Chapter 12

Exponents and Powers

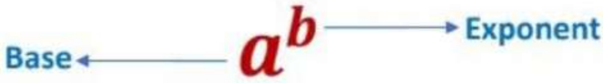
Exponents

Exponents

Exponents help in writing large numbers in a shorter form which we can read, understand and compare easily. A number placed in a superscript position to the right of

another number or variable indicates repeated multiplication.

Very large numbers are difficult to read, understand and compare. We can write such large numbers conveniently using Exponents.



a raised to the power *b*

For example:

a^2 indicates a × a

a^3 indicates a × a × a

3 raised to the eighth power indicates 3 × 3 × 3 × 3 × 3 × 3 × 3 × 3 and would look like a 3 with a small 8 on the upper right-hand side of the 3.

In expression 25, 2 is called the base and 5 is called the exponent or power.

$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

i) Express 256 as a power 2.

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8 = 256$

ii) Which one is greater 82 or 28?

$8^2 = 8 \times 8 = 64$

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

So, 2^8 is greater than 8^2.

Powers with Negative Exponents

For any non-zero integer a,

$a^{-m} = \frac{1}{a^m}$, where m is a positive integer. a^{-m} is the reciprocal of a^m.

a^{-m} is the multiplicative inverse of a^m.

$a^m \times a^{-m} = 1$

For example,

Evaluate :

(i) $(-6)^{-2}$ (ii) $(\frac{1}{2})^{-6}$

i) $(-6)^{-2}$ (= $\frac{1}{-6})^2$ = $\frac{1}{36}$

ii) $(\frac{1}{2})^{-6} = \frac{1}{(2)^{-6}} = 2^{6} = 64$

Calculate $(-1)^2, (-1)^3, (-10)^2, (-5)^3$.

$(-1)^2 = -1 \times -1 = 1$
$(-1)^3 = -1 \times -1 \times -1 = -1$
$(-10)^2 = -10 \times -10 = 100$
$(-5)^3 = -5 \times -5 \times -5 = -125$

$(-1)^{\text{odd number}} = -1$
$(-1)^{\text{even number}} = +1$

$(\text{-ve integer})^{\text{odd number}} = \text{-ve integer}$
$(\text{-ve integer})^{\text{even number}} = \text{+ve integer}$

Laws of Exponents

(a) First Law

If a is any non-zero integer (base) and m, n are integers (powers), then $a^m\, a^n = a^m \times a^n = a^{m+n}$

for example,
(i) $(-2)^5 \times (-2)^{10} = (-2)5+10 = (-2)15$

(ii) Multiply 32 and 34.

$\Rightarrow 3^2 \times 3^4$
$\Rightarrow 3^{2+4}$
$\Rightarrow 3^6$
$\Rightarrow 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$\Rightarrow 729$

(iii) Multiply -3^2 and -3^3.

$\Rightarrow (-3)^2 \times (-3)^3$
$\Rightarrow (-3)^{2+3}$
$\Rightarrow (-3)^5$
$\Rightarrow (-3) \times (-3) \times (-3) \times (-3) \times (-3)$
$\Rightarrow -243$

(b) Second Law

If a is any non-zero integer (base) and m, n are whole numbers (powers).

If m > n, then $a^m \div a^n = a^{m-n}$

If m < n, then $a^m \div a^n = \frac{1}{(a)^{n-m}}$

For example,
(i) $3^5 \div 3^{-6} = 3^{5-(-6)} = 3^{5+6} = 3^{11}$

(ii) Divide 36 by 32.

$\Rightarrow 3^6 \div 3^2$
$\Rightarrow 3^{6-2}$
$\Rightarrow 3^4$
$\Rightarrow 3 \times 3 \times 3 \times 3$
$\Rightarrow 81$

(iii) Divide 3^2 by 3^6.

$\Rightarrow 3^2 \div 3^6$
$\Rightarrow 3^{2-6}$
$\Rightarrow 3^{-4}$

$\Rightarrow \frac{1}{(3)^4}$

$\Rightarrow \frac{1}{81}$

(c) Third Law

If a and b are two non-zero integers (base) and m is a whole number (power), then $a^m \times b^m = (ab)^m$

For example,
(i) $(5^2)^3 = 5^{2\times3} = 5^6$

(ii) Calculate (33)2.

$\Rightarrow 3^3 \times 3^3$
$\Rightarrow 3^{3+3}$
$\Rightarrow 3^6$
$\Rightarrow 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$\Rightarrow 729$

(iii) Which one is greater $(5^2) \times 3$ or $(5^2)^3$?

$(5^2) \times 3$ means 5^2 is multiplied by 3

i.e., $5 \times 5 \times 3 = 75$

but $(5^2)^3$ means 5^2 is multiplied by itself 3 times

$5^2 \times 5^2 \times 5^2$

5^{2+2+2}
$5^6 = 15625$
So, $(5^2)^3$is greater than $(5^2) \times 3$.

(d) Fourth Law

If a and b are two non-zero integers (base) and m is a whole number (power), then $a^m \times b^m = (ab)^m$.

For example,

(i) $(3 \times 5)^4 = 3^4 \times 5^4$

$= (3 \times 3 \times 3 \times 3) \times (5 \times 5 \times 5 \times 5)$
$= 81 \times 625$
$= 50625$
(ii) Multiply 3^3 and 4^3.

$\Rightarrow 3^3 \times 4^3$
$\Rightarrow (3 \times 4)^3$
$\Rightarrow 12^3$
$\Rightarrow 12 \times 12 \times 12$
$\Rightarrow 1728$

(e) Fifth Law

If a and b are two non-zero integers (base) and m is a whole number (power), then

$a^m \div b^m = (\frac{a}{b})^m = \frac{a^m}{b^m} = (a \div b)^m$

For example,

(i) $(\frac{2}{3})^4$

$= \frac{2^4}{3^4}$

$= \frac{16}{81}$

(ii) Divide 3^3 by 4^3.

$\Rightarrow 3^3 \div 4^3$

$\Rightarrow \frac{3^3}{4^3}$
$\Rightarrow (\frac{3}{4})^3$

(iii) Divide $(-7)^4$ by $(-4)^4$.

$\Rightarrow (-7)^4 \div (-4)^4$

$\Rightarrow (\frac{-7^4}{-4^4})$

$\Rightarrow (\frac{-7}{-4})^4$

$\Rightarrow (\frac{7}{4})^4$

(f) Sixth Law

If a is an integer (base) then $a^0 = 1$

For example,
Multiply 3^4 and 3^{-4}.

$\Rightarrow 3^4 \times 3^{-4}$
$\Rightarrow 3^{4 + (-4)}$
$\Rightarrow 3^{4 - 4}$
$\Rightarrow 3^0$
$\Rightarrow 1$
So, we can say that any number (except 0) raised to the power (or exponent) 0 is 1.

Simplify and write the answer in the exponential form
i) $(3^5 \div 3^8)^5 \times 3^{-5}$

$(3^{5-8})^5 \times 3^{-5}$ (Using the law $a^m \div a^n = a^{m-n}$)

$= (3^{-3})^5 \times 3^{-5}$ (Using the law $(a^m)^n = a^{mn}$)

$= (3)^{-15} \times 3^{-5}$

$= 3^{(-15-5)}$ (Using the law am × an = am+n)

$= 3^{-20}$

ii) $(-2)^{-3} \times 5^{-3} \times (-5)^{-3}$

$[(-2) \times 5 \times (-5)]^{-3}$ (Using the law $(ab)^n = a^n \times b^n$)

$= (50)^{-3}$

$= \frac{1}{50^3}$

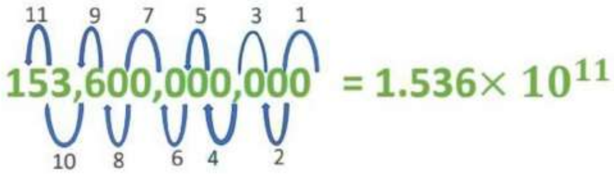
Expressing Numbers in Standard Form

Expressing numbers in Standard Form

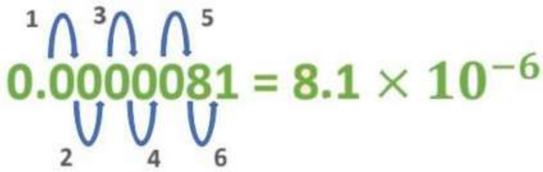
Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its standard form.

A number is said to be in the Standard Form if it is expressed as the product of a number between 1 and 10 and the integral power of 10.

For example,



Decimal is moved 11 places towards left, therefore the power is positive.



Decimal is moved 6 places towards the right, therefore the power is negative.

Large numbers in Standard Form

Express the following numbers in standard form.

i) 14320000000

ii) 205000000

i) $1432000000 = 1432 \times 10000000$

$= 1.432 \times 1000 \times 10000000$

$= 1.432 \times 10^3 \times 10^7$

$= 1.432 \times 10^{(3+7)}$

$= 1.432 \times 10^{10}$

ii) $205000000 = 205 \times 1000000$

$= 2.05 \times 100 \times 1000000$

$= 2.05 \times 10^2 \times 10^6$

$= 2.05 \times 10^{(2+6)}$

$= 2.05 \times 10^8$

Small Numbers in Standard Form

Express following numbers in Standard Form:

i) 0.000000041

ii) 0.00000000537

i) 0.000000041

$= \frac{41}{10^9}$

$= \frac{4.1 X 10}{10^9}$

$= 4.1 \times 10 \times 10^{-9}$

$= 4.1 \times 10^{(1-9)}$

$= 4.1 \times 10^{-8}$

ii) 0.00000000537

$= \frac{537}{10^{11}}$

$\frac{537X10^{2}}{10^{11}}$

$= 5.37 \times 10^{2} \times 10^{-11}$

$= 5.37 \times 10^{(2-11)}$

$= 5.37 \times 10^{-9}$

Express 15360000000 in standard form.
15360000000 = 1536×10000000
= 1.536×1000×10000000
= 1.536 × 103 × 107
= 1.536 × 10(3+7)
= 1.536 × 1010