4.6 Perturbation theory

Time-independent perturbation theory

Unperturbed states	$\hat{H}_0\psi_n = E_n\psi_n$ (ψ_n nondegenerate)	(4.152)	$\hat{H}_0$ unperturbed Hamiltonian ψ_n eigenfunctions of $\hat{H}_0$ E_n eigenvalues of $\hat{H}_0$ n integer ≥ 0
Perturbed Hamiltonian	$\hat{H} = \hat{H}_0 + \hat{H}'$	(4.153)	$\hat{H}$ perturbed Hamiltonian $\hat{H}'$ perturbation ($\ll \hat{H}_0$)
Perturbed eigenvalues[a]	$E_k' = E_k + \langle\psi_k\|\hat{H}'\|\psi_k\rangle + \sum_{n\neq k}\frac{\|\langle\psi_k\|\hat{H}'\|\psi_n\rangle\|^2}{E_k - E_n} + \dots$	(4.154)	E_k' perturbed eigenvalue ($\simeq E_k$) $\langle\|\|\rangle$ Dirac bracket
Perturbed eigen-functions[b]	$\psi_k' = \psi_k + \sum_{n\neq k}\frac{\langle\psi_k\|\hat{H}'\|\psi_n\rangle}{E_k - E_n}\psi_n + \dots$	(4.155)	ψ_k' perturbed eigenfunction ($\simeq \psi_k$)

[a]To second order.
[b]To first order.

Time-dependent perturbation theory

Unperturbed stationary states	$\hat{H}_0\psi_n = E_n\psi_n$	(4.156)	$\hat{H}_0$ unperturbed Hamiltonian ψ_n eigenfunctions of $\hat{H}_0$ E_n eigenvalues of $\hat{H}_0$ n integer ≥ 0
Perturbed Hamiltonian	$\hat{H}(t) = \hat{H}_0 + \hat{H}'(t)$	(4.157)	$\hat{H}$ perturbed Hamiltonian $\hat{H}'(t)$ perturbation ($\ll \hat{H}_0$) t time
Schrödinger equation	$[\hat{H}_0 + \hat{H}'(t)]\Psi(t) = \mathrm{i}\hbar\frac{\partial\Psi(t)}{\partial t}$	(4.158)	Ψ wavefunction
	$\Psi(t=0) = \psi_0$	(4.159)	ψ_0 initial state $\hbar$ (Planck constant)/(2π)
Perturbed wave-function[a]	$\Psi(t) = \sum_n c_n(t)\psi_n\exp(-\mathrm{i}E_n t/\hbar)$ where	(4.160)	c_n probability amplitudes
	$c_n = \frac{-\mathrm{i}}{\hbar}\int_0^t \langle\psi_n\|\hat{H}'(t')\|\psi_0\rangle\exp[\mathrm{i}(E_n - E_0)t'/\hbar]\,\mathrm{d}t'$	(4.161)	
Fermi's golden rule	$\Gamma_{i\to f} = \frac{2\pi}{\hbar}\|\langle\psi_f\|\hat{H}'\|\psi_i\rangle\|^2\rho(E_f)$	(4.162)	$\Gamma_{i\to f}$ transition probability per unit time from state i to state f $\rho(E_f)$ density of final states

[a]To first order.