Integers

Closure Property of Integers over Addition and Subtraction

Consider two integers 9 and −7. What is the value of the expression 9 + (−7)?

Yes, you are right. It is 2, which is an integer. Thus, we say that the integers 9 and −7 are closed under addition.

Is this true for all the integers?

Yes, this is true for all integers, i.e., the addition of any two integers gives an integer again. Thus, we can say that **all integers are closed under addition.**

Thus, according to the **closure property of addition,**

> **If x and y are any two integers and $x + y = z$, then z will always be an integer.**

Now, **can we say that the closure property is valid for subtraction also?** Let us now consider the integers 9 and -7. **What will we get if we subtract these two integers, i.e., what is the value of the expression 9 - (-7)?**

The value of 9 - (-7) is 16, which is again an integer. In fact, when we subtract any two integers, we always get an integer. Thus, we can say that **integers are closed under subtraction also.**

Thus, according to the **closure property of subtraction,**

> **If p and q are two integers and $p - q = r$, then r will always be an integer.**

Let us now look at some examples now.

Example 1:

Verify the closure properties of addition and subtraction for the integers 5 and 8.

Solution:

We have 5 − 8 = −3, which is an integer. Thus, the closure property for subtraction is verified for the integers 5 and 8.

Similarly, 5 + 8 = 13, which is also an integer. Thus, the closure property for addition is also verified for the integers 5 and 8.

Commutative and Associative Properties of Integers over Addition

Consider the two integers 3 and 6. What will be the value of 3 + 6?

3 + 6 = 9

What will happen if we interchange the positions of 3 and 6? Will the value remain the same?

Let us see this.

Now, the value of 6 + 3 is 9.

Thus, 3 + 6 = 6 + 3

This property is true for all integers. It is known as the commutative property of integers over addition, which states that the sum of two integers does not change even if we change the order in which they are added. Thus, **integers are commutative under addition.**

According to the commutative property of integers over addition,

If *a* and *b* are any two integers, then *a* + *b* = *b* + *a*.

Does the commutative property hold true for subtraction also?

Let us see this.

Consider the integers 8 and 3.

Now, 8 − 3 = 5

But, 3 − 8 = − 5

Therefore, 8 − 3 ≠ 3 − 8

Now, consider the integers 7 and − 5.

We have, 7 − (− 5) = 12

But (− 5) − 7 = −12

Therefore, 7 − (− 5) ≠ (−5) − 7

Thus, we can say that the **integers are not commutative under subtraction*.***

That is, **if *a* and *b* are any two integers, then *a* − *b* ≠ *b* − *a*.**

Now, consider the integers 3, 7, and 8.

What will be the value of the expressions 3 + (7 + 8) and (3 + 7) + 8?

Let us see.

Now, 3 + (7 + 8) = 3 + 15 = 18 and (3 + 7) + 8 = 10 + 8 = 18

Thus, 3 + (7 + 8) = (3 + 7) + 8

This property is also true for all integers. It is known as the associative property of integers over addition, which states that when we are adding three integers, we can group them in any order. This does not have any effect on their total sum. Thus, **integers are associative under addition.**

According to the associative property of integers under addition,

If *a*, *b* and *c* are any three integers, then *a* + (*b* + *c*) = (*a* + *b*) + *c*.

Let us now look at some examples.

Example 1:

Use any one of the commutative and associative properties of integers over addition

to fill in the blanks.

(a) (−21) + (_) = (−7) + (_)

(b) [6 + (−20)] + (_) = (_) + [(−20) + (−8)]

(c) 5 + [(_) + 9] = [(_) + 15] + (_)

(d) (_) + 35 = −5 + (_)

Solution:

(a) Integers are commutative under addition.

∴ (−21) + (−7) = (−7) + (−21)

(b) Integers are associative under addition.

∴ [6 + (−20)] + (−8) = 6 + [(−20) + (−8)]

(c) Integers are associative under addition.

∴ 5 + [15 + 9] = [5 + 15] + 9

(d) Integers are commutative under addition.

∴ (−5) + 35 = −5 + 35

Example 2:

Verify the associative property for the integers 5, −22, and 21.

Solution:

We have to prove that 5 + (−22 + 21) = [5 + (−22)] + 21

Now, 5 + (−22 + 21) = 5 + (−1) = 5 − 1 = 4

Also, [5 + (−22)] + 21 = −17 + 21 = 4

Thus, 5 + (−22 + 21) = [5 + (−22)] + 21

Thus, the associative property is verified for the integers 5, −22, and 21.

Multiplication of Integers

Consider the following example.

Suppose you have five bags that contain 6 balls each. **Can you tell the total number of balls in all the bags?**

To calculate this, we have to add 6 five times, i.e. $\underbrace{6+6+6+6+6}_{5 \text{ times}}$.

Now, we know that multiplication of whole numbers is repeated addition. Therefore, instead of adding 6 five times, we can simply multiply 6 by 5, i.e., 5 × 6 = 30

Hence, the bags contain a total number of 30 balls.

Just like whole numbers, the multiplication of integers is also their repeated addition.

For example:

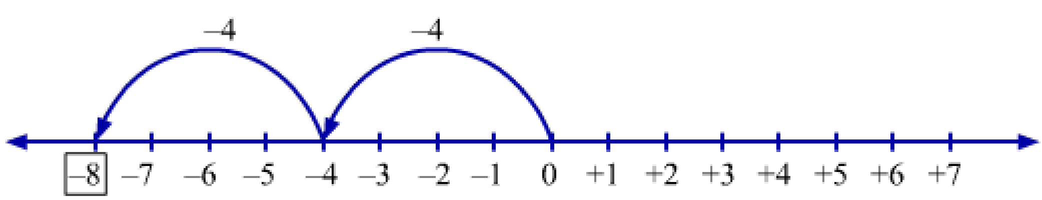
$$4\times(-7)=\underbrace{(-7)+(-7)+(-7)+(-7)}_{4 \text{ times}}=-28$$

Multiplication of two numbers can also be performed on the number line.
Let us multiply (−4) with 2 on number line.
We know that multiplication is a repeated addition.

∴(−4) × 2 = (−4) + (−4)

Now, in order to multiply (−4) with 2, jump 4 steps to the left at a time. This is continuing for 2 times.

This can be done as,



Here, the tip of the finale arrow is at −8.

∴(−4) × 2 = (−4) + (−4) = −8

Now, let us discuss some more examples based on the above concept.

Example 1:

Find the values of the following expressions.

(i) 5×18

(ii) $6 \times (-9)$

(iii) $(-42) \times 12$

(iv) $(-25) \times (-8)$

(v) $6 \times (-8) \times 3$

(vi) $2956 \times 1902 \times 0$

(vii) $(-7) \times (-3) \times 2 \times (-5) \times (-10)$

(viii) $(-2) \times 5 \times (-2) \times (-20) \times (-9) \times (-8)$

Solution:

(i) $5 \times 18 = 90$

(ii) $6 \times (-9) = -(6 \times 9) = -54$ (One positive and one negative integer)

(iii) $(-42) \times 12 = -(42 \times 12) = -504$ (One positive and one negative integer)

(iv) $(-25) \times (-8) = (25 \times 8) = 200$ (Two negative integers)

(v) $6 \times (-8) \times 3 = -(6 \times 8 \times 3)$

$= -144$ (One negative integer)

(vi) $2956 \times 1902 \times 0 = 0$ (Product of any integer with zero is zero)

(vii) $(-7) \times (-3) \times 2 \times (-5) \times (-10) = 7 \times 3 \times 2 \times 5 \times 10$

$= 2100$ (Even number of negative integers)

(viii) $(-2) \times 5 \times (-2) \times (-20) \times (-9) \times (-8) = -(2 \times 5 \times 2 \times 20 \times 9 \times 8)$

$= -28800$ (Odd number of negative integers)

Example 2:

Examine whether the following statements are correct or incorrect. Give reasons.

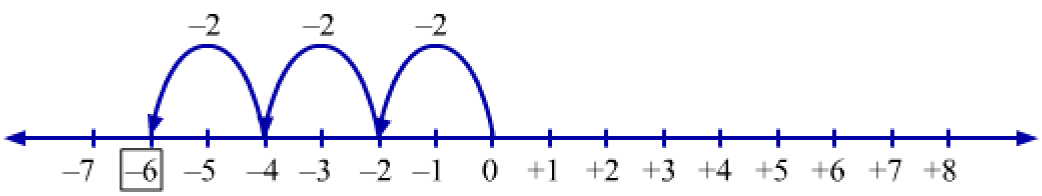
1. When −5 is multiplied *n* number of times, where *n* is even, then the sign of the product is negative.
2. The sign of the product is negative if we multiply 11 negative and 5 positive integers.
3. The product of 295 and 0 is 295.

Solution:

1. False. Since *n* is even, the sign of the product should be positive.
2. True. Since we are multiplying an odd number of negative integers, the sign of the product will be negative.
3. False. The product of any integer and zero is zero.

Example 3:
Multiply −2 with 3 on number line.

Solution:
−2 × 3 = (−2) + (−2) + (−2)
Now, in order to multiply −2 with 3, jump 2 steps to the left at a time. This is continuing for 3 times.
This can be done as,



Here, the tip of the finale arrow is at −6.
So, −2 × 3 = (−2) + (−2) + (−2) = −6
Closure Property of Integers over Multiplication

Consider the integers 2, 8, −4, and −3.

Multiply these integers two at a time. What will you get? Let us see.

2 × 8 = 16

2 × (−4) = −8

2 × (−3) = −6

(−4) × 8 = −32

$8 \times (-3) = -24$

$(-4) \times (-3) = 12$

What do you observe?

Observe that the new number obtained as a result of multiplication is always an integer. This implies that the given set of integers is **closed under multiplication i.e.,** ***the product of any two integers is always an integer.***This property is true for all integers.

Thus, according to the closure property of integers over multiplication:

If *x* and *y* are any two integers, and *x* × *y* = *z*, then *z* will always be an integer.

Commutative, Associative and Distributive Properties of Integers

Consider the integers −12 and 7. **What is the value of (−12) × 7?**

Yes, $(-12) \times 7 = -84$

Now, what will you get if you interchange the positions of −12 and 7? Will the product remain the same? Let us see.

We have $7 \times (-12) = -84$

$\therefore (-12) \times 7 = 7 \times (-12)$

Thus, we see that even when we change the positions of the integers, it makes no difference to their product. This property of integers is known as the **commutative property.** This is true for all integers. Hence, all integers are commutative under multiplication.

Thus, according to the commutative property of integers under multiplication:

If *x* and *y* are any two integers, then *x* × *y* = *y* × *x*.

Now, consider the integers −3, −5, and −6.

Let us find **the value of the expression [(−3) × (−5)] × (−6) and (−3) × [(−5) × (−6)].**

We have, $[(-3) \times (-5)] \times (-6) = 15 \times (-6) = -90$

Also, $(-3) \times [(-5) \times (-6)] = -3 \times 30 = -90$

$\therefore [(-3) \times (-5)] \times (-6) = (-3) \times [(-5) \times (-6)]$

Thus, even when we group the three integers differently, their product remains the same. This property is known as the **associative property** and it is true for all integers. Hence, all integers are associative under multiplication.

Thus, according to the associative property of integers under multiplication:

If a, b, and c are any three integers, then $(a \times b) \times c = a \times (b \times c)$.

Now, let us check whether the integers −7, −9, and −16 are distributive under multiplication.

We have, $(-7) \times [(-9) + (-16)] = (-7) \times (-25) = 175$

Also, $(-7) \times (-9) + (-7) \times (-16) = 63 + 112 = 175$

$\therefore (-7) \times [(-9) + (-16)] = (-7) \times (-9) + (-7) \times (-16)$

This verifies the **distributive property** of multiplication for integers −7, −9, and −16. In fact, all integers are distributive under multiplication.

Thus, according to the distributive property of integers under multiplication:

If x, y, and z are any three integers, then $x \times (y + z) = (x \times y) + (x \times z)$.

Can we also say that the distributive property of integers over subtraction under multiplication i.e., $x \times (y - z) = (x \times y) - (x \times z)$ is also true?

Yes. This is also true for any integer x, y, and z. Let us now verify this with an example.

Consider the integers 5, 8, and 7.

Now, $5 \times (8 - 7) = 5 \times 1 = 5$ and $(5 \times 8) - (5 \times 7) = 40 - 35 = 5$

Thus, $5 \times (8 - 7) = (5 \times 8) - (5 \times 7)$

Thus, the distributive property of multiplication of integers over subtraction is also true.

Thus, according to the distributive property of integers over subtraction under multiplication:

If x, y, and z are any three integers, then $x \times (y - z) = (x \times y) - (x \times z)$.

Cancellation law:

If x, y, and z are any three integers such that $x \neq 0$ and $xy = xz$, then $y = z$.

Let us now look at some examples.

Example 1:

Verify the associative and distributive property for the integers 5, 19, and − 27.

Solution:

We have, $(5 \times 19) \times (-27) = 95 \times (-27) = -2565$

And, $5 \times [19 \times (-27)] = 5 \times (-513) = -2565$

$\therefore (5 \times 19) \times (-27) = 5 \times [19 \times (-27)]$

Thus, the associative property for the given integers is verified.

Next, we have to verify that $5 \times [19 + (-27)] = 5 \times 19 + 5 \times (-27)$

Now, $5 \times [19 + (-27)] = 5 \times (-8) = -40$

And, $5 \times 19 + 5 \times (-27) = 95 - 135 = -40$

$\therefore 5 \times [19 + (-27)] = 5 \times 19 + 5 \times (-27)$

Thus, the distributive property for the given integers is verified.

Example 2:

Fill in the blanks using any of the commutative and associative property of integers under multiplication.

1. **5 × __ = − 2 × __**

2. **− 20 × __ = 8 × __**

3. **(p×__) ×__ = __× (r×s)**

4. **(20 × − 7) × __ = __ × (__× − 9)**

Solution:

1. The commutative property holds for integers under multiplication.

   Hence, 5 × − 2 = − 2 × 5

2. The commutative property holds for integers under multiplication.

   Hence, − 20 × 8 = 8 × − 20

3. The associative property holds for integers under multiplication.

   Hence, $(p \times r) \times s = p \times (r \times s)$

4. The associative property holds for integers under multiplication.

   Hence, (20 × − 7) × − 9 = 20 × (− 7 × − 9)

Multiplicative and Additive Identities for Integers

Consider the integers 5, 7, −8, and −9. **What do we get when we multiply each of these integers with 1?**

5 × 1 = 5

7 × 1 = 7

−8 × 1 = −8

−9 × 1 = −9

What do you observe?

Yes, you are right. The multiplication of an integer with 1 gives the same number again.

This is true for all integers. Thus, we can conclude that the multiplication of any integer with 1 gives the same integer again. Therefore, **1 is called the multiplicative identity of all integers.**

Thus, according to the multiplicative identity of integers:

For any integer x, $x \times 1 = x$

What happens when we add each of these integers to 0? Let us see.

$5 + 0 = 5$

$7 + 0 = 7$

$-8 + 0 = -8$

$-9 + 0 = -9$

What do you observe?

We observe here that the result of the sum of the integers with 0 is the same integer. This property of integers is known as **additive identity**. Just like multiplicative identity, ***additive identity*** **is also valid for all integers. Thus, 0 is called the additive identity of all integers.**

Thus, according to the additive identity of integers:

For any integer x, $x + 0 = x$

The additive inverse of an integer x is an integer which, when added to x, gives the sum as 0. In general, the additive inverse of an integer x is $-x$

$[x + (-x)] = 0$.

The additive inverse of an integer can be found by multiplying the integer by −1.

Let us find the additive inverses of the integers 5, 7, −8, and −9.

For this, we need to multiply each of the integers by −1.

Integer	**Additive inverse**
5	$5 \times (-1) = -5$
7	$7 \times (-1) = -7$
−8	$-8 \times (-1) = 8$
−9	$-9 \times (-1) = 9$

Let us now look at a few examples.

Example 1:

Fill in the blanks in the following expressions:

(i) $1292 \times 1 =$ __

(ii) $7 \times (-5) \times$ __ $\times (-2) = -70$

Solution:

(i) $1292 \times 1 = 1292$

(ii) $7 \times (-5) \times (-1) \times (-2) = -70$

Example 2:

Determine the integer whose product with (−1) is 25.

Solution:

If we multiply any integer with (−1), then we get the additive inverse of that integer.

∴ $25 = (-1) \times (-25)$.

Hence, the required integer is −25.

Using Properties of Integers to Facilitate Multiplication

Properties of numbers such as the associative and distributive properties can help us in our calculations in many cases. Let us try and understand this by taking some examples.

Let us look at some more examples to understand this concept better.

Example 1:

Find the value of the following expressions using the properties of multiplication of integers.

(i) 18×53

(ii) 12×95

(iii) $4 \times (5 \times 39) - 5 \times (2 \times 88)$

Solution:

(i) 18×53

$= 18 \times (50 + 3)$

$= (18 \times 50) + (18 \times 3)$ (Using distributive property)

$= 900 + 54$

$= 954$

(ii) 12×95

$= 12 \times (100 - 5)$

$= (12 \times 100) - (12 \times 5)$ (Using distributive property)

$= 1200 - 60$

$= 1140$

(iii) $4 \times (5 \times 39) - 5 \times (2 \times 88)$

= $(4 \times 5) \times 39 - (5 \times 2) \times 88$ (Using associative property)

$= 20 \times 39 - 10 \times 88$

$= 780 - 880$

$= -100$

Example 2:

A company has ten offices across the country. Out of them, three offices have 19 employees each, four offices have 31 employees each, and the remaining offices have 20 employees each. How many employees are there in the company?

Solution:

It is given that three offices have 19 employees each, four offices have 31 employees each, and the remaining offices [i.e., $10 - (3 + 4) = 10 - 7 = 3$] have 20 employees each.

$\therefore$ Total number of employees $= (3 \times 19) + (4 \times 31) + (3 \times 20)$

$= 3 \times (20 - 1) + 4 \times (30 + 1) + 60$

$= (3 \times 20) - (3 \times 1) + (4 \times 30) + (4 \times 1) + 60$

(Using distributive property)

$= 60 - 3 + 120 + 4 + 60$

$= 241$

Thus, there are 241 employees in the company.

Example 3:

There are 7 containers filled with eggs. Each container contains 22 eggs. If 4 eggs are taken out from any 4 containers, then how many eggs are left in the containers?

Solution:

Number of eggs left in the containers $= (22 \times 7) - (4 \times 4)$

$= (20 + 2) \times 7 - 16$

$= (20 \times 7) + (2 \times 7) - 16$

(Using distributive property)

$= 140 + 14 - 16$

$= 138$

Thus, a total of 138 eggs are left in the containers.

Division of Integers

Suppose Manmohan wants to distribute Rs 1000 equally among his 5 grandchildren. What amount will each child receive?

To answer this question, we are required to divide 1000 by 5. The value of the expression $1000 \div 5$ is found to be 200. Thus, Manmohan gives Rs 200 to each grandchild.

We were able to answer this question easily because it involved division of whole numbers.

Sometimes, we come across the situations when we need to perform division of integers.

Properties of Division

Property 1: If a and b are two integers, then $a \div b$ might not be an integer.
Example: Let a = 27 and b = 4, then $a \div b$ = 27 ÷ 4 = 274274 which is not an integer.

Property 2: If a is an integer and $a \neq 0$, then $a \div a = 1$.
Example: Let a = 7. Then $a \div a$ = 7 ÷ 7 = 77=177=1

Property 3: If a is an integer and $a \neq 0$, then $a \div 1 = a$
Example: Let a = −3. Then $a \div 1$ = (−3) ÷ 1 = −31=−3-31=-3

Property 4: If a is an integer and $a \neq 0$, then $0 \div a = 0$
Example: Let a = −8. Then $0 \div a$ = 0 ÷ (−8) = 0−8=00-8=0

Property 5: If a is a non-zero integer, then $a \div 0$ is not defined.
Example: If a = 25, then $a \div 0$ = 25 ÷ 0 = not defined

Property 6: If a, b and c are non-zero integers, then
$(a \div b) \div c \neq a \div (b \div c)$ except when c = 1
Note: when c = 1, $(a \div b) \div c = a \div (b \div c)$

Property 7: If *a, b* and *c* are integers, such that
(i) $a > b$ and c is positive, then $(a \div c) > (b \div c)$

Example: a = 28, b = 18 and c = 2 then $(a \div c) = 28 \div 2 = 14$ and $(b \div c) = 18 \div 2 = 9$. So, $(a \div c) > (b \div c)$.

(ii) $a > b$ and c is negative, then $(a \div c) < (b \div c)$

Example: a = 28, b = 18 and c = −2 then $(a \div c) = 28 \div -2 = -14$ and $(b \div c) = 18 \div -2 = -9$. So, $(a \div c) < (b \div c)$.

Let us now solve some examples based on these properties.

Example 1:

Find the values of the following expressions.

(i) 26 ÷ 13

(ii) (−44) ÷ 11

(iii) 36 ÷ (−2)

(iv) $\frac{-28}{-4}$

(v) $\frac{23+(-7)}{8}$

(vi) 48 ÷ [(−6) − (+2)]

Solution:

(i) 26 ÷ 13 = 2

(ii) (−44) ÷ 11 = −4

(iii) 36 ÷ (−2) = −18

(iv) $\frac{-28}{-4} = 7$

(v) $\frac{23+(-7)}{8}=\frac{16}{8}=2$

(vi) 48 ÷ [(−6) − (+2)] = 48 ÷ (−8) = −6

Example 2:

Fill in the boxes to satisfy the following equations.

(i) 54 ÷ $\square$ = − 6

(ii) 1 ÷ (−1) = $\square$

(iii) $\frac{\mathbf{145}}{\square}$ **= −145**

(iv) $\frac{\square}{\mathbf{-21}}$ **= 3**

(v) $\square$ **÷ 4 = 7**

Solution:

(i) 54 ÷ $\boxed{-9}$ = −6

(ii) 1 ÷ (−1) = $\boxed{-1}$

(iii) $\frac{145}{\boxed{-1}}$ = −145

(iv) $\frac{\boxed{-63}}{-21}$ = 3

(v) $\boxed{28}$ ÷ 4 = 7

Example 3:

If the cost of 6 pens is Rs 84, then what is the cost of one pen?

Solution:

Cost of 6 pens = Rs 84

∴ Cost of 1 pen = Rs $\frac{84}{6}$

= Rs 14

Example 4:

Isha took a test. Each question in the test carried (+4) marks for the correct answer and (−2) marks for the wrong one. Isha answered 10 questions correctly, but scored 10 marks in total. Find the number of questions that Isha answered incorrectly, if she attempted all the questions.

Solution:

Marks for one correct answer = +4

Isha attempted 10 questions correctly.

Therefore, marks which she scored for the correct answers = $10 \times 4 = 40$

Total marks = 10

∴ Marks which she scored for the incorrect answers = $10 - 40 = -30$

Marks for 1 incorrect answer = −2

Thus, number of questions attempted incorrectly by Isha = $(-30) \div (-2) = 15$

Division of Integers is Not Commutative

Consider two integers −14 and 7.

Are these integers commutative under division?

Let us see.

We will have to find the value of the expressions $(-14) \div (7)$ and $(7) \div (-14)$.

Now, $(-14) \div (7) = -2 \neq (7) \div (-14)$

∴ $(-14) \div (7) \neq (7) \div (-14)$

Let us look at one more example. Consider the integers −24 and −3.

Now, $(-24) \div (-3) = 8$

∴$(-24) \div (-3) \neq (-3) \div (-24)$

In the above two examples, we took the integers by our choice. If we take any other integers, we will see that on reversing their order, we will not get the same answer.

Thus, we can say that integers are not commutative under division.

If a and b are any two integers, then $a \div b \neq b \div a$.

Division of Integers by Zero

Suppose we divide the number 10 by 2. **Then what result will we obtain?**

We will obtain $10 \div 2 = 5$

Now, what do we obtain, if we divide 10 by 0?

Let us see.

We know that division by a number means **subtracting that number repeatedly until we obtain zero.**

Let us try to divide 10 by 0. Let us see what we obtain?

$$\begin{array}{r l} 10 & \\ -0 & \\ \hline 10 & \ldots\ldots\ldots\ldots 1 \\ -0 & \\ \hline 10 & \ldots\ldots\ldots\ldots 2 \\ -0 & \\ \hline 10 & \ldots\ldots\ldots\ldots 3 \\ \vdots & \end{array}$$

We can observe that the above process will never end. Therefore, we can say that we cannot divide 10 by zero.

In the same way, we cannot divide any integer by 0.

> **"The division, where the denominator is zero, is not defined.**
>
> **That is, if *a* is any integer, then $\frac{a}{0}$ is not defined".**

For example, $\frac{1}{0}$, $\frac{5}{0}$, and $\frac{-27}{0}$ etc. are not defined.

Division of Integers by 1 and -1

Consider the following division of integers.

$(-18) \div 1 = -18$

$9 \div 1 = 9$

$(-27) \div 1 = -27$

$16 \div 1 = 16$

What did you notice?

Observe that when we divide any integer by 1, then the result is the same integer again.

We can write the division rule of any integer by 1 as follows.

> **"If *a* is any integer, then $a \div 1 = a$ always".**

Now, consider the following division of integers.

$(-25) \div (-1) = 25$

$10 \div (-1) = -10$

$(-29) \div (-1) = 29$

$14 \div (-1) = -14$

In these examples, we notice that **division of integers by −1 does not result in the same integer.**

But if we observe the above divisions carefully, we will observe that on dividing any integer by −1, we will obtain the quotient same as the dividend but with reversed sign.

We can write the division rule of any integer by −1 as follows.

> **"If a is any integer, then $a \div (-1) = (-a)$".**

Let us look at an example based on the division of integers by 1 and −1.

Example1:

Fill in the blanks in the following expressions.

1. $(-32) \div (-1) =$ ____
2. $(-1) \div$ ____ $= 1$
3. ____ $\div 1 = 20$
4. $(-15) \div$ ____ $= -15$

Solution:

1. We know that for any integer a, we have $a \div (-1) = (-a)$.

   $\therefore (-32) \div (-1) = 32$

2. We know that for any integer a, we have $a \div (-1) = (-a)$.

   $\therefore (-1) \div (-1) = 1$

3. Division of any integer by 1 gives the same integer again.

   $\therefore (20) \div 1 = 20$

4. Division of any integer by 1 gives the same integer again.

   $\therefore (-15) \div 1 = -15$