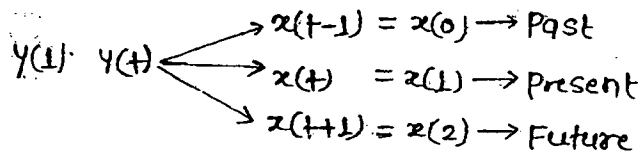
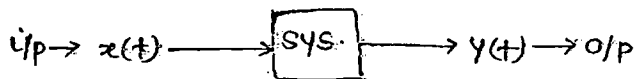


Chapter-03 Basic system properties



1) Static & dynamic sys. →

Static → If o/p of sys. depends only on present values of i/p at each & every instant of time then sys. will be static.

* These sys. are also known as memoryless system.

Dynamic → * If o/p of sys. depends on past (or) future values of i/p at any instant of time then sys. will be dynamic.

* This sys. are also known as sys. with memory.

Q. → Check static/dynamic sys.

(1) $y(t) = x(t) + x(t-1)$

(5.) $y(t) = \text{Even}[x(t)]$

(2) $y(t) = x(-t)$

(6.) $y(t) = \text{Real}[x(t)]$

(3) $y(t) = x(\sin t)$

(7.) $y(t) = \int_{-\infty}^t x(z) dz$

(4) $y(t) = x(t-1)$

(8.) $y(t) = e^{-(t+1)} x(t)$

Ans. → (1) Dynamic.

(2) Dynamic.

(3) $y(t) = x(\sin t)$

$y(-\pi) = x(0)$

$-3.14 \text{ Sec} = x(0)$

Future system is dynamic.

(4) Dynamic

(5) $y(t) = \frac{x(t) + x(-t)}{2}$

$y(1) = \frac{x(1) + x(-1)}{2}$

Past system is dynamic.

(6) $y(t) = \frac{x(t) + x(t)}{2}$

system is static

(7) system is dynamic

Note →

- (1) Integral & derivative sys. are dynamic sys.
- (2) In case of time scaling (or) time shifting system will be dynamic.

(2) Causal & Non-Causal system →

* causal → * If o/p of sys. is independent of future value of i/p at each & every instant of time then sys. will be causal.

* This sys. are practical (or) physically reliable sys.

Eg:- (1) $y(t) = x(t)$

(2) $y(t) = x(t-1)$

(3) $y(t) = x(t) + x(t-1)$

* Non-Causal system → * If o/p of sys. depends on future value of i/p at any instant of time then sys. will be non-causal.

Eg:- (1) $y(t) = x(t+1)$

(2) $y(t) = x(t) + x(t+1)$

(3) $y(t) = x(t+1) + x(t+1)$

(4) $y(t) = x(t) + x(t-1) + x(t+1)$

* Anti Causal System → * If o/p of sys. depends only on future value of i/p then sys. will be anticausal.

Eg:- $y(t) = x(t+1)$

* All anti-causal systems are non-causal but converse of this statement is not true.

Que. → Check Causal & Non-Causal system.

(1) $y(t) = x(2t)$

(2) $y(t) = x(-t)$

(3) $y(t) = x(\sin t)$

(4) $y(t) = \begin{cases} x(2t); & t < 0 \\ x(t-1); & t \geq 0 \end{cases}$

(5) $y(t) = \text{Odd}[x(t)]$

(6) $y(t) = \sin(t+2) \cdot x(t-1)$

(7) $y(t) = \int_{-\infty}^t x(\tau) d\tau$

(8) $y(t) = \int_{-\infty}^{t+1} x(\tau) d\tau$

(9) $y(t) = \int_{-\infty}^{2t} x(\tau) d\tau$

Soln $\rightarrow$ (i) $y(t) = x(2t)$

$(t=1) \downarrow$

$y(1) = x(2)$ (system is non-causal)

(ii) $y(t) = x(-t)$

$(t=-1) \downarrow$

$y(-1) = x(1)$ (system is non-causal)

(iii) $y(t) = x(\sin t)$

$(t=-\pi) \downarrow$

$y(-\pi) = x(0)$

$-3.14 = x(0)$ (system is non-causal)

(iv) $y(t) = \begin{cases} x(2t), & t < 0 \rightarrow \text{past} \\ x(t-1), & t \geq 0 \rightarrow \text{past} \end{cases}$

(system is causal)

(v) $y(t) = \text{odd } x(t)$

$= \frac{x(t) - x(-t)}{2}$

$(t=-1) \downarrow$

$y(-1) = \frac{x(-1) - x(1)}{2}$ (system is non-causal)

(vi) $y(t) = \sin(t+2) \cdot x(t-1)$

(coefficient) $\downarrow$ past

(system is causal)

(vii) $y(t) = \int_{-\infty}^t x(z) dz$

$= \int_{-\infty}^t x(t) dz$ (system is causal)

(viii) $y(t) = \int_{-\infty}^{t+1} x(z) dz$

(system is non-causal)

(ix) $y(t) = \int_{-\infty}^{2t} x(z) dz$

(system is non-causal)

(2.) Linear & Non-linear system →

linear → * A linear sys. follows the law of superposition.

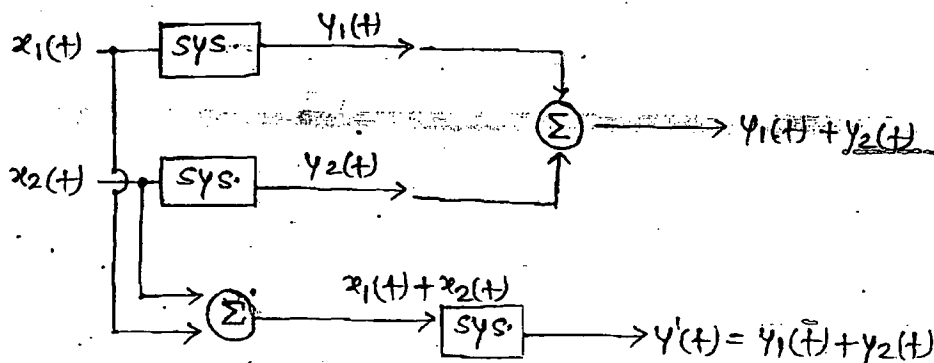
* This law is necessary & sufficient to prove linearity of system.

* It is a combination of two laws:-

(i) Law of additivity.

(ii) Law of Homogeneity.

(1.) Law of additivity →



Eq:- $y(t) = x(t) + 10$

o/p = $y/p + 10$

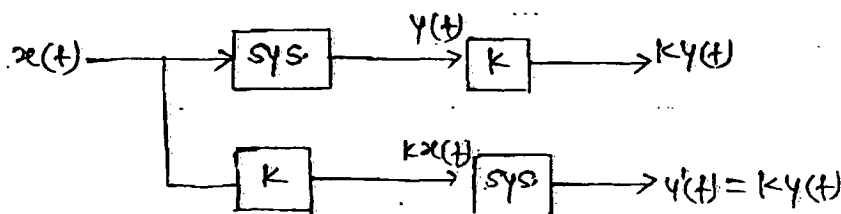
$y_1(t) = x_1(t) + 10$

$y_2(t) = x_2(t) + 10$

$y'(t) = x_1(t) + x_2(t) + 10$

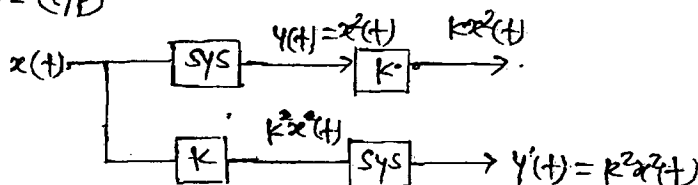
$y(t) \neq y'(t)$ [sys. is NL]

(2.) Law of Homogeneity →



Eq:- $y(t) = x^2(t)$

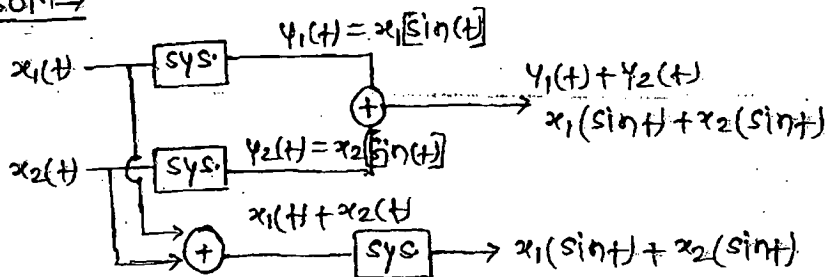
o/p = $(i/p)^2$



Que. → Check linear / Non-linear sys.

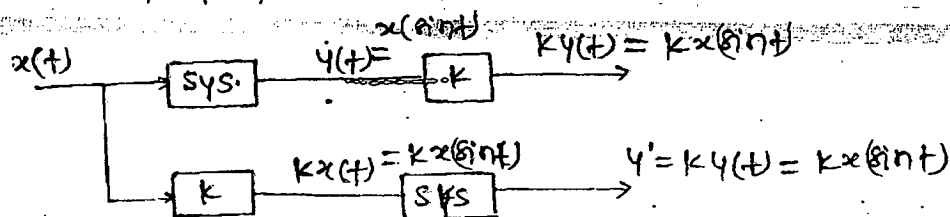
(1) $y(t) = x(\sin t)$ (2) $y(t) = x(t \sin t)$ (3) $y(t) = x(t^2)$

Soln →



Note →

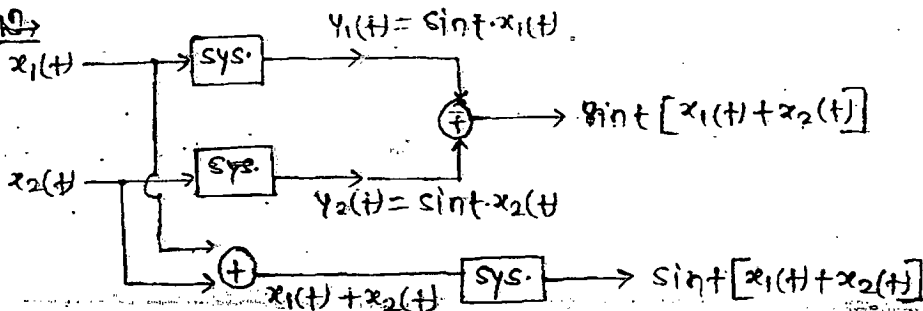
Linearity of sys. is independent of time scaling.



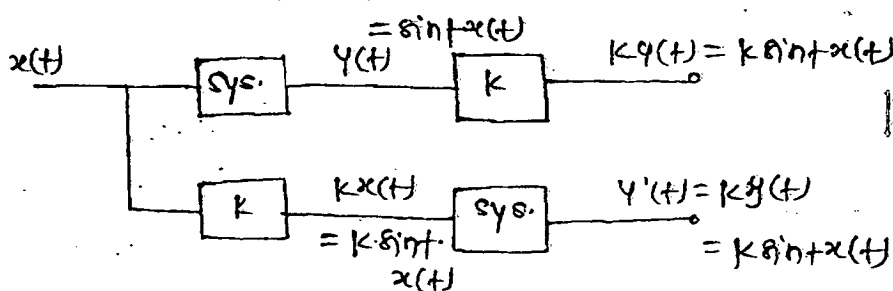
(2) $y(t) = \sin t \cdot x(t)$

(3) $y(t) = \log t \cdot x(t)$

Soln →



Note:- linearity of sys. is independent of coefficient used in sys. relationship.



2nd method →

For linearity :-

(i) O/p should be 0 for 0 i/p.

(ii) There should be ^{not} any 'NL' operation.

eg:- $\left[\begin{array}{l} \sin, \cos, \tan, \sec, \csc, \cot, \dots \\ \log, \text{exponential, modulus, sq, cube,} \dots \\ \dots \text{root,} \dots \text{sq}(\cdot), \text{sinc}(\cdot), \dots \text{sgn}(\cdot) \text{ etc} \\ \text{either on 'x' or 'y'}. \end{array} \right]$

Q → Check linear/NL sys.

(i) $y(t) = x(t) + 2 \rightarrow \text{put } t=0 \text{ then } y(0) \neq x(0) + 2 \rightarrow \text{NL}$

(ii) $y(t) = e^{x(t)} \rightarrow \text{Because of } e^{x(t)} \text{ it is NL \& also both condn not satisfying.}$

(iii) $y(t) = x(\tan t) \rightarrow \text{Linear}$

(i) If $t=0$, then $y(0) = x(0)$ means no i/p no o/p

(ii) above fn is not operating on 'x', it is operating on the 't'.

(iv) $y(t) = \tan[x(t)]$
system is **NL**

(v) $y(t) = x(t-1) + x(t+1)$

i/p $\rightarrow$ sys. $\rightarrow$ o/p = past i/p + future i/p

No any NL fn so this is **linear**

(vi) $y(t) = \text{even}[x(t)]$

$$y(t) = \frac{x(t) + x(-t)}{2}$$

No non-linear operator so **linear**

(vii) $y(t) = \int_{-\infty}^t x(\tau) d\tau$

$$y(t) = \int_{-\infty}^t x(\tau) d\tau \quad \text{linear}$$

(viii) $y(t) = \begin{cases} x(t-1), & t < 0 \\ x(t+1), & t \geq 0 \end{cases} = \begin{cases} \text{past i/p}, & t < 0 \\ \text{future i/p}, & t \geq 0 \end{cases} \quad \text{linear}$

Note →

(1) Integral & derivative operators are linear.

(2) Even & odd operators are linear.

(ix) $y(t) = \int_{-\infty}^t x^2(z) dz$

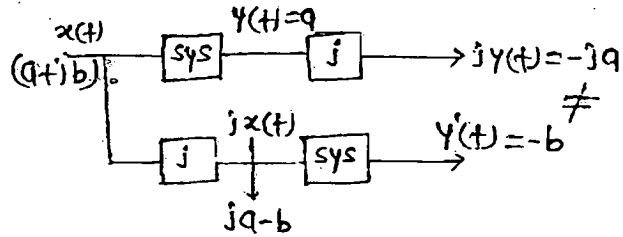
$y(t) = \int_{-\infty}^t x^2(z) dz$ (NL)

(xi) $y(t) = \text{Re} q L[x(t)]$

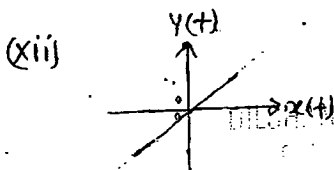
$y(t) = \frac{x(t) + x^*(t)}{2}$ (NL)

(x) $y(t) = e^t x(t)$

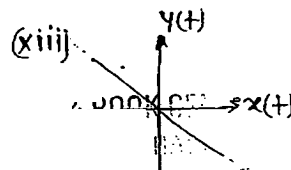
$y(t) = e^t x(t)$ (linear)



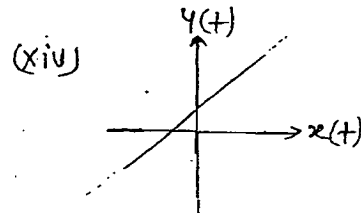
Note → Real & imaginary operators are NL.



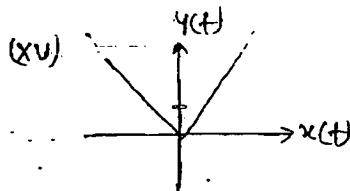
$y(t) = m x(t)$
system is linear



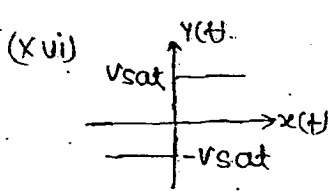
$y(t) = -m x(t)$
system is (L)



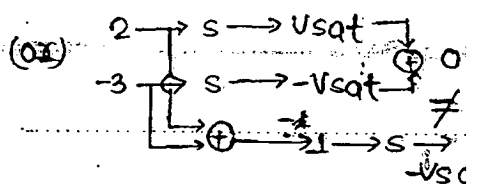
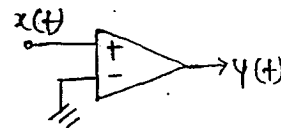
When i/p 0 then we got o/p (NL)



$y(t) = |x(t)|$
(NL)

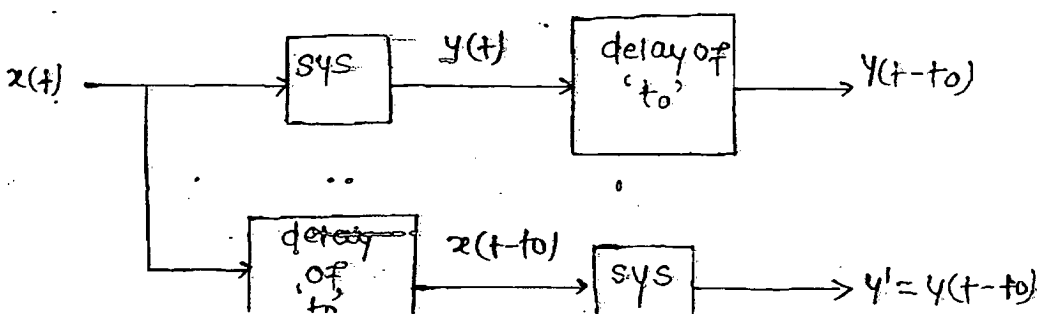


$y(t) = V_{sat} \text{sgn}[x(t)]$ (NL)



(4) Time invariant & time variant sys. →

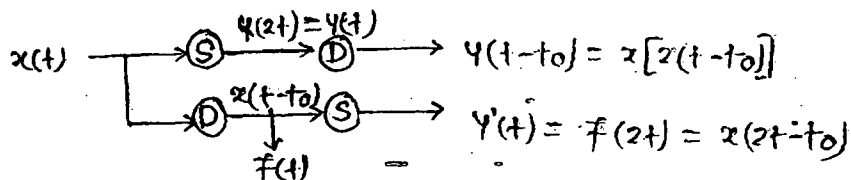
Time invariant →



Note:- Any delay provided in i/p must be reflected in o/p for a time invariant system.

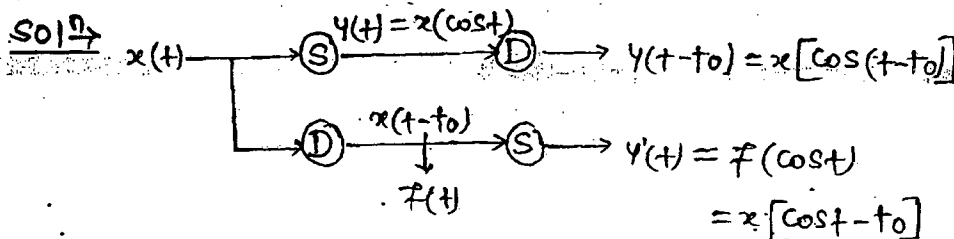
Que.→ Check time invariant/variant sys.

(1) $y(t) = x(2t)$



system is **(TV)**

(2) $y(t) = x(\cos t)$



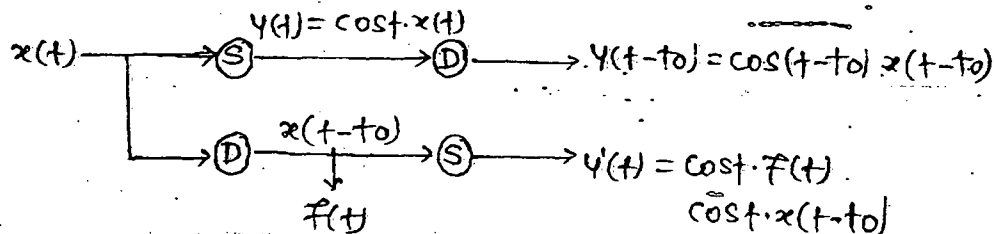
system is **(TV)**

Note:- In case of time scaling sys. will be time variant.

(3) $y(t) = \cos t \cdot x(t)$

(4) $y(t) = \log t \cdot x(t)$

Solⁿ→



system is **(TV)**

Note:- If coefficient in sys. relationship is fⁿ of time then sys. will be time variant.

(5) $y(t) = \text{odd}[x(t)]$

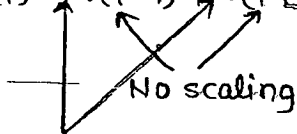
(6) $y(t) = \text{CS}[x(t)]$

Solⁿ→ $y(t) = \frac{x(t) - x(-t)}{2}$
 Time scaling (TV)

$y(t) = \frac{x(t) + x^*(t)}{2}$
 Time scaling (TV)

$$(vi) y(t) = x(t-1) + x(t+1)$$

$$\text{Soln} \rightarrow y(t) = x(t-1) + x(t+1)$$



coefficient are independent of time so TIV

$$(v) y(t) = \int_{-\infty}^{3t} x(z) dz$$

$$\text{Soln} \rightarrow y(t) = \int_{-\infty}^{3t} x(z) dz \rightarrow x(3t) \text{ (TIV)}$$

$$(viii) y(t) = \int_{-\infty}^t x(z) dz$$

$$\text{Soln} \rightarrow y(t) = \int_{-\infty}^t x(z) dz \rightarrow x(t)$$

TIV

$$(x) y(t) = \int_{-\infty}^t \cos z \cdot x(z) dz$$

$$\text{Soln} \rightarrow y(t) = \int_{-\infty}^t \cos z \cdot x(z) dz \rightarrow \cos t \cdot x(t) \text{ (TV)}$$

$$(xi) y(t) = \begin{cases} x(t-1) & t \leq 0 \\ x(t+1) & t \geq 0 \end{cases}$$

$$\text{Soln} \rightarrow = a(t) \cdot x(t-1) + b(t) \cdot x(t+1)$$

split systems are time variant system.

(5) Stable/Unstable sys $\rightarrow$

Stable $\rightarrow$ Bounded i/p bounded o/p (BIBO) criteria.

BIBO $\rightarrow$ For stable system, o/p should be bounded (or) finite for finite (or) bounded i/p at each & every instant of time.

eg:- Bounded i/p are $u(t)$, dc-signal, $\sin t$, $\cos t$, $\text{sgn}(t)$

Que. $\rightarrow$ Check stable/Unstable sys

$$(1) y(t) = x(t) + 2$$

$x(t)$	$y(t)$
10	12

(Stable)

$$(2) y(t) = t x(t)$$

$x(t)$	$y(t)$
10	10t

(Unstable)

$$(3) y(t) = \frac{x(t)}{\sin t}$$

$x(t)$	$y(t)$
2	$\frac{2}{\sin t}$ ($t=0, \pi$)

(Unstable)

(4.) $y(t) = \sin t \cdot x(t)$

Soln $\rightarrow$ $y(t) = \sin t \cdot x(t)$ $\begin{array}{c|c} x(t) & y(t) \\ \hline 2 & (-2, 2) \end{array}$

$\downarrow$ $(-1, 1)$

$\downarrow$ $-x(t), x(t)$

(stable)

(5.) $y(t) = \sin[x(t)]$

Soln $\rightarrow$ $y(t) = \sin[x(t)]$

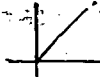
$\downarrow$ $(-1, 1)$

(stable)

(6.) $y(t) = \int_{-\infty}^t x(z) dz$

Soln $\rightarrow$ $y(t) = u(t) \rightarrow$ bounded

$y(t) = x(t) \rightarrow t u(t)$



(Unstable)

(7.) $y(t) = \frac{dx(t)}{dt}$

Soln $\rightarrow$ $x(t) = u(t) \rightarrow$ bounded

$y(t) = \delta(t) \rightarrow$ Unbounded

(Unstable)

(8.) $\int_{-\infty}^t \cos z \cdot x(z) dz = y(t)$

Soln $\rightarrow$ $y(t) = \int_{-\infty}^t \cos z \cdot x(z) dz$

$x(t) \rightarrow \cos t =$ bounded sig.

$y(t) = \int_{-\infty}^t \cos z \cdot \cos z dz$

$= \int_{-\infty}^t \cos^2 z dz$

$= \int_{-\infty}^t \left(\frac{1 + \cos 2z}{2} \right) dz$

$= \frac{1}{2} \left[\left(\frac{z}{1} \right) + \left(\frac{\sin 2z}{2} \right) \right]_{-\infty}^t$

Finite

(Unbounded)

(Unstable)

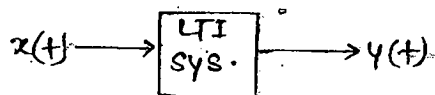
Note:- so the integration & differentiation signals are unstable sys.

Static/Dynamic - $D \rightarrow I, D, TS \& TS$

linear/NL - $L \rightarrow I, D, E, O$

NL $\rightarrow R \& I$

* Linear time invariant (LTI) System →



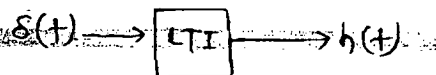
$h(t) \rightarrow$ Impulse-Response of sys.

$H(\omega)$ (or) $H(s) \rightarrow$ TF of sys.

* Impulse Response & TF turns are used only for LTI system.

* Impulse Response is used for defining LTI sys in time domain & TF is used for defining LTI sys in freq. domain.

Impulse Response →



* If i/p to LTI sys is unit impulse then o/p of sys is known as impulse response.

Transfer function →

* It is the ratio of Laplace X form of o/p to Laplace X form of i/p when all initial condⁿ are assumed to be 0.

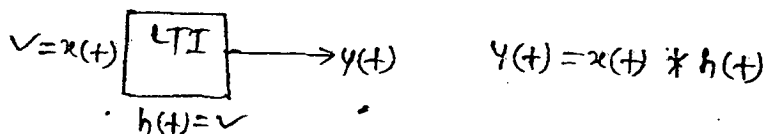
$$H(s) = \frac{Y(s)}{X(s)} \Big|_{\text{zero initial cond}^n}$$

* ~~TF~~ Total o/p = Zero i/p response + zero state response

$$\text{Total o/p} = \underbrace{\text{ZIR}}_{\substack{\text{due to} \\ \text{initial cond}^n \\ \text{states}}} + \underbrace{\text{ZSR}}_{\substack{\text{due to applied} \\ \text{i/p}}}$$

* For linearity of sys, initial condⁿ are assumed to be zero, because non-zero initial condⁿ make the sys. non-linear.

Convolution →



* Convolution is a linear time invariant operator & it is used only for LTI system.

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(z) \cdot x(t-z) dz$$

* The above relation is both linear & TIV. So it is LTI system.

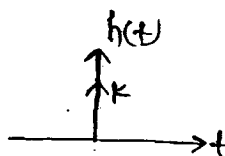
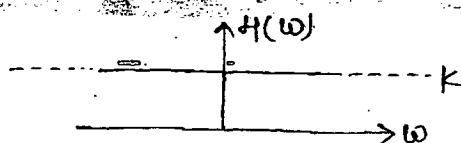
Condition for LTI system to be static $\rightarrow$

$$y(t) = k x(t)$$

$$h(t) \downarrow y(t) \quad \downarrow x(t) = \delta(t)$$

$$h(t) = k \delta(t)$$

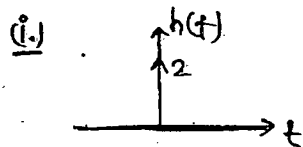
$$H(\omega) = k$$



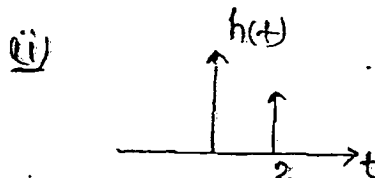
* Impulse $\delta(t)$ is the fn whose all x form is one.

* For static LTI system, impulse response should be impulse at origin & TF should be independent of freq.

Q. $\rightarrow$ check s/d LTI sys.

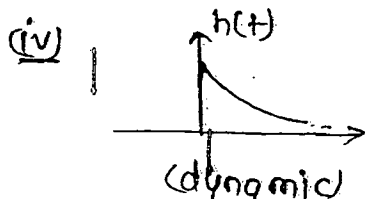


(static)



Impulse is not at origin so dynamic

(iii) $h(t) = u(t)$
(dynamic)



(v) $H(s) = 2$
static (free of freq.)

(vi) $H(s) = \frac{1}{s+1}$
(dynamic)

* Filters are dynamic system because their TF depends on freq.

* Condⁿ for LTI system to be causal →

$$y(t) = x(t) * h(t) \rightarrow \text{Future } y \neq 0$$

$$y(t) = \int_{-\infty}^{\infty} h(z) x(t-z) dz$$

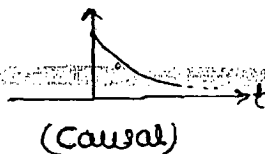
$$h(z) = 0, z < 0$$

$$\downarrow z=t$$

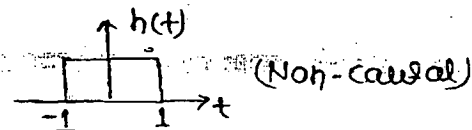
$$h(t) = 0, t < 0$$

Que → Check C/Nc LTI system.

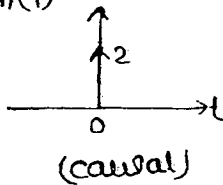
(1) $h(t) = e^{-2t} u(t)$



(2) $h(t) = u(t+1) - u(t-1)$

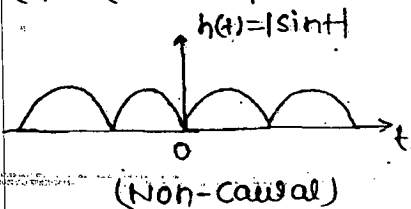


(3) $h(t)$



(4) $h(t) = e^{-(t+1)} u(t)$
(causal)

(5) $h(t) = |\sin t|$



* Condⁿ for LTI sys. to be stable → If impulse response of LTI sys. is absolutely integrable then sys. will be stable. i.e.

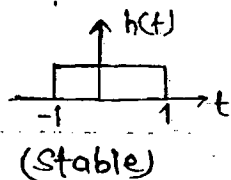
$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

* A sign If impulse response of LTI sys. is represented by energy signal (or) unit impulse fn then sys. will be stable.

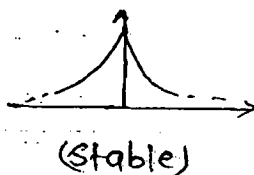
i.e. $h(t) \rightarrow \text{Energy} / \delta(t) \rightarrow \text{Stable}$

Que → Check s/us system.

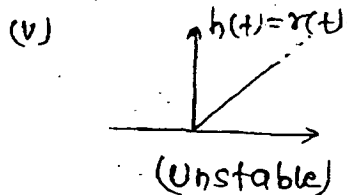
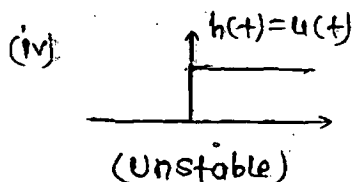
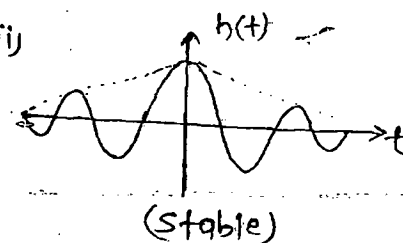
(1) $h(t) = u(t+1) - u(t-1)$



(2) $h(t) = e^{-2t} u(t)$



(iii)



(vi) $H(s) = \frac{1}{s^2 + 1}$

Poles = $s = \pm j$

* Because of imaginary axis lying so it is marginally stable

* $h(t) = \sin t u(t)$
 $\downarrow$
(Unstable)

(vii) $H(s) = \frac{1}{s}$



Pole $\rightarrow s = 0$

* marginally stable

* $h(t) = u(t) \rightarrow$ power sig.
(Unstable)

$H(s) = \frac{1}{s} = \frac{Y(s)}{X(s)}$

$Y(s) = \frac{X(s)}{s}$

Inverse LT.

$Y(t) = \int_{-\infty}^t x(t) dt$] Integrator

According to BIBO criteria:-

$x(t) = u(t) =$ bounded $\hat{y}_p$

$\hat{y}(t) = \int_{-\infty}^t u(t) dt = r(t)$

$r(t) =$ Unbounded sig.

So it is Unstable.

* All marginally stable are BIBO Unstable.

Types:- (i) Magnitude/Amplitude distortion

(ii) Delay / phase distortion.

Note:-

If $x(t) = \sin \omega_0 t$ then $y(t) = \sin \omega_0 t + \sin^2 \omega_0 t$
 $\quad \quad \quad \downarrow \quad \quad \quad \downarrow$
 $\quad \quad \quad \omega_0 \quad \quad \quad \omega_0, 2\omega_0$
 $\quad \quad \quad = \sin \omega_0 t + \frac{1 - \cos 2\omega_0 t}{2}$

$$\begin{array}{cc} x(t) = \sin \omega_0 t & y(t) = \sin \omega_0 t + \sin 2\omega_0 t \\ \downarrow & \downarrow \\ \omega_0 & \omega_0, 2\omega_0 \end{array}$$

for production of harmonics, nature of sys should be either NL (or) T.V.

(1.) Magnitude/Amplitude distortion → If sys. provides unequal amount of amplification (or) attenuation to two diff. freq. components present in i/p sys., then sys. is having magnitude distortion.

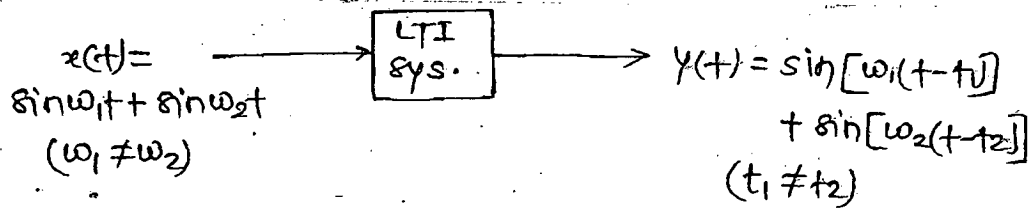
Block diagram of an LTI system:

Input: $x(t) = \sin \omega_1 t + \sin \omega_2 t$
 $\omega_1 \neq \omega_2$

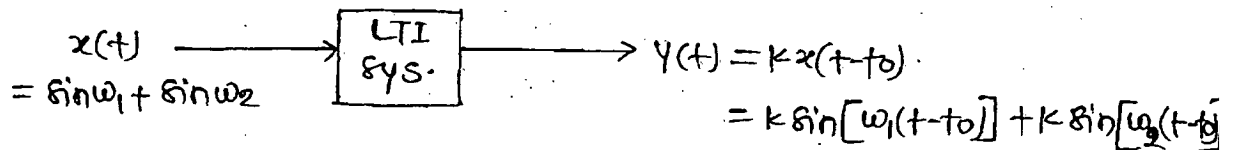
System: LTI sys.

Output: $y(t) = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$
 $A_1 \neq A_2$

(2) Delay (or) phase distortion → If sys. provides unequal amount of time delays to diff. freq. components present in i/p signal then sys. is having delay (or) phase distortion.



* Condⁿ for LTI sys. to be distortionless $\rightarrow$



$$y(t) = kx(t-t_0)$$

So Laplace transform of above eqⁿ

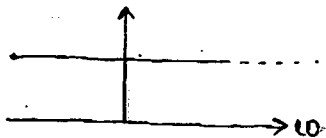
$$Y(s) = kX(s)e^{-st_0}$$

$$H(s) = \frac{Y(s)}{X(s)} = ke^{-st_0}$$

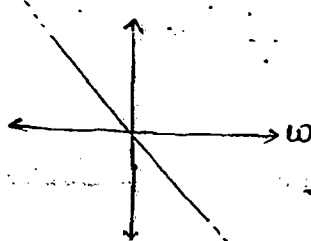
$$(s = j\omega)$$

$$H(j\omega) = ke^{-j\omega t_0}$$

$$|H(\omega)| = k$$



$$\angle H(\omega) = -\omega t_0$$



* For distortionless LTI sys., magnitude of TF should be independent of freq. & phase of TF should be linear.

* Differential eqⁿ for LTI sys. $\rightarrow$

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_0 y(t)$$

$$= b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + \dots + b_0 x(t)$$

For linearity →

All initial condⁿ should be zero.

Time-invariance →

Coefficients $a_n, a_{n-1}, \dots, a_0, b_m, b_{m-1}, \dots, b_0$ should be independent of time.

Que. → Check time invariance & linearity of sys. (initial condⁿ are zero).

(1) $\frac{2d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) = x(t)$

(2) $\frac{2d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + y(t) = x(t)$

(3) $2\left[\frac{dy(t)}{dt}\right]^2 + 3\frac{dy(t)}{dt} + y(t) = x(t)$

Ans. → (1) L, IV

(2) L, TV

(3) NL, TIV