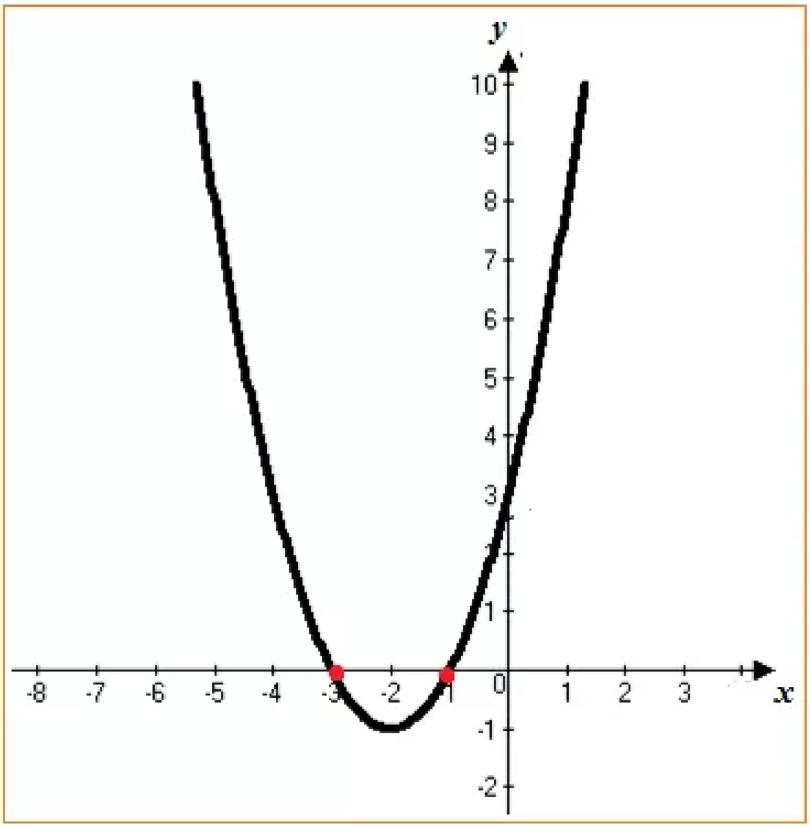
Chapter 10. Quadratic And Exponential Functions

Ex. 10.2

Answer 1CU.

Consider the following graph:



The roots of the graph of the quadratic function are the values of x where the curve meets x -axis, i.e the x – intercepts of the graph.

Observe that the graph cuts x- axis at $(-1,0)$ and $(-3,0)$.

Therefore the x - intercepts of the graph are -3 and -1.

Therefore, the real roots of a quadratic equation are $\boxed{-1}$ and $\boxed{-3}$.

Answer 3CU.

The quadratic equation need not necessarily has two different solutions.

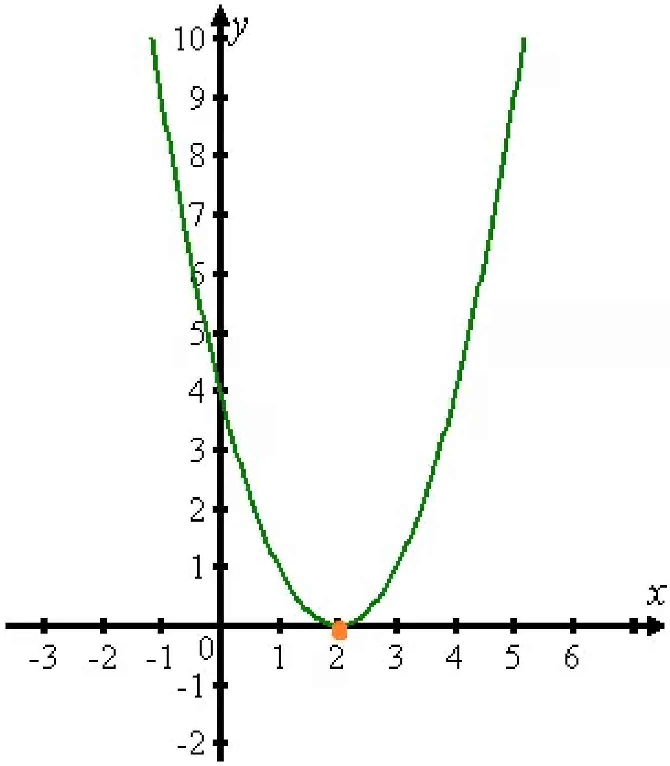
It may have two equal solutions.

Counter **example**:

Consider the equation $x^2-4x+4=0$ and solve for *x*.

$$x^2-2(2x)+2^2=0$$
$$(x-2)^2=0$$
$$(x-2)(x-2)=0$$
$$x=2,2 \text{ (equal roots)}$$

The graph of $f(x)=x^2-4x+4$ is as shown below.



From the graph, the curve meets (touches) *x*-axis only at one point, which confirms that the roots of $x^2-4x+4=0$ are equal, but not different.

Answer 4CU.

Let us consider the equation $x^2 - 7x + 6 = 0$

Step1: Rewrite the consider related function $f(x) = x^2 - 7x + 6$

Now we construct the table for the function $f(x)$

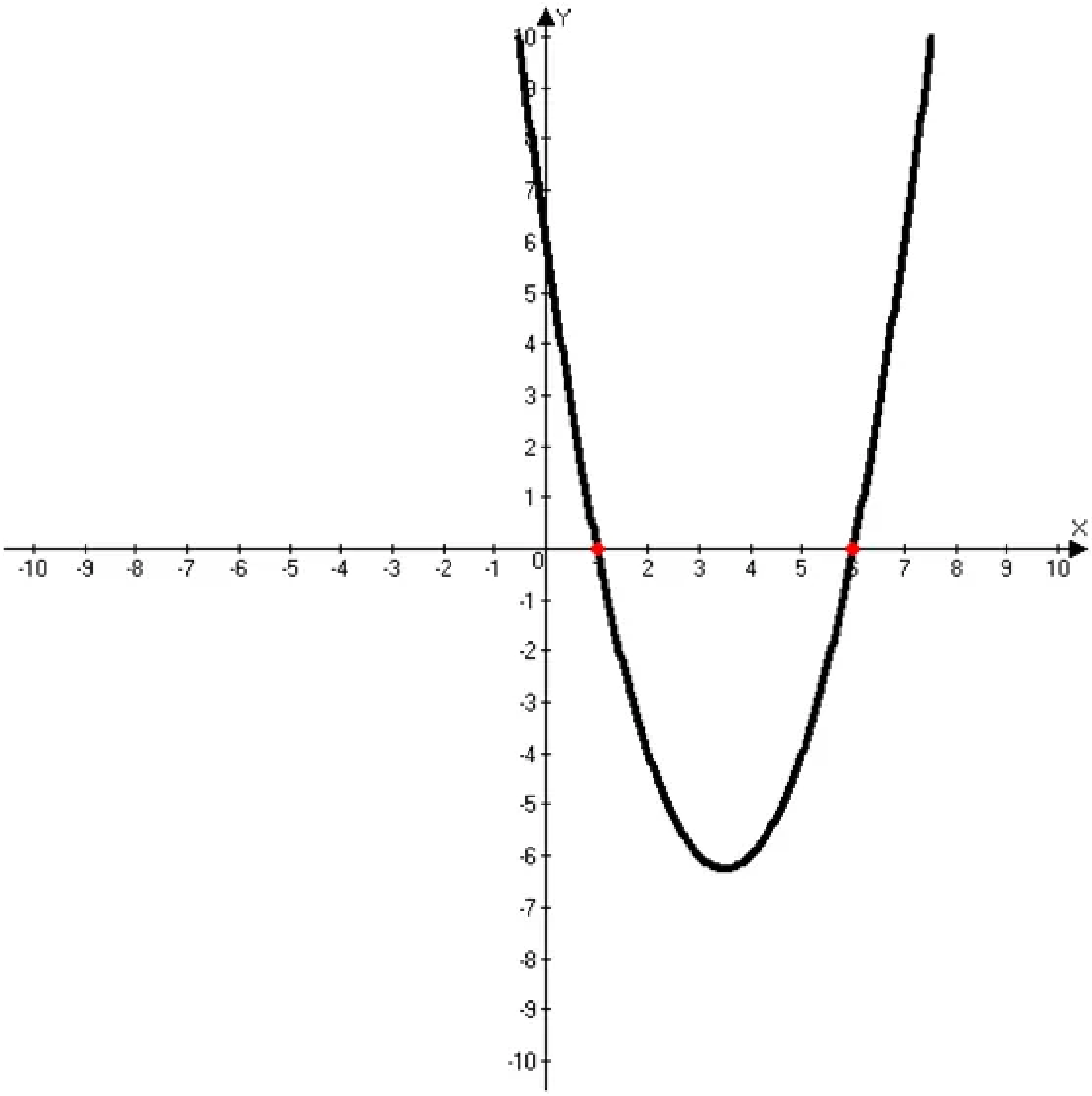
x	$f(x) = x^2 - 7x + 6$
0	$f(0) = 6$
1	$f(1) = 0$
2	$f(2) = -4$
3	$f(3) = -6$
4	$f(4) = -6$
5	$f(5) = -4$
6	$f(6) = 0$
7	$f(7) = 6$

We use the rule that '*x*' is a solution of the equation if $f(x) = 0$. Graphically $f(x)$ intersect *x* – axis at *x* = *x*, then *x* is solution of $f(x)$

From the table, $f(1) = 0, f(6) = 0$

1 and 6 are the roots of $f(x)=0$ in the graph $f(x)$ intersect the x – axis at x = 1, x = 6

The roots of equation $\{1,6\}$ (red dots in the below figure)



Answer 5CU.

Let us consider the equation $a^2 - 10a + 25 = 0$

Step1: Rewrite the consider related function $f(a) = a^2 - 10a + 25$

Now we construct the table for the function $f(a)$

x	$f(x) = a^2 - 10a + 25$
0	$f(0) = (0)^2 - 10(0) + 25 = 25$
1	$f(1) = (1)^2 - 10(1) + 25 = 16$
2	$f(2) = (2)^2 - 10(2) + 25 = 9$
3	$f(3) = (3)^2 - 10(3) + 25 = 4$
4	$f(4) = (4)^2 - 10(4) + 25 = 1$
5	$f(5) = (5)^2 - 10(5) + 25 = 0$
6	$f(6) = (6)^2 - 10(6) + 25 = 1$

From the table we can observe that

$f(5) = 0$

We use the rule an integer 'a' is said to be solution of $f(x) = 0$, if $f(a) = 0$. By this rule we can say that 5 is the solution of the symmetry.

$$x = -\frac{b}{2a}$$

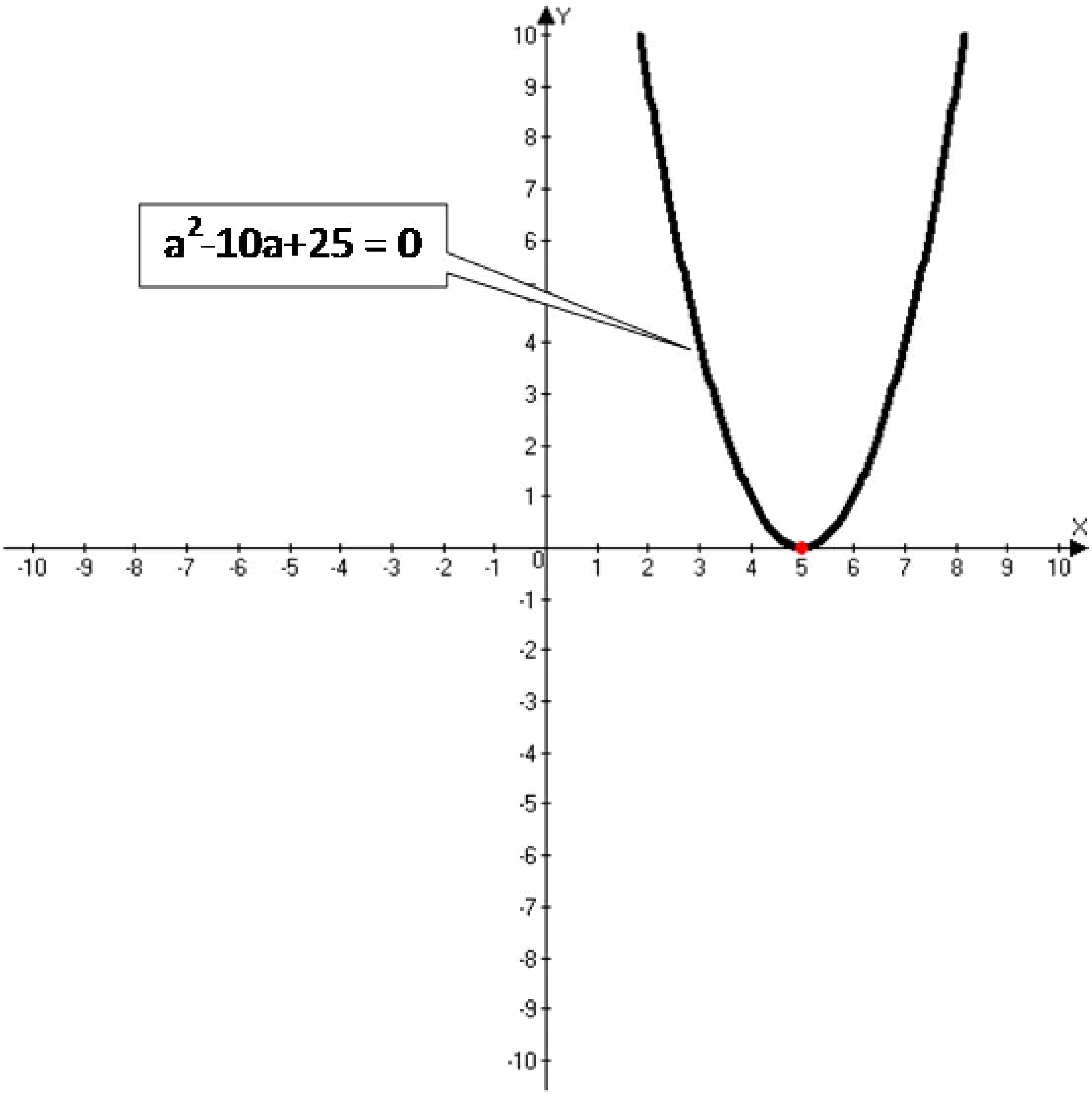
$$x = -\frac{-10}{2(1)} \qquad \left(\begin{array}{l}\text{Compare the given equation } ax^2 + bx + c = 0, \\ a = 1, b = -10 \text{ and } c = 25\end{array}\right)$$

$$x = \frac{10}{2}$$

$$x = \frac{5 \cdot 2}{2} \qquad (\text{factor } 10 = 5 \cdot 2)$$

$$\boxed{x = 5} \qquad (\text{Cancellation of the numerator and the denominator})$$

5, 5 are the equal roots



Answer 6CU.

Consider the equation,

$c^2+3=0$.

Solve the equation $c^2+3=0$ by graphing.

Let the graph the related function $f(c)=c^2+3$

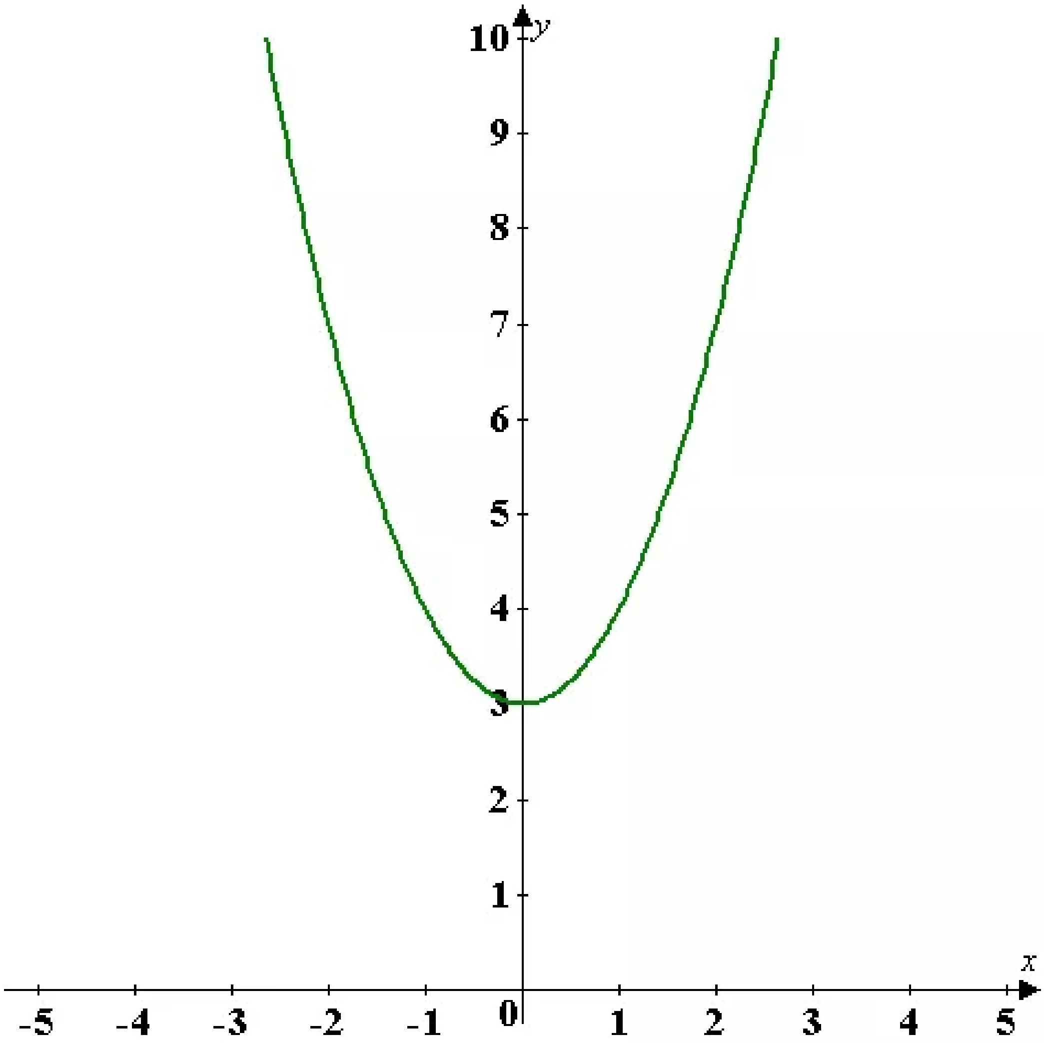
To construct the table for $f(c)=c^2+3$

Now substitute different value of *c* in the original function $f(c)=c^2+3$. We obtain the $f(c)$ values. Plotting these ordered pairs and connected them, we obtain the smooth curve.

Table for $f(c)=c^2+3$ is shown below:

c	c^2+3	$f(c)$	$(c,f(c))$
-3	$(-3)^2+3=12$	12	$(-3,12)$
-2	$(-2)^2+3=7$	7	$(-2,7)$
-1	$(-1)^2+3=4$	4	$(-1,4)$
0	$(0)^2+3=3$	3	$(0,3)$
1	$(1)^2+3=4$	4	$(1,4)$
2	$(2)^2+3=7$	7	$(2,7)$
3	$(3)^2+3=12$	12	$(3,12)$

The graph of the equation is shown below:



From the graph, observe that the graph has no *c*- intercept.

Thus, there are no real number solutions for this equation $c^2+3=0$.

Hence, the solution of the equation $c^2+3=0$ is $\boxed{\{\phi\}}$.

Answer 7CU.

Let us consider the equation $t^2+9t+5=0$

Step1: Rewrite the consider related function $f(t)=t^2+9t+5$

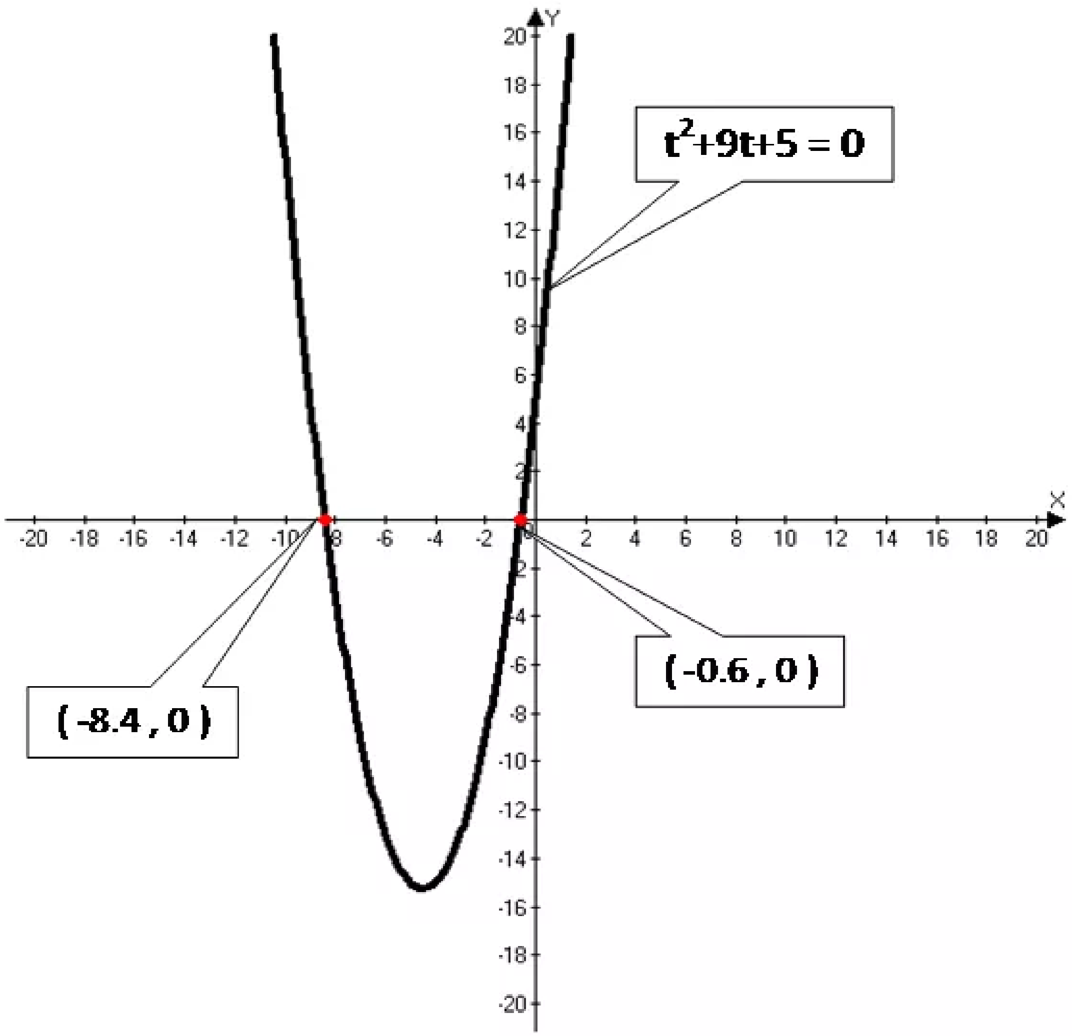
Now we construct the table for the function $f(t)$

t	$f(t)=t^2+9t+5$
-4	$f(-4)=(-4)^2+9(-4)+5=-15$
-3	$f(-3)=(-3)^2+9(-3)+5=-13$
-2	$f(-2)=(-2)^2+9(-2)+5=-9$
-1	$f(-1)=(-1)^2+9(-1)+5=-3$
0	$f(0)=(0)^2+9(0)+5=5$
1	$f(1)=(1)^2+9(1)+5=15$

From the table we can observe that $f(0)=5>0$, $f(-1)=-3<0$

We use the rule an integer 't' is said to be solution of $f(x)=0$, if $f(t)=0$.

By this rule we can say that -1, 0 are the solution of the equations $t^2+9t+5=0$



Answer 8CU.

Let us consider the equation $x^2 - 16 = 0$

Step1: Rewrite the consider related function $f(x) = x^2 - 16$

Now we construct the table for the function $f(x)$

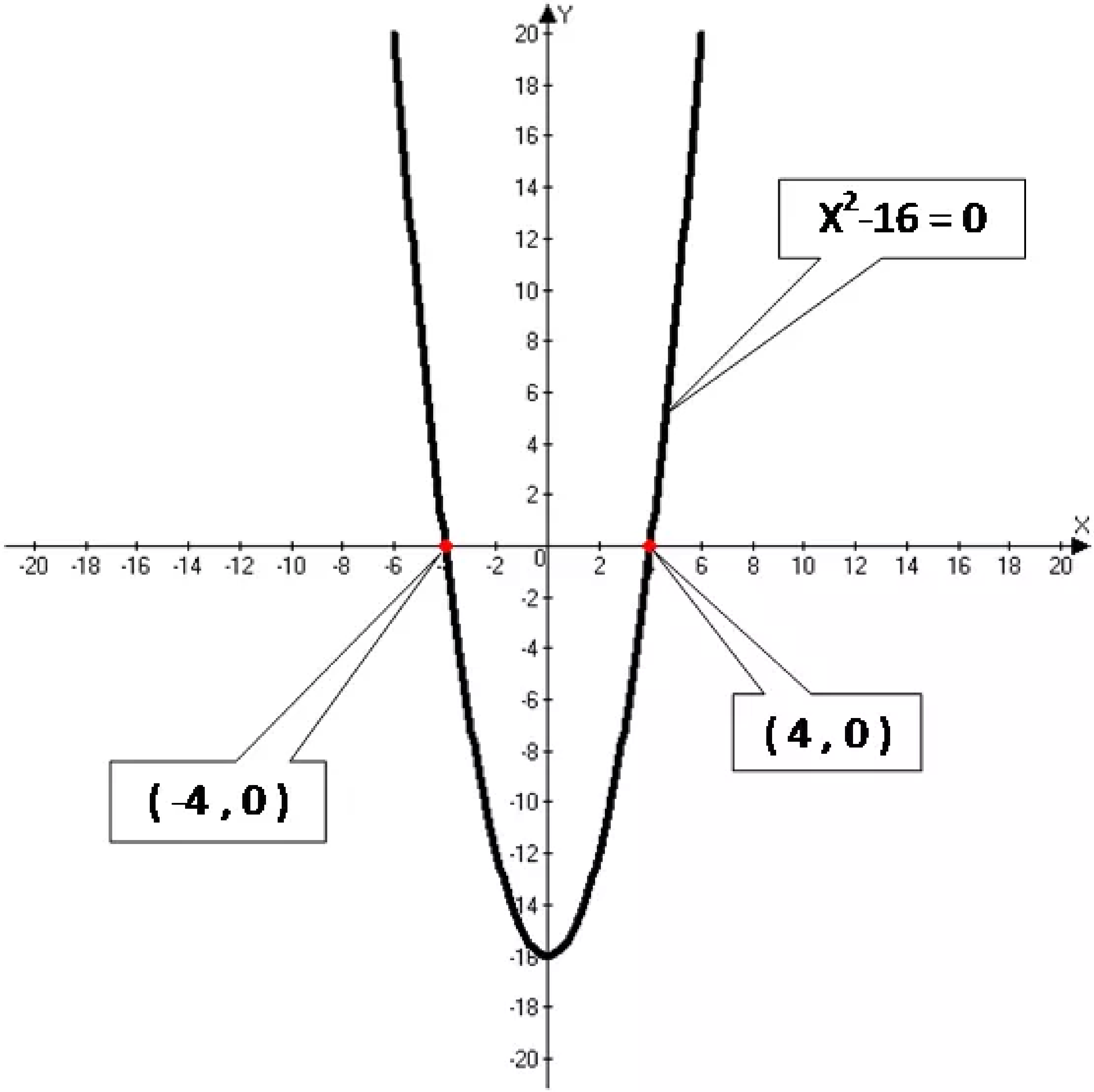
x	$f(x) = x^2 - 16$
-4	$f(-4) = (-4)^2 - 16 = 0$
-3	$f(-3) = (-3)^2 - 16 = -7$
-2	$f(-2) = (-2)^2 - 16 = -12$
-1	$f(-1) = (-1)^2 - 16 = -15$
0	$f(0) = (0)^2 - 16 = -16$
1	$f(1) = (1)^2 - 16 = -15$
2	$f(2) = (2)^2 - 16 = -12$
3	$f(3) = (3)^2 - 16 = -7$
4	$f(4) = (4)^2 - 16 = 0$
5	$f(5) = (5)^2 - 16 = 9$

We use the rule an integer '*x*' is said to be solution of $f(x) = 0$, Graphical, $f(x)$ interest *x* – axis at *x* = *x*, then *x* is solution of $f(x)$

From the table $f(-4) = 0, f(4) = 0$

-4 and 4 are the roots of $f(x) = 0$ in the graph $f(x)$ intersect the *x* – axis at $x = -4$, *x* = 4

The roots of equation $\{-4, 4\}$



Answer 9CU.

Consider the equation,

$w^2 - 3w = 5$.

Solve the equation $w^2 - 3w = 5$ by graphing.

Rewrite the equation $w^2 - 3w = 5$ as a standard quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$.

$w^2 - 3w = 5$ Consider the original equation

$w^2 - 3w - 5 = 5 - 5$ Subtract each side by 5

$w^2 - 3w - 5 = 0$ Simplify

Consider the equation,

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Solve the equation $w^2 - 3w = 5$ by graphing.

Rewrite the equation $w^2 - 3w = 5$ as a standard quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$.

$w^2 - 3w = 5$ Consider the original equation

$w^2 - 3w - 5 = 5 - 5$ Subtract each side by 5

$w^2 - 3w - 5 = 0$ Simplify

Solve the equation $w^2 - 3w = 5$ by graphing.

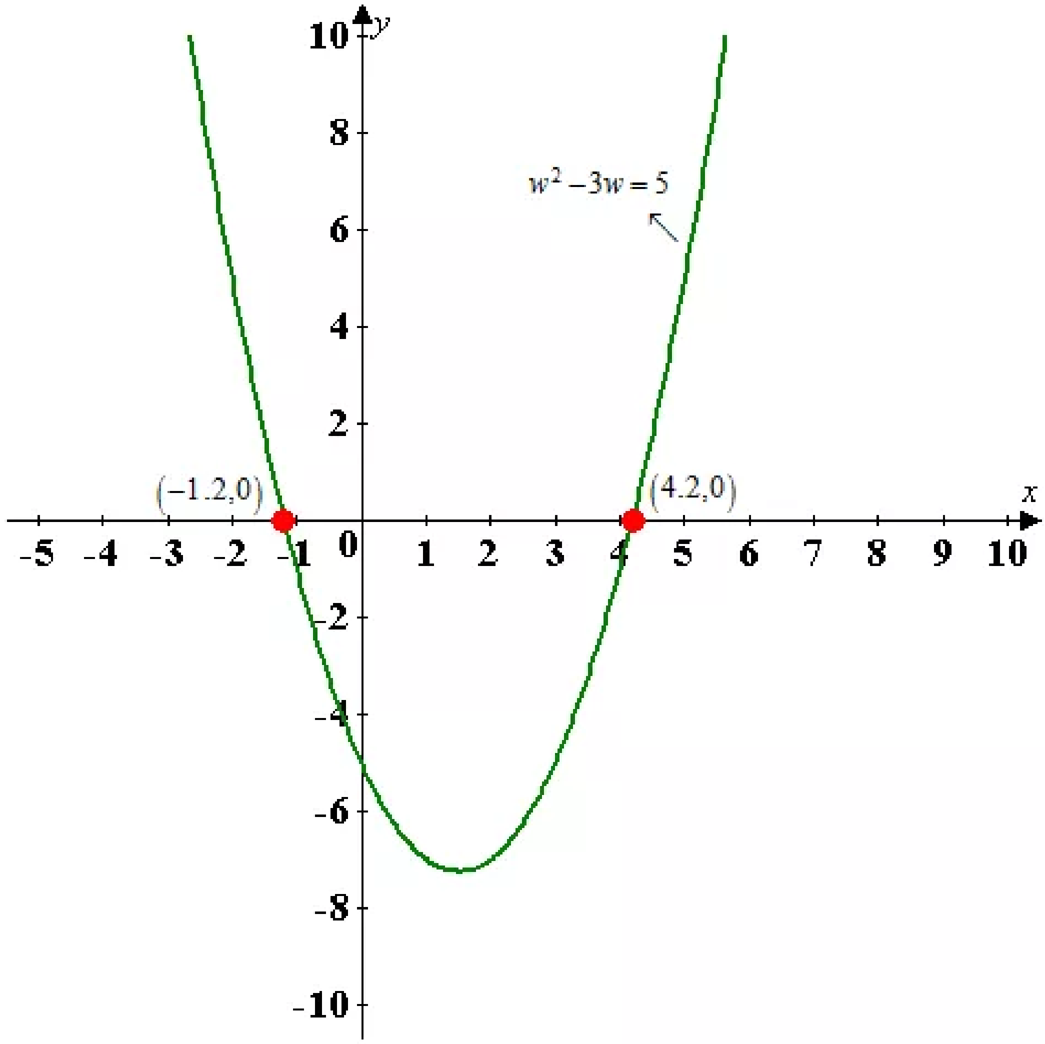
Let graph the related function

$f(w) = w^2 - 3w = 5$

Table for $f(w) = w^2 - 3w = 5$ is shown below:

w	$w^2 - 3w$	$f(w)$	$(w, f(w))$
-3	$(-3)^2 - 3(-3) - 5 = 13$	13	$(-3, 13)$
-2	$(-2)^2 - 3(-2) - 5 = 5$	5	$(-2, 5)$
-1	$(-1)^2 - 3(-1) - 5 = -1$	-1	$(-1, -1)$
0	$(0)^2 - 3(0) - 5 = -5$	-5	$(0, -5)$
1	$(1)^2 - 3(1) - 5 = -7$	-7	$(1, -7)$
2	$(2)^2 - 3(2) - 5 = -7$	-7	$(2, -7)$
3	$(3)^2 - 3(3) - 5 = -5$	-5	$(3, -5)$

Use the table a point, the sketch of the equation is shown below:



From the graph, it is observed that the $x-$intercepts are between -1 and -2 and between 4 and 5.

So, one root is between -1 and -2 and the other root is between 4 and 5.

Therefore, the roots of the equation are

$\boxed{-2 < w < -1 \text{ and } 4 < w < 5}$.

Answer 10CU.

Let the two numbers be *a* and *b*

According to the problem, the sum of two numbers is 4, it follows that

$a+b=4$

Their product of two numbers is -12, it follows that

$ab=-12$

Find the values of *a* and *b*.

Solve for "*b*" in the equation $ab=-12$

$ab=-12$ Consider the original equation

$\frac{ab}{a}=\frac{-12}{a}$ Divide each side by a

$b=\frac{-12}{a}$ Simplify

Substitute $b=\frac{-12}{a}$ in the equation $a+b=4$, to obtain

$$a+b=4 \qquad (\text{Original equation})$$

$$a+\left(\frac{-12}{a}\right)=4 \qquad \left(\text{Replace } b \text{ by } \frac{-12}{a}\right)$$

$$a-\frac{12}{a}=4$$

$$a\cdot\left[a-\frac{12}{a}\right]=4a \qquad (\text{Multiply "}a\text{" on both sides})$$

$$a\cdot a-a\cdot\frac{12}{a}=4a \qquad (\text{Use distrubutive property } a\cdot(b-c)=a\cdot b-a\cdot c)$$

$$a^2-12=4a$$

$$a^2-4a-12=4a-4a \quad (\text{Subtract } 4a \text{ on each side})$$

$$a^2-4a-12=0$$

Solve the equation $a^2-4a-12=0$ by factoring.

$$a^2-4a-12=0 \quad \text{(Original equation)}$$
$$a^2-6a+2a-12=0$$
$$a\cdot a-6a+2a-2\cdot 6=0$$
$$a\cdot[a-6]+2\cdot[a-6]=0 \quad \text{(Use distributive property)}$$
$$[a-6][a+2]=0 \quad \text{(Factoring)}$$
$$a-6=0 \text{ or } a+2=0 \quad \text{(Use zero product property)}$$
$$a=6 \text{ or } a=-2 \quad \text{(Solve for "}a\text{")}$$

Substitute the each value in $b=\frac{-12}{a}$ to obtain the *b* value.

$$b=\frac{-12}{a} \quad \text{(Original equation)}$$
$$=\frac{-12}{6} \quad \text{(Replace } a \text{ by 6)}$$
$$=\frac{-2\cdot 6}{2\cdot 3}$$
$$=\frac{-2}{1}$$
$$=-2$$

Therefore, the two numbers are *a* = 6 and $b=-2$.

<u>Check</u>:

The sum of the two numbers is $6-2=4$

The product of the two numbers is $6\cdot(-2)=-12$.

So, the two number 6 and -2 satisfies the given conditions.

Hence, the two real numbers is $\boxed{6}$ and $\boxed{-2}$.

Answer 11PA.

Construct table for the following function,

$f(c)=c^2-5c-24$.

By taking different value of 'c' we obtain different value of $f(c)$

The table is shown below:

x	$c^2-5c-24$	$f(c)$	$(c,f(c))$
-4	$c^2-5c-24=12$	12	$(-4,12)$
-3	$c^2-5c-24=0$	0	$(-3,0)$
-2	$c^2-5c-24=-10$	10	$(-2,10)$
0	$c^2-5c-24=-24$	-24	$(0,-24)$
1	$c^2-5c-24=-28$	-28	$(1,-28)$
4	$c^2-5c-24=-28$	-28	$(4,-28)$
8	$c^2-5c-24=0$	0	$(8,0)$

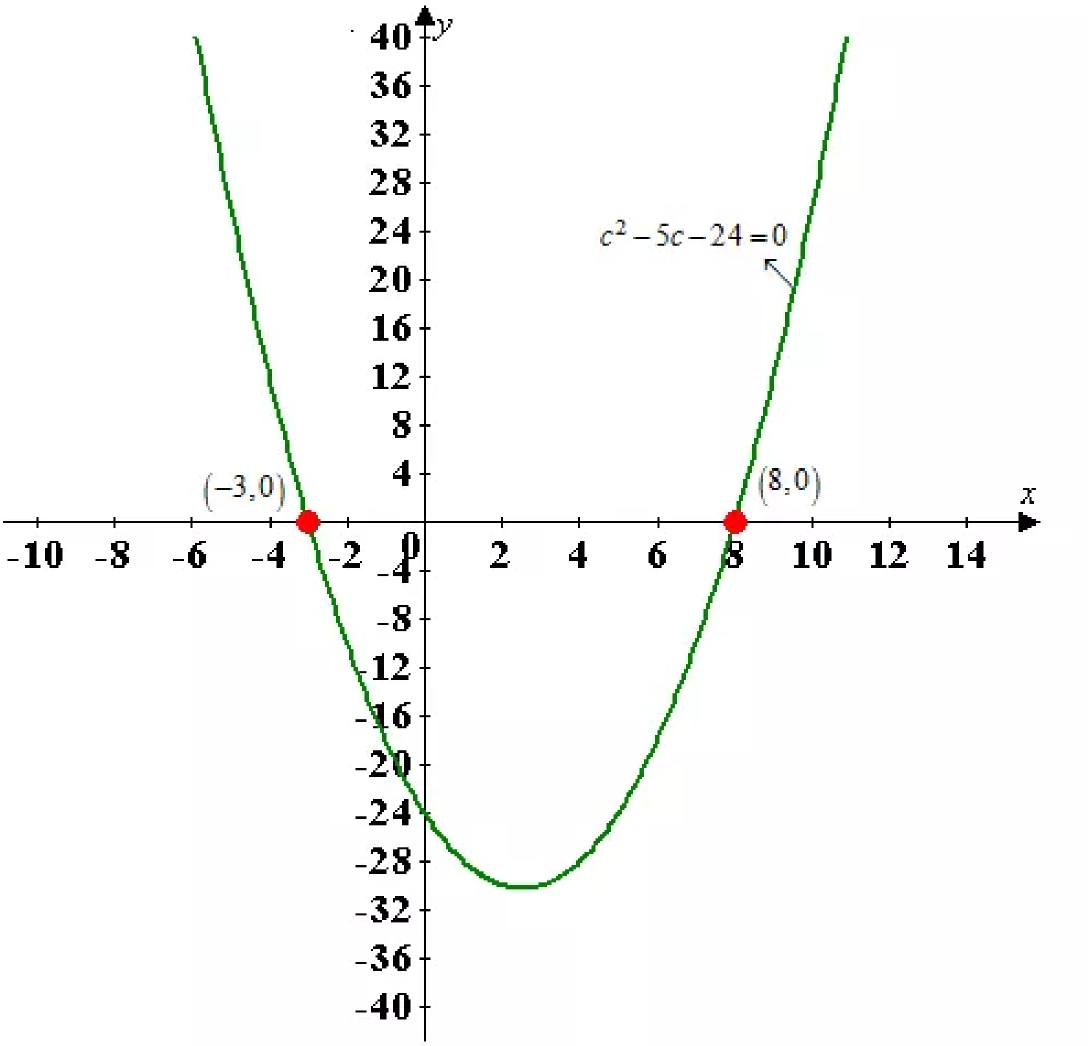
Plot the points on graph and connect the points then to get smooth curve which intersect the x – axis at $(-3,0)$ and $(8,0)$.

The rules the graph of the function $f(x)$ is intersect x – axis at point $(a,0)$ and $(b,0)$ then we can say that the roots of the equation $f(x)=0$ are a and b.

By this rule from the above information the roots of the equation

$c^2-5c-24=0$ are $\boxed{c=-3}$ and $\boxed{c=8}$.

The sketch of the equation is shown below:



Verification:

Given equation $c^2-5c-24=0$ original equation

$$c^2-5c-24=0$$

$$c^2-8c+3c-24=0$$

$$c\cdot c-8\cdot c+3\cdot c-8\cdot 3=0 \qquad (\text{Factos } 24=8\cdot 3)$$

$$c(c-8)+3(c-8)=0$$

$$(c-8)(c+3)=0$$

$$c-8=0 \quad \text{or} \quad c+3=0$$

$$c-8+8=0 \quad \text{or} \quad c+3-3=0-3$$

Add 8 on both sides (Substract 3 on both sides)

$$c=8 \quad \text{or} \quad c=-3$$

So, the roots of the equation $c^2-5c-24=0$ is $c=-3$ and $c=8$

Hence, the result is verified.

Answer 12PA.

Consider the equation,

$5n^2+2n+6=0$.

Let us consider the related function $f(n)=5n^2+2n+6$

Graph this function by giving different value if 'n' to get value of $f(n)$.

Find the vertex of the graph. We have the rule vertex of the graph $f(x)=ax^2+bx+c$ is

$\left(\frac{-b}{2a},\frac{4ac-b^2}{2a}\right)$.

Compare the given function $f(n)=5n^2+2n+6$ with standard quadratic

$f(x)=ax^2+bx+c$, where $a\neq 0$, to obtain

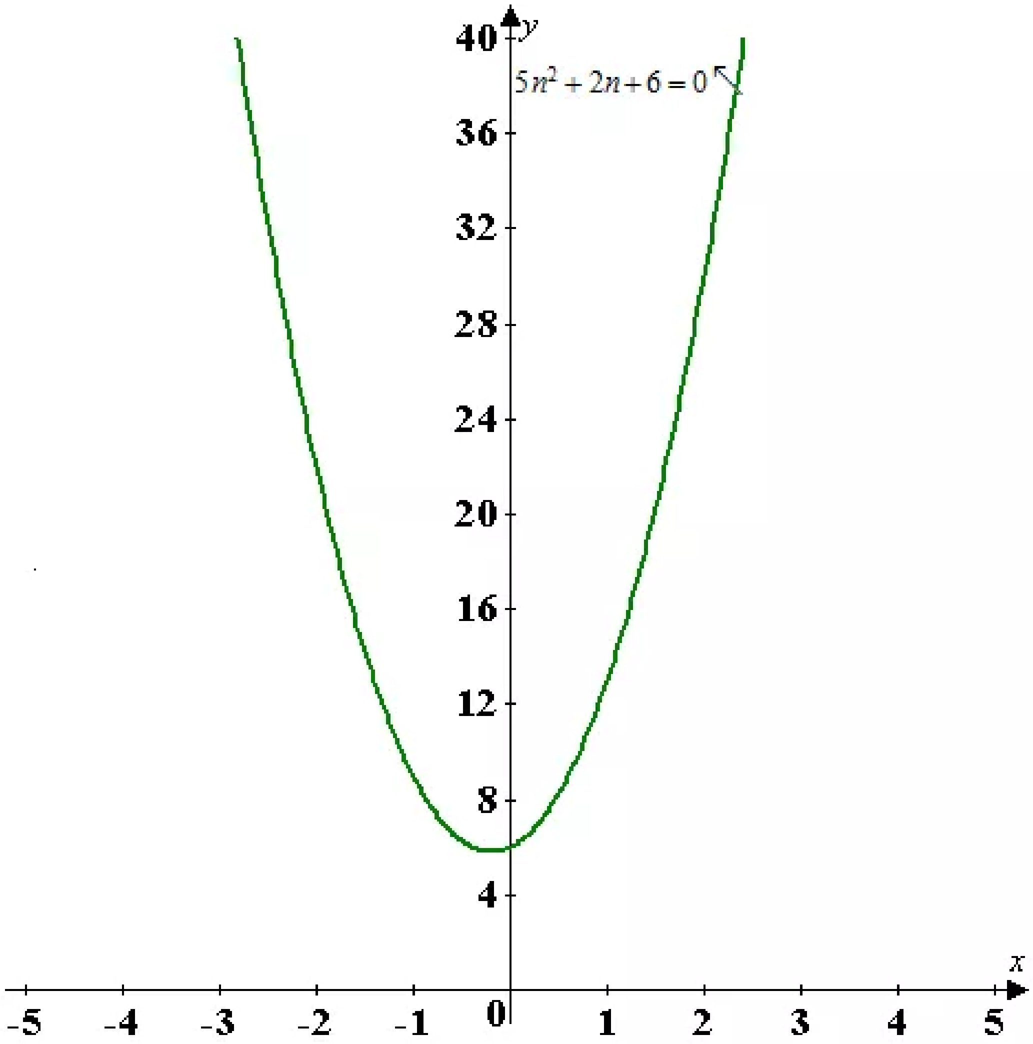
a = 5, b = 2 and c = 6.

By this rule the vertex of the function $f(n)=5n^2+2n+6$ is

$$\left(\frac{-2}{2(5)},\frac{4\cdot(5)\cdot(6)-(2)^2}{2\cdot 5}\right)=\left(\frac{-1}{5},\frac{116}{10}\right)$$
$$=(-0.2,11.6)$$

The vertex $(-0.2,11.6)$ and the graph are open upward.

Graph the function $f(n)=5n^2+2n+6$. By taking different value to 'n' and to obtain different value $f(n)$ plot the points $(n,f(n))$ and connect this we get smooth curve.



Construction of the table

n	$5n^2+2n+6$	$f(n)$	$(n, f(n))$
-1	$5(-1)^2+2(-1)+6=9$	9	$(-1,9)$
0	$5(0)^2+2(0)+6=6$	6	$(0,6)$
1	$5(1)^2+2(1)+6=13$	13	$(1,13)$
4	$5(4)^2+2(4)+6=94$	94	$(4,94)$

From the table it can be concluded that the graph do not intersect the *x*- axis, as there is no point such that four co-ordinate zero.

There is no *x* – intercept for the equation.

Therefore, the equation does not have any real roots.

Answer 13PA.

Consider the equation,

$x^2+6x+9=0$.

Let us consider the related function $f(x)=x^2+6x+9$.

Find the vertex of the graph. We have the rule vertex of the graph

$f(x)=ax^2+bx+c$ is $\left(\frac{-b}{2a},\frac{4ac-b^2}{2a}\right)$

Compare the function $f(x)=x^2+6x+9$ with standard quadratic function

$f(x)=ax^2+bx+c$, where $a\neq 0$

$a = 1$, $b = 6$ and $c = 9$.

By this rule the vertex of the function $f(x)=x^2+6x+9$ is

$$\left(\frac{-6}{2\cdot 1},\frac{4\cdot(1)\cdot(9)-(6)^2}{2\cdot 1}\right)=\left(\frac{3\cdot 2}{2\cdot 1},\frac{36-36}{2\cdot 1}\right)$$
$$=(-3,0) \qquad \text{(Simplification)}$$

The graph $f(x)=x^2+6x+9$ is open upward as coefficient of $x2$ term is positive.

Construct the table for the function $f(x)=x^2+6x+9$ by given values of 'x' and get different values of $f(x)$ and plot the points $(x,f(x))$ and connect the graph.

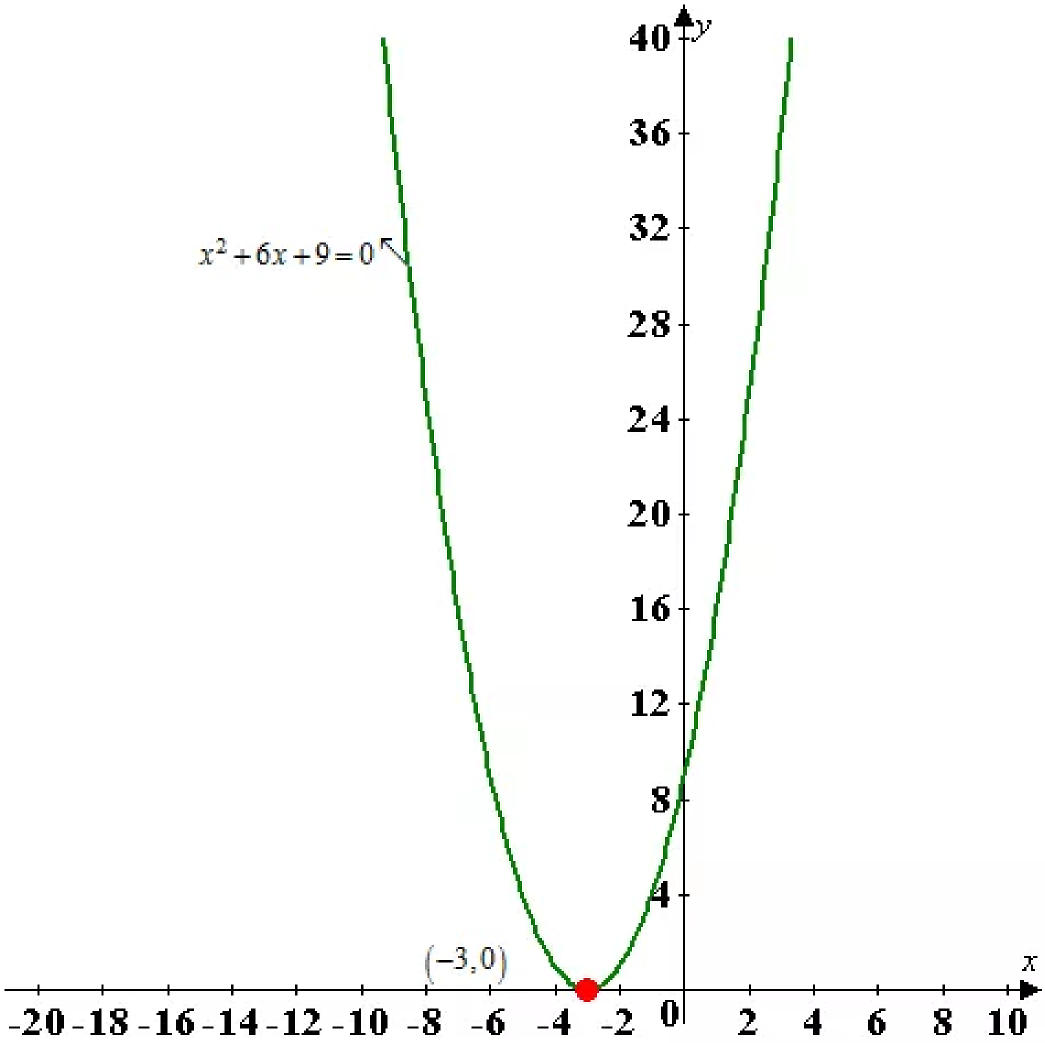
Construction of the table

x	x^2+6x+9	$f(x)$	$(x,f(x))$
-3	$(-3)^2+6(-3)+9=0$	0	$(-3,0)$
-2	$(-2)^2+6(-2)+9=1$	1	$(-2,1)$
-1	$(-1)^2+6(-1)+9=4$	4	$(-1,4)$
0	$(0)^2+6(0)+9=9$	9	$(0,9)$

From the table graph intersect the x – axis at $(-3,0)$ and it is the vertex of the graph.

The equation $x^2+6x+9=0$ have equal roots are $\boxed{-3,0}$.

The graph of the equation is shown below:



From the graph, it is observed that the equation meet the $x-$axis at only one point.

So the solution to the equation is $x=\boxed{\{-3\}}$.

Answer 14PA.

Consider the equation,

$b^2 - 12b + 36 = 0$.

Let us consider the related function $f(b) = b^2 - 12b + 36$

Find the vertex of the graph.

The vertex of the graph $f(x) = ax^2 + bx + c$ is

$\left(\frac{-b}{2a}, \frac{4ac - b^2}{2a}\right)$.

Compare the function $f(b) = b^2 - 12b + 36$ with standard quadratic function

$f(x) = ax^2 + bx + c$, where $a \neq 0$

a = 1, *b* = −12 and *c* = 36.

By this rule the vertex of the function is

$$\left(-\frac{-12}{2\cdot 1}, \frac{4\cdot(1)\cdot(36) - (-12)^2}{2\cdot 1}\right) = \left(-\frac{-6\cdot 2}{2\cdot 1}, \frac{144 - 144}{2\cdot 1}\right)$$
$$= (6, 0) \qquad (\text{Simplification})$$

The graph $f(b) = b^2 - 12b + 36 = 0$ is open upward as coefficient of *b*2 term is positive.

Construct the table for the function $f(b) = b^2 - 12b + 36 = 0$ by given values of '*b*' and get different values of $f(b)$ and plot the points $(b, f(b))$ and connect then.

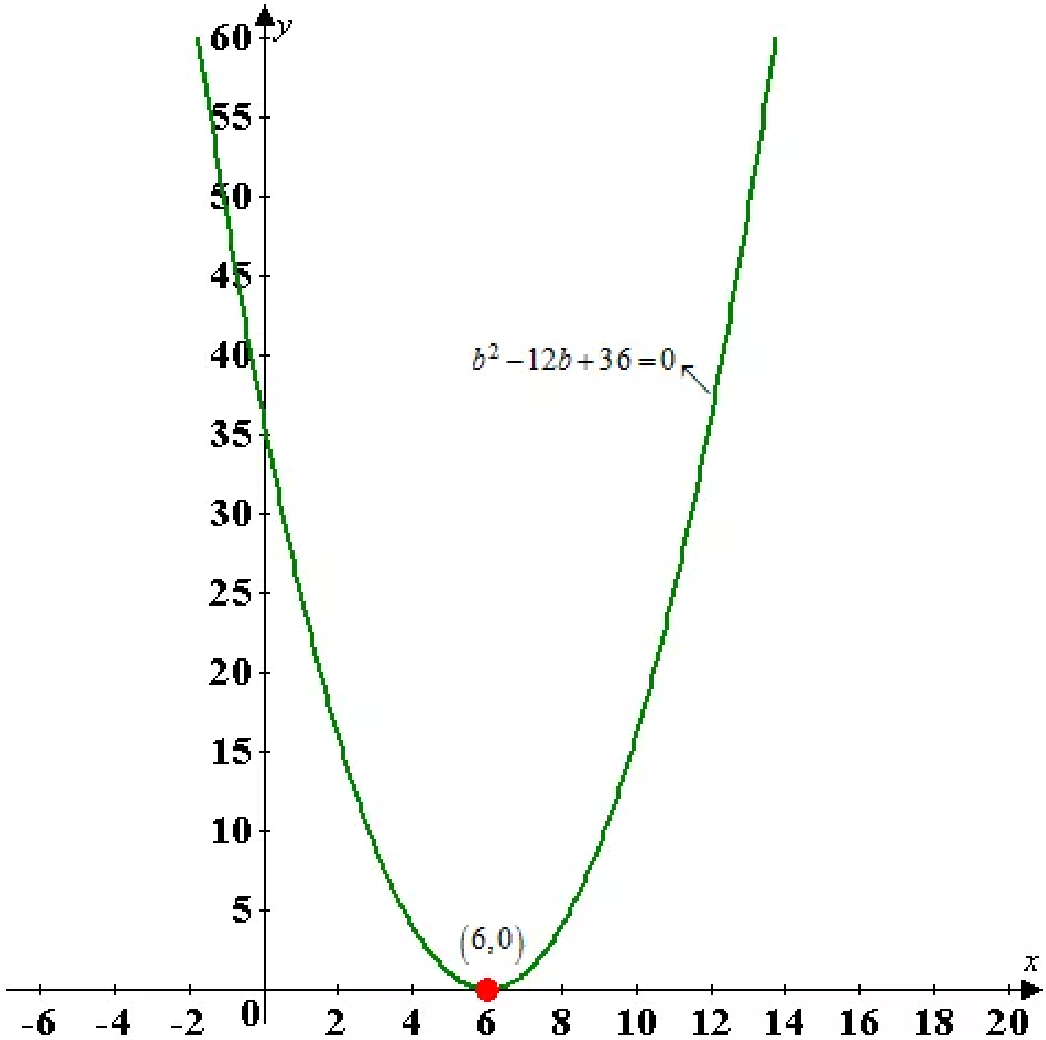
<u>Construction of the table</u>

b	$b^2 - 12b + 36$	$f(b)$	$(b, f(b))$
-1	$(-1)^2 - 12(-1) + 36 = 49$	49	$(-1, 49)$
0	$(0)^2 - 12(0) + 36 = 36$	36	$(0, 36)$
1	$(1)^2 - 12(1) + 36 = 25$	25	$(1, 25)$
2	$(2)^2 - 12(2) + 36 = 16$	16	$(2, 16)$
3	$(3)^2 - 12(3) + 36 = 9$	9	$(3, 9)$
4	$(4)^2 - 12(4) + 36 = 4$	4	$(4, 4)$
5	$(5)^2 - 12(5) + 36 = 1$	1	$(5, 1)$
6	$(6)^2 - 12(6) + 36 = 0$	6	$(6, 6)$
7	$(7)^2 - 12(7) + 36 = 1$	1	$(7, 1)$

Plot the table values the sketch of $b^2 - 12b + 36 = 0$ is shown below:

From the table graph intersect the *x* – axis at (6, 0) and it is the vertex of the graph. The equation $b^2 - 12b + 36 = 0$ have equal roots are $\boxed{6, 6}$.

The graph of the equation is shown below:



From the graph, I is observed that the equation touch the $x-$axis a only one point.

Therefore, the solution to the equation is

$x=\boxed{\{6\}}$.

Answer 15PA.

Consider the equation $x^2+2x+5=0$.

The equation of the axis of symmetry is,

$$\begin{aligned} x &= \frac{-b}{2a} \\ &= \frac{-2}{2\cdot 1} \\ &= -1 \end{aligned}$$

The value of the function at $x=-1$ is,

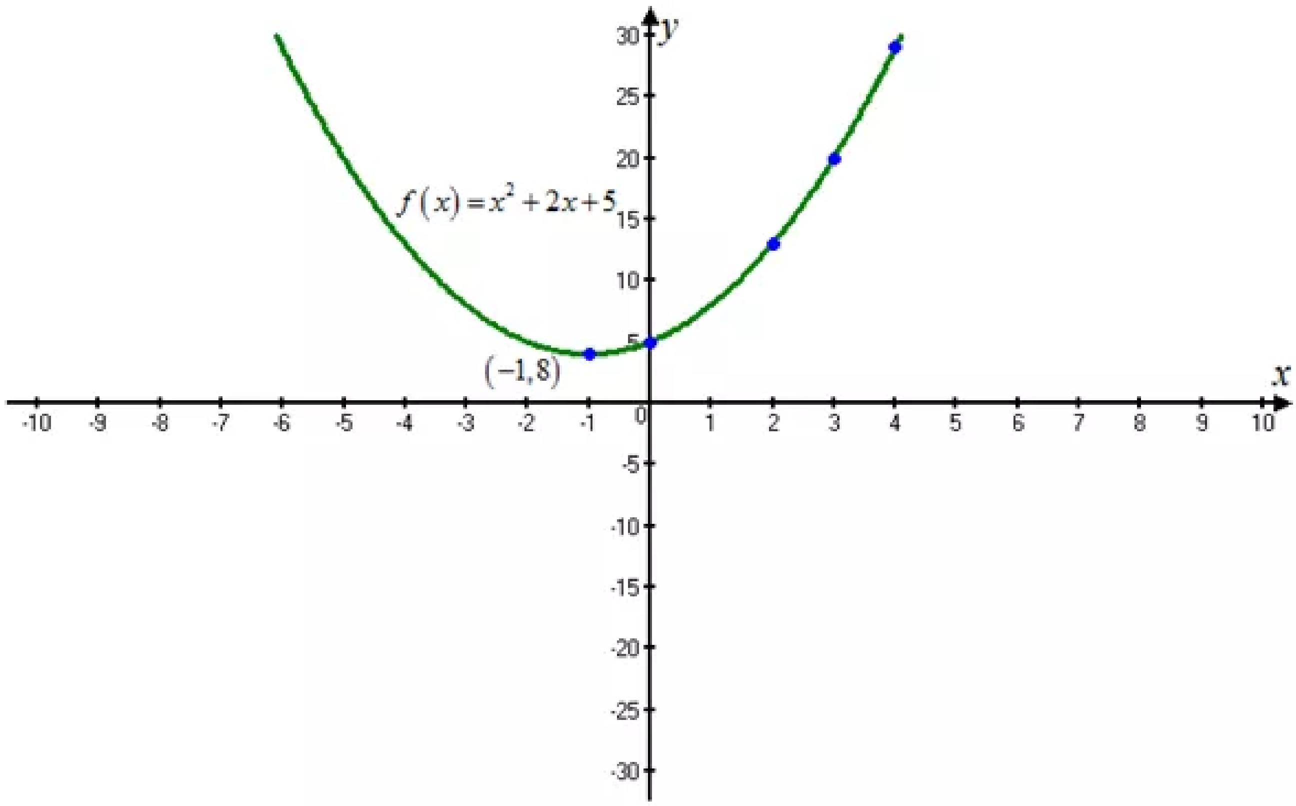
$$\begin{aligned} f(x) &= x^2+2x+5 \\ &= (-1)^2+2(-1)+5 \\ &= 1-2+5 \\ &= 4 \end{aligned}$$

The vertex of the equation is $(-1,8)$ which is in second quadrant and clearly the graph does not intersect the x – axis

The table of values is,

x	$f(x)=x^2+2x+5$	$f(x)$	$(x,f(x))$
-1	$(-1)^2+2(-1)+5=4$	4	$(-1,4)$
0	$(0)^2+2(0)+5=5$	5	$(0,5)$
2	$(2)^2+2(2)+5=13$	13	$(2,13)$
3	$(3)^2+2(3)+5=20$	20	$(3,20)$
4	$(4)^2+2(4)+5=29$	29	$(4,29)$

The graph of the equation is as shown below.



Make $f(x)=0$ to find the x-intercept,

$$f(x)=x^2+2x+5$$
$$0=x^2+2x+5$$

We are unable to factor the equation $x^2+2x+5=0$.

Hence, the equation $x^2+2x+5=0$ does not have x-intercepts.

Answer 16PA.

Consider the equation $r^2+4r-12=0$.

The equation of the axis of symmetry is,

$$\begin{aligned} r &= \frac{-b}{2a} \\ &= \frac{-4}{2\cdot 1} \\ &= -2 \end{aligned}$$

The value of the function at $r=-2$ is,

$$\begin{aligned} f(x) &= r^2+4r-12 \\ &= (-2)^2+4(-2)-12 \\ &= 4-8-12 \\ &= -16 \end{aligned}$$

The vertex of the equation is $(-2,-16)$.

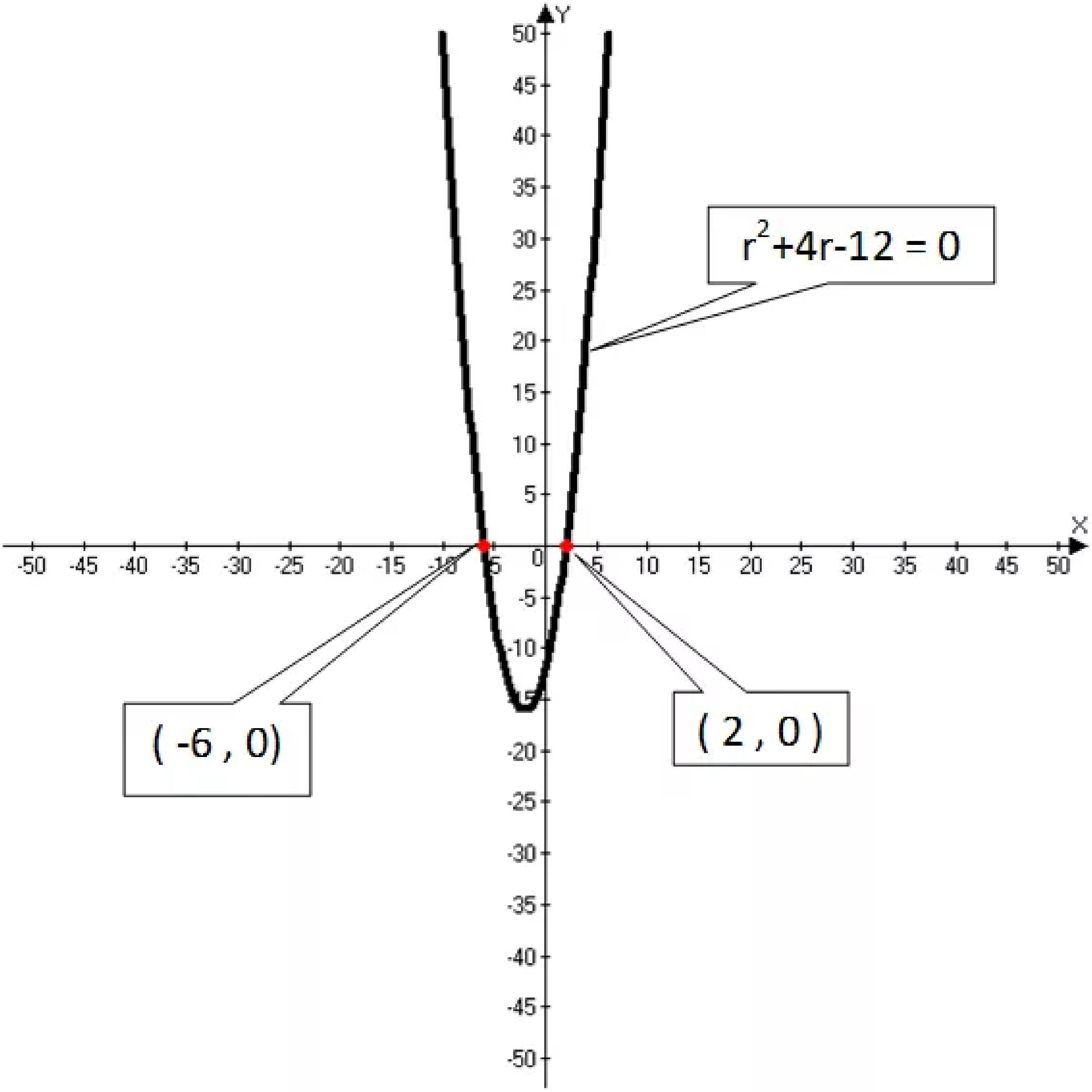
The table of values is,

r	$f(r)=r^2+4r-12$	$f(r)$	$(r,f(r))$
-3	$f(-3)=(-3)^2+4(-3)-12$	-15	$(-3,-15)$
-1	$f(-1)=(-1)^2+4(-1)-12$	-15	$(-1,-15)$
0	$f(0)=(0)^2+4(0)-12$	-12	$(0,-12)$
1	$f(1)=(1)^2+4(1)-12$	-7	$(1,-7)$
2	$f(2)=(2)^2+4(2)-12$	0	$(2,0)$
3	$f(3)=(3)^2+4(3)-12$	9	$(3,9)$

Make the function $f(r)$ equals to zero, the values of r are 2 and -6.

Hence, $r-$intercepts are $(2,0)$ and $(-6,0)$.

The graph of the equation is as shown below.



Verify the equation by factoring.

$$r^2+4r-12=0 \quad (\text{original equation})$$
$$r^2+6r-2r-12=0$$
$$r\cdot r+6\cdot r-2\cdot r-2\cdot 6=0 \quad (\text{factors})$$
$$r\cdot(r+6)-2\cdot(r+6)=0$$
$$(r-2)(r+6)=0 \quad (\text{factorization})$$
$$r-2=0 \text{ or } r+6=0$$
$$r=2 \text{ or } \quad r=-6$$

The roots of the equation $r^2+4r-12=0$ are $(2,0)\text{and}(-6,0)$.

Hence, $r-$intercepts are $(2,0)\text{and}(-6,0)$.

Answer 19PA.

Consider the two real numbers a and b.

The sum of numbers is $a+b=9$ and the product of numbers is $a\cdot b=20$.

Step1,

Solve for b in the equation $a+b=9$,

$$a+b=9 \quad (\text{Original equation})$$
$$-a+a+b=9-a \quad (\text{Substract "}a\text{" on both side})$$
$$b=9-a$$

Step2,

Substitute $b=9-a$ in the equation $a\cdot b=20$,

$$a\cdot b=20 \quad (\text{Original equation})$$
$$a\cdot(9-a)=20 \quad (\text{Replace } b \text{ by } 9-a)$$
$$a\cdot 9-a\cdot a=20$$
$$9a-a^2=20$$
$$-a^2+9a-20=0 \quad (\text{Substract 20 on both side})$$
$$-1\left[-a^2+9a-20\right]=-1\cdot 0 \quad (\text{Multiply } -1 \text{ on both sides})$$
$$a^2-9a+20=0 \quad (\text{Use distributive property})$$

Step3,

Solve the equation $a^2-9a+20=0$ by factoring,

$$a^2-9a+20=0 \quad (\text{Original equation})$$
$$a^2-4a-5a+20=0$$
$$a(a-4)-5(a-4)=0 \quad (\text{Use distributive property})$$
$$(a-4)(a-5)=0 \quad (\text{Factor})$$
$$a-4=0 \quad \text{or} \quad a-5=0 \quad (\text{Use zero product rule})$$
$$a=4 \quad \text{or} \quad a=5 \quad (\text{Solve for } a)$$

Step4,

Substitute each value of a in $b=9-a$.

Case –I: $a=4$

The value is,

$$\begin{aligned} b &= 9-a && \text{(Original equation)} \\ &= 9-4 && \text{(Replace } a \text{ by } 4) \\ &= 5 \\ b &= 5 \end{aligned}$$

If the value *a* = 4 then *b* = 5.

Case-II: $a=5$

$$\begin{aligned} b &= 9-a && \text{(Original equation)} \\ &= 9-5 && \text{(Replace } a \text{ by } 5) \\ &= 4 \\ b &= 4 \end{aligned}$$

If the value *a* = 5 then *b* = 4.

Verify the results.

Case – I: Suppose *a* = 4 and *b* = 5.

The sum of two numbers 4 and 5 is $4+5=9$. True

The product of two numbers 4 and 5 is $4\cdot5=20$. True

Therefore, the two numbers are $\boxed{4}$ and $\boxed{5}$.

Case- II: Suppose *a* = 5 and *b* = 4.

The sum of two numbers 4 and 5 is $4+5=9$. True

The product of two numbers 4 and 5 is $4\cdot5=20$. True

Hence, the two numbers are $\boxed{5}$ and $\boxed{4}$

Answer 20PA.

Consider the two real numbers a and b.

The sum of numbers is $a+b=5$ and the product of numbers is $a \cdot b=-24$.

Step 1:

Solve for b in the equation $a+b=5$

$$\begin{aligned} a+b&=5 && \text{(Original equation)} \\ -a+a+b&=5-a && \text{(Substract "}a\text{" on both side)} \\ b&=5-a \end{aligned}$$

Step 2:

Substitute $b=5-a$ in the equation $a \cdot b=-24$

$$\begin{aligned} a \cdot b&=-24 && \text{(Original equation)} \\ a \cdot (5-a)&=-24 && \text{(Replace } b \text{ by } 5-a\text{)} \\ a \cdot 5-a \cdot a&=-24 \\ 5a-a^2&=-24 \\ 5a-a^2-24&=-24+24 && \text{(Substract 24 on both side)} \\ -a^2+5a+24&=0 \end{aligned}$$

Step 3:

Solve the equation $-a^2+5a+24=0$ by factoring

$$\begin{aligned} -a^2+5a+24&=0 && \text{(Original equation)} \\ -a^2+8a-3a+24&=0 \\ -a(a-8)-3(a-8)&=0 && \text{(Use distributive property)} \\ (-a-3)(a-8)&=0 \\ -a-3=0 \quad &\text{or} \quad a-8=0 && \text{(Use zero product rule)} \\ -a=3 \quad &\text{or} \quad a=8 \\ a=-3 \quad &\text{or} \quad a=8 && \text{(Solve for } a\text{)} \end{aligned}$$

Step 4:

Substitute each value of "a" in $b=5-a$.

Case –I: $a = -3$

$$\begin{aligned} b&=5-a && \text{(Original equation)} \\ &=5-(-3) && \text{(Replace } a \text{ by 8)} \\ &=5+3 \\ b&=8 \end{aligned}$$

If the value $a = -3$ then $b = 8$.

Case-II: $a = 8$

$$\begin{aligned} b&=5-a && \text{(Original equation)} \\ &=5-8 && \text{(Replace } a \text{ by 8)} \\ &=5-8 \\ b&=-3 \end{aligned}$$

If the value $a=8$ then the value is $b=-3$.

Verify the result,

Case – I:

Suppose $a = 8$ and $b = -3$.

The sum of two numbers −3 and 8 is $-3 + 8 = 5$. True

The product of two numbers −3 and 8 is $-3 \cdot 8 = -24$. True

Therefore, the two numbers are $\boxed{-3}$ and $\boxed{8}$.

Case- II:

Suppose $a = -3$ and $b = 8$

The sum of two numbers 8 and −3 is $8 - 3 = 5$. True

The product of two numbers 8 and −3 is $8 \cdot -3 = -24$. True

Hence, the two numbers are $\boxed{8}$ and $\boxed{-3}$

Answer 21PA.

Let us consider the equation $a^2 - 12 = 0$

Now consider related function $f(a) = a^2 - 12$

Now we contract the table for the function $f(a)$

a	$f(a) = a^2 - 12$	$(a, f(a))$
0	$0 - 12 = -12$	$(0, -12)$
1	$1 - 12 = -1$	$(1, -11)$
2	$4 - 12 = -8$	$(2, -8)$
3	$9 - 12 = -3$	$(3, -3)$
4	$16 - 12 = 4$	$(4, 4)$
5	$25 - 12 = 13$	$(5, 13)$

We use the rule '*a*' is a solution. If $f(a)$ = 0 and the root lies between '*b*' and '*c*' if $f(b) < 0$ and $f(c) > 0$

From the table $f(3) = -3 < 0, f(4) = 4 > 0$

The root of equation $a^2 - 12 = 0$ lies between the integers 3 and 4 now and

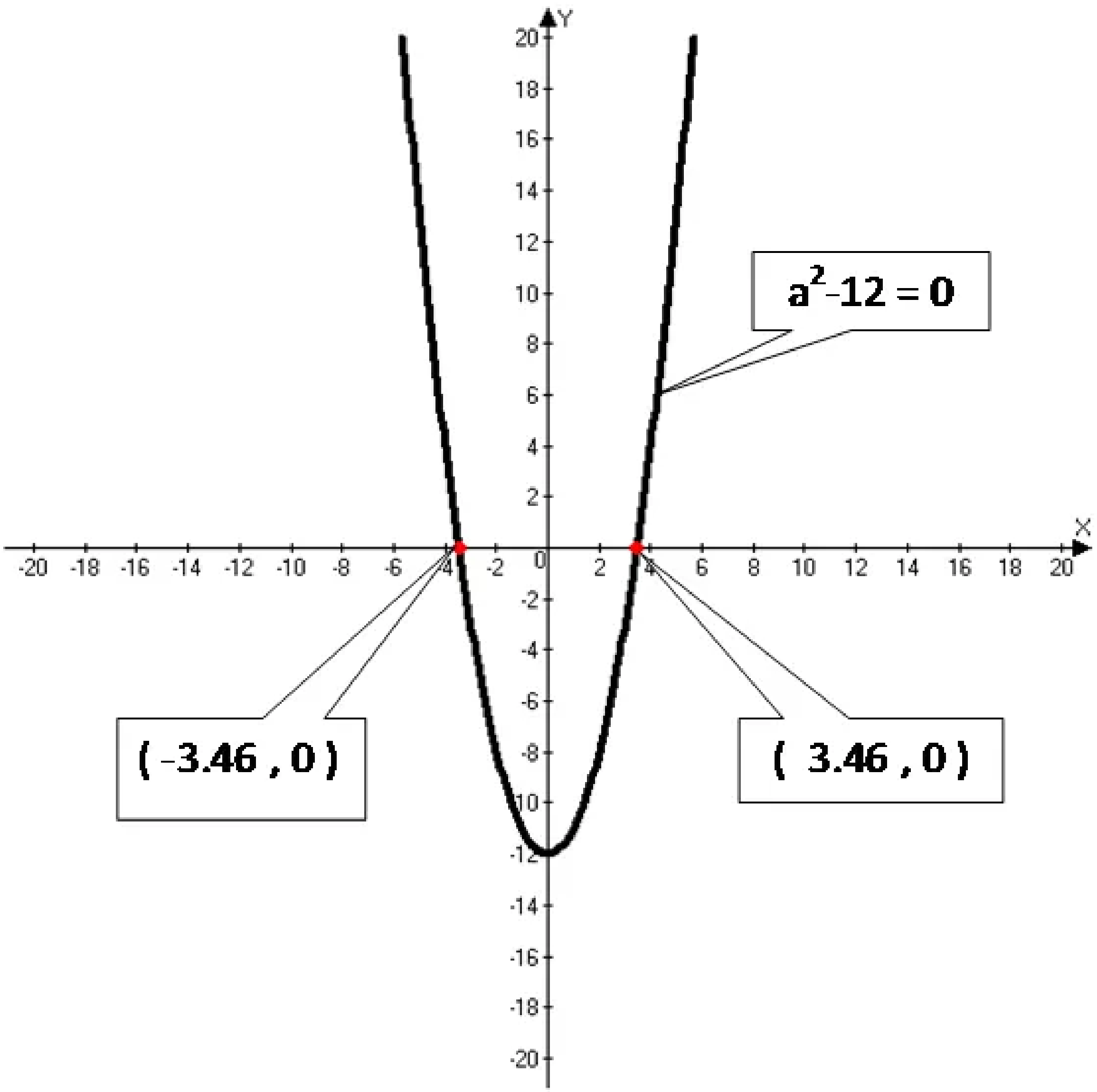
$$f(3.4) = (3.4)^2 - 12$$
$$= 11.56 - 12$$
$$= -0.44$$

$$f(3.42) = (3.42)^2 - 12$$
$$= 11.696 - 12$$
$$= -0.303$$

$$f(3.47) = (3.47)^2 - 12$$
$$= 12.049 - 12$$
$$= 0.04$$

$$f(3.46) = -0.03 \simeq 0$$

Approximately the roots of the equation $a^2 - 12 = 0$ is + 3.46 and - 3.46



Answer 22PA.

Let us consider the equation $n^2 - 7 = 0$

Now consider related function $f(n) = n^2 - 7 = 0$

Now we contract the table for the function $f(n)$

n	$f(n) = n^2 - 7$
0	$f(0) = -7$
1	$f(1) = -6$
2	$f(2) = -3$
3	$f(3) = 2$
4	$f(4) = 9$

We use the rule '*a*' is a solution. If $f(a)$ = 0 and the root lies between '*b*' and '*c*' if $f(b) < 0$ and $f(c) > 0$ by graphically '*a*' is root if at *x* = *a*, $f(x)$ intersect *x* – axis. If the root is non – integer, $f(x)$ have negative and positive values of a two consecutive integers

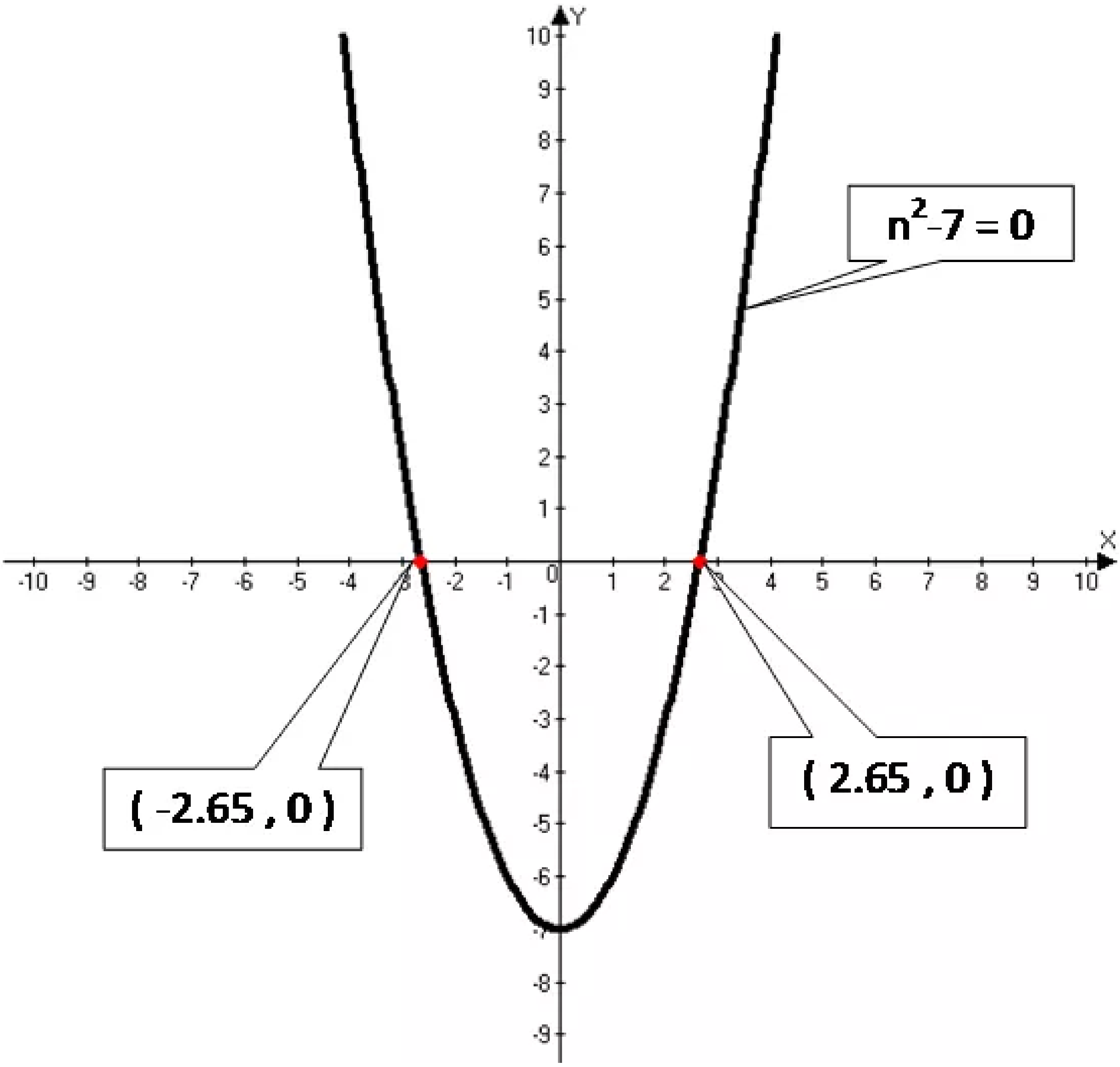
From the table $f(2) = -3, f(3) = 2$ 2 and 3 are consecutive integers and they have −ve, +ve values

$$f(2) = -3 < 0, f(3) = 2 > 0$$

The roots of $f(n)=0$ lies between integers 2 and 3,

$$f(2.64)=6.9696-7$$
$$=-0.0304$$
$$f(2.645)=7.0003-7$$
$$=0.003\simeq 0$$

Approximately the roots of the equation $n^2-7=0$ is 2.645



Answer 23PA.

Let us consider the equation $2c^2+20c+32=0$

Now consider related function $f(c)=2c^2+20c+32$

Now we contract the table for the function $f(c)$

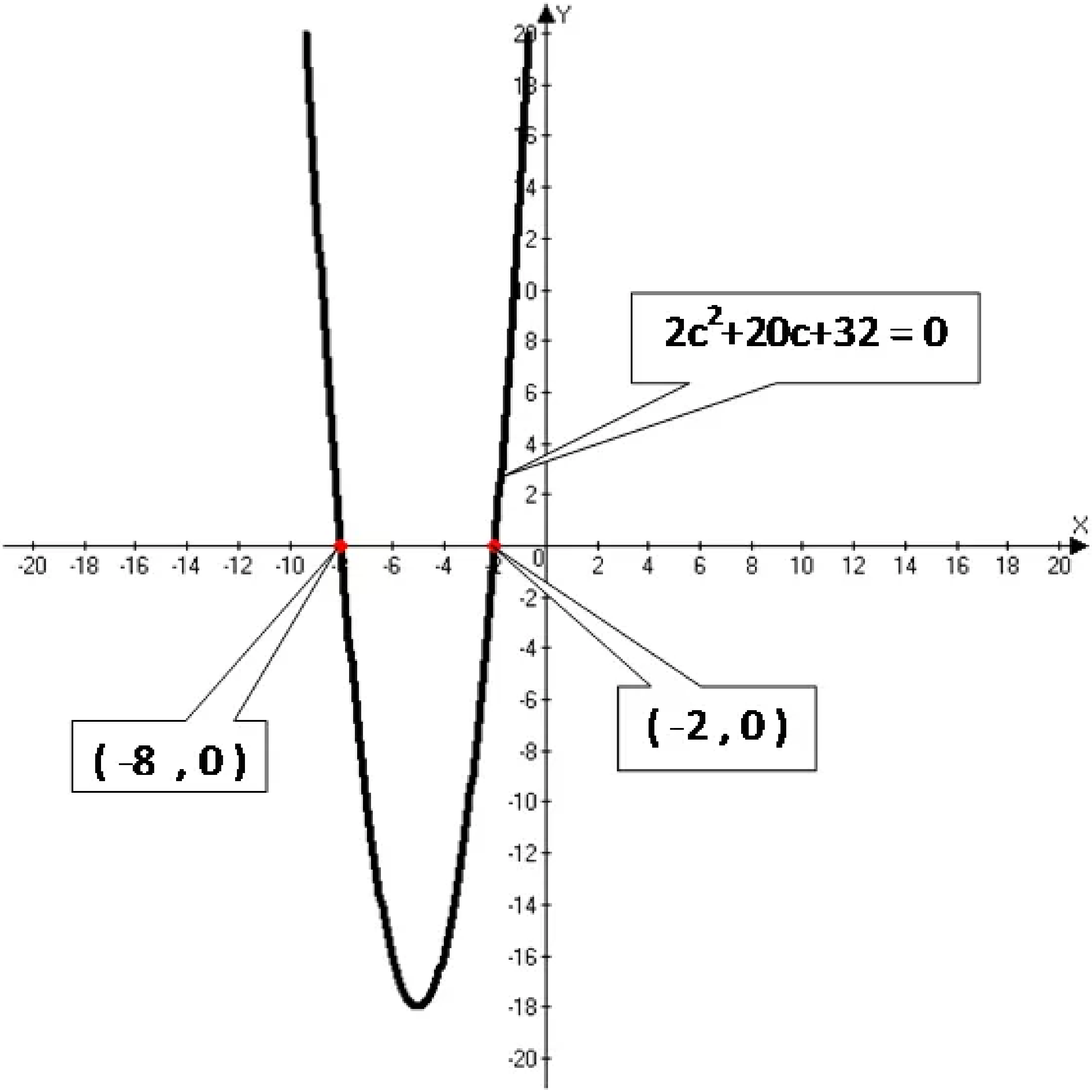
c	$f(c)=2c^2+20c+32$
-9	$f(-9)=14$
-8	$f(-8)=0$
-7	$f(-7)=-10$
-6	$f(-6)=-16$
-5	$f(-5)=-18$
-4	$f(-4)=-16$
-3	$f(-3)=-10$
-2	$f(-2)=0$

We use the rule '*a*' is a solution of the equation of $f(a)$ = 0 graphically $f(x)$ intersect *x* – axis at *x* = *a*, then '*a*' solution of $f(x)$.

From the table $f(-8)=0, f(-2)=0$

-8, -2 are the roots of $f(n)=0$ in the graph $f(n)$ intersect the x – axis at $x=-8, x=-2$

The roots are $-8, -2$



Answer 24PA.

Let us consider the equation $3s^2 + 9s - 12 = 0$

Now consider related function $f(s) = 3s^2 + 9s - 12$

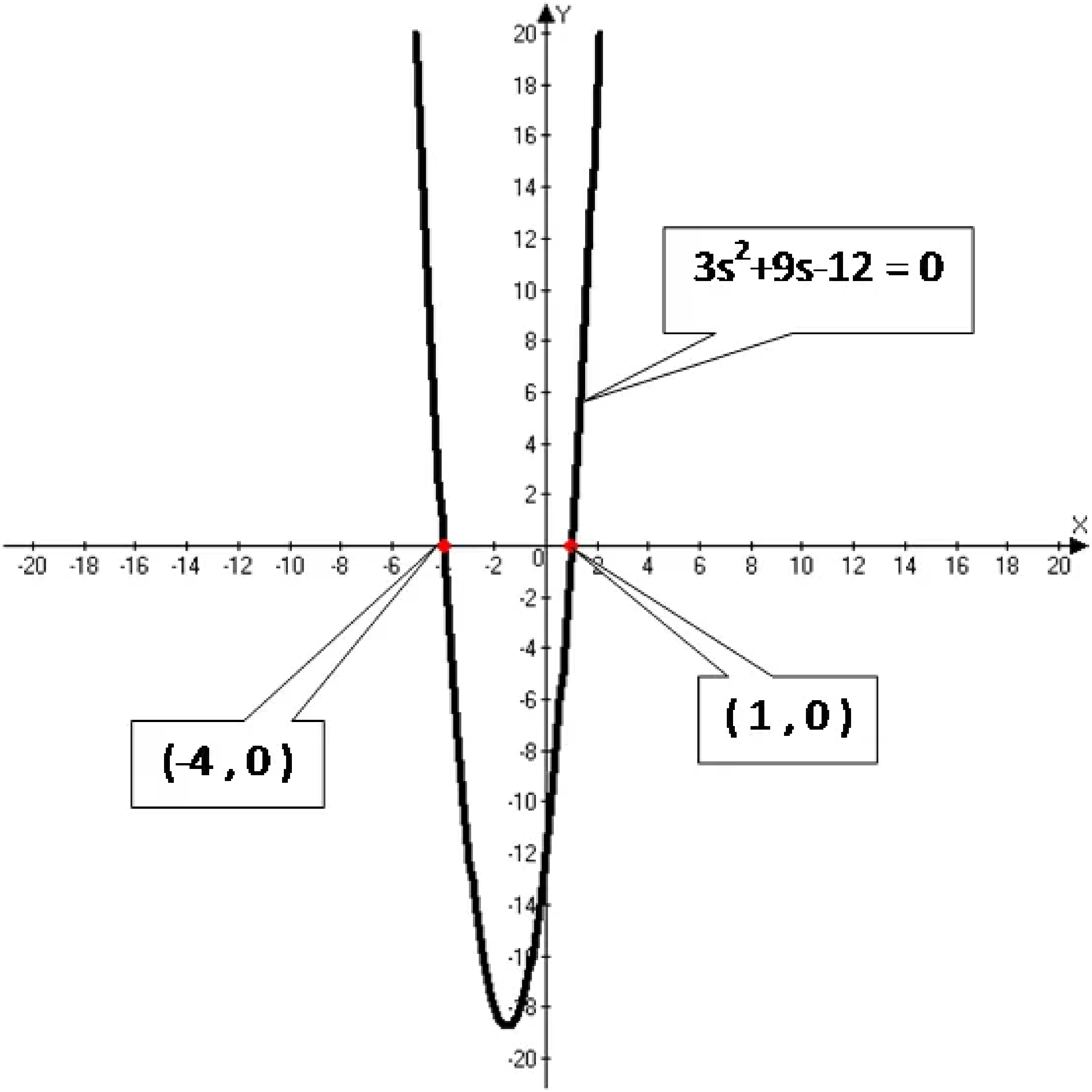
Now we contract the table for the function $f(s)$

s	$f(s)$
-5	$f(-5) = 18$
-4	$f(-4) = 0$
-3	$f(-3) = -12$
-2	$f(-2) = -18$
-1	$f(-1) = -18$
0	$f(0) = -12$
1	$f(1) = 0$
2	$f(2) = 18$

We use the rule's' is a solution of the equation of $f(s)$ = 0 graphically $f(s)$ intersect *x* – axis at *x* = *s*, then 's' solution of $f(s)$.

From the table $f(-4) = 0, f(1) = 0$ -4 and 1 are the roots of $f(s) = 0$ in the graph $f(s)$ intersect the x – axis at $s = -4, s = 1$

The roots are $\{-4,1\}$



Answer 25PA.

Let us consider the equation $x^2 + 6x + 6 = 0$

Now consider related function $f(x) = x^2 + 6x + 6$

Now we contract the table

x	$f(x)$
-2	$f(-2) = -2$
-1	$f(-1) = 1$
0	$f(0) = 6$
1	$f(1) = 13$
2	$f(2) = 22$

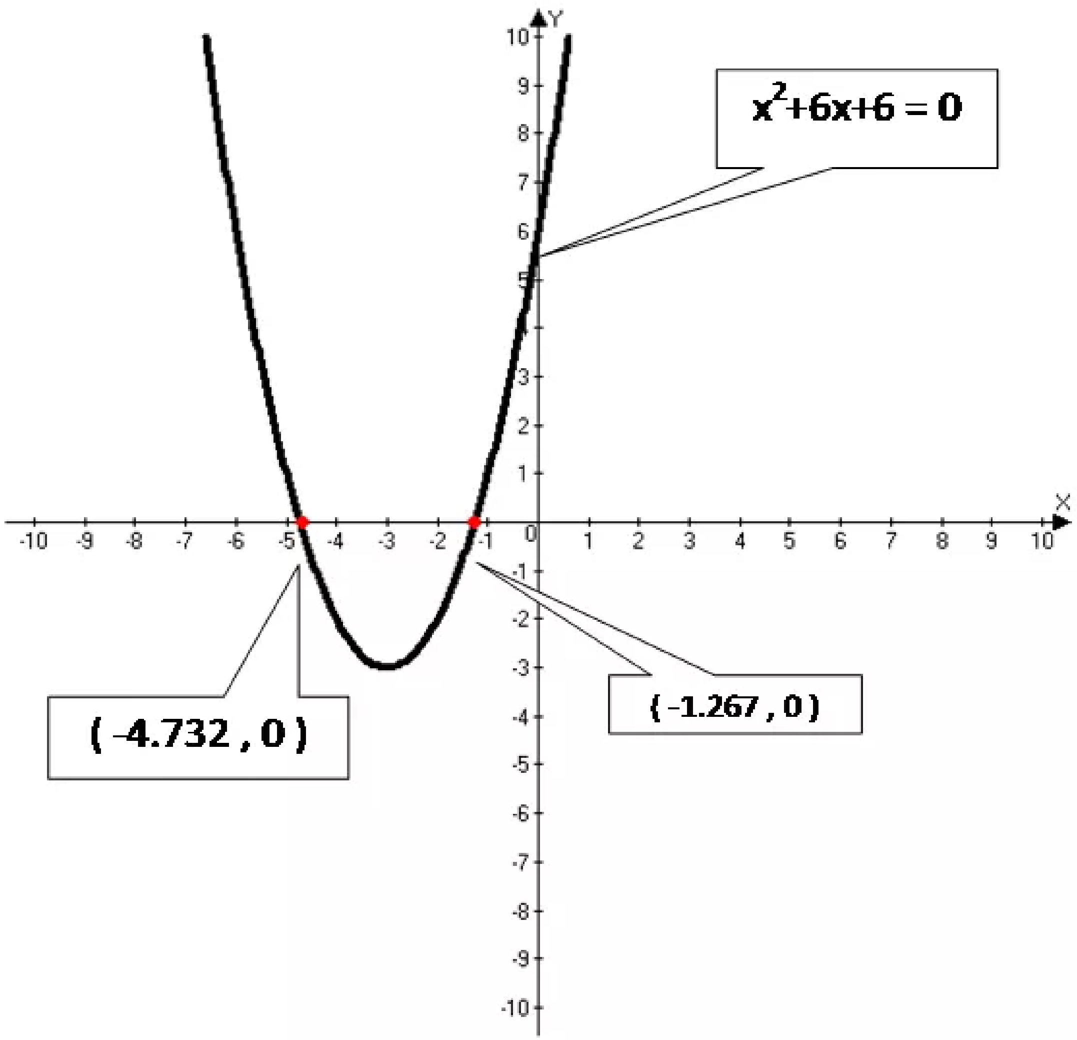
We use the rule 'a' is a solution. If $f(a)$ = 0 and the root lies between 'b' and 'c' if $f(b) < 0$ and $f(c) > 0$ by graphically 'a' is root if at x = a, $f(x)$ intersect x – axis. If the root is non – integer, $f(x)$ have negative and positive values of a two consecutive integers

From the table $f(-2) = -2, < 0 \; f(-1) = 1 > 0$

The roots of $f(x)$ = 0 lies between integers now

$f(-1.267) = 0.0003 \approx 0$

The root is -1.267



Answer 26PA.

Let us consider the equation $y^2-4y+1=0$

Now consider related function $f(y)=y^2-4y+1$

Now we contract the table for the function $f(y)$

y	$f(y)=y^2-4y+1$
-1	$f(-1)=6$
0	$f(0)=1$
1	$f(1)=-2$
2	$f(2)=-3$
3	$f(3)=-2$
4	$f(4)=1$

From table we can observe that $f(0)=1, f(1)=-2$

$f(1)=-2<0, f(0)=1>0$ and also $f(3)=-2<0, f(4)=1>0$

We use the rule " Solution of an equation lies between any two consecutive integers '*b*' and '*c*', if $f(b)<0$ and $f(c)>0$"

By using the rule, the root of the equation lies between 0 and 1 and also lies between 3 and 4.

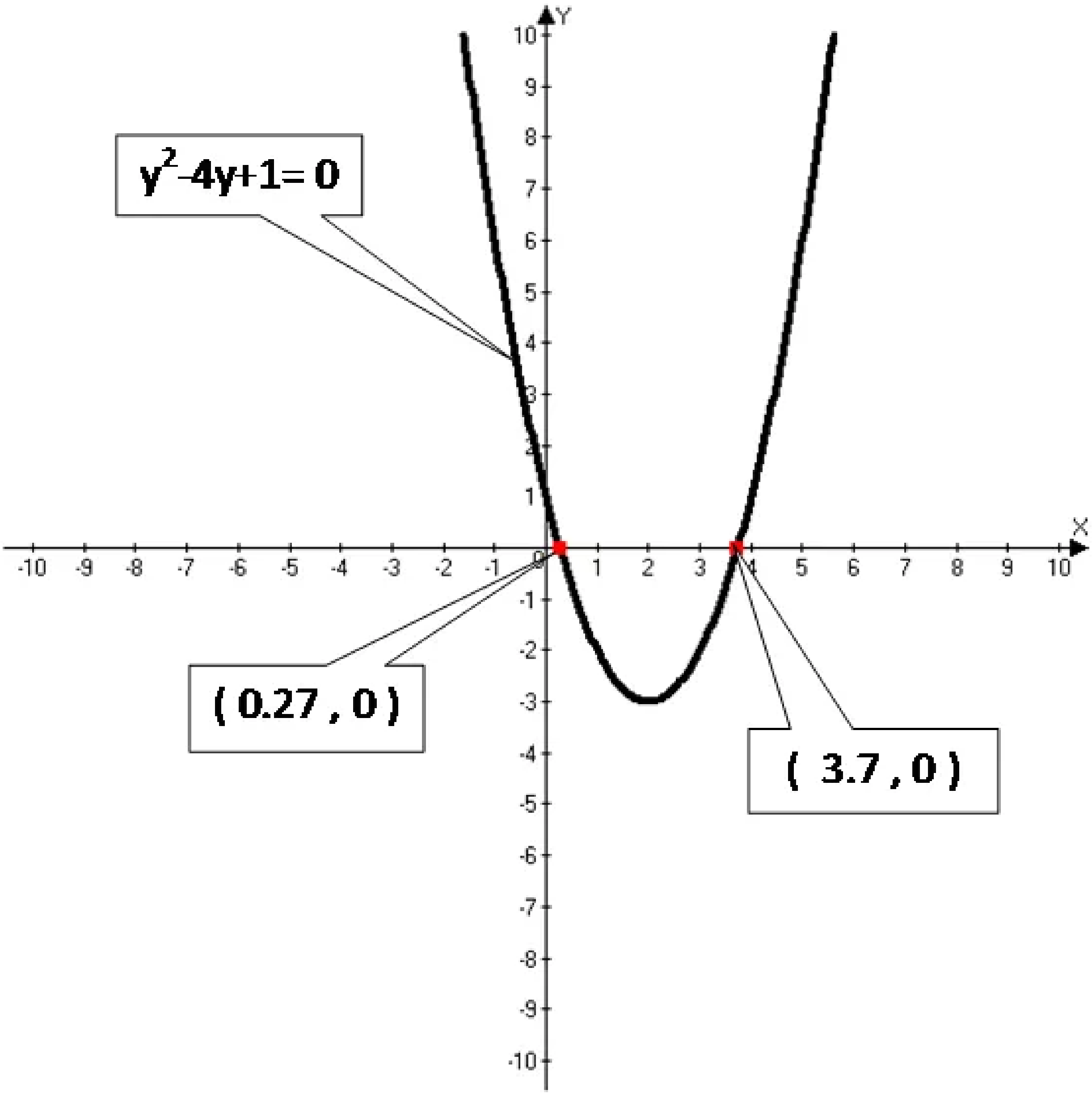
Now,

$$f(3.7321)=0.00017\simeq 0$$

$$f(3.7321=00.00017)\simeq 0$$

Now we use the rule ‘*a*’ is root of the equation $f(x) = 0$. If $f(a) = 0$

The roots of the equation approximately 0.2679 and 3.7321



Answer 27PA.

Consider the equation

$$a^2 - 8a = 4$$
$$a^2 - 8a - 4 = 4 - 4$$
$$a^2 - 8a - 4 = 0$$

Let us consider the related function $f(a) = a^2 - 8a - 4$

Now we construct the table for the function $f(a)$

a	$f(a)$
-1	$f(-1) = 5$
0	$f(0) = -4$
1	$f(1) = -11$
-	
6	$f(6) = -16$
7	$f(7) = -11$
8	$f(8) = -4$
9	$f(9) = 5$

From the table we can observe that $f(-1) = 5 > 0. f(0) = -4 < 0.$ $-1, 0$ are consecutive numbers and $f(8) = -4 < 0, f(9) = 5 > 0$. 8, 9 are consecutive integers.

We have the rule "solution of an equation lies between any two consecutive integers *b* and *c*. If $f(b) < 0, f(c) > 0 \text{ or } f(b) > 0, f(c) < 0$

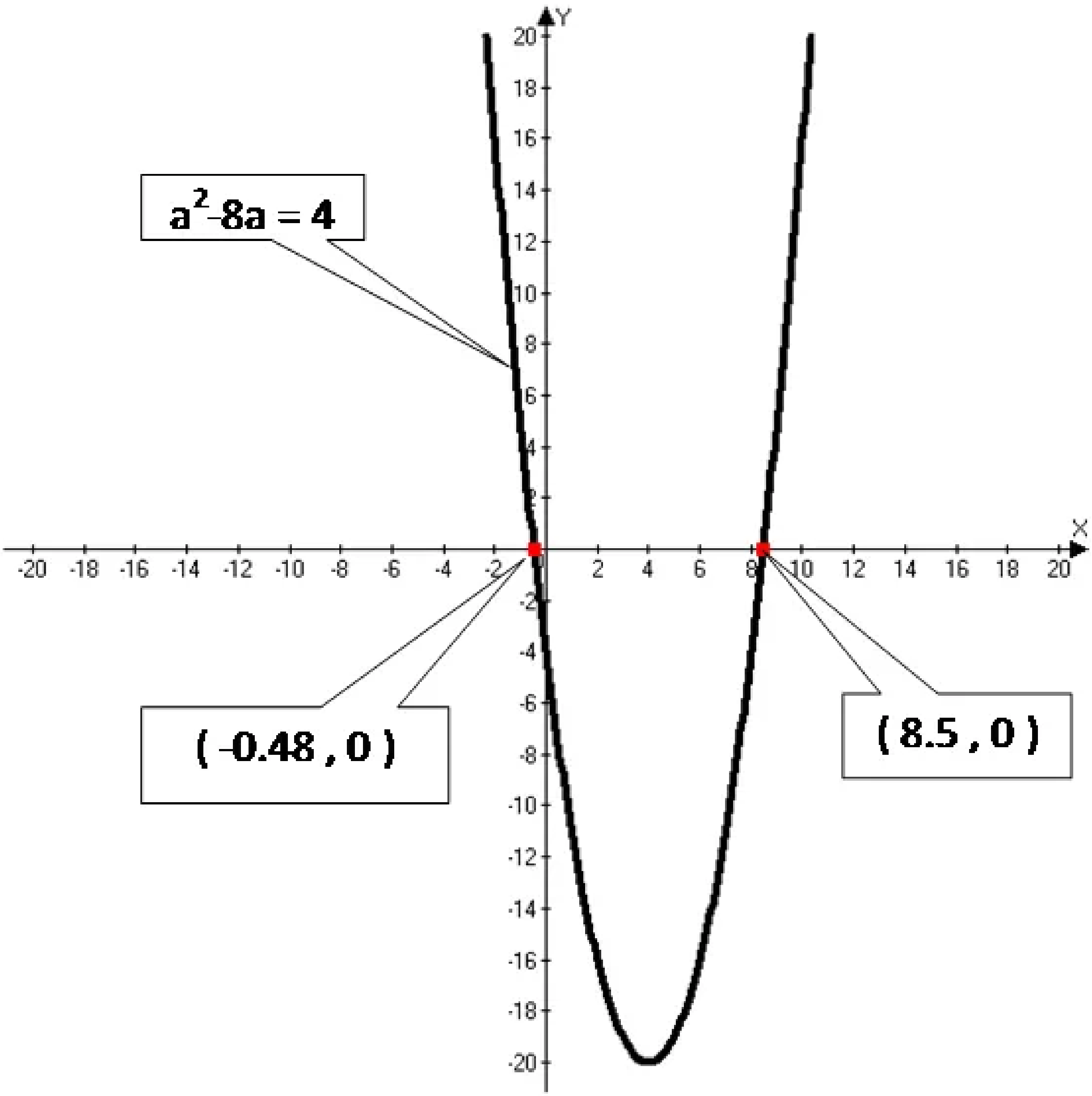
By using this rule we can say that the roots of the equation lies between -1, 0 and 9.

We have the rule '*a*' is the solution of the equation $f(x)$ $\text{if } f(a) = 0$

Now,

$$f(-0.472) = -0.001 \simeq 0$$
$$f(8.472) = -0.0012 \simeq 0$$

By the above rule we can say that the roots of the equation $f(a) = a^2 - 8a - 4 = 0$ are $-0.472, 8.472$ approximately.



Y
X
$a^2-8a = 4$
(-0.48 , 0)
(8.5 , 0)

Answer 28PA.

Consider the equation

$$x^2 + 6x = -7$$

$$x^2 + 6x + 7 = -7 + 7 \qquad \text{(Adding 7 on both sides)}$$

$$x^2 + 6x + 7 = 0$$

Let us consider the related function $f(x) = x^2 + 6x + 7$

Now we construct the table for the function $f(x)$

x	$f(x) = x^2 + 6x + 7$
-5	$f(-5) = 2$
-4	$f(-4) = -1$
-3	$f(-3) = -2$
-2	$f(-2) = -1$
-1	$f(-1) = 2$

From the table we can observe that $f(-5) = 2 > 0, f(-4) = -1 < 0$ and -5, -4 are consecutive integer and also $f(-2) = -1 < 0, f(-1) = 2 > 0$. -2, -1 are consecutive integers.

We have the rule "solution of an equation $f(x) = 0$ lies between any two consecutive $-$ve integers b and c. If $f(b) < 0, f(c) > 0 \text{ or } f(b) > 0, f(c) < 0$

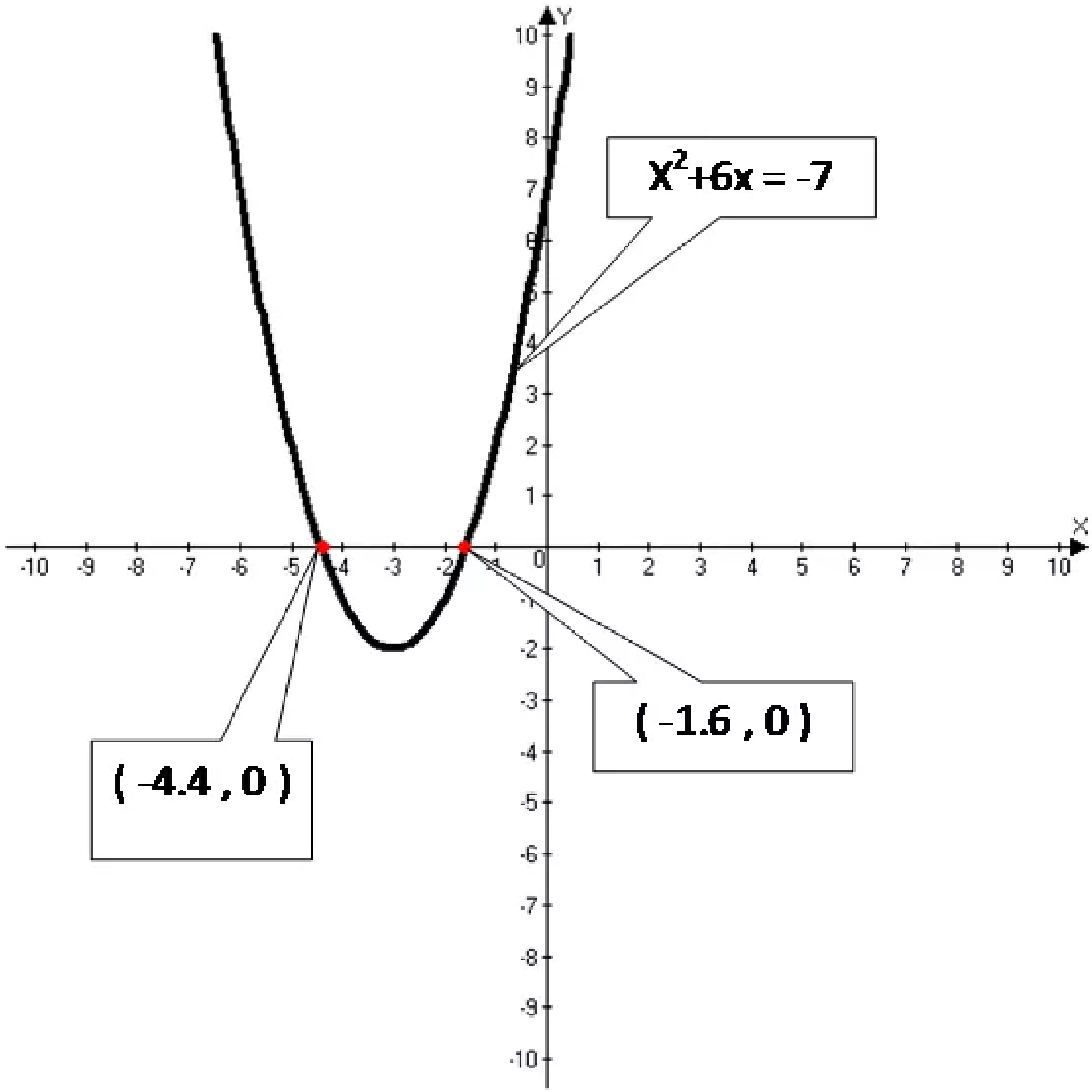
By using this rule we can say that the roots of the equation lies between -5, -4 and -2, -1.

We have the rule 'a' is the solution of the equation $f(x) = 0 \text{ if } f(a) = 0$

Now, we take a number -1.586 which lies between -2, -1 and $f(-1.586) = -0.0006 \simeq 0$ and also take a number -4.4142 which is lies between -5, -4 and

$$f(-4.4142) = -0.006 \simeq 0$$

By the above rule the roots of the equation $f(x)=0$ are $-1.586, -4.4142$ approximately.



Answer 29PA.

Consider the equation $m^2 - 10m = -21$

$m^2 - 10m + 21 = -21 + 21 \qquad (\text{Add } 21 \text{ on both sides})$

$m^2 - 10m + 21 = 0$

Let us consider the related function $f(m) = m^2 - 10m + 21$

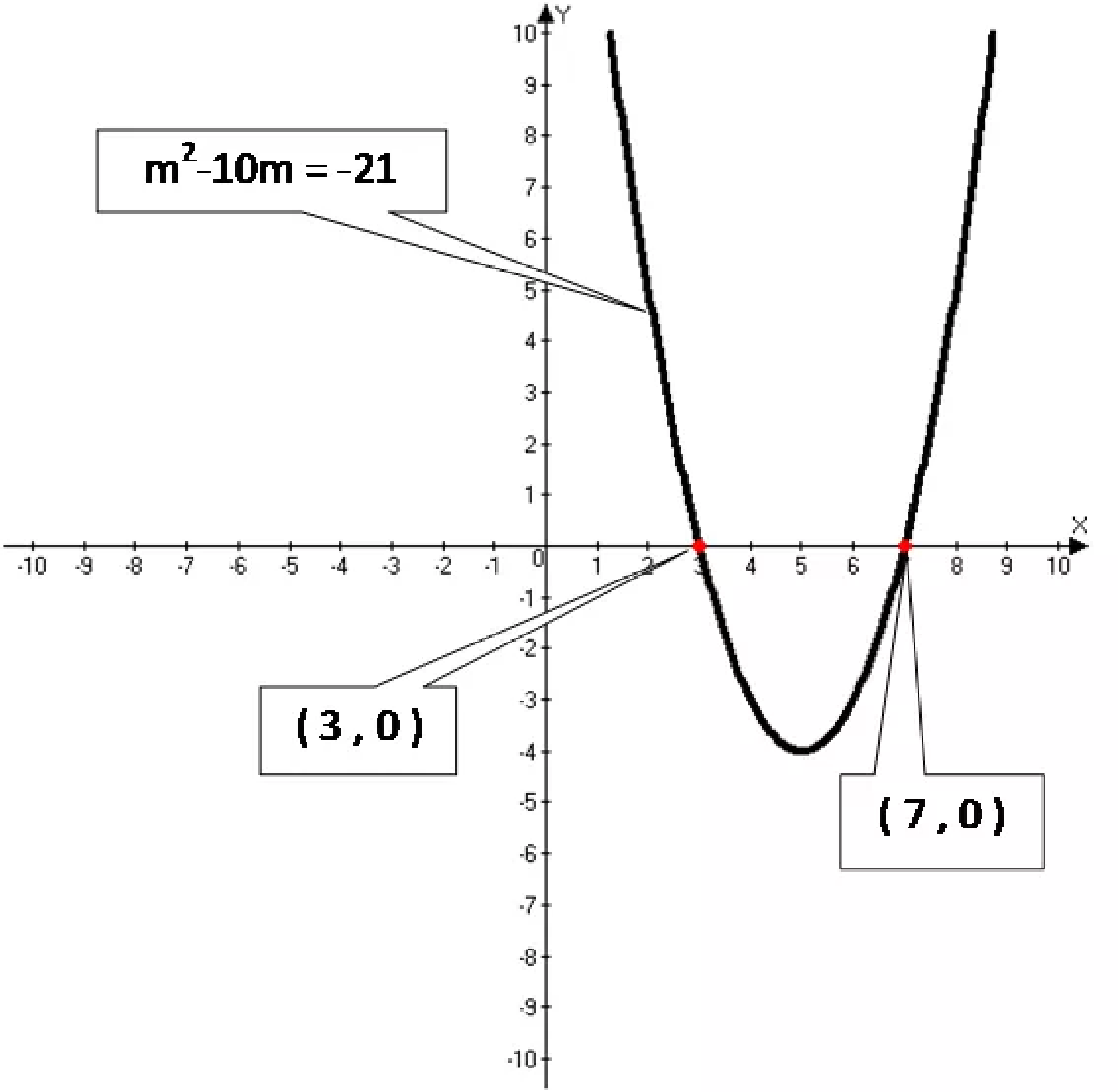
Now we construct the table for the function $f(m)$

m	$f(m) = m^2 - 10m + 21$
0	$f(0) = 21$
-1	$f(-1) = 32$
1	$f(1) = 12$
2	$f(2) = 5$
3	$f(3) = 0$
4	$f(4) = -3$
5	$f(5) = -4$
6	$f(6) = -3$

From the table we can observe that $f(3) = 0, f(7) = 0$

We have the rule an integer *a* is said to be solution of $f(x)=0$ if $f(a)=0$

By using this rule we can say that $(3,7)$ are the solutions of equations $m^2-10m+21=0$



Answer 30PA.

Consider the equation $p^2+16=8p$

$$p^2+16-8p=8p-8p \qquad \text{(Subtract } 8p \text{ on both sides)}$$
$$p^2+16-8p=0$$

Let us consider the related function $f(p)=p^2+16-8p$

Now we construct the table for the function $f(p)$

p	$f(p)=p^2-8p+16$
-1	$f(-1)=25$
0	$f(0)=16$
1	$f(1)=9$
2	$f(2)=4$
3	$f(3)=1$
4	$f(4)=0$
5	$f(5)=1$

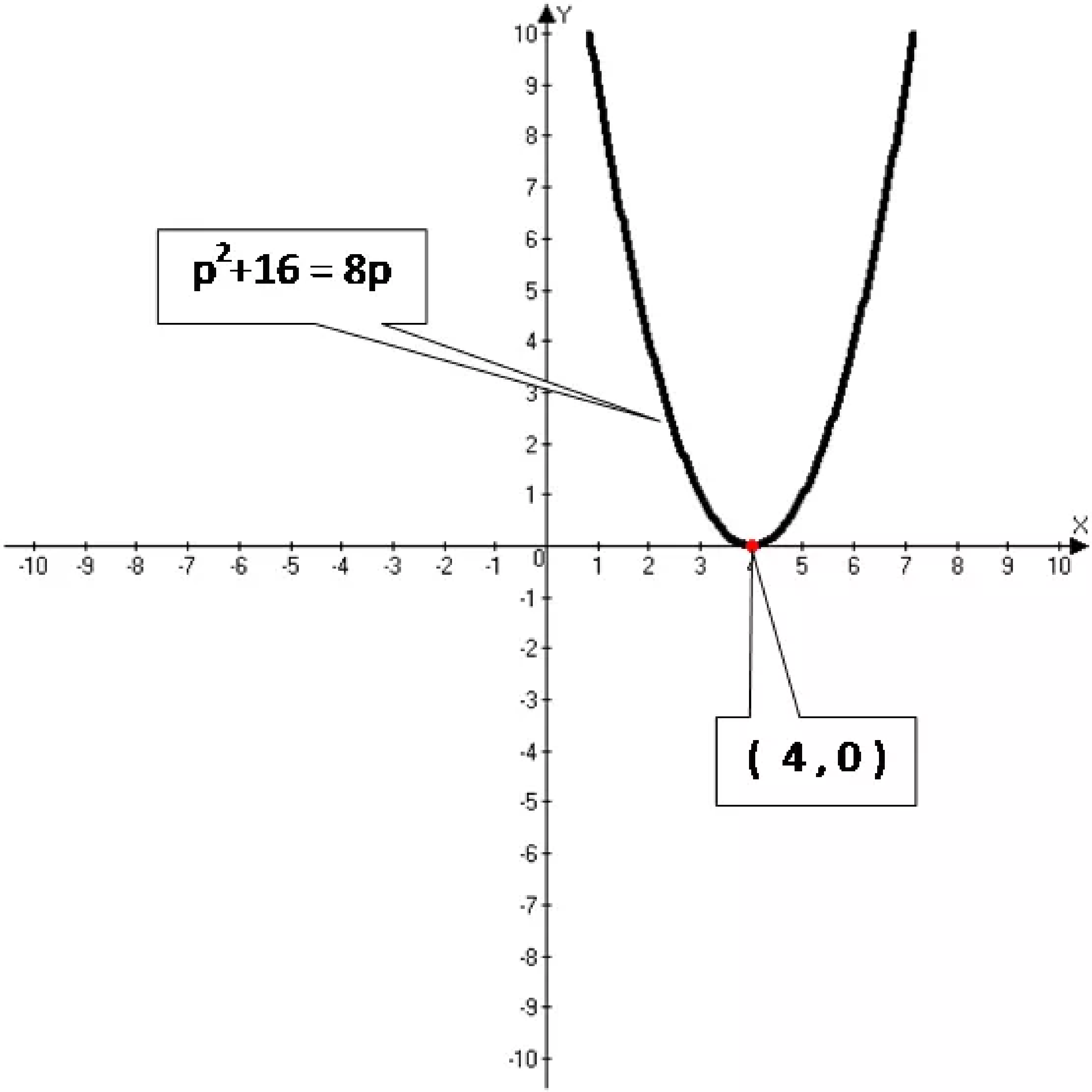
From the above table we can observe that $f(4)=0$

We have the rule an integer 'a' is said to be solution of $f(x)=0$ if $f(a)=0$

By using this rule we can say that 4 is the solutions the symmetry

$$x=\frac{-b}{2a}$$
$$=\frac{8}{2}$$
$$=4$$

4, +4 are the equal roots.



$p^2+16 = 8p$
(4 , 0)
Y
X

Answer 31PA.

Consider the equation $12n^2 - 26n = 30$

$12n^2 - 26n - 30 = 30 - 30$ (Subtract 30 on both sides)

$12n^2 - 26n - 30 = 0$

Let us consider the related function $f(n) = 12n^2 - 26n - 30$

n	$f(n) = 12n^2 - 26n - 30$
-1	$f(-1) = 8$
0	$f(0) = -30$
1	$f(1) = -44$
2	$f(2) = -34$
3	$f(3) = 0$
4	$f(4) = 58$

From the above table we can observe that $f(3) = 0$

We have the rule an integer '*a*' is said to be solution of $f(n) = 0$ if $f(a) = 0$

We can say that 3 is the root of $f(n) = 0$.

Symmetry,

$$x = \frac{-b}{a} = \frac{26}{2(12)} = \frac{13}{12} \neq 3 (b = -26, a = 12, c = -30)$$

3 is not equal root from the table $f(-1) = 8 > 0, f(0) = -30 < 0$

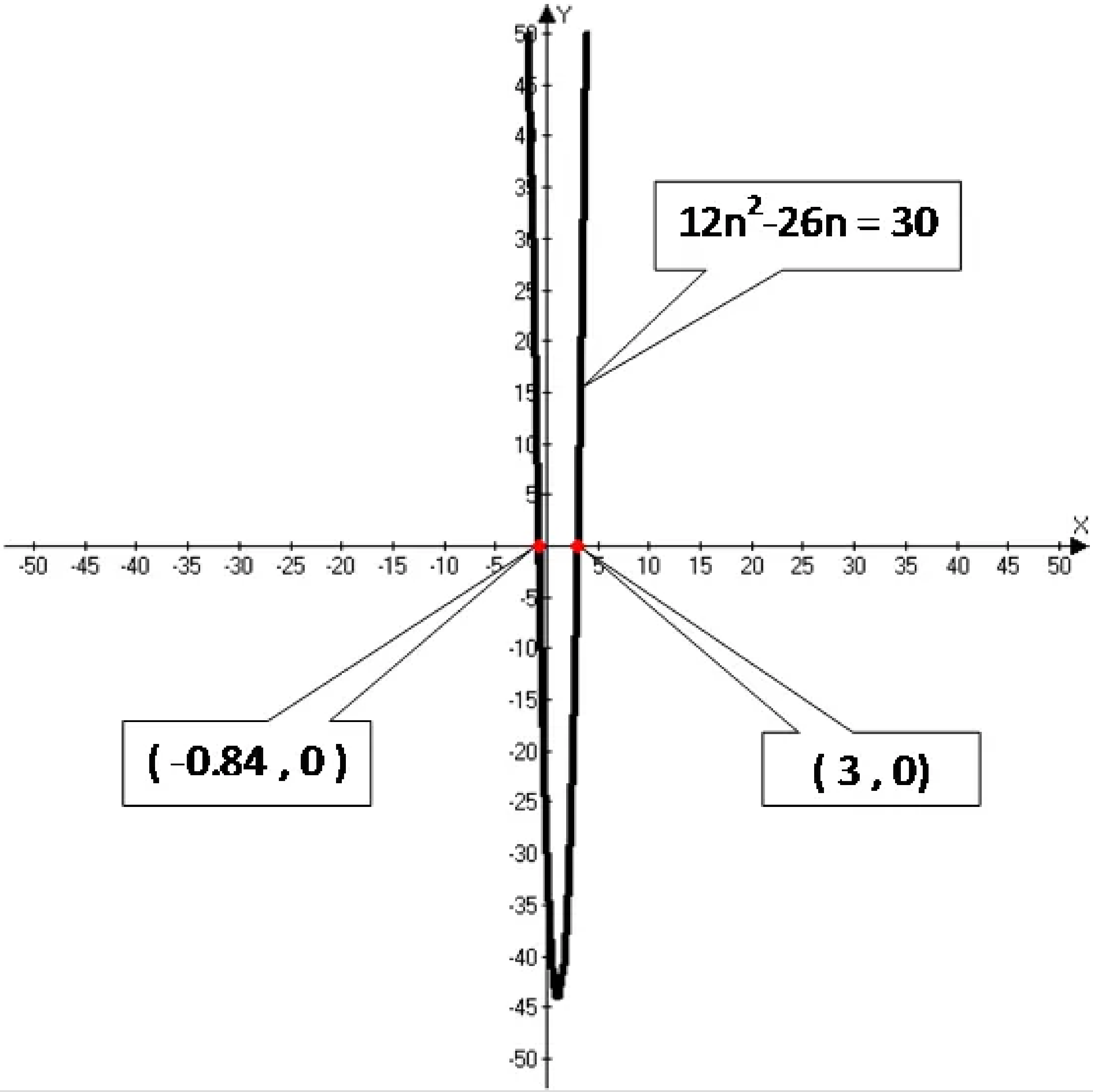
The root of the equation $f(x)=0$, lies between any two consecutive integers *b* and *c* if

$f(b)>0, f(c)<0$

Take the number -0.833 which lies between -1, 0

$f(-0.833)=-0.0153 \simeq 0$

-0.833 is root approximately



Answer 32PA.

Consider the equation $4x^2 - 35 = -4x$

$$4x^2 - 35 + 4x = -4x + 4x \qquad (\text{Add } 4x \text{ on both sides})$$

$$4x^2 - 35 + 4x = 0$$

Let us consider the related function $f(x)$

$$f(x) = 4x^2 + 4x - 35 = 0$$

Now, construct the table for the function $f(x)$

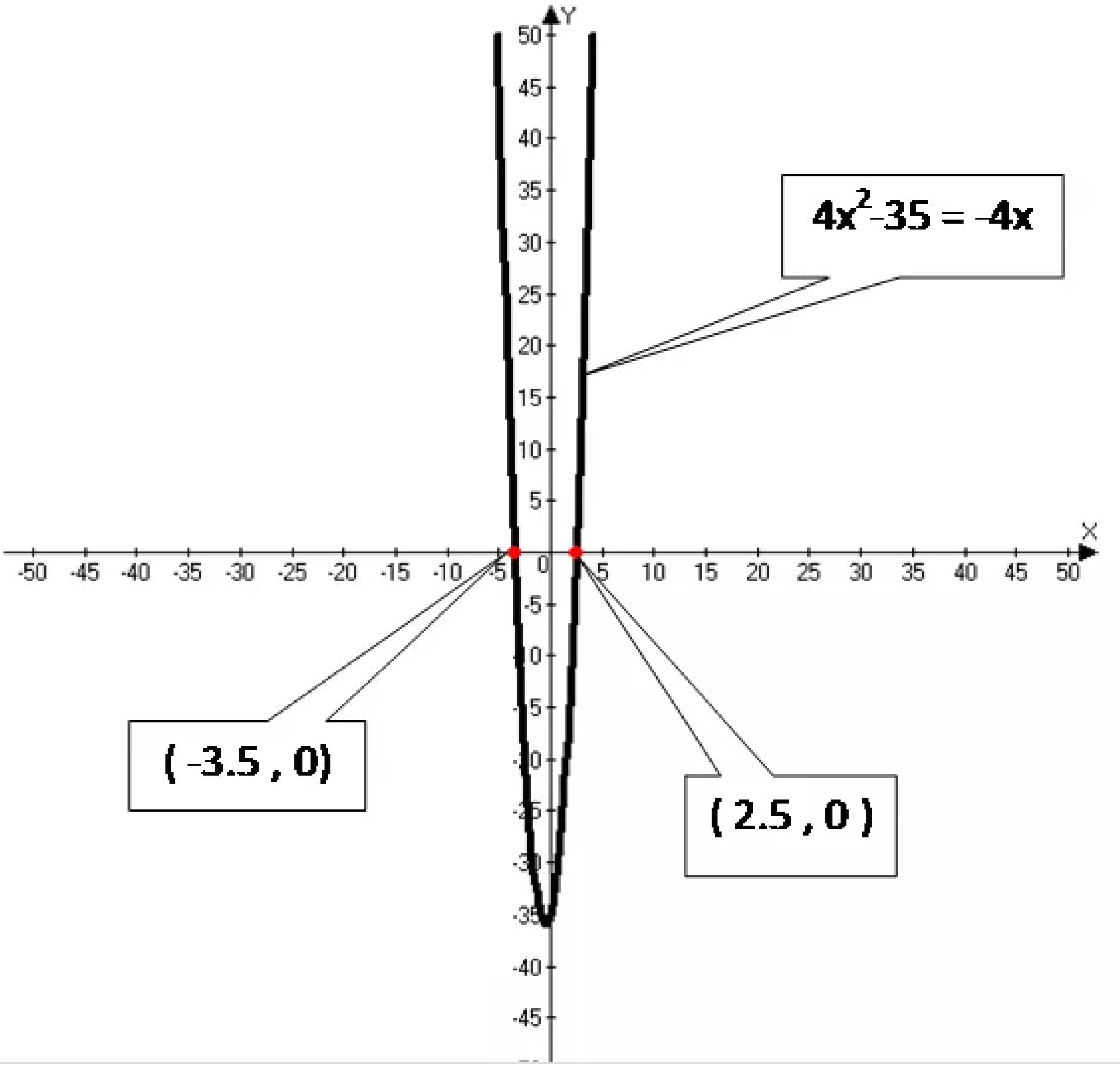
x	$f(x)$
-4	$f(-4) = 4(-4)^2 + 4(-4) - 35 = 13$
-3	$f(-3) = 4(-3)^2 + 4(-3) - 35 = -11$
-2	$f(-2) = 4(-2)^2 + 4(-2) - 35 = -27$
-1	$f(-1) = 4(-1)^2 + 4(-1) - 35 = -35$
0	$f(0) = 4(0)^2 + 4(0) - 35 = -35$
1	$f(1) = 4(1)^2 + 4(1) - 35 = -27$
2	$f(2) = 4(2)^2 + 4(2) - 35 = -11$
3	$f(3) = 4(3)^2 + 4(3) - 35 = 13$
4	$f(4) = 4(4)^2 + 4(4) - 35 = 45$

From the above table, we can observe that $f(-4) = 13 > 0; f(-3) = -11 < 0$ and -4, -3 are consecutive integers and also $f(2) = -11 < 0, f(3) = 13 > 0$ and 2, 3 are consecutive integers. We have the rule "The roots of equation negative integers '*b*' and '*c*'. $f(b) < 0 \text{ and } f(c) > 0 \text{ or } f(c) < 0 \text{ and } f(b) > 0$". By using this rule the roots of the equation lies $-4, -3$ and 2, 3

Let us take the number -3.5 and 2.5 which lies between -4, -3 and 2, 3respectives

$$f(2.5) = 4(2.5)^2 + 4(2.5) - 35$$
$$= 0$$
$$f(-3.5) = 4(-3.5)^2 + 4(-3.5) - 35$$
$$= 0$$

The roots are 2.5 and -3.5



Answer 35PA.

Consider the quadratic function $f(x)=-x^2-4x+12$.

The equation of the axis of symmetry is,

$$\begin{aligned}x&=\frac{-b}{2a}\\&=\frac{-(-4)}{2\cdot -1}\\&=-2\end{aligned}$$

The value of the function at $x=-2$ is,

$$\begin{aligned}f(x)&=-x^2-4x+12\\&=-(-2)^2-4(-2)+12\\&=-4+8+12\\&=16\end{aligned}$$

The vertex of the equation is $(-2,16)$.

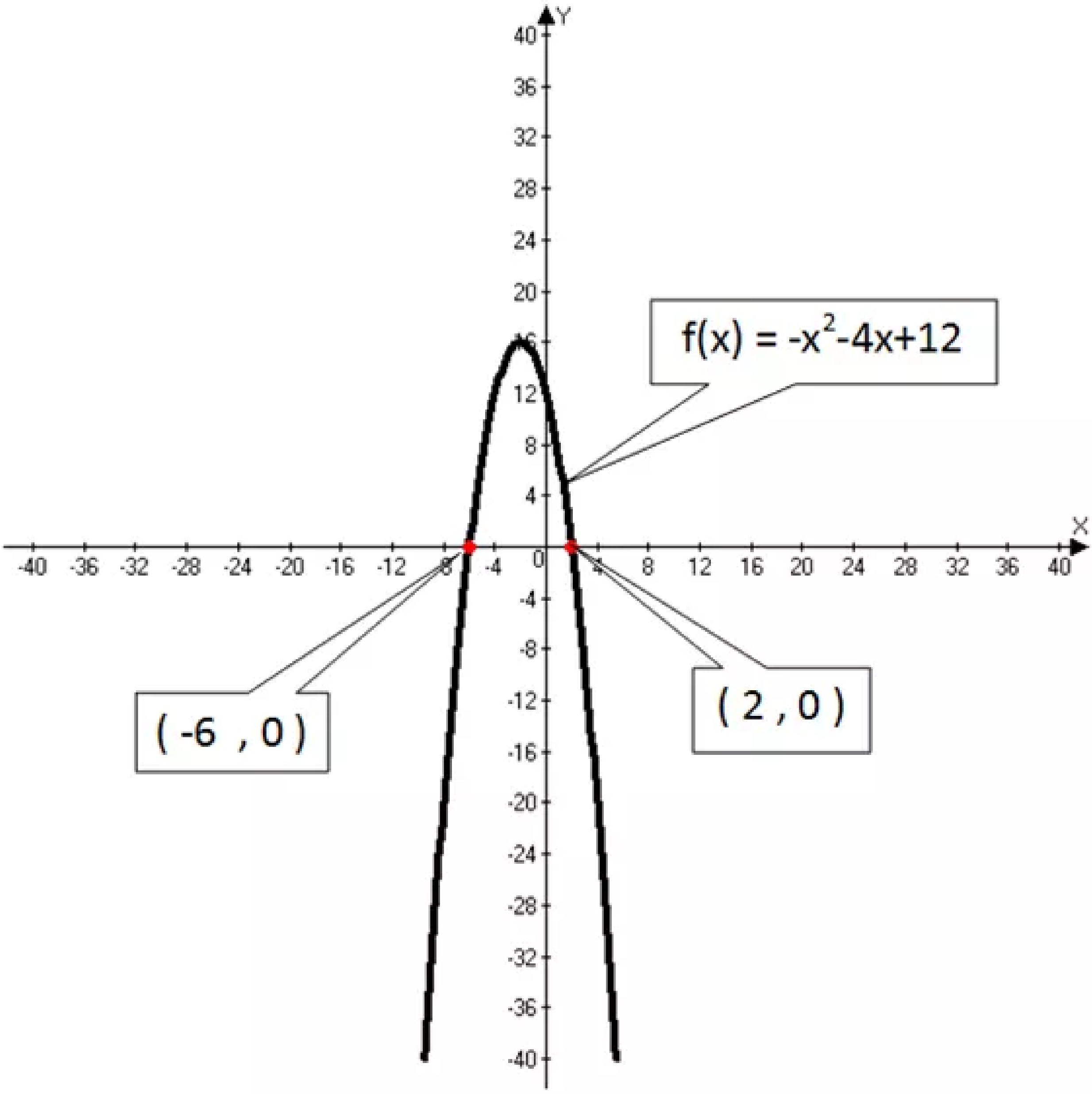
The table of values is,

x	$f(x)=-x^2-4x+12$	$f(x)$	$(x,f(x))$
-3	$f(-3)=-(-3)^2-4(-3)+12$	15	$(-3,15)$
-1	$f(-1)=-(-1)^2-4(-1)+12$	15	$(-1,15)$
0	$f(0)=-(0)^2-4(0)+12$	12	$(0,12)$
1	$f(1)=-(1)^2-4(1)+12$	7	$(1,7)$
2	$f(2)=-(2)^2-4(2)+12$	0	$(2,0)$
3	$f(3)=-(3)^2-4(3)+12$	-9	$(3,-9)$

Make the function $f(r)$ equals to zero, the values of r are 2 and -6.

Hence, $r-$intercepts are $(2,0)$ and $(-6,0)$.

The graph of the equation is as shown below.



Verify the equation by factoring.

$$-x^2 - 4x + 12 = 0 \quad (\text{original equation})$$
$$x^2 + 4x - 12 = 0$$
$$x^2 + 6x - 2x - 12 = 0$$
$$x \cdot (x+6) - 2 \cdot (x+6) = 0$$
$$(x-2)(x+6) = 0 \quad (\text{factorization})$$
$$x - 2 = 0 \text{ or } x + 6 = 0$$
$$x = 2 \text{ or } \quad x = -6$$

The roots of the equation $-x^2 - 4x + 12 = 0$ are $(2,0)\text{and}(-6,0)$.

Hence, $x-$intercepts are $\boxed{(2,0) \text{ and } (-6,0)}$.

Answer 36PA.

Consider the quadratic function $f(x) = -x^2 - 4x + 12$

The graph of the function is open down ward as coefficient of *x*2 is negative. The length of the segment along the floor of each segment / arch is the distance between the points where the curve intersect the x – axis.

We have the rule that the curve $f(x)$ intersect the x- axis then $f(x)$ = 0;

$f(x)$ = 0 then $f(x)$ intersects the x – axis.

Let $f(x) = 0$

$$-x^2 - 4x + 12 = 0$$

Now, construct the graph for the equation table form.

x	$f(x)$	$(x, f(x))$
-7	$(-7)^2-4(-7)+12=0$	$(-7,-9)$
-6	$(-6)^2-4(-6)+12=0$	$(-6,0)$
-5	$(-5)^2-4(-5)+12=0$	$(-5,7)$
-4	$(-4)^2-4(-4)+12=0$	$(-4,12)$
-3	$(-3)^2-4(-3)+12=0$	$(-3,15)$
-2	$(-2)^2-4(-2)+12=0$	$(-2,16)$
-1	$(-1)^2-4(-1)+12=0$	$(-1,15)$
0	$(0)^2-4(0)+12=0$	$(0,12)$
1	$(1)^2-4(1)+12=0$	$(1,17)$
2	$(2)^2-4(2)+12=0$	$(2,0)$

From the table, we can observe that the points $(-6,0)$ and $(2,0)$, $f(x)=0$ then graph intersect the x – axis at the points $(-6,0)$ and $(2,0)$.

Length of the segment of the arch along the floor = distance between the points $(-6,0)$ and $(2,0)$

$=2-(-6)=8$ units ,we use the rule the distance between the points (*a*, 0) and (*b*, 0) is $b-a$

When $b>a$

The length of the segment of the arch along the floor is 8 units.

Consider the given equation of the quadratic equation $f(x) = -x^2 - 4x + 12$

We have the rule, the vertex of the quadratic function $ax^2 + bx + c$ is $x = \frac{-b}{2a}$; $y = \frac{4ac - b^2}{4a}$

Vertex $\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$

Now, compare the given equation $f(x) = -x^2 - 4x + 12$ with $ax^2 + bx + c = f(x)$

Then $a = -1, b = -4, c = 12$

The vertex of the given function $x = \frac{-b}{2a}$

$$\begin{aligned} x &= \frac{-(-4)}{2(-1)} \\ &= \frac{4}{-2} \\ &= -2 \end{aligned}$$

$$\begin{aligned} y &= \frac{4(-1)(12) - (-4)}{4(-1)} \\ &= \frac{-48 - 16}{-4} \\ &= \frac{-64}{-4} \\ &= 16 \end{aligned}$$

The vertex of the graph $f(x) = -x^2 - 4x + 12$ is $(x, y) = (-2, 16)$

We have the rule “The height of the arch is the length between the vertex of the graph and midpoint of the segment of the arch along the floor”

Now, the midpoint of the intersecting point of x – axis an segment of the arch = The midpoint between the intersecting point of x – axis and the graph of the function.

The intersecting point of x – axis and the graph of the function, we can find from the table.

Table form

x	$f(x)$	$(x, f(x))$
-6	$(-6)^2 - 4(-6) + 12 = 0$	$(-6, 0)$
-5	$(-5)^2 - 4(-5) + 12 = 0$	$(-5, 7)$
-4	$(-4)^2 - 4(-4) + 12 = 0$	$(-4, 12)$
-1	$(-1)^2 - 4(-1) + 12 = 0$	$(-1, 15)$
0	$(0)^2 - 4(0) + 12 = 0$	$(0, 12)$
1	$(1)^2 - 4(1) + 12 = 0$	$(1, 7)$
2	$(2)^2 - 4(2) + 12 = 0$	$(2, 0)$

In intersection, points of x – axis and the graph of the function, the y – coordinates will be zero.

From the table, we can observe that the intersecting points of x – axis and the graph of the function are $(-6, 0), (2, 0)$

Now, midpoint of the points

$$=\left(\frac{-6+2}{2},\frac{0+0}{2}\right) \qquad \left(\begin{array}{l}\text{Use the rule midpoints of two points} \\ (a,b),(c,d)=\left(\frac{a+c}{2},\frac{b+d}{2}\right)\end{array}\right)$$

$$=\left(\frac{-4}{2},0\right)$$

$$=(-2,0)$$

The length of the arch = The distance between the vertex and midpoint.

We have the rule the distance between two points (x_1, y_1) and (x_2, y_2)

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

The distance between the vertex $(-2,16)$ and midpoints $(-2,0)$

$$=\sqrt{(-2+2)^2+(0-16)^2}$$

$$=\sqrt{0+(16)^2}$$

$$=\sqrt{256}$$

$$=\pm 16$$

Distance will not be in negative direction or negative values.

Vertex $(-2,16)$ and midpoints of the segment $(-2,0)$ is 16 units

$$\boxed{\text{The length of the arch } = \text{ 16 units}}$$

Answer 38PA.

Consider the given equation of the quadratic equation $f(x) = -x^2 - 4x + 12$

Now construct the graph from the table by giving different values of 'x' then we get different values of $f(x)$ and by plotting the points $(x, f(x))$ and connect them. We get smooth curve.

Construction of table

x	$f(x)$	$(x, f(x))$
-6	$(-6)^2 - 4(-6) + 12 = 0$	$(-6, 0)$
-5	$(-5)^2 - 4(-5) + 12 = 0$	$(-5, 7)$
-4	$(-4)^2 - 4(-4) + 12 = 0$	$(-4, 12)$
-1	$(-1)^2 - 4(-1) + 12 = 0$	$(-1, 15)$
0	$(0)^2 - 4(0) + 12 = 0$	$(0, 12)$
1	$(1)^2 - 4(1) + 12 = 0$	$(1, 7)$
2	$(2)^2 - 4(2) + 12 = 0$	$(2, 0)$

From the table, we can find the graph, *x* – intersecting points. *y* –coordinates '0' points.

x – Intersecting points = $(-6, 0)$ and $(2, 0)$

According the problem, the length of arch segment along the floor = distance between $(-6, 0)$ and $(2, 1)$

We use the formula, distance between the points $(a, 0), (b, 0)$ when *b* > *a*

The distance between the points

$$(b, 0), (2, 0) = 2 - (-6)$$
$$= 8 \text{ units}$$

The length of base of the arch = 8 units

We have, the rule that the vertex of the quadratic equation $ax^2 + bx + c = 0$ is $\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a}\right)$

By using this rule, we find the vertex of the given parabola.

Now, compare the given quadratic equation $-x^2-4x+12=0$ with $ax^2+bx+c=0$

$a=-1; b=-4; c=12$

$$\frac{-b}{2a}=\frac{-(-4)}{2(-1)}$$
$$=\frac{2}{-1}$$
$$=-2$$

$$\frac{4ac-b^2}{4a}=\frac{4(-1)(12)-(-4)^2}{4(-1)}$$
$$=\frac{-48-16}{-4}$$
$$=\frac{-64}{-4}$$
$$=16$$

The vertex of parabola $\left(\frac{-b}{2a},\frac{4ac-b^2}{4a}\right)=(-2,16)$

Now, we find midpoint of the x – intersecting points of the x – intersecting points $(-6,0)(2,0)$

Use the rule, midpoint of two points (a,b) and (c,d) is $\left(\frac{a+c}{2},\frac{b+d}{1}\right)$

Height of the arch is distance between vertex and the midpoint of the intersecting points. We use the rule distance between two points (x_1,y_1) and (x_2,y_2)

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Therefore, the distance between the vertex $(-2,16)$ and $(-2,0)$ is

$$\sqrt{(-2+2)^2+(16-0)^2}=16$$

Hence, The height of the arch is 16 units

Use the rule "The area of the parabola $A=\frac{2}{3}bh$ where *b* represents the base, *h* represents the height of the parabola.

We have the length of base = the arch *b* = 8 units and height of the arch is 16 units.

Therefore, The area of the parabola $A=\frac{2}{3}bh$

$$A=\frac{2}{3}\cdot 8\cdot 16 \qquad \text{(Replace } b \text{ by 8, } h \text{ by 16)}$$

$$=\frac{256}{3}$$

The area of parabola $A=\frac{256}{3}$

Answer 39PA.

Consider the given equation of the quadratic equation $f(x) = -x^2 - 4x + 12$

Consider there is 12 arches which is in form of above function. He wants to paints total arches and the wants to apply 2 coats.

According to manufacture a gallon of paint will cover 200 square feets.

The cost of gallon paints is $27

We have the rule "The area under a parabola $A = \frac{2}{3}bh$ where *b* represents the base, *h* represents the height of the parabola".

We have to rule

"The length of the base of the parabola is distance between the *x* – intersecting points".

The *x* – intercepting points we will get from the table

x	$f(x)$	$(x, f(x))$
-6	$(-6)^2 - 4(-6) + 12 = 0$	$(-6, 0)$
-5	$(-5)^2 - 4(-5) + 12 = 0$	$(-5, 7)$
-4	$(-4)^2 - 4(-4) + 12 = 0$	$(-4, 12)$
-1	$(-1)^2 - 4(-1) + 12 = 0$	$(-1, 15)$
0	$(0)^2 - 4(0) + 12 = 0$	$(0, 12)$
1	$(1)^2 - 4(1) + 12 = 0$	$(1, 7)$
2	$(2)^2 - 4(2) + 12 = 0$	$(2, 0)$

From the table, we can find the graph, *x* – intersecting points. *y* –coordinates '0' points.

x – Intersecting points = $(-6, 0)$ and $(2, 0)$

Therefore, The length of the base = distance between the points $(-6, 0)$ and $(2, 0)$

$$= \sqrt{(2+6)^2 - (0-0)^2}$$
$$= \sqrt{8^2}$$
$$= 8$$

Length of the base of parabola is 8 units

We have the rule “The height of the arches is distance between vertex and the midpoint of the base”

Vertex of a parabola is $\left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right)$

Now compare the function $f(x) = -1x^2 - 4x + 12$ with the quadratic function with $f(x) = ax^2 + bx + c$, we have $a = -1, b = -4$ and $c = 12$

Vertex = $\left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right)$

$= \left(\frac{-(-4)}{2(-1)}, \frac{4(-1)(12)-(-4)^2}{4(-1)}\right)$

$= \left(\frac{4}{-2}, \frac{-48-16}{-4}\right)$

$= \left(-2, \frac{-64}{-4}\right)$

$= (-2, 16)$

The vertex is $(-2, 16)$

Now, we find midpoint of the x – intersecting points of the x – intersecting points $(-6,0)(2,0)$

We have "The rule the midpoint of (a,b) and (c,d) is $\left(\frac{a+c}{2},\frac{b+d}{2}\right)$

The midpoint of $(-6,0)(2,0)$ is $\left(\frac{-6+2}{2},\frac{0+0}{2}\right)$

$$=\left(\frac{-4}{2},\frac{0}{2}\right)$$
$$=(-2,0)$$

The height of the arches = distance between the vertex and the midpoint of the intersecting points we use the use rule distance between two points (x_1,y_1) and (x_2,y_2) is

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

The distance between the vertex $(-2,16)$ and midpoint is $(-2,0)$ is

$$\sqrt{(-2+2)^2+(16-0)^2}=16$$

Height of the arch = 16, length of the base *b* = 8 feet

Area of the parabola is = Area of the Arch = $A=\frac{2}{3}bh$

$$A=\frac{2}{3}\cdot 8\cdot 16 \qquad (\text{Replace } b \text{ by } 8,\ h \text{ by } 16)$$
$$=\frac{256}{3}$$

Area of the arch is $\frac{256}{3}$ square units

Now, Total area of 12 arches $=\frac{256}{3}\times 12$

= 1024 square feet

To paint 200 square feet we required 1 gallon of paint

Total require paint 12 arches is $\frac{1024}{200}$ gallon (per 200 square feet we required 1 gallon of point)

$$= \frac{2 \cdot 512}{2 \cdot 100}$$
$$= \frac{512}{100}$$
$$= 5.12 \text{ gallon}$$

For two cost double coating required paints $2 \cdot (5.12) = 10.24 \approx 11$ gallons approximately.

We cannot buy the part of gallon. So we have to by 11 gallons.

The cost of the paint per a gallon = \$27

The cost of 11 gallons = \$ (11.27)

= \$297

Hence, The total cost of paint 12 arches is double coating is $\boxed{\$297}$

Answer 41PA.

M and K are hiking in the mountains and the equation $h = -16t^2 + 30t + 1000$ represents the height, in feet, of the apple t seconds after it was thrown.

To find the duration for the apple to reach the ground, make the equation equals to 0 i.e. the height is zero at the ground.

The equation is,

$$h = -16t^2 + 30t + 1000$$
$$0 = -16t^2 + 30t + 1000$$

The quadratic formula for the quadratic equation $ax^2 + bx + c = 0$ is,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{(Equation for quadratic formula)}$$

$$t = \frac{-30 \pm \sqrt{(30)^2 - 4 \cdot (-16) \cdot 1000}}{2 \cdot (-16)} \qquad \text{(Replace } a \text{ by } -16,\ b \text{ by } 30, c \text{ by } 1000\text{)}$$

$$= \frac{-30 \pm \sqrt{900 + 64000}}{-32}$$

$$= \frac{-30 \pm \sqrt{64900}}{-32}$$

$$= \frac{-30 \pm 254.75}{-32}$$

$$= \frac{-30 + 254.75}{-32} \text{ or } \frac{-30 - 254.75}{-32}$$

$$= 8.89 \qquad \text{or } -7.02 \qquad \text{(Time is not in negatives)}$$

Hence, the time taken the apple to reach to the ground is $\boxed{9 \sec}$ approximately.

Answer 42PA.

Let the apple was thrown from the K and K was strop at level 1020 feet.

The sound levels 1000feet per second

The time taken the sound to reach the ground level is,

$$\frac{1020}{1000} \text{ seconds} = 1.02 \text{ seconds}$$

But to reach the apple to the ground it will take 8.89 seconds and 3 seconds to reach them.

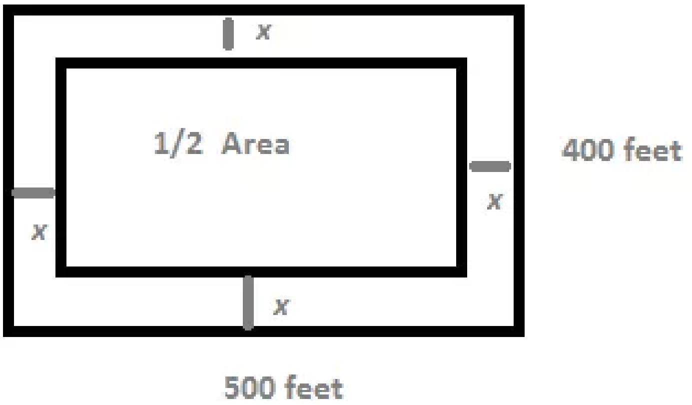
If it takes 3 seconds to reach them then $8.89-(3+1)=4.89$ seconds time will be there.

Hence, the girls have 4.89 seconds time to call down and warn any hikers below.

Answer 43PA.

K and M have accepted a job moving the soccer playing fields.

They must mow an area 500 feet long and 400 feet wide. They agree that each will mow half the area. They decide that K will mow around the edge in a path of equal width until half the area is left.



The length of field = 500 feet and the width of the field = 400 feet.

Total area that both will move is equals to the total area of the field.

Total area of the field is,

$$\begin{aligned} \text{length} \cdot \text{width} &= 500 \cdot 400 \\ &= 200000 \text{ sq feet} \end{aligned}$$

But, each will move half of the area.

Each will move the area is,

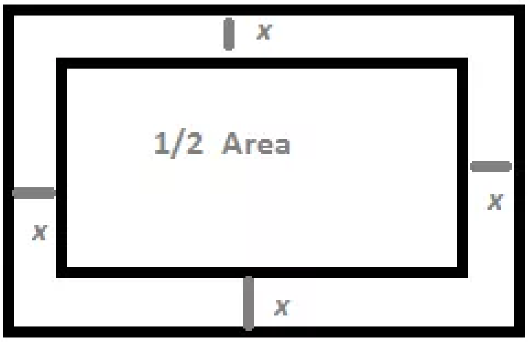
$$\begin{aligned} \frac{1}{2} \cdot \text{area} &= \frac{200000}{2} \\ &= 100000 \text{ sq feet} \end{aligned}$$

Hence, the rea each person will mow is $\boxed{100{,}000 \text{ sq feet}}$.

Answer 44PA.

K and M have accepted a job moving the soccer playing fields.

They must mow an area 500 feet long and 400 feet wide. They agree that each will mow half the area. They decide that K will mow around the edge in a path of equal width until half the area is left.



The area of the path is, the difference of area of the outer rectangle and area of the inner rectangle.

Length of inner rectangle $= 500 - 2x$ $\left(\begin{array}{l}\text{Substract 2 times of hte width 'x'} \\ \text{from the outer rectangle length}\end{array}\right)$

Width of inner rectangle $= 400 - 2x$ $\left(\begin{array}{l}\text{Substract 2 times of the width 'x'} \\ \text{from the outer rectangle width}\end{array}\right)$

So,

The area of the outer rectangle is,

$$\begin{aligned}\text{Area} &= \text{length} \cdot \text{width} \\ &= 500 \cdot 400 \\ &= 2,00,000 \text{ square feet}\end{aligned}$$

The area of the inner rectangle is,

$$\begin{aligned}\text{Area} &= \text{length} \cdot \text{width} \\ &= (500 - 2x) \cdot (400 - 2x) \\ &= 2,00,000 - 1000x - 800x + 4x^2\end{aligned}$$

$$= 2,00,000 - 1800x + 4x^2$$
$$= 4x^2 - 1800x + 2,00,000$$

The area of path is,

$$\begin{aligned} \text{Area } &= 2,00,000 - \left(4x^2 - 1800x + 2,00,000\right) \\ &= 200000 - 4x^2 + 1800x - 2,00,000 \\ &= 1800x - 4x^2 \end{aligned}$$

$$100,000 = 1800x - 4x^2 \qquad \left(\text{Since, area of the path} = 100,000\right)$$

$$4x^2 - 1800x + 100000 = 0 \qquad \left(\text{Make variables one side}\right)$$

$$4x^2 - 4 \cdot 450x + 4 \cdot 25000 = 0$$

$$4\left(x^2 - 450x + 25000\right) = 0$$

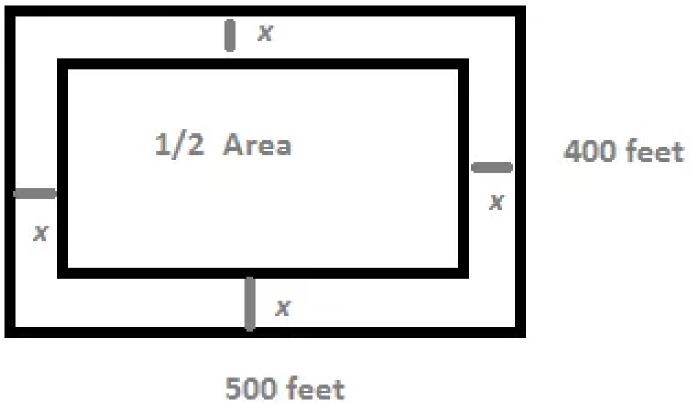
$$x^2 - 450x + 25000 = 0$$

Hence, the required quadratic equation is $x^2 - 450x + 25,000 = 0$.

Answer 45PA.

K and M have accepted a job moving the soccer playing fields.

They must mow an area 500 feet long and 400 feet wide. They agree that each will mow half the area. They decide that K will mow around the edge in a path of equal width until half the area is left.



The quadratic equation represents the area of the path is $x^2 - 450x + 25,000 = 0$.

To find the width (K mow), solve the quadratic equation $x^2 - 450x + 25,000 = 0$.

The quadratic formula for the quadratic equation $ax^2 + bx + c = 0$ is,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{450 \pm \sqrt{202,500 - 1,00,000}}{2}$$

$$= \frac{450 \pm \sqrt{102,500}}{2}$$

$$= \frac{450 \pm \sqrt{2500 \cdot (41)}}{2}$$

$$= \frac{450 \pm 50\sqrt{41}}{2}$$

$$= 225 \pm 25\sqrt{41}$$

$$= 225 + 160.078 \text{ or } 225 - 160.078$$

$$\approx 385 \text{ or } 65$$

Hence, the K should move the field about $\boxed{65 \text{ feet}}$ or with $\boxed{385 \text{ feet}}$.

Answer 46PA.

Let '*x*' be the width of the path that *K* moving.

According to the problem area of the path = $\frac{1}{2}$ Area of the field

Area of the path = $\frac{1}{2}(500 \times 400)$

Area of rectangle field = length $\times$ width

$$= \frac{500 \times 4000}{2}$$

$$= \frac{500 \times 2 \times 200}{2} \qquad \text{Factor } 400 = 2 \cdot 200$$

$$= 500 \times 200$$

$$= 1,00,000 \text{ square feet}$$

Here, the mover will move the path with width $x = 5\text{ft}$.

Then the length will be,

$$\begin{aligned} 500 - 2x &= 500 - 2\cdot 5 \\ &= 500 - 10 \\ &= 490 \text{ ft} \end{aligned}$$

The area of the path is,

$$A = 5\cdot 490$$
$$A = 2450 \text{ square feet}$$

By the person *K* has to move 1, 00,000 square feet.

To move 1, 00, 000 square feet has go around number of times is

$$= \frac{100000}{2450}$$
$$\approx 40$$

Hence, the person *K* will go $\boxed{40 \text{ times}}$ with width $x = 5\text{ft}$.

Answer 47PA.

Consider the function $f(x) = \dfrac{x^3 + 2x^2 - 3x}{x+5}$.

The objective is to find the *x* – intercept of the graph $f(x) = \dfrac{x^3 + 2x^2 - 3x}{x+5}$.

To find the *x* intercept put $y = f(x) = 0$ in the function $f(x) = \dfrac{x^3 + 2x^2 - 3x}{x+5}$.

$$f(x) = \frac{x^3 + 2x^2 - 3x}{x+5}$$

$$\frac{x^3 + 2x^2 - 3x}{x+5} = 0 \qquad \text{Substitute } f(x) = 0$$

$$x^3 + 2x^2 - 3x = 0 \qquad \text{Multiply with } (x+5) \text{ on both sides}$$

$$x\cdot x^2 + 2\cdot x\cdot x - 3\cdot x = 0$$

$$x\left(x^2 + 2x - 3\right) = 0 \qquad \text{Use Distributive property } ab + ac = a(b+c)$$

$$x\left(x^2 + 2x - 3\right) = 0$$

$$z\left(x^2 - x + 3x - 3\right) = 0$$

$$x\left(x(x-1) + 3(x-1)\right) = 0$$

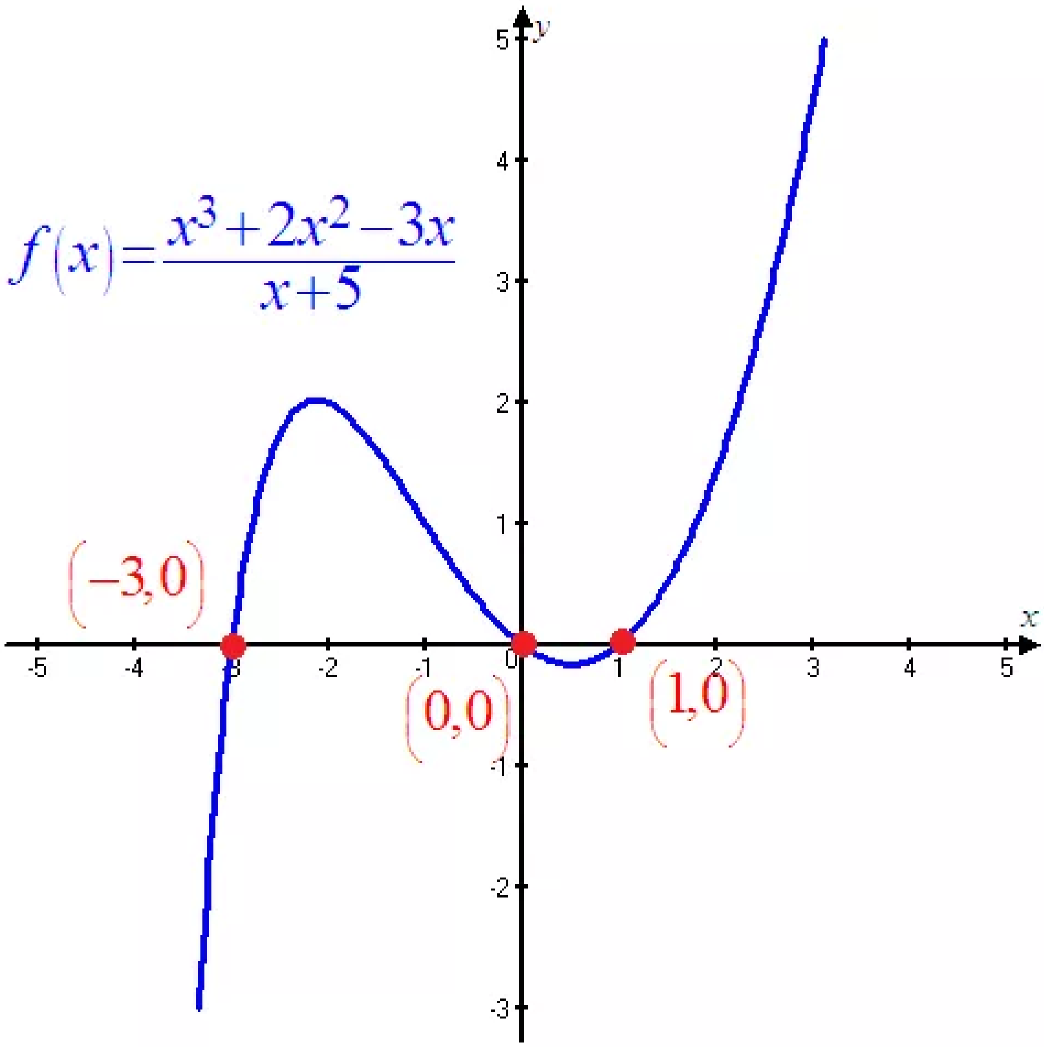
$$x(x+3)(x-1) = 0 \qquad \text{Factor}$$

$$x = 0 \quad \text{or} \quad (x+3) = 0 \quad \text{or} \quad (x-1) = 0 \qquad \text{Use zero product rule}$$

$$x = 0 \quad \text{or} \quad x = -3 \quad \text{or} \quad x = 1$$

Hence, the *x*- intercepts of the function are $\boxed{x = -3, 0, \text{ and } 1}$.

To check the intercepts, sketch the graph and plot the points $(-3,0),(0,0)$, and $(1,0)$.



Answer 49PA.

If the graph of a quadratic equation touch the *x*-axis or intersection point of the graph with $x-$ axis exists then the quadratic equation has real solution.

From the graphs in the options (A),(B),(C), and (D) observe that,

Option (A):

The graph in the option (A) touches the *x*- axis at the origin $(0,0)$.

So, $x=0$ is the solution of the quadratic equation of the graph.

Option (B):

The graph in the option (B) touches the *x*- axis at the point $(-2,0)$.

So, $x=-2$ is the solution of the quadratic equation of the graph.

Option (C):

The graph in the option (C) does not touch the *x*- axis.

That is there is no intersection point of the graph and *x*- axis.

Hence, the quadratic equation of the graph has no real solutions.

Option (D):

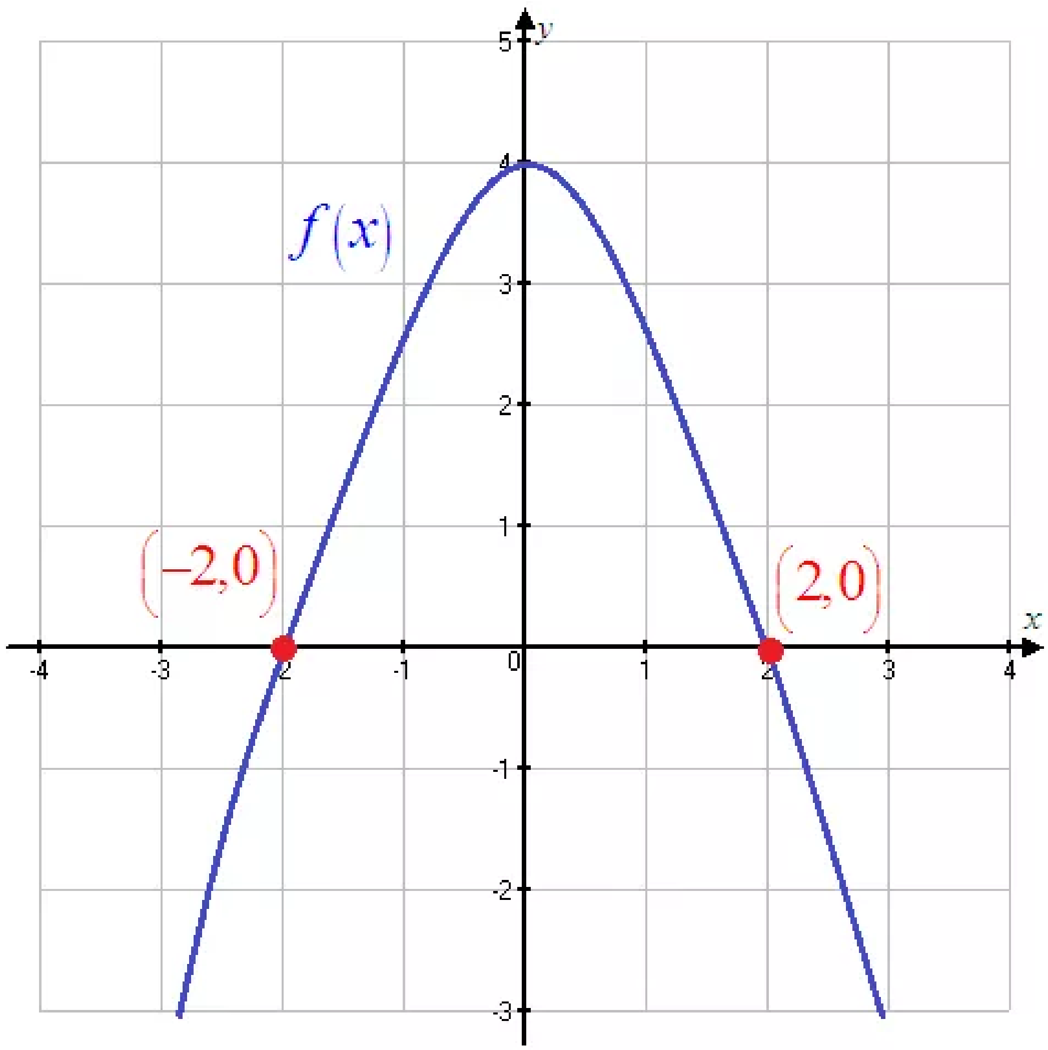
The graph in the option (D) touches the *x*- axis at the points $(-2,0)$, and $(1,0)$.

So, $x=-2$, and $x=1$ are solutions of the quadratic equation of the graph.

Hence, from the given four options, the graph in the **option (C) has no solution.**

Answer 50PA.

Consider the following graph.



If the graph of a quadratic equation touch the *x*-axis or intersection points of the graph with $x-$ axis exists then the quadratic equation has real solutions.

From the figure, the graph $f(x)$ intersects *x*- axis at $(-2,0)$, and $(2,0)$.

Hence, the roots of the quadratic equation are $\boxed{x=-2,\text{ and } x=2}$.

Answer 51PA.

Consider the equation is $x^3-x^2-4x+4=0$.

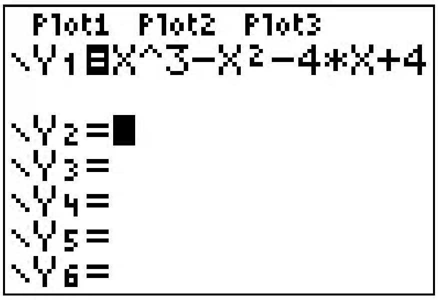
The objective is to find the roots of the function by using graph.

Enter the function in graphing calculator as follows.

Press $\boxed{Y=}$

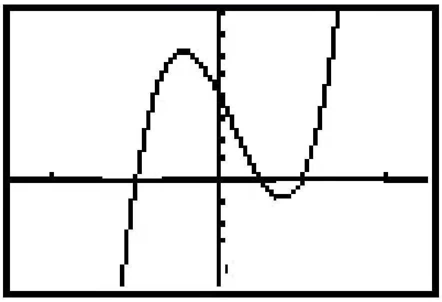
$Y_1=x^3-x^2-4x+4$

The screen short shows as follows.



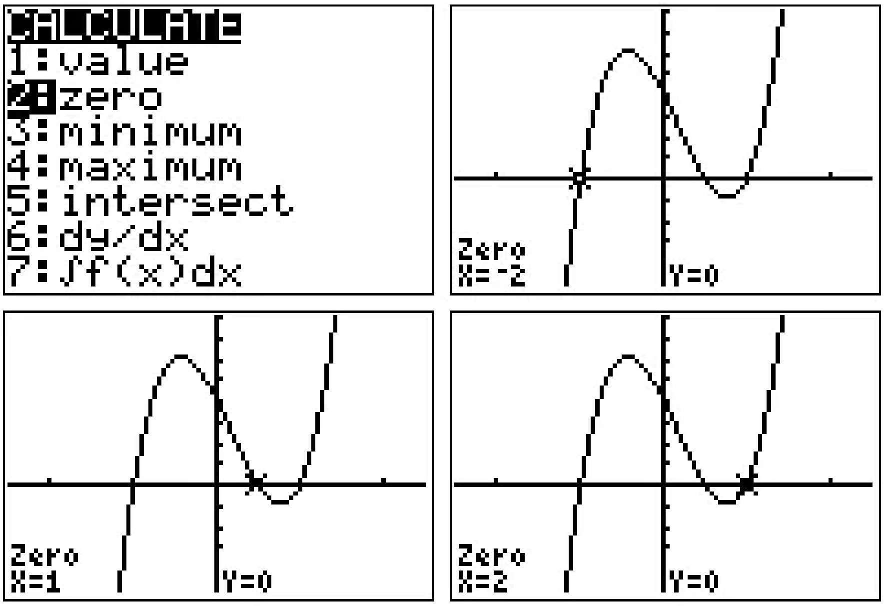
To get the graph of the function, press GRAPH key.

$y=x^3-x^2-4x+4$



To find the zeros of the function press $\boxed{2^{nd} + TRACE}$ then confirm the left end and right end the press enter.

The zeros of the function are



Hence, from the graph, the roots of the function are $\boxed{x = -2, 1, \text{ and } 2}$.

Answer 52PA.

Consider the equation is $2x^3 - 11x^2 + 13x - 4 = 0$.

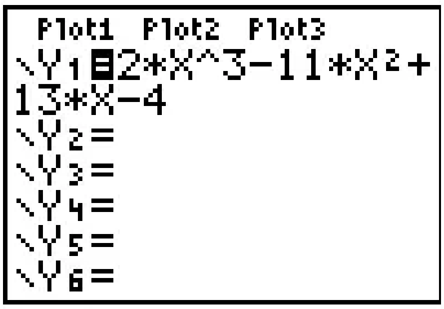
The objective is to find the roots of the function by using graph.

Enter the function in graphing calculator as follows.

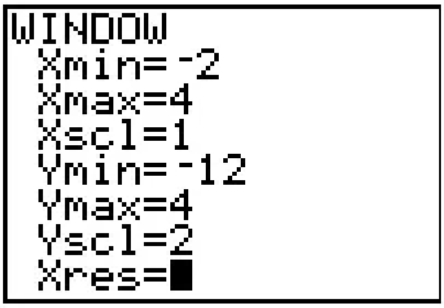
Press $\boxed{Y =}$

$Y_1 = 2x^3 - 11x^2 + 13x - 4$

The screen short shows as follows.

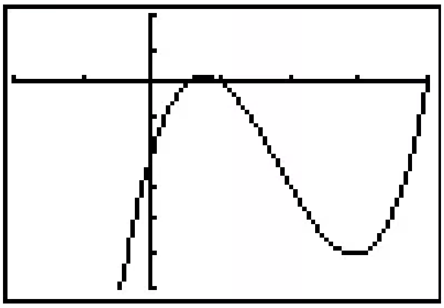


Set the windows as follows.



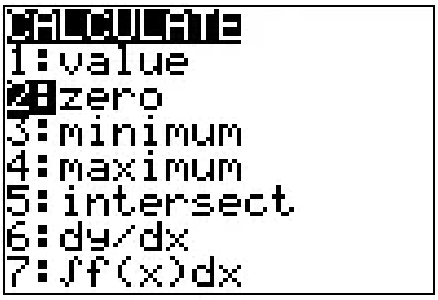
To get the graph of the function, press GRAPH key.

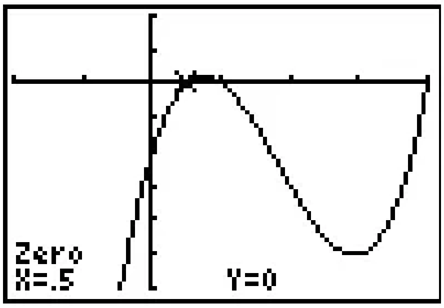
$$Y_1 = 2x^3 - 11x^2 + 13x - 4$$

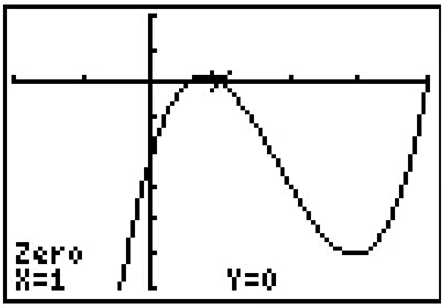


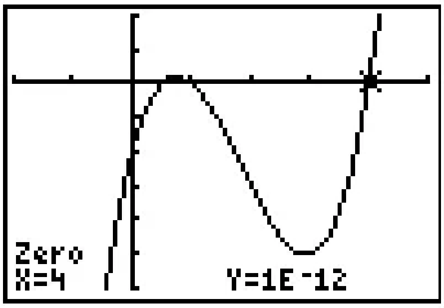
To find the zeros of the function press $\boxed{2^{nd} + TRACE}$ then confirm the left end and right end the press enter.

The zeros of the function are









Hence, from the graph, the roots of the function are $\boxed{x = 0.5, 1, \text{ and } 4}$.

Answer 53MYS.

Consider the equation $y = x^2 + 6x + 9$

Step 1: Write the equation of the axis of symmetry

The given equation $y = x^2 + 6x + 9$

Use the rule: "The equation of the axis of symmetry.

Foe the graph $y = ax^2 + bx + c$ where $a \neq 0$ is $x = -\frac{b}{2a}$"

Now compare the equation $y = 1 \cdot x^2 + 6 \cdot x + 9$ with $y = ax^2 + bx + c$. We have *a* = 1, *b* = 6 and *c* = 9

$$x = -\frac{b}{2a} \quad \text{(Equationm for the axis of symmetry of a parabola)}$$

$$x = -\frac{6}{2 \cdot (1)} \quad (\text{Replace } a \text{ by } 1 \text{ and } b \text{ by } 6)$$

$$x = -\frac{3 \cdot 2}{2 \cdot 1} \quad (\text{Factors } 6 = 3 \cdot 2)$$

$$x = -3 \quad \text{(Cancellation of the nemerator and the denominator)}$$

$$\boxed{x = -3}$$

Hence, the equation of the axis of symmetry is $\boxed{x = -3}$

Step2: Find the coordinates of the vertex, since the equation of the axis of symmetry is $x = -3$ and the vertex on the axis, the x – coordinate for the vertex is -3

$$y = x^2 + 6x + 9 \quad \text{(Original equation)}$$

$$y = (-3)^2 + 6(-3) + 9 \quad (\text{Replace } x \text{ by } -3)$$

$$y = 9 - 18 + 9$$

$$y = 18 - 18$$

$$y = 0$$

Hence, the vertex is $\boxed{-3, 0}$

Step3: Indentify the maximum or minimum

The equation is $y = x^2 + 6x + 9$

Use the rule "The equation of the parabola is $y = ax^2 + bx + c$. Suppose the coefficient of *x*2 term is positive, the parabola open upwards and the vertex is a minimum point.

Suppose the coefficient of *x*2 term is negative, the parabola open downward and the vertex is a maximum point"

Since the coefficient of the *x*2 terms is positive, the parabola open upwards and vertex is a minimum point.

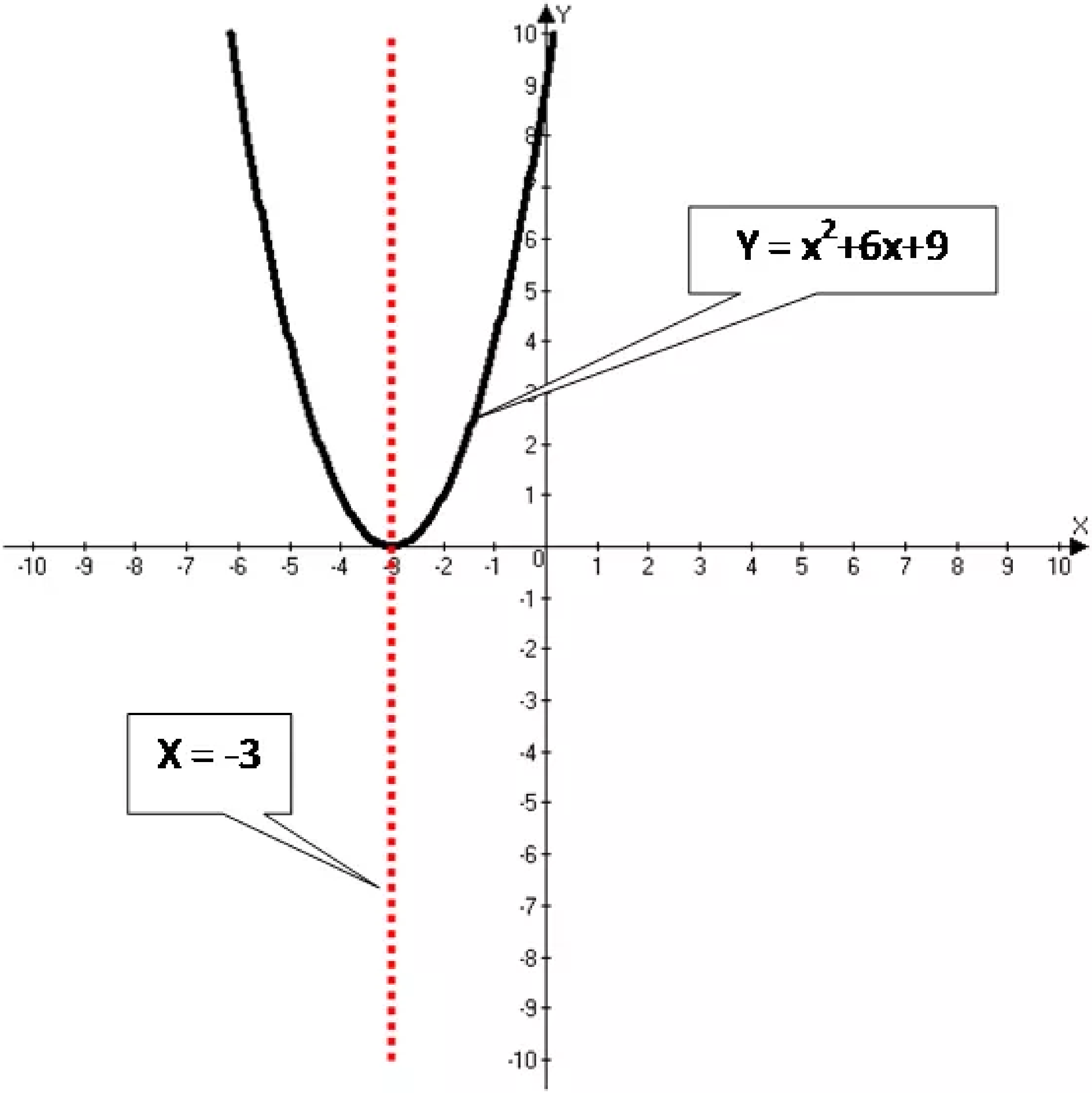
Hence, the maximum point of the parabola is $\boxed{-3,0}$

Step4: Graph the function $y = x^2 + 6x + 9$

Now, we consider the table for $y = x^2 + 6x + 9$. We can substitute the different values of *x* is $y = x^2 + 6x + 9$, we get the *y* – values. Graph these ordered pairs and connect them, we get the smooth curve.

x	$x^2 + 6x + 9$	y	(x, y)
-3	$(-3)^2 + 6(-3) + 9 = 0$	0	$(-3, 0)$
-2	$(-2)^2 + 6(-2) + 9 = 1$	1	$(-2, 1)$
-1	$(-1)^2 + 6(-1) + 9 = 4$	4	$(-1, 4)$
0	$(0)^2 + 6(0) + 9 = 9$	9	$(0, 9)$
1	$(1)^2 + 6(1) + 9 = 16$	16	$(1, 16)$
2	$(2)^2 + 6(2) + 9 = 25$	25	$(2, 25)$
3	$(3)^2 + 6(3) + 9 = 36$	36	$(3, 36)$

Add these all ordered pairs; we get a parabola open upward.



Answer 654MYS.

Consider the equation $y = -x^2 + 4x - 3$

Step 1: Write the equation of the axis of symmetry

The given equation $y = -x^2 + 4x - 3$

Use the rule: "The equation of the axis of symmetry.

Foe the graph $y = ax^2 + bx + c$ where $a \neq 0$ is $x = -\frac{b}{2a}$"

Now compare the equation $y = -x^2 + 4 \cdot x - 3$ with $y = ax^2 + bx + c$. We have a = −1, b = 4 and c = −3

$$x = -\frac{b}{2a} \quad \text{(Equationm for the axis of symmetry of a parabola)}$$

$$x = -\frac{4}{2 \cdot (-1)} \quad \text{(Replace } a \text{ by } -1 \text{ and } b \text{ by } 4\text{)}$$

$$x = -\frac{2 \cdot 2}{2 \cdot (-1)} \quad \text{(Factors } 4 = 2 \cdot 2\text{)}$$

$$x = -\frac{1 \cdot 2}{1 \cdot (-1)} \quad \text{(Cancellation of the nemerator and the denominator)}$$

$$x = 2$$

Hence, the equation of the axis of symmetry is $\boxed{x = 2}$

Step2: Find the coordinates of the vertex, since the equation of the axis of symmetry is $x = 2$ and the vertex on the axis, the x – coordinate for the vertex 2.

$$y = -x^2 + 4x - 3 \quad \text{(Original equation)}$$

$$y = -(2)^2 + 4(2) - 3 \quad \text{(Replace } x \text{ by } 2\text{)}$$

$$y = -4 + 8 - 3$$

$$y = -7 + 8$$

$$\boxed{y = 1}$$

Hence, the vertex is $\boxed{(2,1)}$

Step3: Indentify the maximum or minimum

The equation is $y=-x^2+4x-3$

Use the rule "The equation of the parabola is $y=ax^2+bx+c$. Suppose the coefficient of *x*2 term is positive, the parabola opens upwards and the vertex is a minimum point.

Suppose the coefficient of *x*2 term is negative, the parabola open downward and the vertex is a maximum point"

Since the coefficient of the *x*2 terms is negative, the parabola open downwards and vertex is a maximum point.

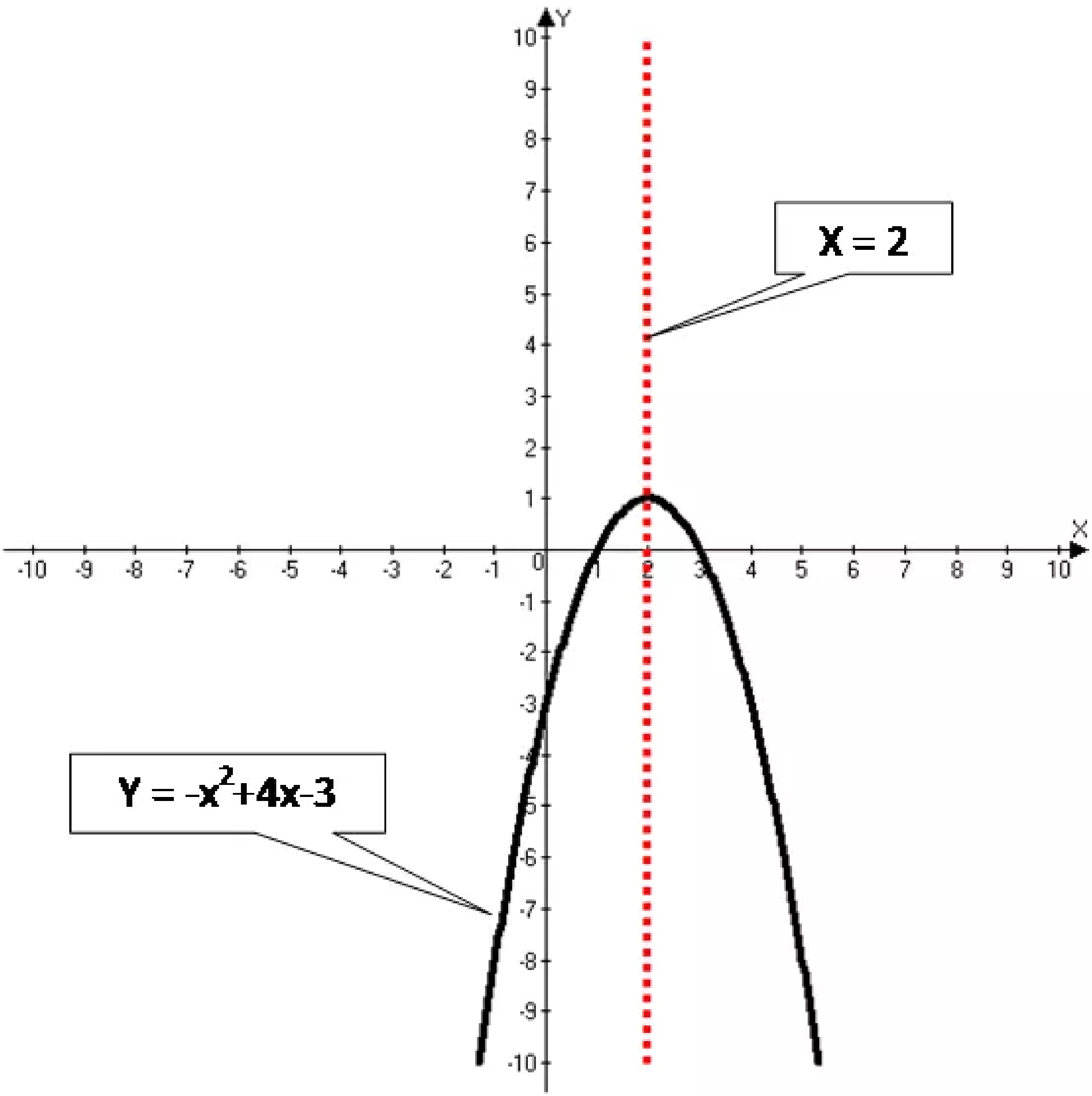
Hence, the maximum point of the parabola is $\boxed{(2,1)}$

Step4: Graph the function $y=-x^2+4x-3$

Now, we consider the table for $y=-x^2+4x-3$. We can substitute the different values of *x* is $y=-x^2+4x-3$, we get the *y* – values. Graph these ordered pairs and connect them, we get the smooth curve.

x	$-x^2+4x-3$	y	(x,y)
-3	$-(-3)^2+4(-3)-3=-24$	-24	$(-3,-24)$
-2	$-(-2)^2+4(-2)-3=-15$	-15	$(-2,-15)$
-1	$-(-1)^2+4(-1)-3=-8$	-8	$(-1,-8)$
0	$-(0)^2+4(0)-3=-3$	-3	$(0,-3)$
1	$-(1)^2+4(1)-3=0$	0	$(1,0)$
2	$-(2)^2+4(2)-3=1$	1	$(2,1)$
3	$-(3)^2+4(3)-3=0$	0	$(3,0)$

Add these all ordered pairs, we get a parabola open downward.



Answer 55MYS.

Consider the equation $y = 0.5x^2 - 6x + 5$

Step 1: Write the equation of the axis of symmetry

The given equation $y = 0.5x^2 - 6x + 5$

Use the rule: "The equation of the axis of symmetry.

Foe the graph $y = ax^2 + bx + c$ where $a \neq 0$ is $x = -\frac{b}{2a}$"

Now compare the equation $y = 0.5x^2 - 6x + 5$ with $y = ax^2 + bx + c$. We have *a* = 0.5, *b* = −6 and *c* = 5

$$x = -\frac{b}{2a} \quad \text{(Equationm for the axis of symmetry of a parabola)}$$

$$x = -\frac{-6}{2\cdot(0.5)} \quad \text{(Replace } a \text{ by 0.5 and } b \text{ by } -6\text{)}$$

$$x = -\frac{-3\cdot 2}{2\cdot(0.5)} \quad \text{(Factors } 6 = 3\cdot 2\text{)}$$

$$x = \frac{3}{0.5} \quad \text{(Cancellation of the nemerator and the denominator)}$$

$$x = 6$$

Hence, the equation of the axis of symmetry is $\boxed{x = 6}$

Step2: Find the coordinates of the vertex, since the equation of the axis of symmetry is *x* = 6 and the vertex on the axis, the x – coordinate for the vertex 6.

$$y = 0.5x^2 - 6x + 5 \quad \text{(Original equation)}$$

$$y = 0.5(6)^2 - 6(6) + 5 \quad \text{(Replace } x \text{ by 6)}$$

$$y = 18 - 36 + 5$$

$$y = 23 - 36$$

$$y = -13$$

Hence, the vertex is $\boxed{(6, -13)}$

Step3: Indentify the maximum or minimum

The equation is $y = 0.5x^2 - 6x + 5$

Use the rule "The equation of the parabola is $y = ax^2 + bx + c$. Suppose the coefficient of *x*2 term is positive, the parabola opens upwards and the vertex is a minimum point.

Suppose the coefficient of *x*2 term is negative, the parabola open downward and the vertex is a maximum point"

Since the coefficient of the *x*2 terms is positive, the parabola open upward and vertex is a minimum point.

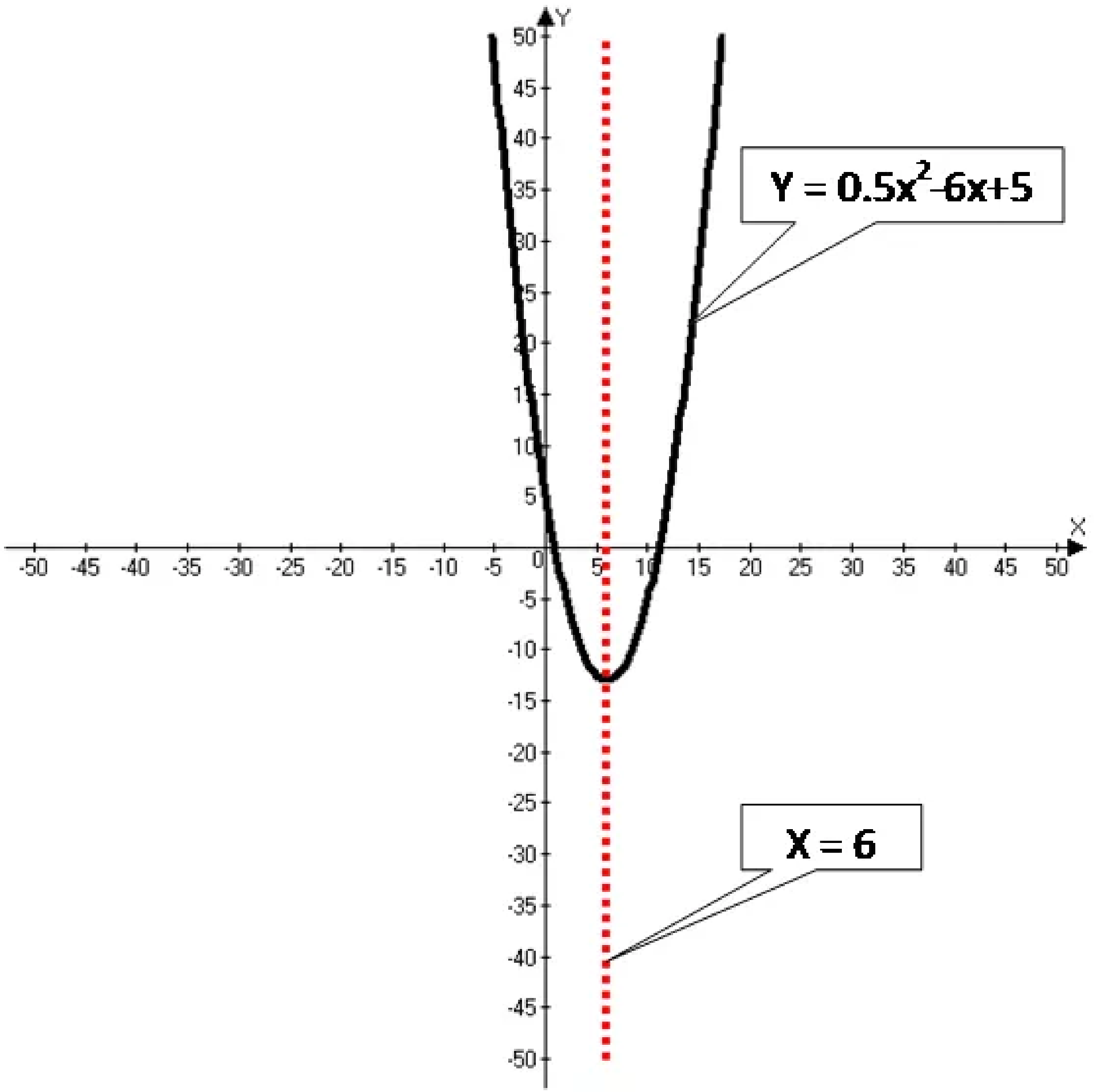
Hence, the minimum point of the parabola is $\boxed{(6,-13)}$

Step4: Graph the function $y = 0.5x^2 - 6x + 5$

Now, we consider the table for $y = 0.5x^2 - 6x + 5$. We can substitute the different values of *x* is $y = 0.5x^2 - 6x + 5$, we get the *y* – values. Graph these ordered pairs and connect them, we get the smooth curve.

x	$0.5x^2 - 6x + 5$	y	(x, y)
-4	$0.5(-4)^2 - 6(-3) + 5 = 37$	37	$(-4, 37)$
-2	$0.5(-2)^2 - 6(-2) + 5 = 19$	19	$(-2, 19)$
0	$0.5(0)^2 - 6(0) + 5 = 5$	5	$(0, 5)$
2	$0.5(2)^2 - 6(2) + 5 = -5$	-5	$(2, -5)$
4	$0.5(4)^2 - 6(4) + 5 = -11$	-11	$(4, -11)$
6	$0.5(6)^2 - 6(6) + 5 = -13$	-13	$(6, -13)$

Add these all ordered pairs, we get a parabola open upward.



Answer 56MYS.

Consider the given equation $m^2 - 24m = -144$

Claim: Solve the equation $m^2 - 24m = -144$ by using factoring.

Step1: Rewrite the equation $m^2 - 24m = -144$ in the standard form of the quadratic equation $ax^2 + bx + c$ where $a \neq 0$

$$m^2 - 24m = -144 \qquad \text{(Original equation)}$$

$$m^2 - 24m + 144 = -144 + 144 \qquad \text{(Add 144 on both sides)}$$

$$m^2 - 24m + 144 = 0$$

Step2: To solve the equation by factoring

$$m^2 - 24m + 144 = 0 \qquad \text{(Original equation)}$$

$$m^2 - 12m - 12m + 12 \cdot 12 = 0 \qquad (\text{Factors } 24m = 12m + 12m)$$

$$m(m-12) - 12(m-12) = 0$$

$$(m-12)(m-12) = 0$$

Step3: Substitute each value of *m* in original equation

$$m^2 - 24m = -144$$

$$m^2 - 24m = -144$$

$$(12)^2 - 24(12) \overset{?}{=} -144 \qquad (\text{Replace } m \text{ by } 12)$$

$$144 - 288 \overset{?}{=} -144$$

$$-144 = -144 \qquad \text{true}$$

$$m^2 - 24m = -144$$

$$(12)^2 - 24(12) \overset{?}{=} -144 \qquad (\text{Replace } m \text{ by } 12)$$

$$144 - 288 \overset{?}{=} -144$$

$$-144 = -144 \qquad \text{true}$$

Therefore *m* = 12 and *m* = 12 satisfies the equation $m^2 - 24m = -144$

Hence the solution set is $\boxed{\{12\}}$

Answer 57MYS.

Consider the given equation $7r^2 = 70r - 175$

Claim: Solve the equation $7r^2 = 70r - 175$ by using factoring.

Step1: Rewrite the equation $7r^2 = 70r - 175$ in the standard form of the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$

$$7r^2 = 70r - 175 \qquad \text{(Original equation)}$$

$$7r^2 - 70r = 70r - 175 - 70r \qquad \text{(Subtract } 70r \text{ on both sides)}$$

$$7r^2 - 70r = -175$$

$$7r^2 - 70r + 175 = -175 + 175 \qquad \text{(Add 175 on both sides)}$$

$$7r^2 - 70r + 175 = 0$$

$$\frac{7r^2 - 70r + 175}{7} = \frac{0}{7} \qquad \text{(Divide 7 on both sides)}$$

$$\frac{7r^2}{7} - \frac{70r}{7} + \frac{175}{7} = 0$$

$$r^2 - 10r + 25 = 0$$

Step2: To solve the equation by factoring

$$r^2 - 10r + 25 = 0 \qquad \text{(Original equation)}$$

$$r^2 - 5r - 5r + 25 = 0 \qquad \left(\begin{matrix} \text{Factors } r^2 = r \cdot r \\ 25 = 5 \cdot 5 \end{matrix}\right)$$

$$r \cdot r - 5 \cdot r - 5 \cdot r + 25 = 0$$

$$r(r-5) - 5(r-5) = 0$$

$$(r-5)(r-5) = 0$$

$$r - 5 = 0 \quad \text{or} \quad r - 5 = 0$$

$$r = 5 \quad \text{or} \quad r = 5$$

Step3: each value of *r* in original equation

$7r^2 = 70r - 175$

$7r^2 = 70r - 175$

$7(5)^2 \stackrel{?}{=} 70(5) - 175$ (Replace r by 5)

$7 \cdot 25 \stackrel{?}{=} 350 - 175$

$175 = 175$ true

$7r^2 = 70r - 175$

$7(5)^2 \stackrel{?}{=} 70(5) - 175$ (Replace r by 5)

$7 \cdot 25 \stackrel{?}{=} 350 - 175$

$175 = 175$ true

Therefore *r* = 5 and *r* = 5 satisfies the equation $7r^2 = 70r - 175$

Hence the solution set is $\boxed{\{5\}}$

Answer 58MYS.

Consider the given equation $4d^2 + 9 = -12d$

Claim: Solve the equation $4d^2 + 9 = -12d$ by using factoring.

Step1: Rewrite the equation $4d^2 + 9 = -12d$ in the standard form of the quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$

$4d^2 + 9 = -12d$ (Original equation)

$4d^2 + 9 + 12d = -12d + 12d$ (Subtract $12d$ on both sides)

$4d^2 + 12d + 9 = 0$

Step2: To solve the equation by factoring

$$4d^2+12d+9=0 \qquad \text{(Original equation)}$$

$$4d^2+6d+6d+9=0$$

$$2d\cdot 2d+2d\cdot 3+2d\cdot 3+3\cdot 3=0$$

$$2d(2d+3)+3(2d+3)=0$$

$$(2d+3)(2d+3)=0$$

$$2d+3=0 \quad \text{or} \quad 2d+3=0$$

$$2d+3-3=0-3 \quad \text{or} \quad 2d+3-3=0-3 \qquad \text{(Subtract 3 on both sides)}$$

$$2d=-3 \quad \text{or} \quad 2d=-3$$

$$\frac{2d}{2}=\frac{-3}{2}\ \left(\begin{array}{l}\text{Divide 2 on}\\ \text{both sides}\end{array}\right) \quad \text{or} \quad \frac{2d}{2}=\frac{-3}{2} \qquad \text{(Divide 2 on both sides)}$$

$$d=\frac{-3}{2} \quad \text{or} \quad d=\frac{-3}{2}$$

Step3: Each value of *d* in original equation

$$4d^2+9=-12d$$

$$4d^2+9=-12d$$

$$4\left(\frac{-3}{2}\right)^2+9\overset{?}{=}-12\left(\frac{-3}{2}\right) \qquad \left(\text{Replace } d \text{ by } \frac{-3}{2}\right)$$

$$4\cdot\frac{9}{4}+9\overset{?}{=}-12\cdot\left(\frac{-3}{2}\right)$$

$$1\cdot 9+9\overset{?}{=}-6\cdot 2\cdot\left(\frac{-3}{2}\right)$$

$$9+9\overset{?}{=}18 \qquad \text{(Simplification)}$$

$$18=18 \qquad \text{true}$$

$$4d^2+9=-12d$$

$$4\left(\frac{-3}{2}\right)^2+9\overset{?}{=}-12\left(\frac{-3}{2}\right) \qquad \left(\text{Replace } d \text{ by } \frac{-3}{2}\right)$$

$$4\cdot\frac{9}{4}+9\overset{?}{=}-12\cdot\left(\frac{-3}{2}\right)$$

$$1\cdot 9+9\overset{?}{=}-6\cdot 2\cdot\left(\frac{-3}{2}\right)$$

$$9+9\overset{?}{=}18 \qquad (\text{Simplification})$$

$$18=18 \qquad \text{true}$$

Therefore $d=\frac{-3}{2}$ and $d=\frac{-3}{2}$ satisfies the equation $4d^2+9=-12d$

Hence the solution set is $\boxed{\left\{\frac{-3}{9}\right\}}$

Answer 63MYS..

Consider the expression $a^2+14a+49$

Claim: To write the trinomial $a^2+14a+49$ as a perfect square.

$$a^2+14a+49=a^2+2\cdot a\cdot 7+7\cdot 7$$

$$=(a+7)^2 \qquad \left(\text{Use the rule } a^2+2ab+b^2=(a+b)^2\right)$$

The perfect square of $a^2+14a+49$ is $\boxed{(a+7)^2}$

Answer 64MYS.

Consider trinomial $m^2-10m+25$

Claim: To write the trinomial $m^2-10m+25$ as a perfect square.

$$m^2-10m+25=m^2-10m+25$$

$$=m\cdot m-5m-5m+25$$

$$=m\cdot(m-5)-5\cdot(m-5)$$

$$=(m-5)(m-5)$$

$$=(m-5)^2$$

The perfect square of $m^2-10m+25$ is $\boxed{(m-5)^2}$

Answer 65MYS.

Consider the trinomial $t^2 + 16t - 64$.

The objective is to determine whether the trinomial is perfect square.

The trinomial can be written as,

$$t^2 + 16t - 64 = t^2 + 2 \cdot 8 \cdot t - 8^2$$

$$= (t-8)^2 \qquad \text{Use } (a-b)^2 = a^2 - 2ab + b^2$$

Thus, the **trinomial is a perfect square.**

Answer 66MYS.

Consider trinomial $4y^2 + 12y + 9$

Claim: To write the trinomial $4y^2 + 12y + 9$ as a perfect square.

$$4y^2 + 12y + 9 = 4y^2 + 12y + 9$$

$$= 2y \cdot 2y + 6y + 6y + 3 \cdot 3$$

$$= 2y \cdot (2y+3) + 3 \cdot (2y+3)$$

$$= (2y+3)(2y+3)$$

$$= (2y+3)^2$$

The perfect square of $4y^2 + 12y + 9$ is $\boxed{(2y+3)^2}$

Answer 67MYS.

Consider the trinomials $9d^2 - 12d - 4$.

The objective is to determine whether the trinomial is perfect square.

The trinomial can be written as,

$$9d^2 - 12d - 4 = (3d)^2 - 2 \cdot 3d \cdot 4 - 2^2$$

$$\neq (3d)^2 - 2 \cdot 3d \cdot 4 + 2^2$$

Thus, the coefficient is negative, so it can't be written as a perfect square.

Hence, the **trinomial is not a perfect square.**

Answer 68MYS.

Consider trinomial $25x^2 - 10x + 1$

Claim: To write the trinomial $25x^2 - 10x + 1$ as a perfect square.

$$\begin{aligned} 25x^2 - 10x + 1 &= 25x^2 - 10x + 1 \\ &= (5x)^2 - 10x + 1 \\ &= 5x \cdot 5x - 5x - 5x + 1 \cdot 1 \\ &= 5x \cdot 5x - 5x \cdot 1 - 5x \cdot 1 + 1 \cdot 1 \\ &= 5x \cdot (5x - 1) - 1 \cdot (5x - 1) \end{aligned}$$

$$= (5x - 1)^2$$

The perfect square of $25x^2 - 10x + 1$ is $\boxed{(5x-1)^2}$