

Topics : Fundamentals of Mathematics, Function, Limits

Type of Questions

Single choice Objective (no negative marking) Q.1

(3 marks, 3 min.)

M.M., Min.

[3, 3]

Subjective Questions (no negative marking) Q.2,3,4,5,6,7,8

(4 marks, 5 min.)

[28, 35]

1. Given $x^2 - xy + y^2 = 4(x + y - 4)$, where x, y both are real numbers. The number of pairs (x, y) satisfying the equation is

(A) only one (B) only two (C) three (D) None of these

2. Evaluate

(i) $\lim_{n \rightarrow \infty} (2^n + 3^n)^{1/n}$

(ii) $\lim_{x \rightarrow -1} \frac{\cos 2 - \cos 2x}{x^2 - |x|}$

3. Evaluate

(i) $\lim_{x \rightarrow 0} \frac{\tan \sqrt[3]{x} \ln(1+3x)}{(\tan^{-1} \sqrt{x})^2 (e^{5\sqrt[3]{x}} - 1)}$

(ii) $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} + \sqrt{x} + x\sqrt{x} - 3}{x^3 - 1}$

4. (a) $\lim_{x \rightarrow \infty} \left(\frac{x-2}{x^2 - 4x + 3} \right)^x$ is equal to

- (b) $\lim_{x \rightarrow 2} [x]$ (where $[.]$ denotes greatest integer function) is equal to

- (c) $\left[\lim_{x \rightarrow 2} x \right]$ (where $[.]$ denotes greatest integer function) is equal to

5. Solve $\frac{1}{[x]} + \frac{1}{[2x]} = \{x\} + \frac{1}{3}$, where $[.]$ denotes greatest integral function and $\{x\}$ denotes fractional part of x .

6. (a) Whether function $f(x) = \begin{cases} x & 0 \leq x < 1 \\ 3-x & 1 \leq x \leq 2 \end{cases}$ is invertible? If yes, then find its inverse.

- (b) If Domain of $f(x)$ is $[\pi, 3\pi]$ & $g(x) = \pi + x + \sin x$, then find domain of $f(g(x))$.

7. Evaluate :

(i) $\lim_{x \rightarrow \infty} \frac{(2+x)^{40} (4+x)^5}{(2-x)^{45}}$

(ii) $\lim_{x \rightarrow 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$

(iii) $\lim_{x \rightarrow 0} \frac{\ln(1+2x) - 2\ln(1+x)}{x^2}$

8. Evaluate

(i) $\lim_{x \rightarrow 0} \frac{x^2 2^{2x} - x^2 \cdot 2^{x+1} + x^2}{\cos 2x - 4 \cos x + 3}$

(ii) $\lim_{x \rightarrow \infty} \frac{(x+1)^4 - (x-1)^4}{(x+1)^4 + (x-1)^4}$

Answers Key

1. (A)
2. (i) 3 (ii) $2 \sin 2$
3. (i) $\frac{3}{5}$ (ii) $\frac{7}{9}$
4. (a) 0 (b) does not exist (c) 2
5. $\frac{29}{12}, \frac{19}{6}, \frac{97}{24}$
6. (a) Yes, $f^{-1}(x) = \begin{cases} x, & 0 \leq x < 1 \\ 3 - x, & 1 \leq x \leq 2 \end{cases}$ (b) $[0, 2\pi)$
7. (i) -1 (ii) $\frac{3}{2}$ (iii) -1
8. (i) $2(\ln 2)^2$ (ii) 0