt{21}\times\sqrt{62}}$
$\alpha = \cos^{-1} 0$
$\alpha = \frac{\pi}{2}$

S20. (a) The required plane is: $(\vec{r} - \vec{r_1}) \cdot (\vec{b} \times \vec{d}) = 0$
Now $\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & -1 \\ -5 & 4 & 0 \end{vmatrix} = (0 + 4)\hat{i} -$
$(0 - 5)\hat{j} + (8 - 25)\hat{k} = 4\hat{i} + 5\hat{j} - 17\hat{k}$
$(\vec{r} - (3\hat{i} - \hat{j} + 2\hat{k})) \cdot (4\hat{i} + 5\hat{j} - 17\hat{k}) = 0$
$\vec{r} \cdot (4\hat{i} + 5\hat{j} - 17\hat{k}) - (3 \cdot 4 + (-1) \cdot 5 + 2 \cdot (-17)) = 0$
$\vec{r} \cdot (4\hat{i} + 5\hat{j} - 17\hat{k}) + 27 = 0$

S21. (c) The given plane is $2x-3x+4z-6=0$ ... (1)
Direction ratio of the normal to the plane is
<2, -3,4>.
The equation of the line through origin O (0,0,0) and perpendicular to the plane (1) are
$\frac{x-0}{2} = \frac{y-0}{-3} = \frac{z-0}{4}$ ... (2)
Any point on the line (2) is
N $(2\lambda, -3\lambda, 4\lambda)$, where λ is an arbitrary real number.
If it lies on the line (1), then
$2 \cdot 2\lambda - 3 \cdot (-3\lambda) + 4 \cdot 4\lambda - 6 = 0$
$\Rightarrow 29\lambda = 6$
$\Rightarrow \lambda = \frac{6}{29}$
$\therefore$ The foot of perpendicular from origin to the given plane is
$\left(2 \cdot \frac{6}{29}, -3 \cdot \frac{6}{29}, 4 \cdot \frac{6}{29}\right)$ i.e. $\left(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29}\right)$.

S22. (d) $\frac{x-5}{0} = \frac{y-0}{3-\alpha} = \frac{z-0}{-2}$
$\frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$
will be coplanar if shortest distance is zero
$\Rightarrow \begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$
$(5 - \alpha)(\alpha^2 - 5\alpha + 4) = 0, \alpha = 1,4,5$
so $\alpha = 1,4$

S23. (a) Equation of line AB

$\frac{x-2}{2}=\frac{y-1}{3}=\frac{z+3}{-6}=\lambda$

Let B $(2\lambda+2,3\lambda+1,-6\lambda-3)$

B satisfy plane $2x+y+z+8=0$

$2(2\lambda+2)+(3\lambda+1)-6\lambda-3+8=0$

$\lambda=-10$

B $(-18,-29,57)$

Distance AB $=\sqrt{400+900+3600}=70$

S24. (d) Now let $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}=\lambda$

Let P $(3\lambda+2,4\lambda-1,12\lambda+2)$ be a point on the line.

If point P lies also on the plane $x-y+z=16$

Then,

$3\lambda+2-4\lambda+1+12\lambda+2=16$

$\Rightarrow 11\lambda+5=16$

$\Rightarrow \lambda$=1

Therefore P (5,3,14) the distance of the point (1, 0, 2) from P (5,3,14) using distance formula

$=\sqrt{(1-5)^2+(0-3)^2+(2-14)^2}$

$=\sqrt{16+9+144}=\sqrt{169}=13$

S25. (a) Let lines are $l_1x+m_1y+n_1z+d=0\ldots(i)$ and

$l_2x+m_2y+n_2z+d=0$...(ii)

If $lx+my+nz+d=$

0 is perpendicular to (i) and (ii), then,

$l_1+mm_1+nn_1=0, l_2+mm_2+nn_2=0$

$\Rightarrow \frac{l}{m_1n_2-m_2n_1}=\frac{m}{n_1l_2-l_1n_2}=\frac{n}{l_1m_2-l_2m_1}=d$

Therefore,

direction cosines are $(m_1n_2-m_2n_1)$,

$(n_1l_2-l_1n_2),(l_1m_2-l_2m_1)$

S26. (d) Any point B on line is $(2\lambda-2,-\lambda-1,3\lambda)$

Point B lies on the plane for some λ.

$\Rightarrow (2\lambda-2)+(-\lambda-1)+3\lambda=3$

$\Rightarrow 4\lambda=6\Rightarrow\lambda=\frac{3}{2}\Rightarrow B\equiv\left(1,\frac{-5}{2},\frac{9}{2}\right)$

The foot of the perpendicular from point $(-2,-1,0)$ on the plane is the point $A(0,1,2)$

$\Rightarrow$ D.R. of $AB=\left(1,\frac{-7}{2},\frac{5}{2}\right)\equiv(2,-7,5)$

Hence $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

S27. (a) We know that $\sin\theta=\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}=\frac{(\hat{\imath}-\hat{\jmath}+\hat{k}).(2\hat{\imath}-\hat{\jmath}+\hat{k})}{\sqrt{3}.\sqrt{6}}=\frac{2\sqrt{2}}{3}$

$\theta=\sin^{-1}\frac{2\sqrt{2}}{3}$

S28. (b) Vector equation of a line that passes through two given points whose position vector are $\vec{a}$ *and* $\vec{b}$ is $\vec{r}=\vec{a}+\lambda\vec{b}$

S29. (d) Let $\vec{u}=u_1\hat{\imath}+u_2\hat{\jmath}+u_3\hat{k}$ .As $\vec{u}$ is coplanar with the vectors $\vec{a}$ and $\vec{b}$, $\begin{vmatrix}u_1 & u_2 & u_3\\ 2 & 3 & -1\\ 0 & 1 & 1\end{vmatrix}=0$

$4u_1-2u_2+2u_3=0$

$2u_1-u_2+u_3=0\ldots(1)$

$\vec{u}\cdot\vec{a}=0$

$\Rightarrow 2u_1+3u_2-u_3=0\ldots(2)$

$u\cdot b=12$

$\Rightarrow u_2+u_3=12\ldots(3)$

Solving (1),(2) and (3),

$u_1=-4, u_2=8, u_3=16$

Thus , $\vec{u}=-4\hat{\imath}+8\hat{\jmath}+16\hat{k}$

S30. (c)

$\vec{a}=\lambda\hat{\imath}+\hat{\jmath}+4\hat{k}$

$\vec{b}=2\hat{\imath}+6\hat{\jmath}+3\hat{k}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}|=(2^2+6^2+3^2)^{1/2}$

$|\vec{b}|=(4+36+9)^{1/2}=(49)^{1/2}=7$

a unit vector in the direction of the sum of the vectors is given by:

$\hat{b}=\frac{b}{|\vec{b}|}=\frac{2\hat{\imath}+6\hat{\jmath}+3\hat{k}}{7}$

Now it is given that: $\vec{a}.\hat{b}=4$

$\Rightarrow(\lambda\hat{\imath}+\hat{\jmath}+4\hat{k})\cdot\left(\frac{2\hat{\imath}+6\hat{\jmath}+3\hat{k}}{7}\right)=4$

$\Rightarrow 2\lambda+6+(3\times4)=28$

$\Rightarrow\lambda=(28-12-6)/2$

$\Rightarrow\lambda=10/2=5$

SUBJECTIVE QUESTIONS

S1. Let A ≡ (1, 2, 4), B ≡ (2, 4, 6)

Now $\vec{a}=\vec{OA}=\hat{i}+2\hat{j}+4\hat{k}\quad\Rightarrow\vec{b}=\overrightarrow{OB}$

$=2\hat{i}+4\hat{j}+6\hat{k}$

Equation of the line through A($\vec{a}$) and B($\vec{b}$) is $\vec{r}=\vec{a}+t(\vec{b}-\vec{a})$

or $\vec{r}=\hat{i}+2\hat{j}+4\hat{k}+t(\hat{i}+2\hat{j}+2\hat{k})$ ..... (a)

Equation in cartesian form :

Equation of AB is $\frac{x-1}{2-1}=\frac{y-2}{4-2}=\frac{z-4}{6-4}$ or,

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-4}{2}$

S2. Given line is $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{6}$ ..... (a)

Let P ≡ (1, −2, 3)

Direction ratios of line (a) are 2, 3, 6

∴ Direction cosines of line (a) are $\frac{2}{7},\frac{3}{7},\frac{6}{7}$

Equation of line (a) may be written as

$\frac{x-1}{\frac{2}{7}}=\frac{y+2}{\frac{3}{7}}=\frac{z-3}{\frac{6}{7}}$ ..... (b)

Co-ordinates of any point on line (b) may be taken as $\left(\frac{2}{7}r+1, \ \frac{3}{7}r-2, \ \frac{6}{7}r+3\right)$

Let $Q \equiv \left(\frac{2}{7}r+1, \ \frac{3}{7}r-2, \ \frac{6}{7}r+3\right)$

Distance of Q from P = | r |

According to question | r | = 10 $\quad \therefore$

r = ±10 Putting the value of r, we have

$Q \equiv \left(\frac{27}{7}, \ \frac{16}{7}, \ \frac{81}{7}\right)$ or $Q \equiv \left(-\frac{13}{7}, \ -\frac{44}{7}, \ -\frac{39}{7}\right)$

S3. Given line is $\frac{x+1}{3} = \frac{y-2}{-2} = \frac{z+1}{-1}$ ..... (a)

Let P ≡ (3, 0, 1)

Co-ordinates of any point on line (a) may be taken as Q ≡ (3r – 1, – 2r + 2, – r – 1)

Direction ratios of PQ are 3r – 4, – 2r + 2, – r – 2

Direction ratios of line AB are 3, – 2, – 1

Since PQ AB

$\therefore$ 3 (3r – 4) – 2 (– 2r + 2) – 1 (– r – 2) = 0

$\Rightarrow$ 9r – 12 + 4r – 4 + r + 2 = 0

$\Rightarrow$ 14r = 14 $\Rightarrow$ r = 1

Therefore, direction ratios of PQ are 1, 0, 3 or, –1, 0, –3

Equation of line PQ is $\frac{x-3}{1} = \frac{y-0}{0} = \frac{z-1}{3}$

or, $\frac{x-3}{-1} = \frac{y-0}{0} = \frac{z-1}{-3}$

S4. Given lines are $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ ..... (a)

and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$ ..... (b)

Any point on line (a) is P (2r + 1, 3r + 2, 4r +3)

and any point on line (b) is Q (5λ + 4, 2λ + 1, λ)

Lines (a) and (b) will intersect if P and Q coincide for some value of λ and r.

$\therefore$ 2r + 1 = 5λ + 4 $\Rightarrow$ 2r – 5λ = 3 ..... (c)

3r + 2 = 2λ + 1 $\Rightarrow$ 3r – 2λ = – 1 ..... (d)

4r + 3 = λ $\Rightarrow$ 4r – λ = – 3 ..... (5)

Solving (c) and (d), we get r = – 1, λ = – 1

Clearly these values of r and λ satisfy eqn. (5)

Now P ≡ (– 1, – 1, – 1)

Hence lines (a) and (b) intersect at (– 1, – 1, – 1).

S5. Given line is $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+2}{-1}$ ..... (a)

P ≡ (2, – 3, 1)

Co-ordinates of any point on line (a) may be taken as Q ≡ (2r – 1, 3r + 3, – r – 2)

Direction ratios of PQ are 2r – 3, 3r + 6, – r – 3

Direction ratios of AB are 2, 3, – 1

Since PQ AB

$\therefore$ 2 (2r – 3) + 3 (3r + 6) – 1 (– r – 3) = 0

or, 14r + 15 = 0 $\quad \therefore \quad r = \frac{-15}{14}$

$\therefore \quad Q \equiv \left(\frac{-22}{7}, \ \frac{-3}{14}, \ \frac{-13}{14}\right) \quad \therefore \quad PQ = \sqrt{\frac{531}{14}}$ units.

NUMERICAL TYPE QUESTIONS

S1. ($\frac{1}{3}$) Given planes are 2x – y + 2z – 4 = 0.....(a)

and 6x – 3y + 6z – 9 = 0 .....(b)

We find that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence planes (a) and (b) are parallel.

Plane (b) may be written as 2x – y + 2z – 3 = 0 ..... (c)

$\therefore$ Required distance between the planes = $\frac{|4-3|}{\sqrt{2^2+(-1)^2+2^2}} = \frac{1}{3}$

S2. (7) Given plane is x – y – z = 9 ..... (a)

Given line AB is $\frac{x-2}{2} = \frac{y+2}{3} = \frac{z-6}{-6}$ ..... (b)

Equation of a line passing through the point Q (1, 0, – 3) and parallel to line (b) is

$\frac{x-1}{2} = \frac{y}{3} = \frac{z+3}{-6}$ = r. ..... (c)

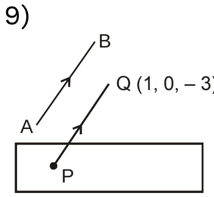
Co-ordinates of any point on line (c) may be taken as

P (2r + 1, 3r, – 6r – 3)

If P is the point of intersection of line (c) and plane (a), then P lies on plane (a),

$\therefore$ (2r + 1) – (3r) – (– 6r – 3) = 9 , r = 1 or P ≡ (3, 3, – 9)

Distance between points Q (1, 0, – 3) and P (3, 3, – 9)



$PQ = \sqrt{(3-1)^2+(3-0)^2+(-9-(-3))^2} = \sqrt{4+9+36} = 7.$

S3. **(90)** Given lines are $\left.\begin{array}{l} x-3y-4=0 \\ 4y-z+5=0 \end{array}\right\}$ ..... (a)

And $\left.\begin{array}{l} x+3y-11=0 \\ 2y-z+6=0 \end{array}\right\}$ ..... (b)

Let λ_1, m_1, n_1 and λ_2, m_2, n_2 be the direction cosines of lines (a) and (b) respectively

Since line (a) is perpendicular to the normals of each of the planes

x − 3y − 4 = 0 and 4y − z + 5 = 0

$\therefore \lambda_1 - 3m_1 + 0.n_1 = 0$ ..... (c)

And $0\lambda_1 + 4m_1 - n_1 = 0$ ..... (d)

Solving equations (c) and (d), we get $\frac{\ell_1}{3-0} = \frac{m_1}{0-(-1)} = \frac{n_1}{4-0}$ or, $\frac{\ell_1}{3} = \frac{m_1}{1} = \frac{n_1}{4}$ = k (let).

Since line (b) is perpendicular to the normals of each of the planes

x + 3y − 11 = 0 and 2y − z + 6 = 0,

$\therefore \lambda_2 + 3m_2 = 0$ ..... (5)

And $2m_2 - n_2 = 0$ ..... (6)

$\therefore \lambda_2 = -3m_2$ or, $\frac{\ell_2}{-3} = m_2$ and

$n_2 = 2m_2$ or, $\frac{n_2}{2} = m_2$.

$\therefore \frac{\ell_2}{-3} = \frac{m_2}{1} = \frac{n_2}{2}$ = t (let).

If θ be the angle between lines (a) and (b), then $\cos\theta = \lambda_1\lambda_2 + m_1m_2 + n_1n_2$

= (3k) (− 3t) + (k) (t) + (4k) (2t) = − 9kt + kt + 8kt = 0

$\therefore \theta = 90°$.

S4. **(7)** Let us take a point on line $(3\lambda + 6, 2\lambda + 7, -2\lambda + 7)$ .Direction ratio's of line which is perpendicular to given line $(3\lambda + 6 - 1, 2\lambda + 7 - 2, -2\lambda + 7 - 3) = (3\lambda + 5, 2\lambda + 5, -2\lambda + 4)$

And the direction ratio's of given line are 3 , 2 , -2

These two lines are perpendicular , so $(3\lambda + 5) \times 3 + (2\lambda + 5) \times 2 + (-2\lambda + 4) + (-2) = 0$

$\lambda = -1$

So, point is (2, 4, 6)

So, distance between (3 , 5, 9) and (1, 2, 3) is 7. (by distance formula)

S5. ($3\sqrt{30}$) $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ ....(i)

and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ ....(ii)

Comparing the equation with ,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

we get

$\Rightarrow x_1 = 3, y_1 = 8, z_1 = 3$ and $a_1 = 3, b_1 = -1, c_1 = 1$

$\Rightarrow x_2 = -3, y_2 = -7, z_2 = 6$ and $a_2 = -3, b_2 = 2, c_2 = 4$

$\Rightarrow \begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} -6 & -15 & 3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$ = -6 (-4 -2) + 15 (12 + 3) + 3(6 -3) = 270

$\sqrt{(a_1b_2 - a_2b_1)^2 + (b_1c_2 - c_1b_2)^2 + (c_1a_2 - c_2a_1)^2}$

$\Rightarrow \sqrt{((3)(2) - (-3)(-1))^2 + ((-1)(4) - (2)(1))^2 + ((1)(-3) - (4)(3))^2}$

$\Rightarrow \sqrt{270}$

Shortest distance between line is d.

$$\Rightarrow d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(a_1b_2-a_2b_1)^2+(b_1c_2-c_1b_2)^2+(c_1a_2-c_2a_1)^2}}$$

$\Rightarrow d = \left|\frac{270}{\sqrt{270}}\right|$ $\Rightarrow d = 3\sqrt{30}$

TRUE AND FALSE

S1. (True) Given planes are $x + y + z - 6 = 0$ ..... (a)
And $2x + 3y + 4z + 5 = 0$ ..... (b)
Given point is P (1, 1, 1).
Equation of any plane through the line of intersection of planes (a) and (b) is
$x + y + z - 6 + k(2x + 3y + 4z + 5) = 0$ ..... (c)
If plane (c) passes through point P, then
$1 + 1 + 1 - 6 + k(2 + 3 + 4 + 5) = 0$ or, $k = \frac{3}{14}$
From (c) required plane is $20x + 23y + 26z - 69 = 0$

S2. (False) Given planes are $x - cy - bz = 0$ ..... (a)
$cx - y + az = 0$ ..... (b)
$bx + ay - z = 0$ ..... (c)
Equation of any plane passing through the line of intersection of planes (a) and (b) may be
taken as $x - cy - bz + \lambda(cx - y + az) = 0$
or, $x(1 + \lambda c) - y(c + \lambda) + z(-b + a\lambda) = 0$ ..... (d)
If planes (c) and (d) are the same, then equations (c) and (d) will be identical.
$\therefore \frac{1+c\lambda}{b} = \frac{-(c+\lambda)}{a} = \frac{-b+a\lambda}{-1}$
(i) (ii) (iii)
From (i) and (ii), $a + ac\lambda = -bc - b\lambda$
or, $\lambda = -\frac{(a+bc)}{(ac+b)}$ ..... (5)
From (ii) and (iii),
or, $\lambda = -\frac{(a+bc)}{(ac+b)}$ ..... (5)
From (ii) and (iii),
$c + \lambda = -ab + a^2\lambda$ or $\lambda = \frac{-(ab+c)}{1-a^2}$ ..... (6)
From (5) and (6), we have $\frac{-(a+bc)}{ac+b} = \frac{-(ab+c)}{(1-a^2)}$.
or, $a - a^3 + bc - a^2bc = a^2bc + ac^2 + ab^2 + bc$ or $a^2bc + ac^2 + ab^2 + a^3 + a^2bc - a = 0$
or, $a^2 + b^2 + c^2 + 2abc = 1$.

S3. (True) The angle θ between the planes $\vec{r}\cdot\vec{n}_1 = d_1$ and $\vec{r}\cdot\vec{n}_2 = d_2$ is given by, $\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}$
Planes are perpendicular if $\vec{n}_1 \cdot \vec{n}_2 = 0$ & planes are parallel if $\vec{n}_1 = \lambda\vec{n}_2$.

S4. (True) The distance between two parallel planes $ax + by + cx + d = 0$ and $ax + by + cx + d' = 0$ is $\frac{|d-d'|}{\sqrt{a^2+b^2+c^2}}$

S5. (True) Work done against a constant force $\vec{F}$ over a displacement $\vec{s}$ is defined as $W = \vec{F}.\vec{s}$

ASSERTION AND REASONING

S1. (a) The image of the point P(1,3,4) in the plane $2x - y + z + 3 = 0$ is given by
$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = -\frac{2(2-3+4+3)}{4+1+1}$
$\Rightarrow \frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = -2$
$\Rightarrow x = -3, y = 5, z = 2$
$\therefore$ Co- ordinate of image of the point P(1 , 3, 4) is (-3 , 5, 2).
Thus A and R are true and R is correct explanation of A

S2. (a) Equation of the sphere are $x^2 + y^2 + z^2 + 2x + 2y + 2z + 5 = 0$ ....(i)
And $x^2 + y^2 + z^2 + 3x + 3y + 7z + 6 = 0$ ....(ii)
$(x^2 + y^2 + z^2 + 2x + 2y + 2z + 5) - (x^2 + y^2 + z^2 + 3x + 3y + 7z + 6) = 0$
$\Rightarrow -x - y - 5z - 1 = 0$
$\Rightarrow x + y + 5z + 1 = 0$
Thus A and R are true and R is correct explanation of A

S3. (c) If the direction cosines of any line is $\langle l, m, n\rangle$, then $l^2 + m^2 + n^2 = 1$
So, for the direction cosines,
$\langle\sqrt{\frac{l^2+m^2}{2}}, \sqrt{\frac{m^2+n^2}{2}}, \sqrt{\frac{n^2+l^2}{2}}\rangle = \left(\sqrt{\frac{l^2+m^2}{2}}\right)^2 + \left(\sqrt{\frac{m^2+n^2}{2}}\right)^2 + \left(\sqrt{\frac{n^2+l^2}{2}}\right)^2 = \frac{l^2+m^2}{2} + \frac{m^2+n^2}{2} + \frac{n^2+l^2}{2} = l^2 + m^2 + n^2 = 1$
While , $l + m + n \neq 1$

S4. (a) **Assertion (A) :** Let us consider the straight line $L_1 = \frac{x+4}{1} = \frac{y-3}{2} = \frac{z+2}{3}$,then DC's of L_1 is $\langle\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$
Reason (R): DR' s of line L_1 is (1, 2 ,3)
$\therefore$ DR's of $L_1 = \langle\frac{1}{\sqrt{1^2+2^2+3^2}}, \frac{2}{\sqrt{1^2+2^2+3^2}}, \frac{3}{\sqrt{1^2+2^2+3^2}}\rangle = \langle\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$
Thus A and R are true and R is correct explanation of A

S5. (a) Assertion (A) : The plane $x + y + z = 4$ is rotated through 90° about its line of intersection with the plane $x + y + 2z = 4$. Its equation in the new position is $-x - y + 2z - 28 = 0$

Reason (R): Given planes are $x + y + z = 4$ ..... (a)

and $x + y + 2z = 4$ ..... (b)

Since the required plane passes through the line of intersection of planes (a) and (b)

$\therefore$ its equation may be taken as

$x + y + 2z - 4 + k(x + y + z - 4) = 0$

or $(1 + k)x + (1 + k)y + (2 + k)z - 4 - 4k = 0$ ..... (c)

Since planes (a) and (c) are mutually perpendicular,

$\therefore (1 + k) + (1 + k) + (2 + k) = 0$

or, $4 + 3k = 0$ or $k = -\frac{4}{3}$

Putting $k = -\frac{4}{3}$ in equation (c), we get $-x - y + 2z - 28 = 0$

This is the equation of the required plane.

Thus A and R are true and R is correct explanation of A

HOMEWORK

MCQ

S1. **(b)** If the direction ratios are given as x , y, z , then the direction cosine are given by
$\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}$
So, the direction cosines for direction ratios 2 , 3, -6 are
$\frac{2}{\sqrt{4+9+36}}, \frac{3}{\sqrt{4+9+36}}, \frac{-6}{\sqrt{4+9+36}}$
$\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$

S2. **(c)** Let l , m ,n be the d.c's of the given line segment PQ.
∴ l = cos α , m = cos β , n = cos ϒ
Where α , β , ϒ are the angles which the line segment PQ makes with the axes .
Suppose length of line segments PQ = r .
Thus , projection of line segment PQ on x – axis = PQ cos α = rl.
Also the projection of line segment PQ on x -axis = 3 (given)
∴ rl = 3 , Similarly , mr = 4 , nr = 5
Now squaring and adding there equations , we get
$(lr)^2 + (mr)^2 + (nr)^2 = r^2[3^2 + 4^2 + 5^2]$
$\Rightarrow r^2(l^2 + m^2 + n^2) = 9 + 16 + 25$
$\Rightarrow r = 5\sqrt{2}$ [since $l^2 + m^2 + n^2 = 1$)
Now direction cosine's are
$\Rightarrow \frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{5}{5\sqrt{2}}$

S3. **(a)** For parallel , lines , DC ; s must be equal .
∴ DC's for 1st $\Rightarrow 5 - 4, \lambda - 1, 0 - 2 \Rightarrow 1, \lambda - 1, -2$
∴ DC's for 2nd $\Rightarrow 3 - 2, 3 - 1, -1 - 1 \Rightarrow 1, 2, -2$
Hence $\lambda - 1 = 2 \Rightarrow \lambda = 3$

S4. **(b)** Given equation is $\frac{x-1}{l} = \frac{y-2}{m} = \frac{z+1}{n}$
The equation of line passing through (1, 2, -1) and (-1 , 0, 1) is
$\frac{x-1}{-1-1} = \frac{y-0}{0-2} = \frac{z+1}{1+1}$
$\Rightarrow \frac{x-1}{-2} = \frac{y-2}{-2} = \frac{z+1}{2}$
$\Rightarrow \frac{x-1}{1} = \frac{y-2}{1} = \frac{z+1}{-1}$ ....(i)
Comparing (i) with given equation , we get
$l = 1, m = 1, n = -1$

S5. **(b)** Since , line equally inclined to the axes.
∴ $l = m = n$ ....(i)
The required equatiom of line is
$\frac{x+3}{l} = \frac{y-2}{l} = \frac{z+4}{l}$ [using (i)]
$\Rightarrow \frac{x+3}{1} = \frac{y-2}{1} = \frac{z+4}{1}$
$\Rightarrow x + 3 = y - 2 = z + 4$

S6. **(a)** Equation of line AB
$\frac{x-4}{3-4} = \frac{y-7}{5-7} = \frac{z-1}{3-1}$
$\frac{x-4}{-1} = \frac{y-7}{-2} = \frac{z-1}{2}$
Consider , $\frac{x-4}{-1} = \frac{y-7}{-2} = \frac{z-1}{2} = r$
$x = -r + 4, y = -2r + 7, z = 2r + 1$
General point on the line AB is (-r + 4 , -2r + 7 , 2r +1)
Direction ratios of perpendicular are -r + 4 -1 , -2r +7 -0 , 2r +1 -3
i.e., - r + 3 , -2r + 7 , 2r -2
Since perpendicular is on line AB
Therefore , -1(-r+3) -2(-2r+7)+2(2r-2) = 0
Then , r = $\frac{7}{3}$
Therefore , co – ordinate of the foot of perpendicular are $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

S7. **(a)** We have , $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}, \vec{c} = -7\hat{\imath} - 3\hat{\jmath} - 5\hat{k}$
Let , $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
Now , using the formula for plane ,
$(\vec{r} - \vec{a}).\left[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})\right] = 0$
$\Rightarrow \{(x-1)\hat{\imath} + (y-1)\hat{\jmath} + (z-1)\hat{k}\}.\{(-2\hat{\jmath}) \times (-8\hat{\imath} - 4\hat{\jmath} - 6\hat{k})\} = 0$
$\Rightarrow \{(x-1)\hat{\imath} + (y-1)\hat{\jmath} + (z-1)\hat{k}\}.\{(-2\hat{\jmath}) \times (12\hat{\imath} - 16\hat{k})\} = 0$
$\Rightarrow 12(x-1) - 16(z-1) = 0$
$\Rightarrow 12x - 16z - 12 + 16 = 0$
$\Rightarrow 3x - 4z + 1 = 0$

S8. **(a)** Given plane passes through (2 , -3, 1)
Diretcion ratio line joining two points (3 , 4, -1) and (2 , -1 , 5)
$\Rightarrow 2 - 3, -1 - 4, 5 - (-1)$
$\Rightarrow -1, -5, 6$
Equation of the plane A $(x - x_1) + B(y - y_1) + C(z - z_1) = 0$
$\Rightarrow (-1)(x-2) + (-5)(y - (-3)) + 6(z-1) = 0$
$\Rightarrow (-1)(x-2) + (-5)(y+3) + 6(z-1) = 0$
$\Rightarrow x + 5y - 6z + 19 = 0$

S9. **(a)** We know, the angle between two planes, $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is,

$$\theta = \cos^{-1}\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}$$

So, the acute angle between the plane 2x – y + z = 6 and x + y + 2z = 3 is

$$\theta = \cos^{-1}\frac{((2\times1)+(-1\times1)+(1\times2))}{\sqrt{2^2+(-1)^2+1^2}\sqrt{1^2+1^2+2^2}}$$

$\theta = 60°$

S10. **(b)** The equaton of the plane through the intersection of the plane x + y + z = 1 and 2x + 3y -z + 4 = 0 is

$\Rightarrow (x+y+z-1)+\lambda\,(2x+3y-z+4)=0$

$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z+4\lambda-1=0$

Since the plane parallel to x – axis,

$\therefore\ 1+2\lambda=0 \Rightarrow \lambda = -\frac{1}{2}$

Hence, the required equation will be y – 3z + 6 = 0

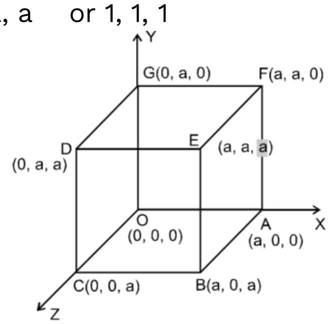
SUBJECTIVE QUESTIONS

S1. Let θ be the required angle, then $\cos\theta = \lambda_1\lambda_2 + m_1m_2 + n_1n_2$

$$= \left(-\frac{\sqrt3}{4}\right)\left(-\frac{\sqrt3}{4}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)+\left(-\frac{\sqrt3}{2}\right).\left(\frac{\sqrt3}{2}\right)$$

$$= \frac{3}{16}+\frac{1}{16}-\frac{3}{4}=-\frac{1}{2} \Rightarrow \theta = 120°,$$

S2. The cube has four diagonals
OE, AD, CF and GB
The direction ratios of OE are
a, a, a or 1, 1, 1



∴its direction cosines are $\frac{1}{\sqrt3}, \frac{1}{\sqrt3}, \frac{1}{\sqrt3}$.

Direction ratios of AD are – a, a, a or – 1, 1, 1.

∴its direction cosines are $\frac{-1}{\sqrt3}, \frac{1}{\sqrt3}, \frac{1}{\sqrt3}$.

Similarly, direction cosines of CF and GB respectively are

$\frac{1}{\sqrt3}, \frac{1}{\sqrt3}, \frac{-1}{\sqrt3}$ and $\frac{1}{\sqrt3}, \frac{-1}{\sqrt3}, \frac{1}{\sqrt3}$.

We take any two diagonals, say OE and AD

Let θ be the acute angle between them, then

$$\cos\theta = \left|\left(\frac{1}{\sqrt3}\right)\left(\frac{-1}{\sqrt3}\right)+\left(\frac{1}{\sqrt3}\right).\left(\frac{1}{\sqrt3}\right)+\left(\frac{1}{\sqrt3}\right).\left(\frac{1}{\sqrt3}\right)\right| = \frac{1}{3}$$

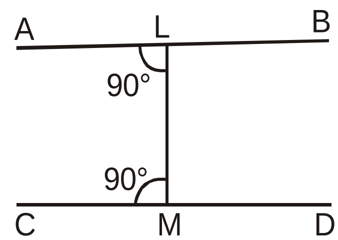
or, $\theta = \cos^{-1}\left(\frac{1}{3}\right)$.

S3. Given lines are

$\vec r = 3\hat i + 8\hat j + 3\hat k + \lambda\ (3\hat i - \hat j + \hat k)$ ..... (a)

And $\vec r = -3\hat i - 7\hat j + 6\hat k + \mu\ (-3\hat i + 2\hat j + 4\hat k)$ ... (b)

Equation of lines (a) and (b) in cartesian form is



AB : $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} = \lambda$

And CD : $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

= μ Let L ≡ (3λ + 3, – λ + 8, λ + 3)

And M ≡ (– 3μ – 3, 2μ – 7, 4μ + 6)

Direction ratios of LM are

3λ + 3μ + 6, – λ – 2μ + 15, λ – 4μ – 3.

Since LM ⊥ AB

∴ 3 (3λ + 3μ + 6) – 1 (– λ – 2μ + 15) + 1 (λ – 4μ – 3) = 0

or, 11λ + 7μ = 0 ..... (5)

Again LM ⊥ CD

∴ – 3 (3λ + 3μ + 6) + 2 (– λ – 2μ + 15) + 4 (λ – 4μ – 3) = 0

or, – 7λ – 29μ = 0 ..... (6)

Solving (5) and (6), we get λ = 0, μ = 0

∴ L ≡ (3, 8, 3), M ≡ (– 3, – 7, 6)

Hence shortest distance LM = $\sqrt{(3+3)^2+(8+7)^2+(3-6)^2} = \sqrt{270} = 3\sqrt{30}$ units

Vector equation of LM is

$\vec r = 3\hat i + 8\hat j + 3\hat k + t\ (6\hat i + 15\hat j - 3\hat k)$

S4. Given planes are y + z = 0 ..... (i)

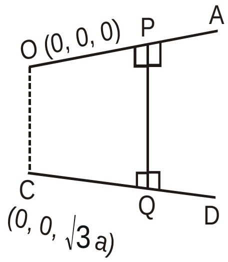
x + z = 0 ..... (ii)

x + y = 0 ..... (iii)

x + y + z = $\sqrt3$ a ..... (iv)

Clearly planes (i), (ii) and (iii) meet at O(0, 0, 0)

Let the tetrahedron be OABC



Let the equation to one of the pair of opposite edges OA and BC be

$y + z = 0, x + z = 0$ ..... (a)

$x + y = 0, x + y + z = \sqrt{3}\,a$ ..... (b)

equation (a) and (b) can be expressed in symmetrical form as

$$\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{-1} \quad \text{..... (c)}$$

and, $$\frac{x-0}{1} = \frac{y-0}{-1} = \frac{z-\sqrt{3}\ a}{0} \quad \text{..... (d)}$$

d. r. of OA and BC are respectively (1, 1, – 1) and (1, – 1, 0).

Let PQ be the shortest distance between OA and BC having direction cosines (λ, m, n)

$\therefore$ PQ is perpendicular to both OA and BC.

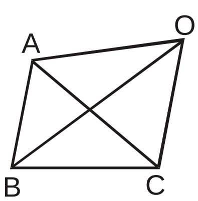
$\therefore$ $\lambda + m - n = 0$

And $\lambda - m = 0$

Solving (5) and (6), we get, $\frac{\ell}{1} = \frac{m}{1} = \frac{n}{2} = k$ (say)

also, $\lambda^2 + m^2 + n^2 = 1$

$\therefore \quad k^2 + k^2 + 4k^2 = 1 \Rightarrow k = \pm\frac{1}{\sqrt{6}}$

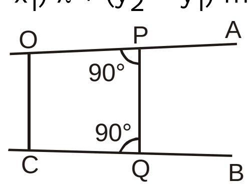


$\therefore \quad \lambda = \pm\frac{1}{\sqrt{6}}, m = \pm\frac{1}{\sqrt{6}}, n = \pm\frac{2}{\sqrt{6}}$

Shortest distance between OA and BC

i.e. PQ = The length of projection of OC on PQ

$= |(x_2 - x_1)\lambda + (y_2 - y_1)m + (z_2 - z_1)n|$



$$= \left|0 \ . \ \frac{1}{\sqrt{6}} + 0 \ . \ \frac{1}{\sqrt{6}} + \sqrt{3}\ a \ . \ \frac{2}{\sqrt{6}}\right| = \sqrt{2}\ a.$$

S5. If p be the length of perpendicular from origin to the plane and λ, m, n be the direction cosines of this normal, then its equation is

$\lambda x + my + nz = p$ ..... (a)

Here p = 10

Direction ratios of normal to the plane are 3, 2, 6

$\sqrt{3^2+2^2+6^2} = 7$

$\therefore$ Direction cosines of normal to the required plane are $\lambda = \frac{3}{7}, m = \frac{2}{7}, n = \frac{6}{7}$

Putting the values of λ, m, n, p in (a), equation of required plane is

$\frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z = 10$ or, $3x + 2y + 6z = 70$

NUMERICAL TYPE QUESTIONS

S1. (0) If the planes are perpendicular to each other , then the dot product of direction ratio's is 0.

$\Rightarrow (3, -6, -2).(2, 1, -k) = 0$

$\Rightarrow 6 - 6 + 2k = 0$

$\Rightarrow k = 0$

S2. ($\frac{\sqrt{7}}{2\sqrt{2}}$) The equation of second plane can be rearrange as $x + 2y + 3z + \frac{7}{2} = 0$

The distance between parallel planes $Ax + By\ Cz + D_1 = 0$ and $Ax + By + Cz + D_2$ us given by

$\frac{|D_1 - D_2|}{\sqrt{A^2+B^2+C^2}}$

Hence , for the given problem distance between plane is given by ; $\frac{\left|7-\frac{7}{2}\right|}{\sqrt{1+5+9}} = \frac{\sqrt{7}}{2\sqrt{2}}$

S3. (-6 or 4) Let d_1 , d_2 be the distance of the point (1, 1, 1) from the origin (0 , 0, 0) and plane $x - y + z + k = 0$

$\therefore \ d_1 = \sqrt{(1-0)^2 + (1-0)^2 + (1-0)^2} = \sqrt{3}$

And $d_2 = \frac{|1-1+1+k|}{\sqrt{3}}$

From the given condition , $d_1 . d_2 = 5$

$\therefore \sqrt{3} \times \frac{|1+k|}{\sqrt{3}} = 5 \Rightarrow |1+k| = 5 \Rightarrow 1 + k = \pm 5$

$\therefore k = -6, 4$

S4. ($\frac{1}{\sqrt{3}}$) $2x + 3y + z = 1$, $x + 3y + 2z = 2$

$$\hat{n} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 3 & 2 \end{vmatrix}$$

$\hat{n} = \hat{\imath}(6-3) - \hat{\jmath}(4-1) + \hat{k}(6-3)$

$\hat{n} = 3\hat{\imath} - 3\hat{\jmath} + 3\hat{k}$

$\therefore\ \vec{a} = \frac{3\hat{\imath}-3\hat{\jmath}+3\hat{k}}{\sqrt{9+9+9}}$

$\vec{a} = \frac{1}{\sqrt{3}}\,\hat{\imath} - \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

$\therefore \cos\alpha = \frac{1}{\sqrt{3}}$

S5. **(-5)** Given , two lines intersect at a point .Therefore , the shortest distance between them is zero .

$\therefore \begin{vmatrix} k & 2 & 3 \\ 3 & k & 2 \\ 1 & 1 & -2 \end{vmatrix} = 0$

$\Rightarrow k\,(-2k-2) - 2(-6-2) + 3(3-k) = 0$

$\Rightarrow 2\,k^2 + 5k - 25 = 0$

$\Rightarrow k = -5\,, \frac{5}{2}$

Hence , integer value of k is -5.

TRUE AND FALSE

S1. (True) If a, b and c are direction ratios of a line, then $\frac{l}{a} = \frac{m}{b} = \frac{n}{c}$

S2. (False) $\overrightarrow{a_1} = \hat{\imath} + \hat{\jmath}$, $\overrightarrow{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$,$\overrightarrow{b_1} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\overrightarrow{b_2} = 3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = \hat{\imath} - \hat{k}$, $\overrightarrow{b_1} \times \overrightarrow{b_2} = 3\hat{\imath} - \hat{\jmath} - 7\hat{k}$ and $\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{59}$

S3. (True) The distance of the point (x_1, y_1, z_1) from the plane Ax + By + Cz = D is

d = $\left|\frac{Ax_1+By_1+Cz_1-D}{\sqrt{A^2+B^2+C^2}}\right|$

S4. **(True)** Since the numbers 1+ 4 × 2 + 3 – 3 = 9 and 2 – 4 + 4 – 3 = – 1 are of opposite sign, then points are on opposite sides of the plane.

S5. **(True)** Given plane is 2x + y + z = 0 ... (a)

Direction ratios of normal to plane (a) are 2, 1, 1

Let Q be the image of point P in plane (a). Let PQ meet plane (a) in R then PQ $\perp$ plane (a)

Let R ≡ (2r + 3, r + 5, r + 7)

Since R lies on plane (a)

∴ 2(2r + 3) + r + 5 + r + 7 = 0

or, 6r + 18 = 0 ∴ r = – 3

∴ R ≡ (– 3, 2, 4)

Let Q ≡ (α, β, γ)

Since R is the middle point of PQ

$\therefore -3 = \frac{\alpha+3}{2} \Rightarrow \alpha = -9$ and $2 = \frac{\beta+5}{2} \Rightarrow \beta = -1$ and $4 = \frac{\gamma+7}{2}$

$\Rightarrow \gamma = 1$

∴ Q = (– 9, – 1, 1).

ASSERTION AND REASONING

S1. **(a)** Equation of any plane passing through (1, 2, 0) may be taken as

a (x – 1) + b (y – 2) + c (z – 0) = 0 .... (a)

where a, b, c are the direction ratios of the normal to the plane. Given line is

$\frac{x+3}{3} = \frac{y-1}{4} = \frac{z-2}{-2}$ ..... (b)

If plane (a) contains the given line, then

3a + 4b – 2c = 0 ..... (c)

Also point (– 3, 1, 2) on line (b) lies in plane (a)

∴ a (– 3 – 1) + b (1 – 2) + c (2 – 0) = 0

or, – 4a – b + 2c = 0 ..... (d)

Solving equations (c) and (d), we get

$\frac{a}{8-2} = \frac{b}{8-6} = \frac{c}{-3+16}$ or, $\frac{a}{6} = \frac{b}{2} = \frac{c}{13} = k$ (say). ..... (5)

Substituting the values of a, b and c in equation (a), we get

6 (x – 1) + 2 (y – 2) + 13 (z – 0) = 0.

or, 6x + 2y + 13z – 10 = 0.

This is the required equation.

Thus A and R are true and R is correct explanation of A.

S2. **(b)** **Assertion(A):** Let λ, m, n be direction ratio of line

then 2λ + 3m + n = 0 and λ + m – n = 0

$\therefore \frac{\ell}{-3-1} = \frac{m}{1+2} = \frac{n}{2-3}$ or $\frac{\ell}{-4} = \frac{m}{3} = \frac{n}{-1}$ or $\frac{\ell}{4} = \frac{m}{-3} = \frac{n}{1}$

let x = 0 then y – z = 3

3y + z = – 4

solving that we get $y = \frac{-1}{4}$; $z = \frac{-13}{4}$

line is passing through point $\left(0, -\frac{1}{4}, -\frac{13}{4}\right)$

and having direction ratios 4, –3 and 1.

$\frac{x-0}{4} = \frac{y+\frac{1}{4}}{-3} = \frac{z+\frac{13}{4}}{1}$

Reason (R): The points (0, – 1, 0), (2, 1, – 1), (1, 1, 1), (3, 1, –3) are coplanar.

Let A ≡ (0, – 1, 0), B ≡ (2, 1, – 1), C ≡ (1, 1, 1) and D ≡ (3, 1, –3)

Equation of a plane through A (0, – 1, 0) is

a (x – 0) + b (y + 1) + c (z – 0) = 0

or, ax + by + cz + b = 0 ..... (a)

If plane (a) passes through B (2, 1, – 1) and C (1, 1, 1)

Then $2a + 2b - c = 0$ ..... (b)

And $a + 2b + c = 0$ ..... (c)

From (b) and (c), we have $\frac{a}{2+2} = \frac{b}{-1-2} = \frac{c}{4-2}$ or, $\frac{a}{4} = \frac{b}{-3} = \frac{c}{2} =$ k (say)

Putting the value of a, b, c, in (a), equation of required plane is

$4kx - 3k(y + 1) + 2kz = 0$

or, $4x - 3y + 2z - 3 = 0$ ..... (b)

Clearly point D (3, 1, –3) lies on plane (b)

Thus point D lies on the plane passing through A, B, C and hence points A, B, C and D are coplanar.

S3. **(b)** **Assertion (A) :** Bisector of acute/obtuse angle: First make both the constant terms positive. Then (i) $a_1a_2 + b_1b_2 + c_1c_2 > 0$ $\Rightarrow$ origin lies on obtuse angle

(ii) $a_1a_2 + b_1b_2 + c_1c_2 < 0$ $\Rightarrow$origin lies in acute angle

A is true.

Reason(R): If θ is the angle between line $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and the plane $ax + by + cz + d = 0$, then $\sin\theta = \left[\frac{a\ell + bm + cn}{\sqrt{(a^2+b^2+c^2)}\sqrt{\ell^2+m^2+n^2}}\right]$.

R is true.

S4. **(c)** **Assertion(A):**

Intercept Form: The equation of a plane cutting intercept a, b, c on the axes is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

A is true

Reason (R): R is false

Because , Vector form: The equation of a plane passing through a point having position vector & normal to vector $\vec{n}$ is $(\vec{r} - \vec{a})\,\vec{n}. = 0$ or $\vec{r}\,.\,\vec{n} = \vec{a}\,.\,\vec{n}$

S5. **(d)** (A) Equation of plane is $x + 2y -3z+4 = 0$ .Its direction rations are (1 ,2, -3)

$\therefore$ Direction cosines are $\langle\frac{1}{\sqrt{14}},\frac{2}{\sqrt{14}},\frac{-3}{\sqrt{14}}\rangle$

Hence A is false.

(R): Shortest distance between the two parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ & $\vec{r} = \vec{a}_2 + \mu\vec{b}$ is $d = \left|\frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|}\right|$.

Hence R is true