Ratio and Proportion

Ratio

The ratio of two quantities in the same units is the fractions that one quantity is of the other.

Or

It is a tool to compare two or more numbers of same quantities. Thus, the ratio a to b is the fraction $\frac{a}{b}$ written as $a : b$.

In the ratio $a : b$, the first term a is antecedent and second term b is consequent.

Properties of Ratio

(i) The value of a ratio remains unchanged, if each one of its term is multiplied or divided by a same non-zero number.

(ii) $a^2 : b^2$ is the duplicate ratio of a : b.

(iii) $a^3 : b^3$ is the triplicate ratio of a : b.

(iv) $\sqrt{a} : \sqrt{b}$ is the sub-duplicate ratio of a : b.

(v) $\sqrt[3]{a} : \sqrt[3]{b}$ is the sub-triplicate ratio of a : b.

(vi) If a : b and c : d are two ratios, then the compounded ratio is ac : bd.

Example 1 If $p : q = 3 : 4$ and $q : r = 8 : 9$. Find the ratio of $p : q : r$ is

(a) 2:4:3 (b) 9:6:8
(c) 6:8:9 (d) 9:8:6

Sol. *(c)* $\frac{p}{q} = \frac{3}{4}$ and $\frac{q}{r} = \frac{8}{9}$

$\Rightarrow \quad \frac{p}{q} = \frac{3\times 2}{4\times 2} = \frac{6}{8}$ and $\frac{q}{r} = \frac{8}{9}$

$\therefore \quad p : q : r = 6 : 8 : 9$

Example 2 If $A : B = 3 : 4$, $B : C = 5 : 6$ and $C : D = 11 : 9$, then find the ratio of $A : D$ is

(a) 55:72 (b) 73:55
(c) 11:9 (d) 55:73

Sol. *(a)* $\frac{A}{D} = \left(\frac{A}{B}\times\frac{B}{C}\times\frac{C}{D}\right) = \left(\frac{3}{4}\times\frac{5}{6}\times\frac{11}{9}\right) = \frac{55}{72}$

$\Rightarrow \quad A : D = 55 : 72$

Proportion

The equality of two ratios is called proportion.

Let a, b, c and d are four quantities, then the proportional are a : b :: c : d.

In the proportion $a : b :: c : d$, a and d are extreme values and b and c are mean values.
i.e. Product of means = Product of extreme

Properties of Proportion

(i) If x is the third proportional to a, b, then a : b :: b : x.

(ii) Mean proportional between a and b is $\sqrt{ab}$.

(iii) If x is the fourth proportional to a, b, c, then a : b :: c : x.

(iv) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ and $\frac{a-b}{a+b} = \frac{c-d}{c+d}$ (Componendo and dividendo)

Example 3 $x : 75 :: 15 : 45$. Find the value of x.

(a) 52 (b) 25
(c) 16 (d) 9

Sol. *(b)* $\frac{x}{75} = \frac{15}{45}$

$\Rightarrow \quad x = \frac{15 \times 75}{45} \Rightarrow x = 25$

Example 4 Find the ratio in between 20 paise and ₹ 3.

(a) 2:3 (b) 15:1 (c) 1:15 (d) 3:2

Sol. *(c)* ₹ 3 = 300 paise

$\therefore$ Required ratio $= \frac{20}{300} = \frac{1}{15} = 1 : 15$

Example 5 Salaries of Vivek and Vimal are ₹ 1400 and ₹ 1600. Find out the ratio of their salaries.

(a) 7:8 (b) 15:16 (c) 4:5 (d) 3:4

Sol. *(a)* Ratio $= \frac{\text{Vivek's salary}}{\text{Vimal's salary}} = \frac{1400}{1600} = \frac{14}{16} = \frac{7}{8}$

$\therefore \quad$ Ratio $= 7 : 8$

Example 6 The ratio of the numbers of boys and girls in a school is 2 : 5. If there are 350 students in the school, find the number of girls in the school.

(a) 252 (b) 260
(c) 245 (d) 250

Sol. *(d)* Number of girls

$= \frac{\text{Ratio term for girls}}{\text{Total sum of ratio}} \times \text{Number of students}$

$= \frac{5}{2+5} \times 350 = \frac{5}{7} \times 350 = 250$

Helping Tips

- If any amount A (say) is distributed among P, Q in the ratio $x : y$ respectively, *then share of each is given as follows*

  Share of $P = \frac{x}{x+y} \times A$

  Share of $Q = \frac{y}{x+y} \times A$

- If any amount A (say) is distributed among P, Q, R in the ratio $x : y : z$ respectively, *then share of each is given as follows*

  Share of $P = \frac{x}{x+y+z} \times A$

  Share of $Q = \frac{y}{x+y+z} \times A$

  and share of $R = \frac{z}{x+y+z} \times A$

Practice Exercise

1. If $a : b = 2 : 3$ and $b : c = 4 : 5$, the ratio $a : b : c$ is equal to
   (a) 4 : 6 : 5 (b) 12 : 8 : 15
   (c) 8 : 12 : 15 (d) 2 : 3 : 4

2. If $0.75 : x :: 5 : 8$, then find x.
   (a) 1.5 (b) 1.2
   (c) 12 (d) 2.1

3. $\frac{3}{48}$ is what part of $\frac{1}{12}$?
   (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

4. $\frac{1}{35}$ is what part of $\frac{2}{7}$?
   (a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) $\frac{1}{5}$ (d) 10

5. What is the ratio in between 7 months and 7 yr?
   (a) 12 : 1 (b) 1 : 12 (c) 4 : 3 (d) 5 : 6

6. The ratio of number of boys and girls in a school is 4 : 3. If there are 480 boys in the school, find the number of girls in the school.
   (a) 360 (b) 336 (c) 300 (d) 367

7. In a ratio which is equal to 3 : 7, if the antecedent is 33, what is the consequent?
   (a) 88 (b) 68 (c) 70 (d) 77

8. The ratio of two numbers is 3 : 8 and their difference is 115. What is the largest number?
   (a) 188 (b) 186 (c) 184 (d) 200

9. Speed of one bus is 80 km/h and other is 60 km/h. What is the ratio of the speeds of two buses?
(a) 4 : 3 (b) 2 : 3 (c) 3 : 2 (d) 3 : 4

10. What must be subtracted from each term of the ratio 3 : 7, so that the ratio becomes 2 : 5?
(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

11. What must be added to each term of the ratio 49 : 68, so that it becomes 3 : 4?
(a) 10 (b) 16
(c) 8 (d) 12

12. The prices of a cycle and a scooter are the ratio of 9 : 5. If a cycle costs ₹ 4200 more than a scooter, what is the price of scooter?
(a) ₹ 5050 (b) ₹ 5500
(c) ₹ 5000 (d) ₹ 5250

Answers

1	(c)	2	(b)	3	(d)	4	(a)	5	(b)	6	(a)	7	(d)	8	(c)	9	(a)	10	(b)
11	(c)	12	(d)																

Hints & Solutions

1. $\frac{a}{b}=\frac{2}{3}$ and $\frac{b}{c}=\frac{4}{5}$

or $\frac{a}{b}=\frac{2\times4}{3\times4}=\frac{8}{12}$ and $\frac{b}{c}=\frac{4\times3}{5\times3}=\frac{12}{15}$,

Then, a:b:c=8:12:15

Short Trick

a:b = 2 : 3

b:c = 4 : 5

8 : 12 : 15

$\therefore$ a : b : c = 8 : 12 : 15

2. 0.75 : x :: 5 : 8

$\frac{0.75}{x}=\frac{5}{8} \Rightarrow x=\frac{0.75\times8}{5} \Rightarrow x=1.2$

3. $\therefore$ Required part $=\frac{3/48}{1/12}=\frac{3}{48}\times\frac{12}{1}=\frac{3}{4}$ part

4. $\therefore$ Required part $=\frac{\frac{1}{35}}{\frac{2}{7}}=\frac{1}{35}\times\frac{7}{2}=\frac{7}{70}=\frac{1}{10}$ part

5. 7 yr = 7×12 months = 84 months

$\therefore$ Ratio $=\frac{7}{84}=\frac{1}{12}=1:12$

6. Let the number of girls be x.

$\therefore$ $4:3=480:x \Rightarrow \frac{4}{3}=\frac{480}{x}$

$\Rightarrow x=\frac{480\times3}{4} \Rightarrow x=360$ girls

7. $\frac{3}{7}=\frac{33}{x}$ [Where, *x* is the consequent]

$\Rightarrow x=\frac{7\times33}{3} \Rightarrow x=77$

8. Let number be 3x and 8x.
According to the question,

$\Rightarrow 8x-3x=115 \Rightarrow 5x=115 \Rightarrow x=\frac{115}{5}=23$

$\therefore$ Largest number $=8\times23=184$

9. Ratio $=\frac{80}{60}=\frac{4}{3}=4:3$

10. Let x is to be subtracted.

$\therefore \frac{3-x}{7-x}=\frac{2}{5}$

$\Rightarrow 15-5x=14-2x \Rightarrow 15-14=5x-2x$

$\Rightarrow 1=3x \Rightarrow x=\frac{1}{3}$

$\therefore$ The number $=\frac{1}{3}$.

11. Let x is to added

$\therefore \frac{49+x}{68+x}=\frac{3}{4}$

$\Rightarrow 196+4x=204+3x$

$\Rightarrow 4x-3x=204-196 \Rightarrow x=8$

$\therefore$ The number = 8.

12. The cost of cycle = ₹ 9x
The cost of scooter = ₹ 5x
According to the question,

$\Rightarrow 9x-5x=4200$

$\therefore 4x=4200 \Rightarrow x=\frac{4200}{4}=1050$

$\therefore$ Cost of scooter $=5\times1050=₹\,5250$

Try Yourself

1) $A : B = 5 : 7$ and $B : C = 6 : 11$. So, find $A : B : C$.
(a) 42:30:77 (b) 30:42:77
(c) 26:40:25 (d) 30:44:77

2) If $A : B = 11 : 12$, $B : C = 13 : 14$ and $C : D = 15 : 16$. So, find $A : D$.
(a) 715:896 (b) 718:890
(c) 896:700 (d) 718:895

3) $x : 3 :: 12 : 4$. Find the value of x.
(a) 12 (b) 8
(c) 10 (d) 9

4) $625 : 100 :: 125 : x$. Find the value of x.
(a) 20 (b) 18
(c) 22 (d) 19

5) $7 : x :: 17.5 : 22.5$. Find the value of x.
(a) 7 (b) 8
(c) 9 (d) 10

6) The ratio between two numbers is 12 : 13. If each number is reduced by 20, the ratio becomes 2 : 3. Find the numbers.
(a) 30, 26 (b) 26, 28
(c) 24, 26 (d) 25, 27

7) Marks obtained by Ramesh and Mahesh in Maths are 65 and 75 respectively, then find ratio of their marks.
(a) 15:13 (b) 12:14 (c) 15:12 (d) 13:15

8) Divide ₹ 108 among *A, B* and *C* in ratio 3 : 4 : 5.
(a) ₹ 25, ₹ 45, ₹ 29 (b) ₹ 27, ₹ 36, ₹ 45
(c) ₹ 36, ₹ 45, ₹ 25 (d) ₹ 28, ₹ 38, ₹ 40

9) A number 351 is divided into two parts in the ratio 2 : 7. Find the product of the number.
(a) 213094 (b) 21296
(c) 21294 (d) 21300

10) Divide ₹ 4000 among *A, B, C*, so that their shares may be in the ratio of 5 : 7 : 8.
(a) ₹ 1400, ₹ 1000, ₹ 1500
(b) ₹ 1000, ₹ 1400, ₹ 1600
(c) ₹ 1024, ₹ 1425, ₹ 1500
(d) ₹ 1050, ₹ 1500, ₹ 1500

Answers

1	*(b)*	2	*(a)*	3	*(d)*	4	*(a)*	5	*(c)*
6	*(c)*	7	*(d)*	8	*(b)*	9	*(c)*	10	*(b)*