PRACTICE PAPER

22

MATHEMATICS

Time allowed: 45 minutes **Maximum Marks: 200**

General Instructions: *As given in Practice Paper – 1.*

Section–A

Choose the correct option:

1. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b are

   (*a*) –6, –12, –18 (*b*) – 6, 4, 9 (*c*) – 6, – 4, – 9 (*d*) – 6, 12, 18

2. The value of $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{vmatrix}$ is

   (*a*) 1 (*b*) –1 (*c*) 0 (*d*) ω

3. If A is a square matrix of order (3×3) such that $|A| = 2$. Then $adj\ (adj\ A)$ is

   (*a*) $2A$ (*b*) A (*c*) $-A$ (*d*) None of these

4. If $f(x) = \log x$ then $f'(x)$ is equal to

   (*a*) $-\frac{1}{x^2}$ (*b*) $\frac{1}{x^2}$ (*c*) $\frac{1}{x}$ (*d*) $-\frac{1}{x}$

5. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$

   (*a*) touch each other (*b*) cut at right angle

   (*c*) cut at an angle $\frac{\pi}{3}$ (*d*) cut at an angle $\frac{\pi}{4}$

6. $\int \frac{dx}{x^2 + 2x + 2}$ is equal to

   (*a*) $x \tan^{-1}(x+1) + C$ (*b*) $\tan^{-1}(x+1) + C$

   (*c*) $(x+1)\tan^{-1} x + C$ (*d*) $\tan^{-1} x + C$

7. $\int \frac{\cos 2x\, dx}{(\sin x + \cos x)^2}$ is equal to

   (*a*) $\frac{-1}{\sin x + \cos x} + C$ (*b*) $\log|\sin x + \cos x| + C$

(c) $\log|\sin x - \cos x| + C$ (d) $\frac{1}{(\sin x + \cos x)^2} + C$

8. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{24}$ (d) $\frac{\pi}{4}$

9. The value of $\int_0^a \log(\cot a + \tan x)\,dx$, where $a \in (0, \pi/2)$ is

(a) $a \log(\sin a)$ (b) $-a \cos a$ (c) $-a \log(\sin a)$ (d) $\log(\sin a)$

10. The area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to

(a) $\pi^2 ab$ sq. units (b) πab sq. units (c) $\pi a^2 b$ sq. units (d) πab^2 sq. units

11. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is

(a) 4 (b) $\frac{3}{2}$ (c) not defined (d) 2

12. The solution of differential equation $x\,dy - y\,dx = 0$ represents

(a) a rectangular hyperbola (b) a parabola whose vertex is at origin
(c) straight line passing through origin (d) a circle whose centre is at origin

13. The maximum value of $Z = 4x + 3y$ subject to constraint $x + y \le 10$, $x, y \ge 0$ is

(a) 36 (b) 40 (c) 20 (d) none of these

14. If X follows a BD (Binomial Distribution) with parameter $n = 6$ and $4P(X = 4) = P(X = 2)$, then p equals

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

15. The probability distribution of a discrete random variable X is given below.

X	2	3	4	5
$P(X)$	$5/k$	$7/k$	$9/k$	$11/k$

The value of k is

(a) 17 (b) 13 (c) 32 (d) 48

Section–B (B1)

16. Let A and B be finite sets containing m and n elements respectively. The number of relations that can be defined from A to B is

(a) 2^{mn} (b) 2^{m+n} (c) mn (d) 0

17. The binary operation * defined on $\mathbb{N}$ by $a * b = a^b \; \forall \; a, b \in \mathbb{N}$ is

(a) Associative (b) Commutative and associative
(c) Commutative (d) None of these

18. If $f : R \to R$ be the functions defined by $f(x) = x^3 + 5$, then $f^{-1}(x)$ is

(a) $(x + 5)^{1/3}$ (b) $(x - 5)^{1/3}$ (c) $(5 - x)^{1/3}$ (d) $(5 - x)$

19. If $f : A \to B$ and $g : B \to C$ be the bijective functions, then $(gof)^{-1}$ is

(a) $f^{-1} o g^{-1}$ (b) fog (c) $g^{-1} o f^{-1}$ (d) gof

20. If $f : [0, 1] \to [0, 1]$ be defined by $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 1 - x & \text{if } x \text{ is irrational} \end{cases}$ then $(fof)(x)$ is

(a) Constant (b) $1 + x$ (c) x (d) None of these

21. Which of the following is the principal value branch of $\text{cosec}^{-1}x$?

(a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (b) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$ (c) $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$ (d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

22. If $\cos\left(\sin^{-1}\frac{3}{5} + \cos^{-1}x\right) = 0$, then x is equal to

(a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) 0 (d) 1

23. The value of $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{5\pi}{2}$ (d) $\frac{7\pi}{2}$

24. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $a, x \in (0, 1)$, then the value of x is

(a) 0 (b) $\frac{\pi}{2}$ (c) a (d) $\frac{2a}{1-a^2}$

25. Assuming that the sums and products given below are defined, which of the following is not true for matrices?

(a) $A + B = B + A$ (b) $AB = AC$ does not imply $B = C$

(c) $AB = 0$ implies $A = 0$ or $B = 0$ (d) $(AB)' = B'A'$

26. If A is a square matrix, then which of the following matrices is not symmetric?

(a) $A + A'$ (b) AA' (c) $A'A$ (d) $A - A'$

27. Using determinants, for what value(s) of x are the three points $A\ (x + 1, x)$, $B\ (x, x + 1)$ and $C\ (3x + 1, 2x - 3)$ collinear?

(a) $\left(-1, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, 1\right)$ (c) $(0, 1)$ (d) 1

28. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(a) A (b) $I - A$ (c) $I + A$ (d) $3A$

29. The value of a if the function $f(x)$ defined by $f(x) = \begin{cases} 2x - 1, & x < 2 \\ a, & x = 2 \\ x + 1, & x > 2 \end{cases}$ is continuous at $x = 2$ is

(a) 3 (b) -3 (c) 0 (d) 4

30. If $x^y = e^{x-y}$ then $\frac{dy}{dx}$ is equal to

(a) $\frac{\log x}{(1 + \log x)^2}$ (b) $\frac{x}{\log x}$ (c) $\frac{\log x}{(1 - \log x)^2}$ (d) None of these

31. If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, $a > 0$ and $-1 < t < 1$, then $\frac{dy}{dx}$ is

(a) $\frac{y}{x}$ (b) $\frac{x}{y}$ (c) $\frac{-y}{x}$ (d) None of these

32. Derivative of $\tan^{-1}\left(\frac{1+2x}{1-2x}\right)$ w.r.t $\sqrt{1+4x^2}$ is

(a) $\frac{1}{2x\sqrt{1+4x^2}}$ (b) $\frac{1}{x\sqrt{1+x^2}}$

(c) $\frac{1}{4x\sqrt{1+2x^2}}$ (d) $\frac{1}{2x\sqrt{1-4x^2}}$

33. The slope of normal to the curve $y = 2x^2 + 3 \sin x$ at $x = 0$ is

(a) 3 (b) $\frac{1}{3}$ (c) -3 (d) $-\frac{1}{3}$

34. $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that $f(2) = 0$. Then $f(x)$ is

(a) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (b) $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(c) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ (d) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

35. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)\,dx$ is equal to

(a) 41 (b) 51 (c) 21 (d) 42

36. $\int_{-\pi}^{\pi} \frac{x \sin x\,dx}{e^x + 1}$ is equal to

(a) $\frac{3\pi}{2}$ (b) $\frac{\pi}{2}$ (c) π (d) 0

37. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is

(a) $\frac{37}{3}$ sq. units (b) $\frac{256}{3}$ sq. units (c) $\frac{64}{3}$ sq. units (d) $\frac{128}{3}$ sq. units

38. The integrating factor of $x\frac{dy}{dx} - y = x^4 - 3x$ is

(a) x (b) $\log x$ (c) $\frac{1}{x}$ (d) $-x$

39. Solution of the equation $x^2y - x^3\frac{dy}{dx} = y^4 \cos x$, when $y(0) = 1$ is

(a) $y^3 = 3x^3 \sin x$ (b) $x^3 = 3y^3 \sin x$ (c) $x^3 = y^3 \sin x$ (d) None of these

40. The vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with y and z-axis, respectively is

(a) $\vec{r} = \pm 4\hat{i} - 5\hat{j}$ (b) $\vec{r} = \pm 3\hat{i} + 3\hat{j}$ (c) $\vec{r} = \pm 5\hat{i} + 5\hat{j}$ (d) $\vec{r} = \pm 4\hat{i} + 5\hat{j}$

41. The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represents the two sides AB and AC, respectively of ΔABC. The length of the median through A is

(a) $\frac{\sqrt{34}}{2}$ (b) $\frac{\sqrt{48}}{2}$ (c) $\sqrt{18}$ (d) None of these

42. If $\vec{a}$ and $\vec{b}$ are unit vectors, then the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\,\vec{a} - \vec{b}$ to be a unit vector is

(a) 30° (b) 45° (c) 60° (d) 90°

43. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$

44. If α, β, γ, are the angles that a line makes with a positive direction of x, y, z axes, respectively, then the direction cosines of the line are

(a) $\sin\alpha, \sin\beta, \sin\gamma$ (b) $\cos\alpha, \cos\beta, \cos\gamma$ (c) $\tan\alpha, \tan\beta, \tan\gamma$ (d) $\cos^2\alpha, \cos^2\beta, \cos^2\gamma$

45. The equation of x-axis in space are

(a) $x = 0, y = 0$ (b) $x = 0, z = 0$ (c) $x = 0$ (d) $y = 0, z = 0$

46. **Distance of the point (α, β, γ) from y-axis is**

(*a*) β (*b*) $|\beta|$ (*c*) $|\beta| + |\gamma|$ (*d*) $\sqrt{\alpha^2 + \gamma^2}$

47. **The reflection of the point (α, β, γ) in the XY-plane is**

(*a*) $(\alpha, \beta, 0)$ (*b*) $(0, 0, \gamma)$ (*c*) $(-\alpha, -\beta, \gamma)$ (*d*) $(\alpha, \beta, -\gamma)$

48. **If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B/A) = 0.6$, then $P(A \cup B)$ is**

(*a*) 0.24 (*b*) 0.57 (*c*) 0.48 (*d*) 0.96

49. **Let A and B be two events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$ then $P(A'/B')$ equals**

(*a*) $\frac{1}{10}$ (*b*) $\frac{3}{10}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{6}{7}$

50. **If $P(A/B) > P(A)$, then which of the following is correct**

(*a*) $P(B/A) < P(B)$ (*b*) $P(A \cap B) < P(A) \cdot P(B)$

(*c*) $P(B/A) > P(B)$ (*d*) $P(B/A) = P(B)$